Motion of an Object due to the Adjusted Rate of Modifications Performed on its Environment

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Abstract

A special statistical model illustrates how an object can move anisotropically in two dimensions by taking advantage of some controlled modifications of its environment, under the condition that the rate of such modifications depends on the position of the object under study.

Introduction

In 2014, I wrote an article [1] that focused nearly exclusively on the quantum mechanical behavior of a certain modification of Szilard's 1929 model[2], showing how such a modification could circumvent the need for any costly "memory erasure". The resulting picture was arguably rather deprived of intuitive features. In the present article, I undertake to discuss a closely related model by starting at a purely classical level (section A). The role of its most fundamental physical ingredients can be understood in a transparent way (section B). The link of the present model with two earlier models I considered respectively in 2005 and 2014 is discussed in section C.

A. A simple model

Let us consider in the present section a simple discrete statistical model, constituted by a two-dimensional grid upon which a single object A and a countable number of different objects B_1, B_2,…B_n…can move from one grid-intersection to the next. Let us further suppose that objet A can only move along the *xx'* direction, whereas all objects B_i can only move along the *yy'* direction. The only authorized motion of all A and B_i consists in jumping from their current position on the grid to a nearest *empty* location at discrete times t_i. This condition of emptiness ensures that the motions of A and of each B_i are correlated.

Let us now describe in more detail the rules according to which our system, whose initial state is randomly chosen (cf. Fig. 1), must evolve. These rules apply for a discrete series of discrete times t_i , with $i \geq 0$:

- at time t_{4i+1}, all objects B_i, except those belonging to the same column as A, are simultaneously translated by one grid unit-length towards y. All objects belonging to the same column as A remain immobile (cf. Fig. 2).

- at time t_{4i+2}, object A is allowed to move by one unit length, either towards the left or the right, provided the corresponding location(s) is(are) empty. Supposing for instance that only the *left* (resp. *right*) location is available for A's move, we expect that the two possible final locations of A will be occupied with pre-defined probabilities p_{left} and p_{middle} (respectively p_{middle} and p_{right}), with $p_{left}+p_{middle}=1$ (resp. : $p_{middle}+p_{right}=1$). One possible resulting configuration is represented in Fig. 3.

- at time t_{4i+3}, all objects B_i located *on the left side* of A are uniformly translated by one unit of length towards y (cf. Fig. 4), whereas all other B_i remain immobile.

- at time t_{4i+4}, A is allowed to move according to exactly the same rules as at time t_{4i+2}. (cf. Fig. 5)

In most cases, the resulting long-term motion of A can be expected to exhibit some rather complex behavior, including some chaotic characteristics. However, at least one limiting case can be analyzed easily without any computer simulation : it corresponds to the situation wherein the average density $\rho_{(B)}$ of B_i objects becomes so close to 1 that nearly all grid locations are occupied by one B_i. In that case, the probability that A may be simultaneously surrounded by two empty locations along the xx' axis becomes negligible, so that its anisotropic average displacement (towards the left) during the interval $t_{4i+1} \leq t \leq t_{4i+4}$ simply becomes approximately equal to :

$$Drift_{(A)} \approx (1-\rho_{(B)})(2p_{left} - p_{right}) \tag{1}$$

Under the most common physical circumstances, supposing that A's motion can be described for instance in terms of Brownian motion, $p_{left}=p_{right}={}^1/_2$.

