A NEW DARK MATTER DENSITY PROFILE FOR M31 GALAXY TO DEMONSTRATE THAT DARK MATTER IS GENERATED BY GRAVITATIONAL FIELD

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1. ABSTRACT

The main target this paper is to check a theory about dark matter nature, which was published by the author in previous papers. It was postulated it that dark matter density is a function which depend on E, gravitational field. Also were proposed several experimental test to check that theory.

In this work has been calculated a new function for DM density for M31. Reader could think, why disturb me with a new DM density profile, called Bernoulli profile in this paper, whose values have relative differences with NFW ones below 10%?

The reason is clear. This DM profile has been got starting from hypothesis that DM is generated by the own gravitational field. Therefore if DM Bernoulli profile fits perfectly to NFW DM profile then it is possible conclude that observational data supports author´s hypothesis about DM nature.

To find reasons that author has to do so daring statement, reader can consult [1] Abarca,M.2014. *Dark matter model by quantum vacuum*. [2] Abarca,M.2015. *Dark matter density function depending on gravitational field as Universal law.* [3] Abarca,M.2015. *A new dark matter density profile for NGC 3198 galaxy to demonstrate that dark matter is generated by gravitational field.* [9] Abarca,M.2016.*A New Dark Matter Density Profile as Power of Gravitational Field for Coma Cluster.*

Briefly will be explained method followed to develop this paper. Firstly are presented rotation curve and table with data about rotational velocity depending on radius inside M31 galaxy. These data come from [5]Sofue,Y.2015.

In fourth epigraph, considering rotation curve of M31 from Sofue data, has been tabulate gravitational field inside a wide region of halo, from 40 kpc to 300 Kpc.

In fifth epigraphs has been tabulated data of NFW DM density profile published by [5] Sofue, Y.2015. for M31.

In sixth epigraph has been fitted data of NFW DM density profile as power of gravitational field, E. Particularly formula found is $\varphi_{DM}(r) = A \cdot E^B$ Where A = 0,0012004275 B= 1.878838501 and correlation coefficient r = 0,9996041653 into I.S. of units.

In seventh epigraph it has been compared DM density as power of E and NFW profiles. Tables and plots show clearly that relative differences between both profiles are mainly below 6%.

In eight epigraph it is considered derivative of gravitational field, E, in halo region where density of baryonic matter is negligible regarding DM density. As consequence it is got a Bernoulli differential equation whose solution allows to get a new DM density profile called hereafter Bernoulli density profile.

In nineth epigraph Bernoulli and NFW DM density profiles have been compared. Its relative differences are below 10% for radius bigger than 100 kpc, and below 12% from 40 kpc to 100 kpc. This is a super result, speciallly if it is considered that Do parameter of NFW profile has 10,7 % as relative error.

In addition Bernoulli density is rewritten with a similar formula to NFW profile thanks a variable change.

In tenth chapter has been calculated total masses under radius 200 kpc, 385 kpc and 475 kpc through NFW profile and Bernoulli profile. Last one profile is very hard to integrate but thanks Wolfram alpha software on line it has been possible.

2. INTRODUCTION

As reader knows M31 is the twin galaxy of Milky Way in Local Group of galaxies. Its radius is approximately 35 kpc and according [5] Sofue, Y. 2015. Baryonic mass of M31 galaxy is $M_{BARYONIC} = 1,61 \cdot 10^{11}\ M_{SUN}$

As radius is 35 kpc is supposed that for radius bigger than 40 kpc baryonic matter density is negligible versus DM density. This is the reason why radius dominion in this work is from 40 kpc to 300 kpc. In chapter eight it will be got a simple Bernoulli differential equation for gravitational field. However to get a so simple differential equation it is needed that $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ .In other words, it is needed that density of baryonic matter would be negligible versus D.M. density.

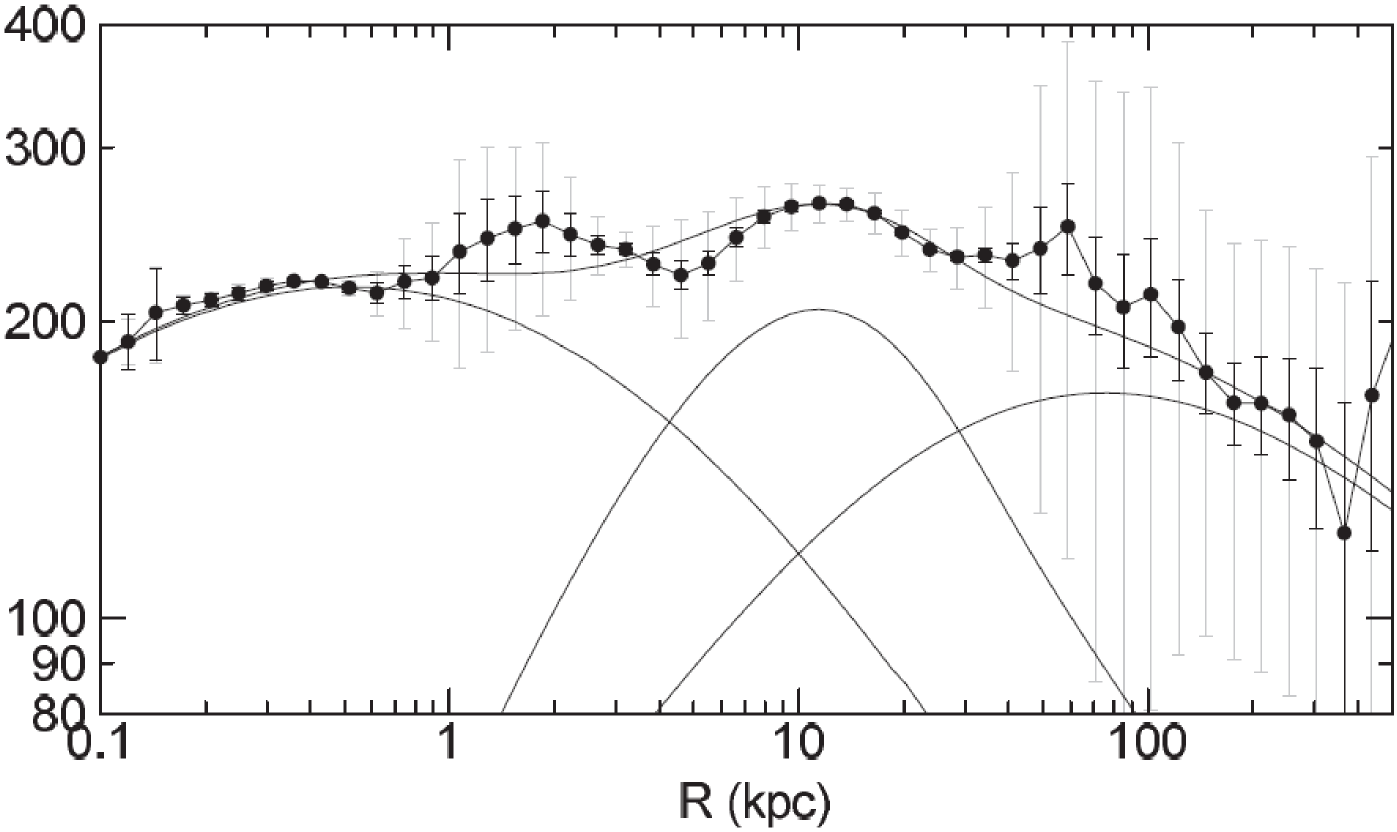
In paper [1] Abarca,M.2014, it was postulated that DM density depends on gravitational field. Further papers has studied DM density as power of gravitational field in several galaxies: M33, NGC3198 and others galaxies. Correlation coefficient of both magnitudes has been always higher than 0,99.

