(3G, 2e) virtual model of final unification

U. V. S. Seshavatharam[1*] & S. Lakshminarayana[2]

[1]Honorary Faculty, I-SERVE, Alakapuri, Hyderabad-35, Telangana, India.
[2]Department of Nuclear Physics, Andhra University, Visakhapatnam-03, AP, India
[*]For correspondence: seshavatharam.uvs@gmail.com

Abstract: In the early publications, with reference to final unification, the authors suggested that, 1) There exists a strong interaction elementary charge of magnitude, *es*~4.72058686E-19 C. 2) Like quarks, the strong interaction elementary charge is experimentally undetectable and can be called as 'invisible elementary nuclear charge'. 3) There exists a gravitational constant associated with strong interaction, *Gs*~3.329560807E28 m3/kg/sec2. 4) There exists a gravitational constant associated with electromagnetic interaction, *Ge*~2.374335472E37 m3/kg/sec2. Based on these concepts, an attempt is made to understand the mystery of origin of 'discrete' angular momentum of electron in hydrogen atom. Proceeding further, estimated value of Newtonian gravitational constant is G_N~6.67985603E-11 m3/kg/sec2.

Keywords: 3 different gravitational constants, 2 different elementary charges, hydrogen atom, *s* - shell, final unification.

1. Introduction

1.1. About 'strong gravity' and 'strong nuclear charge '

Roberto Onofrio says: "It is worth to point out that, with different motivations, the concept of 'strong gravity' has appeared from time to time in the literature, especially in connection with the possibility that gravity plays a role in the confinement of quarks inside hadrons through black-hole analogies, although not within the framework of considering weak interactions as derivable from gravity at short length scale" [1,2].

According to Roberto Onofrio [1], weak interactions are peculiar manifestations of quantum gravity at the Fermi scale, and that the Fermi coupling constant is related to the Newtonian constant of gravitation. In his opinion. at atto-meter scale, Newtonian gravitational constant seems to reach a magnitude of $8.205\times10^{22}\text{m}^3\text{kg}^{-1}\text{sec}^{-2}$. In this context, one can see plenty of papers on 'strong gravity' in physics literature [3-19]. It may be noted that, till date, 'strong gravity' is a non-mainstream theoretical approach to Color confinement/particle confinement having both a cosmological scale and a particle scale gravity. In between ~(1960 to 2000), it was taken up as an alternative to the then young QCD theory by several theorists, including Abdus Salam [3]. Very interesting point to be noted is that, Abdus Salam showed that the 'particle level gravity approach' can produce confinement and asymptotic freedom while not requiring a force behavior differing from an inverse-square law, as does QCD. C. Sivaram published a review of this [4].

Qualitatively and quantitatively, references [1-20] strongly suggest the possible existence of 'Newtonian (like) gravitational constant with very large magnitude' in nuclear and particle physics. Based on this concept and in pursuit of bridging the gap in between 'General theory of relativity' and 'Quantum field theory' [21-24], in the recent publications [25-30], the authors suggested and validated the role of two gravitational constants associated with strong and electromagnetic interactions.

Proceeding further, the authors also suggested and validated the role of a new elementary charge associated with nuclear physics and currently believed strong coupling constant [31,32]. This new elementary charge can be compared with the historical strong interaction elementary charge. It may be noted that, in nuclear physics literature, staring form 1950's scientists supposed the existence of a new type of 'charge' associated with strong interaction. In analogy with electromagnetic interaction strength $\alpha \cong e^2/4\pi$, quantum chromo dynamics [33] presumes the strong interaction strength as $\alpha_s \cong g_s^2/4\pi$. Considering many body nuclear system, strong elementary charge was assumed to be playing a key role [34-39]. In this connection, with reference the old historical idea of 'strong nuclear chare', in this paper, the authors made an attempt in fixing and extending the applications of 'strong nuclear charge' starting from the 'strong coupling constant' to the observable nuclear properties like 'magnetic moments' of nucleons, 'nuclear binding energy' and 'nuclear stability'.

1.2. About 'unification of quantum mechanics' and 'general theory of relativity'

A) Even though 'String theory' and 'Quantum gravity' models [40,41] are having a strong mathematical back ground and sound physical basis, both the models are failing in developing a 'workable' model of final unification. In this context, at fundamental level, starting from sub-nuclear physics to low energy (observable) nuclear physics, along with the proposed 'new nuclear elementary charge' proposed two gravitational constants assumed to be associated with electron and proton seem to play a vital role in understanding the basics of final unification. In an integrated approach the authors also showed that, 'quantum of angular momentum' is a characteristic result of the combined effects of gravitational constants associated with proton and electron. Proceeding further, the authors discovered simple relations that seem to be connected with the three gravitational constants i.e, Newtonian gravitational constant and the proposed two gravitational constants assumed to be associated with proton and electron.