A more useful scenario corresponds to the case when A's displacement towards the left costs an amount of work δW per lattice unit-length. For the sake of simplicity, let us continue to suppose that $\rho_{(B)}$ is so close to 1 that A is practically never surrounded by two empty locations at the same time, and that A's drift is influenced by Brownian collisions with a thermalizer whose uniform and constant temperature is T. We obtain :

$$p_{left} \approx e^{-\delta W/k_B T}/(1+e^{-\delta W/k_B T}) \tag{2a}$$

$$p_{right} \approx e^{+\delta W/k_B T}/(1+e^{+\delta W/k_B T}) \tag{2b}$$

Eq. (1) thus becomes :

$$Drift_{(A)} \approx (1-\rho_{(B)})\left(2e^{-\delta W/k_B T}/(1+e^{-\delta W/k_B T}) - e^{+\delta W/k_B T}/(1+e^{+\delta W/k_B T})\right) \tag{3}$$

Provided δW remains small enough, A continues to drift, on average, towards the left. The amount of thermal energy (random collisions) converted to "useful" energy (derived from A's anisotropic motion) increases linearly with time. If the grid possesses infinite dimensions, energy conversion can reach arbitrarily high levels.

B. Salient features of the model

Imposing that A and B_i should move in discrete steps upon a grid, jumping from one intersection to a neighboring one, provides us with an idealized representation which, admittedly, does not look very physical. A fully realistic picture can be easily constructed, however, without altering its statistical properties, for instance by dividing the xy plane into square boxes of identical dimensions (Fig. 6) within which A and B_i may evolve in the same way as a single molecules would do. The rules enunciated above (§A) for the motion of A and B_i can be physically implemented in the following way : at times t_{4i+1} and t_{4i+3}, suitable columns of boxes are shifted by one unit-length towards y ; at times t_{4i+2} and t_{4i+4}, the separations between the box containing A and its two nearest neighbor boxes (left and right) are removed ; shortly before t_{4i+3} and t_{4i+5}, these separations are reinstated.

Within a classical picture, shifting entire columns of boxes boxes at t_{4i+1} and t_{4i+3} does not cost any energy ; suppressing or reinstating dividing walls at t_{4i+2} and t_{4i+4} does not cost any energy either. Within a quantum picture, the situation becomes slightly different : whereas shifting boxes at t_{4i+1} and t_{4i+3} remains energetically costless, suppressing or reinstating dividing walls at t_{4i+2} and t_{4i+4} involves some energy. Fortunately, this difference between the classical and quantum picture disappears when one works with a sufficiently high temperature : in that case, the energy cost of removing or reinstating dividing walls becomes marginally small in comparison with k_BT.

Yet another complication arises, at least in principle, within a quantum picture : object A may partially occupy different boxes *simultaneously*, with a different occupation probability for each box. Fortunately, this kind of delocalization effect does not raise any energy issue ; what is more, sooner or later, some decoherence process can be expected to localize A somewhere in the lattice, which leads us to recover a classical picture, even if we do not really need such a simplification to occur.

Under more general terms, let me further emphasize that no part of the present article implies the use of any unconventional physical concept ; even less the use of any new mathematical equation. Standard thermodynamic notions, which can be found explained in any textbook, fully suffice to describe what happens in my model. In order to keep my article as short as possible, and to avoid the fastidious discussion of many concrete details that would not be of any fundamental importance, let me focus on two key features of the above model only, starting with the most crucial one :

(i) First of all, it appears obvious that the most original step of the procedure described in §A above occurs at $t=t_{4i+3}$: only the B_i located *on the left side* of A are supposed to be manipulated at $t=t_{4i+3}$. Such a procedure immediately raises the following question : doesn't one need to be informed of A's location in order to displace those B_i located *on the left side* of A only ? If this really had to be the case, handling the corresponding information would become thermodynamically so costly that our entire energy-conversion scheme would be ruined (even if the information needed at $t=t_{4i+3}$ consisted of a few bits only). It is therefore absolutely crucial for us to show that any "knowledge" of A's location by an external machine is not necessary, which can be done independently of A's precise characteristics :

- If A is macroscopic object (in that case, our discussion only corresponds to a "thought experiment" of weak experimental relevance), it can be equipped with a machine whose task consists in displacing surrounding boxes ; obviously, this machine does not need to know where it is located in order to distinguish between its left and its right[1], so that no information-cost problem exists in this case.