In paper [2] Abarca,M.2015 it was justified properly that DM density is a function as power of E. $\varphi_{DM}(r) = A \cdot E^B$ Where A and B may be got by statistical regression and its values depend on each galaxy, although galaxies with similar mass have similar coefficients A & B.

Having formula $\varphi_{DM}(r) = A \cdot E^B$ it is right to get a Bernoulli equation for galactic E, whose solution allows to get a DM density profile called in this paper Bernoulli profile.

The main target this paper is to get Bernoulli profile for M31 and compare its values with NFW profile got by Sofue.2015. Results have been successful because relative differences are under 10 % from 100 kpc to 300 kpc and are under 12% from 40 kpc to 100 kpc.

3. OBSERVATIONAL DATA FROM SOFUE. 2015 PAPER



Graphic come from [5] Sofue,Y. 2015.

Radius kpc	Velocity km/s
40	214
50	209,5
60	202,8
70	197,8
80	194,7
90	191,6
100	188
120,1	182,7
144,7	177,1
174,5	171,6
209	165,7
253,5	159,5
300	151,2

Table data have been got from graphic. As scale axis are logarithmic, calculus have been made carefully to get table results. Velocity data have been got from curve fitted to experimental measures, which are plotted as black points.

4. GRAVITATIONAL FIELD E THROUGH VIRIAL THEOREM

In this work dominion of radius extend from 40 kpc to 300 kpc. despite the fact that rotation curve has accuracy measures for radius lower than 40 kpc. As it is known galactic radius is approximately 35 kpc. Therefore it is supposed that for radius bigger than 40 kpc density of baryonic matter is negligible versus DM density. This hypothesis will be used to get a differential equation for gravitational field in this paper.

As it is known total gravitational field may be calculated through Virial theorem, formula $E = v^2/R$ whose I.S. unit is m/s^2 is well known. Hereafter, gravitational field got through this formula will be called Virial E. In fourth column is shown results of Virial E. Reader can check these data taking into account that 1 Kpc = $3{,}0857 \cdot 10^{19}$ m. Data of velocity has been got from solid line, which fits series of points in above figure.

Radius	radius	Velocity	E virial
Kpc	m	Km/s	m/s^2
40	1,23E+21	214	3,7103E-11
50	1,54E+21	209,5	2,8448E-11
60	1,85E+21	202,8	2,2214E-11
70	2,16E+21	197,8	1,8113E-11
80	2,47E+21	194,7	1,5356E-11
90	2,78E+21	191,6	1,3219E-11
100	3,09E+21	188	1,1454E-11
120,1	3,71E+21	182,7	9,0070E-12
144,7	4,47E+21	177,1	7,0245E-12
174,5	5,38E+21	171,6	5,4687E-12
209	6,45E+21	165,7	4,2574E-12
253,5	7,82E+21	159,5	3,2523E-12
300	9,26E+21	151,2	2,4696E-12

5. NFW DARK MATTER DENSITY PROFILE

According [5] Sofue,Y., 2015. Parameters of NFW profile for M31 are

Dark matter density function profile NFW
Rs = 34.6 ± 2.1 Kpc
Do = $1.50926 \cdot 10^{-22}$ kg/m^3
Do = $2.23 \pm 0.24 \cdot 10^{-3}$ Msolar/pc^3 = 2.23 mMsolar/pc^3

Knowing that mMsolar/pc^3 = $6{,}768 \cdot 10^{-23}$ Kg /m^3

Unit of Do has been changed into mMsolar/pc^3 which is a very common unit for galactic densities.

$$D_{NFW}(R) = \frac{D_0}{x \cdot (1+x)^2}$$

Where x= radius/ Rs Rs is called length scale and Do is density scale.

Below are tabulated NFW DM density depending on radius. This data will be used in next chapter to get a power regression of DM density as power of gravitational field E.

DM NFW	Radius	DM NFW
Kg/m^3	kpc	mMsun/pc^3
2,8092E-23	40	0,41494976
1,7475E-23	50	0,25812046
1,1646E-23	60	0,1720282
8,1651E-24	70	0,12060679
5,9520E-24	80	0,08791722
4,4755E-24	90	0,06610808
3,4517E-24	100	0,05098507
2,1757E-24	120,1	0,03213731
1,3443E-24	144,7	0,01985656
8,1963E-25	174,5	0,01210681
5,0422E-25	209	0,00744787
2,9721E-25	253,5	0,00439004
1,8619E-25	300	0,00275017

6. NFW D.M. DENSITY AS POWER OF VIRIAL FIELD E

Below are tabulated values of gravitational field E and NFW DM density, because DM density will be fitted with a power function of E. Units are International System.
Reason why the author has decided to fit this function is explained in [2] Abarca,M.2015. & [1] Abarca,M.2014. Briefly, the author defends hypothesis that DM is generated by the own gravitational field. Therefore it is right to look for a function of DM density depending on E.

m/s^2	DM NFW	Radius
E virial	Kg/m^3	kpc
3,7103E-11	2,8092E-23	40
2,8448E-11	1,7475E-23	50
2,2214E-11	1,1646E-23	60
1,8113E-11	8,1651E-24	70
1,5356E-11	5,9520E-24	80
1,3219E-11	4,4755E-24	90
1,1454E-11	3,4517E-24	100
9,0070E-12	2,1757E-24	120,1
7,0245E-12	1,3443E-24	144,7
5,4687E-12	8,1963E-25	174,5
4,2574E-12	5,0422E-25	209
3,2523E-12	2,9721E-25	253,5
2,4696E-12	1,8619E-25	300

Doing power regression of DM density versus gravitational field according formula $\text{Density}_{\text{DARK MATTER}} = A \cdot E^B$ through International System of units, it is right to get A = 0,0012004275 and B= 1.878838501 being correlation coefficient r = 0,9996041653. There is a very high correlation between DM density and gravitational field.