1.3. Key topics of this paper

In this paper,

1. The authors revised the third assumption and compiled important characteristic relations pertaining to 'final unification'.
2. Made an attempt to understand the mystery of discrete nature of revolving electron's discrete angular momentum.
3. Proposed three simple semi empirical relations for estimating the Newtonian gravitational constant.

2. Three basic assumptions of final unification

In the recent publications [25-30] the authors proposed and established three assumptions. Here, in this paper the authors revised the third assumption for better understanding.

Assumption-1: Magnitude of the gravitational constant associated with the electromagnetic interaction is, $G_e \cong 2.374335472 \times 10^{37}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$.

Assumption-2: Magnitude of the gravitational constant associated with the strong interaction is, $G_s \cong 3.329560807 \times 10^{28}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$.

(Revised) Assumption-3: There exists a strong elementary charge, $e_s \cong 4.72058686 \times 10^{-19}$ C. Like quarks, the strong interaction elementary charge is experimentally undetectable and can also be called as 'invisible elementary nuclear charge'.

Note-1: It may be noted that, with reference to the operating force magnitudes, protons and electrons cannot be considered as 'black holes'. See section (6) for a detailed discussion. But electrons and protons can be assumed to follow the relations that black holes generally believed to follow. Clearly speaking, in the study of black holes, Newtonian gravitational constant G_N plays a major role, whereas in the study of elementary particles, G_s and G_e play the key role.

Note-2: As 3 gravitational constants and 2 elementary charges seem to be involved in this model, the authors wish to call this model as (3*G*, 2*e*) virtual model of final unification.

3. Important and characteristic unified results

Considering the following semi empirical results one can understand and validate the role of the proposed three assumptions.

$$\text{Here,}\begin{cases} e \cong 1.602\ 176\ 565(35) \times 10^{-19}\ \text{C}, \\ \varepsilon_0 \cong 8.854187817 \times 10^{-19}\ \text{F/m} \\ m_n \cong 1.674\ 927\ 471(21) \times 10^{-27}\ \text{kg}, \\ m_p \cong 1.672\ 621\ 777(74) \times 10^{-27}\ \text{kg} \\ m_e \cong 9.109\ 382\ 91(40) \times 10^{-31}\ \text{kg}, \\ \hbar \cong 1.054\ 571\ 726(47) \times 10^{-34}\ \text{J.sec.} \\ \alpha \cong 7.297\ 352\ 5698(24) \times 10^{-3} \end{cases}$$

$$\text{and}\quad \begin{Bmatrix} G_s \cong 3.329560807 \times 10^{28}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\ G_e \cong 2.374335472 \times 10^{37}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\ e_s \cong 4.72058686 \times 10^{-19}\ \text{C} \end{Bmatrix}$$

1) Nuclear charge radius:

$$R_0 \cong \frac{2G_s m_p}{c^2} \cong 1.239291 \times 10^{-15}\ \text{m} \tag{1}$$

2) **Root mean square radius of proton**:

$$R_p \cong \frac{\sqrt{2}G_s m_p}{c^2} \cong 0.876311\times10^{-15}\ \text{m} \tag{2}$$

3) **Fermi's weak coupling constant**:

If $\dfrac{G_s m_p^2}{R_0^2} \cong \dfrac{c^4}{4G_s}$,

$$F_W \cong \left(\frac{e_e}{e_s}\right)\left[\frac{\left(G_s m_p^2\right)\left(G_s m_e^2\right)}{\left(c^4/4G_s\right)}\right] \cong 1.44021\times10^{-62}\ \text{J.m}^4 \tag{3}$$

4) **Bohr radius of electron**:

$$a_0 \cong \left(\frac{4\pi\varepsilon_0 G_e m_e^2}{e_e^2}\right)\left(\frac{G_s m_p}{c^2}\right) \tag{4}$$

This is one crystal clear result of the proposed $\left(G_e, G_s\right)$. See section-3 for its potential application.

5) **Fine structure ratio**:

$$\alpha \cong \frac{e_e^2}{4\pi\varepsilon_0\hbar c} \cong \frac{e_s e_e}{4\pi\varepsilon_0 G_s m_p^2} \tag{5}$$

6) **Strong interaction strength**:

$$\beta \cong \frac{e_s^2}{4\pi\varepsilon_0\hbar c} \cong \left(\frac{e_s}{e_e}\right)\frac{e_s^2}{4\pi\varepsilon_0 G_s m_p^2} \cong 0.06334853354 \tag{6}$$

7) **Ratio of Strong and electromagnetic interaction strengths**:

$$\frac{\alpha}{\beta} \cong \left(\frac{e_e}{e_s}\right)^2 \cong 0.1151937095 \tag{7}$$

Here, very interesting point to be noted that, $\left(\alpha/\beta\right)$ seems to be matching with the currently believed 'strong coupling constant' α_s [9,10]. Geometric mean strength of strong and electromagnetic interactions i.e. $\sqrt{\alpha\beta}$ seems to play a crucial role in nuclear binding energy scheme [27]. See the following relation (8).