- If A is a microscopic object (such as a single molecule, as in Szilard's 1929 article), shifting surrounding boxes must be controlled by an external operator ; this can be accomplished at $t=t_{4i+3}$ with a Hamiltonian of the following form :

$$\mathcal{H} = W.\ \Sigma_{i,j}\ |\textit{i column located at the left of j shifted towards y}\rangle.\langle \textit{i column located at the left of j}| \otimes |A_j\rangle.\langle A_j| + c.c. \quad (4)$$

In the above Hamiltonian, columns of boxes are indexed by i , and A's locations are indexed by j ; W simply serves as a numerical factor. This Hamiltonian is strongly reminiscent of another Hamiltonian proposed in 2014[1], enabling an operator to monitor a modified version of Szilard's engine in a perfectly mechanical way, without having the need to erase any information in a costly way at all. Using a slightly modified version of Eq. 4 (with a Hamiltonian acting on particles B_i instead of acting on the boxes containing them) could also enable us to avoid the awkward task of having to shift entire columns of boxes (whose idealized size is supposed to be infinite in our model described in §A above), still without any extra thermodynamic cost ; in that case, one should be careful to avoid the possibility that A's trajectory towards the left may be obstructed forever by the same B_i objects ; this could be avoided, for instance, by adding a small source of thermal noise allowing B_i's position to fluctuate, or otherwise a small source of thermal fluctuation allowing B_i objects to be created/destroyed at a certain rate. In any case, let us stress that this kind of technical issue does not affect the most fundamental principles upon which our model is based.

1 This situation is strongly reminiscent of the one I described in 2005[3,4], where a compass (playing a role analogous to a macroscopic version of the object named A in the present article) was supposed to be equipped with all the machinery required to modify its own moment of inertia, thereby enabling it to explore different rotation states.

(ii) Another feature of the model presented in §A should also be at least mentioned : the need for a clock ordering times t_{4i+1}, t_{4i+2}, t_{4i+3}, t_{4i+4} *etc*. Information about current-time, like any other kind of information, should *a priori* be considered thermodynamically costly to erase. As it happens, however, in the present case, we do not even need to examine whether a clever procedure may help us to avoid the cost of erasing information about time. Let us rather simply suppose that we find ourselves experimentally in the worst situation ; in other words, let us suppose that the implementation of our model requires us to handle information concerning time at a cost reaching as much as a few k_BT for each time interval. Instead of studying a single object A, we might then choose to deal simultaneously with several A_i objects. Only one clock would be necessary for dealing simultaneously with all those A_i objects. The kinetic energy derived from the drift of all A_i should increase in a roughly linear fashion with the number of A_i objects, thereby safely exceeding the energy needed for monitoring a single clock.

C. Comparison with two other previously proposed models

The model presented above (§A) presents some very strong links with the modified version of Szilard's single-particle engine[1] which I discussed in 2014. Fundamentally speaking, it appears even justified to consider it as kind of 2-dimensional "illustration" of my 2014 model. This 2-dimensional version represents more than a mere redundancy, however, as the three following arguments can show :

(i) Although, from the point of view of quantum physics, the simplicity of my 2014 model could hardly be surpassed, my new model appears simpler, more intuitive and even self-contained from the point of view of classical physics (even simply "statistics"), which may possibly prove helpful one day for the development of concrete experimental set-ups.

(ii) In 2014, I conjectured that "it is rather likely that a hidden continuity may be found between both of my [2005 and 2014] models". My present model goes a long way into showing how this conjecture must indeed have been correct. The crucial step t_{4i+3}, described in §A above, serves to increase the degrees of freedom available on object *A's* left side only ; it can be paralleled with the procedure serving to temporally decrease the moment of inertia of the "compass" of my 2005 model, thereby increasing the number of its thermally accessible rotational degrees of freedom ; it can also be closely paralleled with the functioning of the Hamiltonian $\mathcal{H}$ described in my 2014 model. Eventually, although the technical details of kinetic energy acquisition, shared by both my 2014 and my present model, do not seem to reproduce closely those of my 2005 model, all of these three models possess a most essential ingredient : an isothermal procedure analogous to the one followed at step t_{4i+3} above (§A), according to which some fluctuating process (describable in terms of spin precession, single molecule dynamics or distribution of Szilard boxes) may be adjusted as a function of the variable location of a given object (described respectively by the "compass" of my 2005 model, the "single molecule" of my 2014 model, and by "object *A*" of my present model)