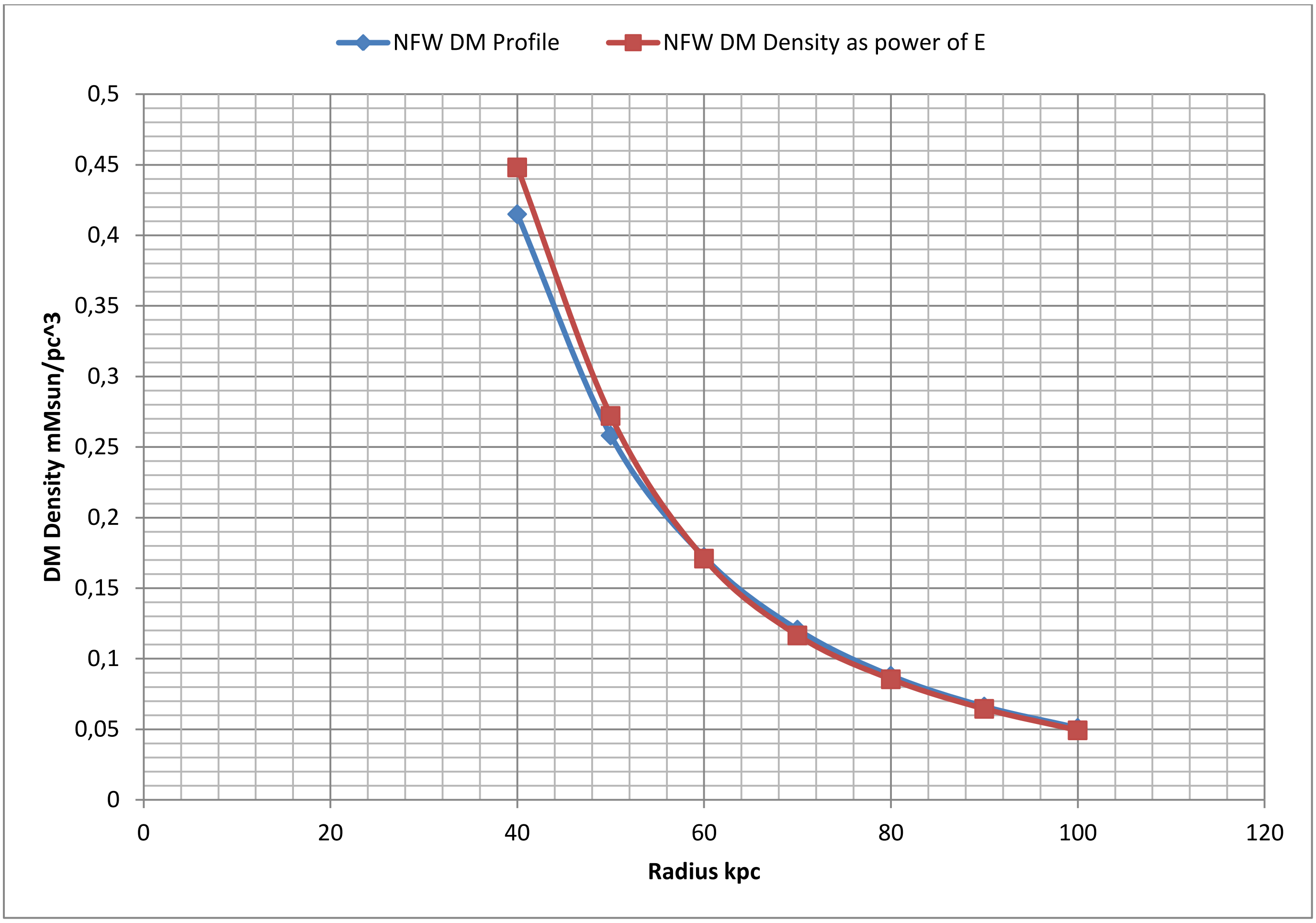
NFW Dark Matter Density as power of Virial E for M31 inside dominion 40 kpc < radius < 300kpc
$D_{DM\ Pw\ VE} = A \cdot E^B$
A = 0,0012004275 B= 1.878838501 and correlation coefficient r = 0,9996041653

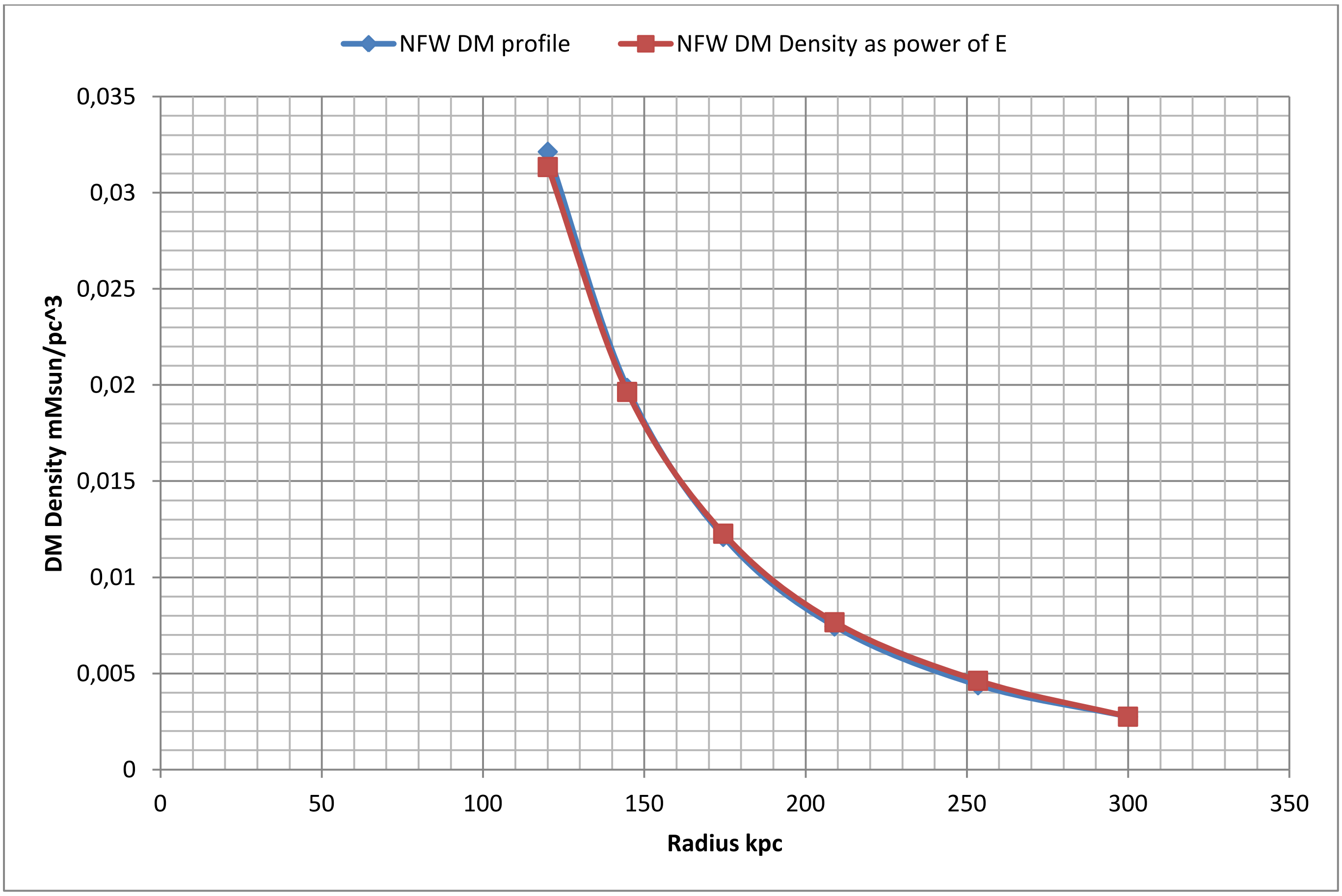
Hereafter dark matter density as power of Virial E will be shortened as $D_{DM\ Pw\ VE} = A \cdot E^B$

7. COMPARISON BETWEEN DM DENSITY AS POWER OF E AND NFW PROFILE

In this paragraph will be compared NFW DM density introduced in chapter 5 and DM density as power of E got in chapter 6.As reader can see relative differences are below 6% everywhere except for 40 kpc whose relative difference is 8%.