(iii) Already in 2005, I wondered why I needed to mimic a quasi "cyclic" evolution of events in order to break the second law, even as my model did not require the use of two different sources of heat characterized by different temperatures, as standard Carnot cycles do. The "quasi-cyclic" features of my 2014 model are also quite prominent. Why couldn't my earlier models operate in a more "continuous", less "cyclic" fashion ? In fact, it now clearly appears that the quasi "cyclic" appearance of my former models was purely accidental. My present model provides a rather convenient way to illustrate the fact that the removal of the quasi "cyclic" features of my previous models is indeed possible.

Conclusion

The idea that a finite being endowed with extremely sharp faculties might serve to illustrate the statistical nature of the second principle of thermodynamics was originally enunciated by Maxwell in his 1871 treatise, *Theory of Heat*[2]. Three years later, such a hypothetical being was nicknamed by William Thomson an "intelligent demon". A humoristic mythology has continued to develop around this term in the literature (even in cartoons[6]) ever since. Perhaps such folklore would not have enjoyed the same degree of popularity if Thomson had used the more antiquated spelling *dæmon*. In any case, Thomson's reaction to Maxwell's treatise can be *a posteriori* credited with the merit of having encouraged more physicists to ponder on the potentially interesting connections existing between entropy and information (even "intelligence"). The title of Szilard's 1929 article *On the decrease of entropy in a system by the intervention of intelligent beings* appears quite emblematic in this respect. Along the years, scientists like Claude Shannon, Rolf Landauer, Charles H. Bennett and many others have helped to clarify a large number of related issues. Ultimately, their answer to the question of whether any concrete "intelligent being" could decrease the entropy of a system in any useful way has become more and more clearly negative. My own answer to the same question is just as negative as theirs. However, as it happens, what an "intelligent being" cannot do, a "perfectly stupid being" can achieve ! Such a "mindless" being can perpetuate its own oriented course of motion without ever needing to bother about erasing the information which it has never even started to record in the first place.

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[2] The statement according to which the 2nd Law of Thermodynamics is based on a kind of "statistical certainty" has been precisely formulated by Maxwell in an undated letter to Tait (Cf. Ref. [5]). In this letter, Maxwell indicates :
Concerning Demons.
1. *Who gave them this name ? Thomson.*
2. *What were they by nature ? Very small BUT lively beings incapable of doing work but able to open and shut valves which move without friction or inertia.*
3. *What was their chief end? To show that the 2nd Law of Thermodynamics has only a statistical certainty.*
4. *Is the production of an inequality of temperature their only occupation? No, for less intelligent demons can produce a difference in pressure as well as temperature by merely allowing all particles going in one direction while stopping all those going the other way. This reduces the demon to a valve. As such value him. Call him no more a demon but a valve like that of the hydraulic ram, suppose.*

Figures

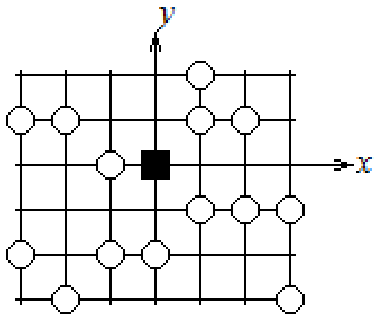

Figure 1
Initial configuration of the system at time t_0=0. This configuration has been randomly chosen. Object A has been represented by a square (■). Objects B_i correspond to empty circles (O).

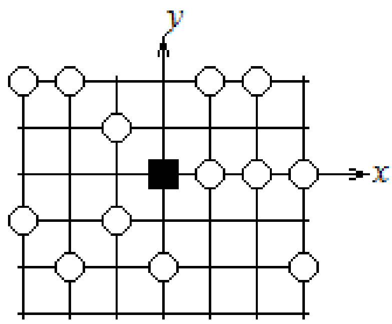

Figure 2
Configuration of the system at time t=t_1. All objects B_i have been shifted upwards, except for the only one located along the same vertical as A.

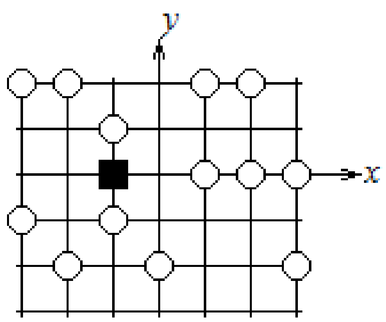

Figure 3
Configuration of the system at time t=t_2. Object A has been shifted towards the left by one lattice unit. According to our rules, A could have either stayed in place with probability p_{middle}, or shifted towards the left with probability p_{left}. Only the second possibility has been represented here.

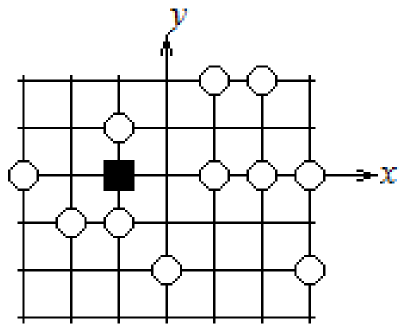

Figure 4
Configuration of the system at time t=t_3. All objects B_i located on the left side of A have been shifted upwards by one lattice unit.

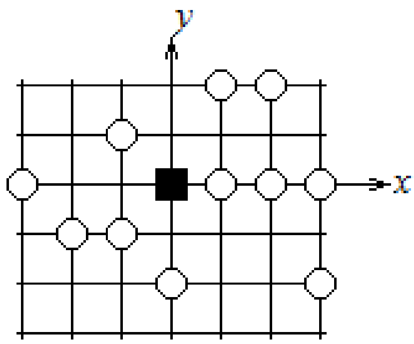

Figure 5
Configuration of the system at time t=t_4. Object A has been shifted towards x by one lattice unit. According to our rules, A could either have moved towards the left, remained in place or moved towards the right by one lattice unit. Only the third possibility has been represented here.

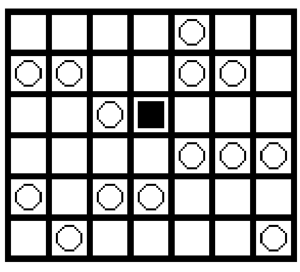

Figure 6
Insertion of our system (the configuration is identical to the initial configuration represented in Fig. 1) within an array of boxes, whose separating walls may supposedly be controlled at will by the operator (or by object A, depending on what kind of specific model one wishes to consider).

References

[1] Barriquand F., *When is Information Erasure Necessary Between Successive Thermodynamic Cycles ?* (2014) http://vixra.org/abs/1401.0009

[2] Szilard L., *On the decrease of entropy in a system by the intervention of intelligent beings*, Z. f. Physik 53, 840-856 (1929). Article reproduced in [5].

[3] Barriquand F., *Perte d'information et irréversibilité en thermodynamique*, Revue des Questions Scientifiques, **176** (**1**), 59-74 (2005). An English version of this article, without figures, is available at http://arxiv.org/abs/physics/0507184 . The figures of the French version can also be found in [4].