E Virial	Radius	DM NFW	DM pow E	Rel. Diff.
m/s^2	kpc	mMsun/pc^3	mMsun/pc^3	%
3,7103E-11	40	0,41494976	4,48E-01	7,99E+00
2,8448E-11	50	0,25812046	2,72E-01	5,38E+00
2,2214E-11	60	0,1720282	1,71E-01	6,45E-01
1,8113E-11	70	0,12060679	1,16E-01	3,42E+00
1,5356E-11	80	0,08791722	8,54E-02	2,85E+00
1,3219E-11	90	0,06610808	6,45E-02	2,51E+00
1,1454E-11	100	0,05098507	4,92E-02	3,43E+00
9,0070E-12	120,1	0,03213731	3,13E-02	2,46E+00
7,0245E-12	144,7	0,01985656	1,96E-02	1,05E+00
5,4687E-12	174,5	0,01210681	1,23E-02	1,40E+00
4,2574E-12	209	0,00744787	7,67E-03	2,97E+00
3,2523E-12	253,5	0,00439004	4,62E-03	5,33E+00
2,4696E-12	300	0,00275017	2,76E-03	2,33E-01


NFW DM Profile
NFW DM Density as power of E
DM Density mMsun/pc^3
0,5
0,45
0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
0
20
40
60
80
100
120
Radius kpc


NFW DM profile
NFW DM Density as power of E
DM Density mMsun/pc^3
0,035
0,03
0,025
0,02
0,015
0,01
0,005
0
0
50
100
150
200
250
300
350
Radius kpc

8. BERNOULLI DIFFERENTIAL EQUATION FOR GRAVITATIONAL FIELD IN M31 HALO

It will be considered the region 40 Kpc < Radius < 300 Kpc where density of baryonic matter is negligible versus baryonic density. So for radius bigger than 40 Kpc, it will be considered that derivative of M(r) depend on dark matter density only.

As it is known in this formula $E = G\frac{M(r)}{r^2}$, M(r) represents mass enclosed by a sphere with radius r. If it is considered radius > 40 Kpc then the derivative of M(r) depend on dark matter density only and therefore $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ As $\varphi_{DM}(r) = A \cdot E^B(r)$ Where A = 0,0012004275 and B= 1.878838501 then $M'(r) = 4\pi r^2 \cdot A \cdot E^B$

Now it will differentiated E(r) when r > 40 Kpc

If $E = G\frac{M(r)}{r^2}$ is differentiated it is got $E'(r) = G\frac{M'(r)\cdot r^2 - 2rM(r)}{r^4}$

If $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ is replaced above it is got $E'(r) = 4\pi G \varphi_{DM}(r) - 2G\frac{M(r)}{r3}$ As $\varphi_{DM}(r) = A \cdot E^B(r)$ it is right to get $E'(r) = 4\pi \cdot G \cdot A \cdot E^B(r) - 2\frac{E(r)}{r}$ which is a Bernoulli differential equation.

$E'(r) = K \cdot E^B(r) - 2\frac{E(r)}{r}$ being $K = 4\pi \cdot G \cdot A$ then K = $1{,}00662552904 \cdot 10^{-12}$ I.S. as A= 0,0012004275

Calling ***y*** to E, the differential equation is written this way $y' = K \cdot y^B - \frac{2 \cdot y}{r}$

Bernoulli family equations $y' = K \cdot y^B - \frac{2 \cdot y}{r}$ may be converted into a differential linear equation with this variable change $u = y^{1-B}$.

General solution is $E(r) = \left(Cr^{2B-2} + \frac{Kr(1-B)}{3-2B} \right)^{\frac{1}{1-B}}$ *with* $B \neq 1$ *and* $B \neq 3/2$ where C is the parameter of initial condition of gravitational field at a specific radius.

Calling $\alpha = 2B - 2$ $\beta = \frac{1}{1-B}$ and $D = \left(\frac{K(1-B)}{3-2B} \right)$ formula may be written as

$E(r) = \left(Cr^{\alpha} + Dr \right)^{\beta}$ Where specifically values for these parameters are the following ones:

$\alpha = 2B - 2$ = 1,7576770020 $\beta = \frac{1}{1-B}$ = -1,1378654882 $D = \left(\frac{K(1-B)}{3-2B} \right)$ = $1{,}167596837 \cdot 10^{-12}$

Initial condition for parameter C calculus

Suppose R_0 *and* E_0 are specific initial conditions for radius and gravitational field then $C = \frac{E_0^{1/\beta} - D \cdot R_0}{R_0^{\alpha}}$

In order to check calculus it will be calculated parameter C for different initial condition.

Radius	radius	E virial	param. C
kpc	m	m/s^2	
40	1,23E+21	3,71034E-11	2,29748E-30
50	1,54E+21	2,84475E-11	3,02986E-30
60	1,85E+21	2,22142E-11	5,93630E-30
70	2,16E+21	1,81134E-11	7,45646E-30
80	2,47E+21	1,53564E-11	7,65928E-30
90	2,78E+21	1,32189E-11	8,03110E-30
100	3,09E+21	1,14541E-11	8,83697E-30
120,1	3,71E+21	9,00700E-12	9,42246E-30
144,7	4,47E+21	7,02449E-12	9,95489E-30
174,5	5,38E+21	5,46872E-12	1,02581E-29
209	6,45E+21	4,25741E-12	1,07202E-29
253,5	7,82E+21	3,25229E-12	1,10194E-29
300	9,26E+21	2,46961E-12	1,24637E-29

As it was expected parameter C is very similar for different initial condition.

Numerically may be checked that data below minimize relative difference between Burket DM density and DM density got through Bernoulli solution therefore these values will be considered as initial condition.

Initial condition values Ro & Eo	
Ro =	253 Kpc
Eo =	$3.25 \cdot 10^{-12}$ m/s^2
C =	$1.13 \cdot 10^{-29}$ units I.S.

Finally it is possible to write formula for DM density profile got through Bernoulli method.

Bernoulli Solution for Gravitational field inside halo 40 kpc < Radius < 300 kpc
$E_{BER}(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$ C = $1.13 \cdot 10^{-29}$ D=$1,167596837 \cdot 10^{-12}$ α = 1,7576770020 β = -1,1378654882

8.1 BERNOULLI PROFILE OF DARK MATTER DENSITY FOR M31 GALAXY

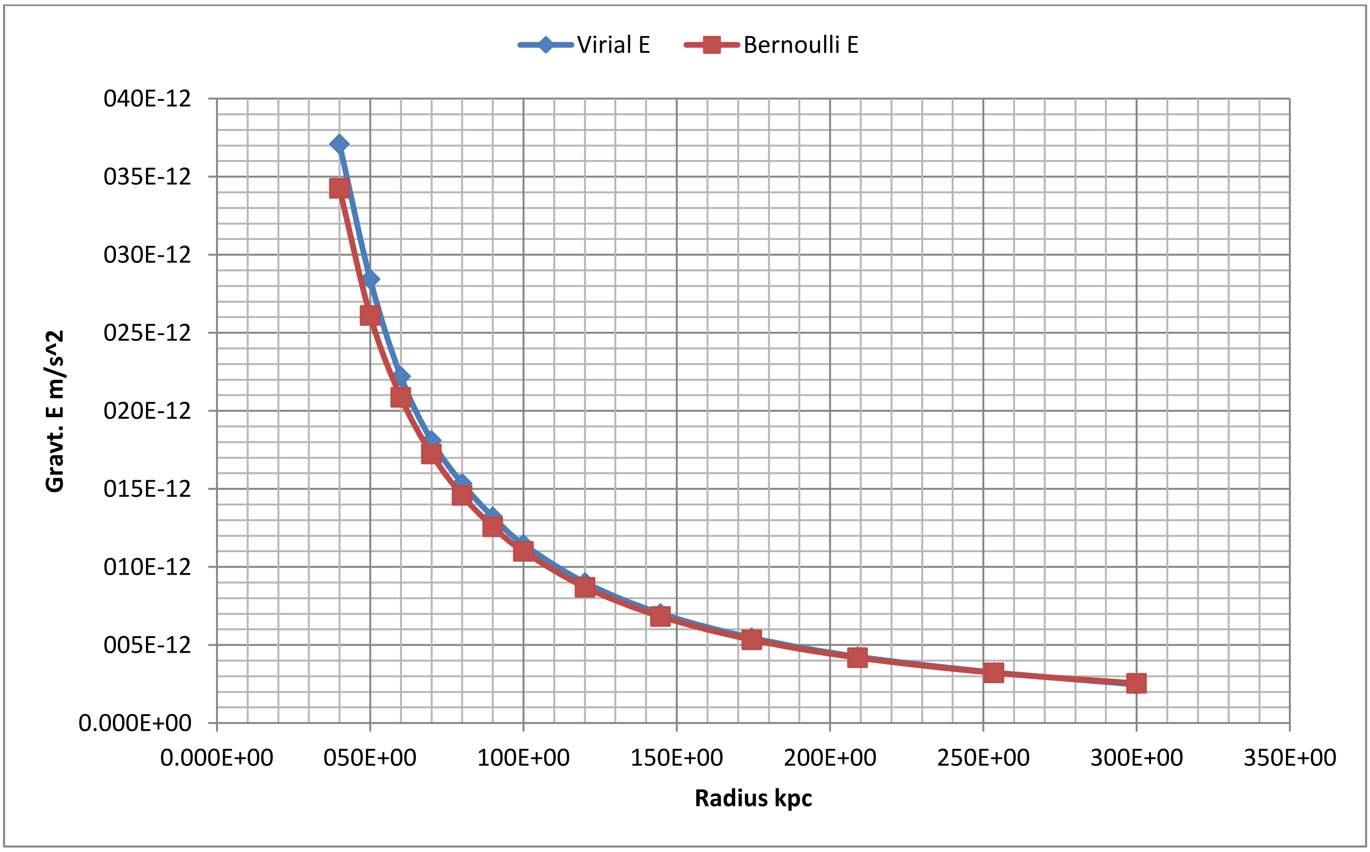
Thanks Bernoulli solution for gravitational field is right to get DM density through power of E formula.

DM Density Bernoulli profile for M31 inside halo 40 kpc < radius < 300 kpc
$E_{BER}(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$ C = $1.13 \cdot 10^{-29}$ D=$1{,}167596837 \cdot 10^{-12}$ α = 1,7576770020 β = -1,1378654882
Density $_{D.M.\ BERNOULLI}$ (r) = $D_{DM\ B}$(r) = $A \cdot E^{B}$ Where A = 0,0012004275 and B= 1.878838501 unit Kg/ m^3

9. COMPARISON BETWEEN BERNOULLI AND NFW PROFILES

9.1 COMPARISON BETWEEN VIRIAL GRAVT. FIELD AND BERNOULLI SOLUTION FOR E

Radius	Virial E	Bernoulli E	Relt. Diff.
kpc	m/s^2	m/s^2	%
4,00E+01	3,71E-11	3,43E-11	8,27E+00
5,00E+01	2,84E-11	2,61E-11	8,91E+00
6,00E+01	2,22E-11	2,09E-11	6,40E+00
7,00E+01	1,81E-11	1,72E-11	5,02E+00
8,00E+01	1,54E-11	1,46E-11	5,20E+00
9,00E+01	1,32E-11	1,26E-11	5,03E+00
1,00E+02	1,15E-11	1,10E-11	4,02E+00
1,20E+02	9,01E-12	8,71E-12	3,42E+00
1,45E+02	7,02E-12	6,84E-12	2,73E+00
1,75E+02	5,47E-12	5,34E-12	2,36E+00
2,09E+02	4,26E-12	4,20E-12	1,45E+00
2,54E+02	3,25E-12	3,23E-12	7,76E-01
3,00E+02	2,47E-12	2,56E-12	3,39E+00