[4] Barriquand F., *Retrospective examination of Three Articles Published in the Revue des Questions Scientifiques in 2005 and 2006*. (2010) http://vixra.org/abs/1006.0057
The main interest of this article resides, in my view, in the thought-experiment illustrated in Fig. 2 of this 2010 article. Having acquired only a rather superficial knowledge of general relativity, I was unaware in 2010 that the "principle of equivalence" between acceleration and gravitation fails to apply if rotational degrees of freedom are taken into consideration (cf. Hans C. Ohanian, Remo Ruffini, *Gravitation and Spacetime*, Cambridge, 2013m p. 41). Technically speaking, the thought-experiment I proposed in Fig. 2 of my 2010 article is therefore invalid if one considers, as I did, the "spin" properties of a pair or particles in order to illustrate the problematic character of the combination of gravitational and non-local quantum effects. Obviously, however, most observables (except "spin"/orbital-moment : I have been unfortunate to pick up an unsuitable observable) are compatible with the equivalence principle. This is why the question raised by Fig. 2 of my 2010 article continues to be fundamentally relevant.
For the sake of completeness, let me mention that my 2010 article also contains another technical error. In part A.4 of this article, I stated that "the condition that initial eigenstate populations should decrease as a function of energy eigenvalues is never exactly fulfilled at the macroscopic level, even when macroscopic equilibrium is attained. This is due to the fact that whenever the spatial length required to distinguish between two different eigenstates exceeds the thermal coherence length of the system, thermalization of such different eigenstates cannot impose their respective occupations to be energetically ordered". Strictly speaking, this statement is simply wrong, for which I wish to apologize.
A.E. Allahverdyan and Th. M. Nieuwenhuizen's 2002 study[7], which I wished to signal as an interesting reference in my 2010 article, proves the validity of the second law under several conditions. Their study requires, in particular, that the initial eigenstate populations of a canonical ensemble should decrease as a function of energy eigenvalues. Whenever decoherence effects occur in a system, eigenstate populations are likely to be "measured" (quantum mechanically speaking) at any time, which shows that A.E. Allahverdyan and Th. M. Nieuwenhuizen's constraint cannot be expected to be universally respected. However, the precise characteristics of the initial eigenstate populations of a given system have nothing to do with the main reason that explains why A.E. Allahverdyan and Th. M. Nieuwenhuizen's proof does not apply to any of the three models I have proposed in 2005, 2014 and at present. A.E. Allahverdyan and Th. M. Nieuwenhuizen's proof rests on calculating the energy variation of a system $\delta W=\mathrm{tr}\{H_0[\rho(t)-\rho(0)]\}$. The result of the calculation proves that $\delta W\geq 0$. In all my models, adopting the most direct way to define a quantity similar to Allahverdyan and Nieuwenhuizen's δW leads to the perfectly satisfying equality $\delta W=0$! In other words, Allahverdyan and Nieuwenhuizen's mathematical theorem is not violated in the least by any of my models. But this does not imply that these models do not violate the second law : in order to check its validity, it does not suffice to compute the energy variation of what Allahverdyan and Nieuwenhuizen define as their "system". For instance, my 2005 model consists of one "compass" C with two different isothermal baths (which I may note here Th_1 and Th_2). As it happens, Th_1 thermalizes the ensemble $\{C + Th_2\}$, whereas Th_2 thermalizes the ensemble $\{C + Th_1\}$. As I have shown in 2005, the entire process results in some energy loss in Th_2 (whereas the average energy variation of C remains zero in the end!). In my 2014 and my present model, the average energy of what Allahverdyan and Nieuwenhuizen's

article would induce us to define as a "system" also remains constant. Energy conversion is not to be searched within such a "system", but within its thermalizer(s), whose history does not develop in a cyclic way at all.

[5] Harvey S. Leff and Andrew F. Rex (editors), *Maxwell's Demon 2*, IOP publishing, 2003, p. 5.

[6] Bennet Charles H., *Information physics in cartoons*, Superlattices and Microstructures **23**, 367-372 (1998), reproduced in Harvey S. Leff and Andrew F. Rex (editors), *Maxwell's Demon 2*, IOP publishing, 2003, p. 346-351.

[7] Allahverdyan A.E. and Nieuwenhuizen Th. M., *A mathematical theorem as the basis for the second law : Thomson's formulation applied to equilibrium*, Physica A **305** (2002), 542-552.