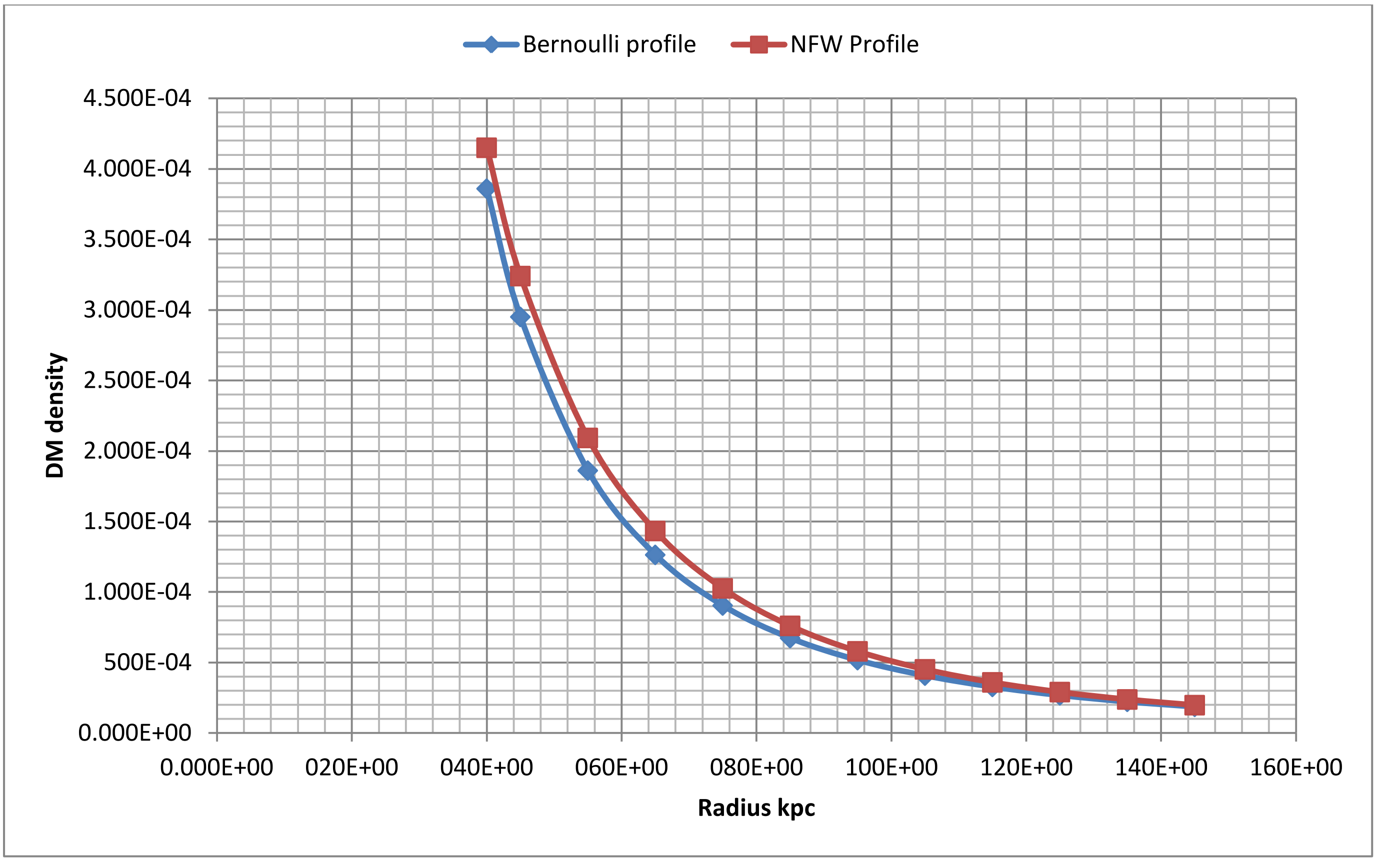


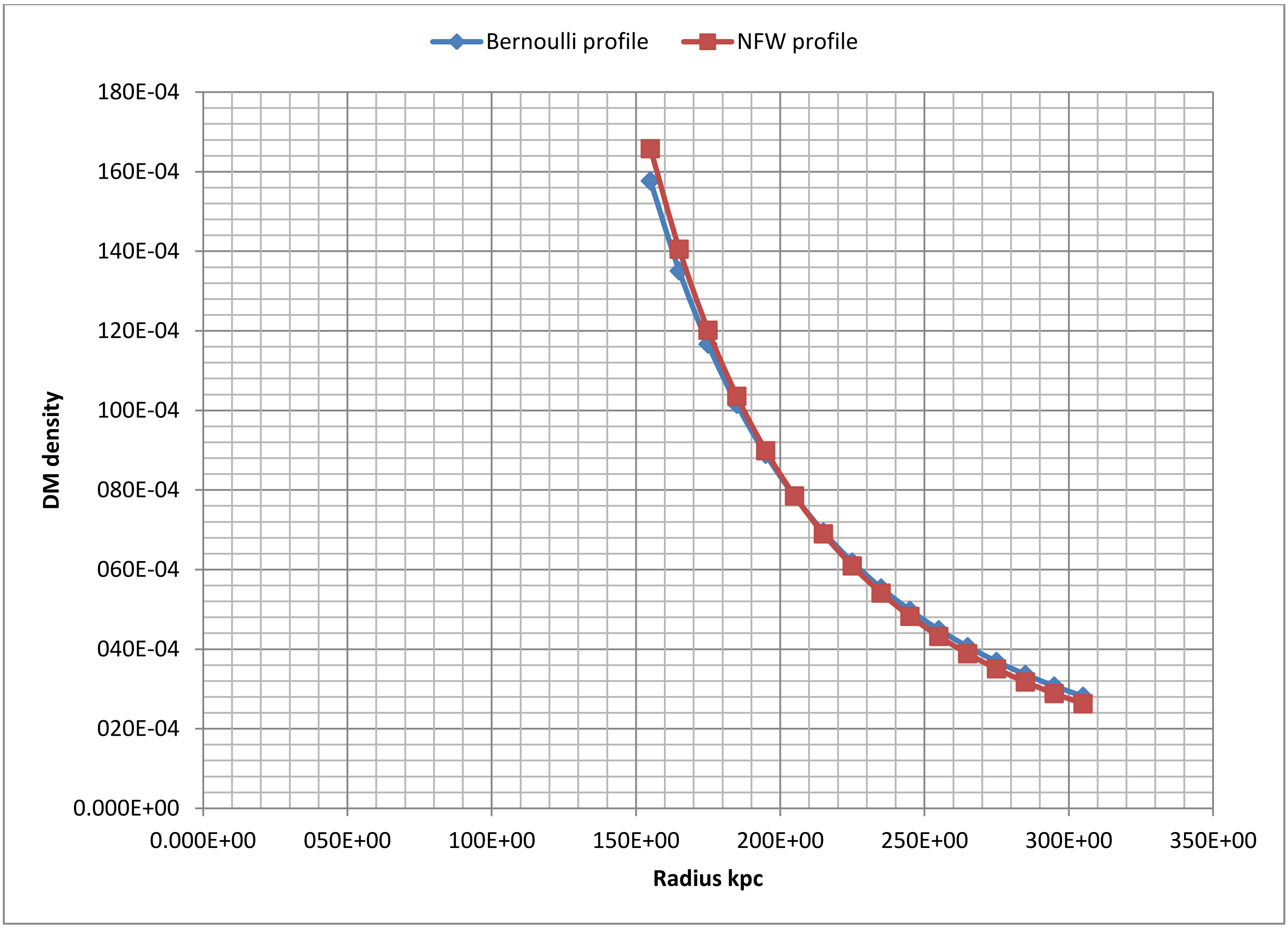
Bernoulli solution for gravitational field fits almost perfectly to Virial gravitational data got through observational values of spin speed of rotational curve of M31 galaxy.

9.2 COMPARISON BETWEEN NFW DM PROFILE AND BERNOULLI DM PROFILE

Radius	Bernoulli DM	NFW DM	Rel diff.
kpc	mMsun/pc^3	mMsun/pc^3	%
4,00E+01	3,86E-01	4,15E-01	6,99488E+00
4,50E+01	2,95E-01	3,24E-01	8,93748E+00
5,50E+01	1,86E-01	2,09E-01	1,10671E+01
6,50E+01	1,26E-01	1,43E-01	1,18333E+01
7,50E+01	9,04E-02	1,03E-01	1,18467E+01
8,50E+01	6,73E-02	7,60E-02	1,14213E+01
9,50E+01	5,17E-02	5,79E-02	1,07324E+01
1,05E+02	4,07E-02	4,51E-02	9,88458E+00
1,15E+02	3,27E-02	3,59E-02	8,94277E+00
1,25E+02	2,67E-02	2,90E-02	7,94889E+00
1,35E+02	2,21E-02	2,38E-02	6,93060E+00
1,45E+02	1,86E-02	1,97E-02	5,90648E+00
1,55E+02	1,58E-02	1,66E-02	4,88914E+00
1,65E+02	1,35E-02	1,41E-02	3,88718E+00
1,75E+02	1,17E-02	1,20E-02	2,90641E+00
1,85E+02	1,02E-02	1,04E-02	1,95074E+00
1,95E+02	8,89E-03	8,99E-03	1,02269E+00
2,05E+02	7,84E-03	7,85E-03	1,23809E-01
2,15E+02	6,95E-03	6,90E-03	7,45049E-01
2,25E+02	6,19E-03	6,09E-03	1,58354E+00
2,35E+02	5,54E-03	5,41E-03	2,39171E+00
2,45E+02	4,98E-03	4,82E-03	3,16983E+00
2,55E+02	4,49E-03	4,32E-03	3,91839E+00
2,65E+02	4,06E-03	3,88E-03	4,63801E+00
2,75E+02	3,69E-03	3,50E-03	5,32939E+00
2,85E+02	3,36E-03	3,17E-03	5,99332E+00
2,95E+02	3,07E-03	2,88E-03	6,63060E+00
3,05E+02	2,82E-03	2,63E-03	7,24206E+00

Reader can check that maximum relative difference is lower than 12% and is lower than 10 % for radius bigger than 100 kpc. This result is superb because error estimated by Sofue in its calculus of density Do are higher to 10 %. According Sofue Do = 2.23 ± 0.24 mMsolar/pc^3 so error is 10.7%.


Bernoulli profile
NFW Profile
4.500E-04
4.000E-04
3.500E-04
3.000E-04
2.500E-04
2.000E-04
1.500E-04
1.000E-04
500E-04
0.000E+00
DM density
0.000E+00
020E+00
040E+00
060E+00
080E+00
100E+00
120E+00
140E+00
160E+00
Radius kpc


Bernoulli profile
NFW profile
180E-04
160E-04
140E-04
120E-04
100E-04
080E-04
060E-04
040E-04
020E-04
0.000E+00
DM density
0.000E+00
050E+00
100E+00
150E+00
200E+00
250E+00
300E+00
350E+00
Radius kpc

9.3 WRITING BERNOULLI DENSITY FORMULA SIMILARLY TO NFW PROFILE

In this paragraph Bernoulli density formula will be rewritten similarly to NFW profile through a variable change. All calculus are into International System of units.

According Sofue NFW parameters
Rs = $1.068 \cdot 10^{21}$ m
Do = $1.50926 \cdot 10^{-22}$ kg/m^3

$$D_{NFW}(R) = \frac{D_0}{x \cdot (1+x)^2}$$ Where x= radius/ Rs

According calculus in chapter eight:

DM Density Bernoulli profile for M31 inside halo 40 kpc < radius < 300 kpc
$E_{BER}(r) = (Cr^{\alpha} + Dr)^{\beta}$ C = $1.13 \cdot 10^{-29}$ D=$1{,}167596837 \cdot 10^{-12}$ α = 1,7576770020 β = -1,1378654882
Density $_{\text{D.M. BERNOULLI}}$ (r) = $D_{DM\,B}$(r) = A· E^B Where A = 0,0012004275 and B= 1.878838501 unit Kg/ m^3

Therefore $D_{BERNI}(r) = A \cdot (Cr^{\alpha} + Dr)^{B \cdot \beta} = A \cdot (Cr^{\alpha} + Dr)^{V}$ *Where* $V = B \cdot \beta$

As x= radius/ Rs is the variable of NFW density, it is needed to do the same change of variable for Bernoulli density in order to compare both densities of DM.

Changing radius r by x through r = x· Rs , after a simple calculus it is got Bernoulli density depending on x.

This is the formula.

$$D_{BERNI}(x) = K \cdot x^{V} \cdot (1 + Mx^{\alpha-1})^{V} \quad Where\, K = A \cdot (D \cdot R_S)^{V} \;\; and \;\; M = \frac{R_S{}^{\alpha-1} \cdot C}{D}$$

Below are parameters for M31 according Sofue data and calculus of this paper.

Units into I.S.	
Do density scale NFW	1,5100000E-22
Rs radius scale NFW	1,0680000E+21
A	1,2000000E-03
B	1,8788385E+00
C	1,1300000E-29
D	1,1675968E-12
α	1,7576770E+00
β	-1,1378655E+00
V=B* β	-2,1378655E+00
$K = A \cdot (D \cdot R_S)^V$	4,2997892E-23
$M = \frac{R_S{}^{\alpha-1} \cdot C}{D}$	8,2918286E-02

Dominion for radius from 40 Kpc to 300 kpc, which is equivalent to

$x \in (1.15\,,\,8.7)$.

In particular formula is

$$D_{BERNI}(X) \approx \frac{4.29 \cdot 10^{-23}}{x^{2,1} \cdot (1 + 0.083 \cdot x^{0,76})^{2,1}}$$ Where $x \in (1.15\,,\,8.7)$

And $D_{NFW}(X) = \frac{1.51 \cdot 10^{-22}}{x \cdot (1+x)^2}$ where $x \in (1.15\,,\,8.7)$

Reader can check that both formulas have a similar structure.

Below is developed both density profiles through the whole dominion. It is remarkable that maximum relative difference is about 11% although in average is under 7 %.

X=R/Rs	R Kpc	D Berni	D NFW	Relt Diff. %
1,16E+00	40	2,61E-23	2,81E-23	7,11E+00
1,73E+00	60	1,03E-23	1,17E-23	1,17E+01
2,31E+00	80	5,26E-24	5,96E-24	1,18E+01
2,89E+00	100	3,09E-24	3,46E-24	1,04E+01
3,47E+00	120	2,00E-24	2,18E-24	8,58E+00
4,04E+00	140	1,37E-24	1,47E-24	6,55E+00
4,62E+00	160	9,86E-25	1,03E-24	4,52E+00
5,20E+00	180	7,36E-25	7,55E-25	2,56E+00
5,78E+00	200	5,65E-25	5,69E-25	7,12E-01
6,36E+00	220	4,43E-25	4,39E-25	-1,02E+00
6,93E+00	240	3,55E-25	3,46E-25	-2,64E+00
7,51E+00	260	2,89E-25	2,77E-25	-4,13E+00
8,09E+00	280	2,38E-25	2,26E-25	-5,51E+00
8,67E+00	300	1,99E-25	1,86E-25	-6,78E+00

Table below show extended dominion up to 5 Mpc and its relative differences, which are under 20 %.

X=R/Rs	Radius kpc	D Berni	D NFW	Rel. Diff. %
1,16E+01	400	9,26E-26	8,29E-26	-1,18E+01
1,44E+01	500	5,04E-26	4,38E-26	-1,49E+01
1,73E+01	600	3,03E-26	2,59E-26	-1,69E+01
2,02E+01	700	1,96E-26	1,66E-26	-1,80E+01
2,31E+01	800	1,33E-26	1,12E-26	-1,84E+01
2,60E+01	900	9,43E-27	7,96E-27	-1,84E+01
2,89E+01	1000	6,91E-27	5,85E-27	-1,81E+01
3,47E+01	1200	4,00E-27	3,42E-27	-1,68E+01
4,04E+01	1400	2,50E-27	2,17E-27	-1,50E+01
4,62E+01	1600	1,65E-27	1,46E-27	-1,29E+01
5,20E+01	1800	1,14E-27	1,03E-27	-1,06E+01
5,78E+01	2000	8,19E-28	7,56E-28	-8,32E+00
7,22E+01	2500	4,00E-28	3,90E-28	-2,69E+00
8,67E+01	3000	2,21E-28	2,27E-28	2,56E+00
1,01E+02	3500	1,33E-28	1,43E-28	7,35E+00
1,16E+02	4000	8,49E-29	9,62E-29	1,17E+01
1,30E+02	4500	5,71E-29	6,77E-29	1,56E+01
1,44E+02	5000	3,99E-29	4,94E-29	1,92E+01

In conclusion it is clear that Bernoulli and NFW profiles are very similar. This fact is important because Bernoulli profile has been got under hypothesis that DM is locally generated by the own gravitational field according a power of E i.e. $D_{BERNOULLI} = A \cdot E^{B}$.

10. MASSES IN M31

10.1 DYNAMICAL MASS UP TO 40 KPC

It has been chosen 40 kpc, because at this radius baryonic density is negligible regarding DM density in M31.

According Sofue at 40 Kpc, rotation velocity v = 214 km/s. As $M_{DYNSMICAL}$ = v^2 *R/G

$M_{DYNAMICAL}$ (< 40 kpc) = 4,25·10^11 Msun

According Sofue data Baryonic matter = 1,6·10^11 so DM (<40 kpc)= 2,65·10^11

M31 masses up to 40 kpc		
$M_{DYNAMICAL}$ = Total mass	Baryonic mass	Dark matter mass
4,25·10^11 Msun	1,6·10^11 Msun	2,65·10^11 Msun

10.2 CORONA HALO MASSES THROUGH BERNOULLI PROFILE

Below is formula Bernoulli profile

DM Density Bernoulli profile for M31 inside halo 40 kpc < radius < 300 kpc
$E_{BER}(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$ C = 1.13· 10 $^{-29}$ D=1,167596837·10^{-12} α = 1,7576770020 β = -1,1378654882
Density $_{D.M.\ BERNOULLI}$ (r) = $D_{DM\ B}$(r) = A· E^B Where A = 0,0012004275 and B= 1.878838501 unit Kg/ m^3

As for radius bigger than 40 kpc baryonic density is negligible regarding DM density, by integration of Bernoulli profile it is possible to calculate DM corona mass.

It is a hard integration but thanks Wolfram alpha software it is possible to calculate on line definite integral needed.

$$M_{DM} = \int_{R1}^{R2} 4\pi r^2 \cdot \rho(r) dr = \int_{R1}^{R2} 4\pi r^2 A E^B dr = 4\pi A \int_{R1}^{R2} r^2 \left[\left(Cr^{\alpha} + Dr\right)^{\beta}\right]^{B} dr = 4\pi A \cdot I = K \int r^2 \left(Cr^{\alpha} + Dr\right)^{V} = K * I$$

where $K = 4\pi A$ and $I = \int r^2 \left(Cr^{\alpha} + Dr\right)^{V}$

It has been chosen following corona distances [40kpc,200kpc] , [40kpc,385kpc] and [40kpc,475kpc]

In table below are data needed for integrations into I.S. of units.

Bernoulli Corona halo data	
R1 40 kpc	1.234*10^21 m
R2 200 kpc	6.17*10^21
R3 385 kpc	1.188·10^22 m
R4 475 kpc	1.466*10^22
K= 4pi*A	0.015085016
C	1.13*10^-29
alpha	1.7576770020
D	1.167596837*10^-12
V= Beta*B	-2.137865488

10.2.1 WOLFRAM ALPHA DEFINITE INTEGRALS

Below there are two copies of on line wolfram alpha integration.

Definite integral:

$$\int_{1.234\times 10^{21}}^{6.17\times 10^{21}} x^2 \left(\frac{1.13\, x^{1.757677002}}{10^{29}} + 1.167596837\times 10^{-12}\, x\right)^{-2.137865488} dx = 1.16597\times 10^{44}$$

Corona [40kpc,200kpc] I= 1,166·10^44. As Mcorona = K*I it is right to calculate DM corona mass.

Definite integral:

$$\int_{1.234\times 10^{21}}^{1.1877\times 10^{22}} x^2 \left(\frac{1.13\, x^{1.757677002}}{10^{29}} + 1.167596837\times 10^{-12}\, x\right)^{-2.137865488} dx = 2.00049\times 10^{44}$$

Enlarge | Data | Customize | Plaintext | Interactive

Corona [40kpc,385kpc] I= 2·10^44

Similarly it has been calculated Corona [40kpc,475kpc] I= 2,31·10^44

In table below there are three corona masses and three total masses by addition of dynamical mass up to 40 kpc i.e. Total mass(<R) = $M_{DYNAMIC}$ (< 40 kpc) + $M_{CORONA.}$

Where $M_{DYNAMIC}$ (< 40 kpc)= 4,25·10^11 Msun

Coronas R1 = 40 kpc	DM Corona halo mass	Total mass = Dyn+ Corona
R2 = 200 kpc	8,838*10^11 Msun	1,3·10^12 Msun
R3= 385 kpc	15,17·10^11 Msun	1,945·10^12 Msun
R4= 475 kpc	17,5*10^11 Msun	2,175·10^12 Msun

10.3 CORONA HALO MASSES THROUGH NFW PROFILE

Below are two different notation for NFW profile.

$$D_{NFW}(R) = \frac{D_0}{x\cdot(1+x)^2} \quad \text{or} \quad D_{NFW}(R) = \frac{4D_S}{x\cdot(1+x)^2} \quad \text{where } x = R / Rs$$

Dark matter density profile NFW-Sofue data
Rs = 34.6 $\pm$ 2.1 Kpc
Do = 2.23 $\pm$ 0.24 · 10^{-3} Msolar/pc^3 = 2.23 mMsolar/pc^3 = 1.50926 ·10^{-22} kg/m^3
Ds= Do/4= 3.77 ·10^{-23} kg/m^3

Total DM enclosed by a sphere with R radius is

$$M(<R) = 16\pi R^3{}_S \cdot D_S \cdot \left[\ln(1+x) - \frac{x}{1+x}\right] \quad \text{or} \quad M(<R) = 4\pi R^3{}_S \cdot D_o \cdot \left[\ln(1+x) - \frac{x}{1+x}\right]$$

Calling $f(x) = \left[\ln(1+x) - \frac{x}{1+x}\right] = \ln(1+r/Rs) - \frac{r}{r+Rs}$ and $Z = 16\pi R^3{}_S \cdot D_S$ then

M(R)= Z·f(x)

According previous data, for M31 galaxy Z= 11.65·10^{11} Msun. Therefore DM corona mass

DM Corona (R2-R1) = Z(f_2 –f_1)

In table below is shown DM corona masses for R= 40 kpc up to different radius. Also in last column is calculated total mass for a sphere with R radius by addition of total mass under 40 kpc i.e. by addition of 4,25·10^11 Msun. As reference, in first column is tabulated f(x) at different radius.

		M(<R)=Z*f(x)	Corona NFW	Total mass
	Radius	NFW	40kpc to R	M < R
f(x)	kpc	M sun	Msun	M sun
2,3209E-01	4,00E+01	2,70E+11	0,0000E+00	4,25E+11
4,3707E-01	7,00E+01	5,09E+11	2,3880E+11	6,64E+11
6,1551E-01	1,00E+02	7,17E+11	4,4668E+11	8,72E+11
7,6987E-01	1,30E+02	8,97E+11	6,2651E+11	1,05E+12
9,0489E-01	1,60E+02	1,05E+12	7,8381E+11	1,21E+12
1,0245E+00	1,90E+02	1,19E+12	9,2318E+11	1,35E+12
1,0615E+00	2,00E+02	1,24E+12	9,6627E+11	1,39E+12
1,2288E+00	2,50E+02	1,43E+12	1,1612E+12	1,59E+12
1,3725E+00	3,00E+02	1,60E+12	1,3286E+12	1,75E+12
1,4744E+00	3,40E+02	1,72E+12	1,4473E+12	1,87E+12
1,5779E+00	3,85E+02	1,84E+12	1,5679E+12	1,99E+12
1,7577E+00	4,75E+02	2,05E+12	1,7773E+12	2,20E+12

10.4 NFW MASSES VERSUS BERNOULLI MASSES

In table below are joined results from previous epigraphs in order to compare both methods to calculate masses in M31 galaxy

In last column are compared total mass < R though NFW & Bernoulli profiles. Relative difference is very little.

		DM NFW M(<R)=Z*f(x)	NFW Corona	Total mass NFW	Bernoulli Corona	Total mass Bernoulli	Rel. Diff. Ber&NFW
f(x)	Radius	M(<R)	40 kpc to R	M <R	40 kpc to R	M<R	
adimensional	kpc	Msun	Msun	M sun	Msun	Msun	%
2,3209E-01	4,00E+01	2,70E+11	0,0000E+00	4,25E+11	0	4,25E+11	0 %
1,0615E+00	2,00E+02	1,24E+12	9,6627E+11	1,39E+12	8,84E+11	1,31E+12	-5,9 %
1,5779E+00	3,85E+02	1,84E+12	1,5679E+12	1,99E+12	1,52E+12	1,95E+12	-2,4 %
1,7577E+00	4,75E+02	2,05E+12	1,7773E+12	2,20E+12	1,75E+12	2,18E+12	-1,2 %

11. CONCLUSION

It seem clear that inner logic of development this paper allow to state that this paper has demonstrated that DM origin is gravitational field.

This is the inner logic: NFW DM density profile, which has been got by meticulous measures of M31 rotation curve, is fitted with a function as power of E with a correlation coefficient bigger than 0.999. Thanks this function it has been possible to state a Bernoulli differential equation for gravitational field E, inside galactic halo where density of baryonic is negligible in comparison with DM density.

Solution of Bernoulli for gravitational field is used to get a new DM profile called Bernoulli DM profile, which has been compared with NFW DM density getting relative differences under 10 % inside main part of dominion, exactly for radius bigger than 100 kpc up to 300 kpc. Calculus of DM halo masses through profile integration for both profiles has shown that Bernoulli profile gives results very closed to NFW results.

In my opinion these results suggest strongly that DM density is generated according a Universal law as power of E $D_{DM} = A \cdot E^B$ where A and B are parameters which depend on each galaxy, more exactly, values of coefficients A and B depend on mass of galaxies.

In addition, taking in consideration results of previous paper, it has been found that the more massive the galaxy the less DM density is at a specific value of E. Results also suggest that two galaxies with similar baryonic mass have similar DM density at a specific value of E.

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