8) **Nuclear binding energy close to stable atomic nuclides' beginning range**:

$$\begin{aligned} BE &\approx \left(Z-2+\sqrt{\frac{Z}{30}}\right)\left(\sqrt{\alpha\beta}-\alpha\beta\right)\left(m_p c^2\right) \\ &\approx \left(Z-2+\sqrt{\frac{Z}{30}}\right)19.74\ \text{MeV} \end{aligned} \tag{8}$$

where $Z \geq 5$

Note that, according to Fermi gas model of nucleus [27], mean kinetic energy of nucleon is roughly 20 MeV and can be fitted with $\sqrt{\alpha\beta}\left(m_p c^2\right) \cong 20.173$ MeV.

9) **Ratio of rest mass of proton and electron**:

$$\left.\begin{aligned} &\left(\frac{m_p}{m_e}\right) \cong \left(\frac{4\pi\varepsilon_0 G_e m_e^2}{e_e^2}\right) \Big/ \left(\frac{4\pi\varepsilon_0 G_s m_p^2}{e_s^2}\right) \\ &\rightarrow \left(\frac{m_p}{m_e}\right) \cong \left(\frac{e_s^2 G_e}{e_e^2 G_s}\right)^{1/3} \cong \left(\frac{\beta G_e}{\alpha G_s}\right)^{1/3} \end{aligned}\right\} \tag{9A}$$

If is G_N the Newtonian gravitational constant, it is noticed that,

$$\left.\begin{aligned} &\left(\frac{m_p}{m_e}\right) \cong \sqrt{\frac{e_e}{e_s}}\sqrt{\frac{G_s}{G_N^{1/3}G_e^{2/3}}} \cong \sqrt{\frac{e_e G_s}{e_s G_N^{1/3} G_e^{2/3}}} \\ &\Rightarrow G_N \cong \left(\frac{e_e}{e_e}\right)^7\left(\frac{G_s^5}{G_e^4}\right) \\ &\cong 6.679856043\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\ &\text{See section-4 for further information.} \end{aligned}\right\} \tag{9B}$$

10) **Specific charge ratio of proton and electron:**

$$\begin{aligned} k &\cong \left(\frac{G_s m_p m_e}{\hbar c}\right) \cong \left(\frac{\hbar c}{G_e m_e^2}\right) \cong \left[\left(\frac{e_s}{m_p}\right)\Big/\left(\frac{e_e}{m_e}\right)\right] \\ &\cong \frac{\text{specific charge of proton associated with } e_s}{\text{specific charge of electron associated with } e_e} \\ &\cong 1.604637101\times10^{-3} \end{aligned} \tag{10}$$

Note that, this ratio seems to play a key role in understanding 'electronic stability' in hydrogen atom and 'proton-neutron stability' in nuclear physics and casts doubt on the independent existence of 'quantum constants'.

11) **Reduced Planck's constant**:

$$\left.\begin{array}{l}\hbar \cong \left[\left(\dfrac{e_s}{m_p}\right)\Big/\left(\dfrac{e_e}{m_e}\right)\right]\left(\dfrac{G_e m_e^2}{c}\right) \\ \cong \left(\dfrac{e_s}{e_e}\right)\left(\dfrac{m_e}{m_p}\right)\left(\dfrac{G_e m_e^2}{c}\right) \cong \left(\dfrac{e_s}{e_e}\right)\left(\dfrac{G_s m_p^2}{c}\right) \\ \cong \left(\dfrac{m_e}{m_p}\right)^{\frac{1}{2}}\left[\dfrac{\sqrt{\left(G_s m_p^2\right)\left(G_e m_e^2\right)}}{c}\right]\end{array}\right\} \quad (11A)$$

Alternatively, with reference to proton-electron rest mass, i.e. from relation (9B), it is also noticed that,

$$\hbar \cong \left(\frac{e_e}{e_s}\right)^2\left(\frac{G_s}{G_N^{1/3}G_e^{2/3}}\right)\left(\frac{G_s m_e^2}{c}\right) \quad (11B)$$

12) **Proton-neutron beta stability line**:

$$\left.\begin{array}{l}A_s \cong 2Z + \left[\left(\dfrac{e_s}{m_p}\right)\Big/\left(\dfrac{e_e}{m_e}\right)\right](2Z)^2 \\ \cong 2Z + \left(\dfrac{e_s m_e}{e_e m_p}\right)(2Z)^2 \cong 2Z + \left(\dfrac{G_s m_p m_e}{\hbar c}\right)(2Z)^2 \\ \cong 2Z + (0.0064185Z)^2 \\ \text{where } A_s \text{ is the stable mass number of } Z.\end{array}\right\} \quad (12)$$

13) **Magnetic moment of electron**:

$$\mu_p \cong \frac{e_e\hbar}{2m_e} \cong \left(\frac{e_e}{e_s}\right)^2\left(\frac{G_s}{G_N^{1/3}G_e^{2/3}}\right)\left(\frac{G_s m_e e_e}{2c}\right) \quad (13)$$

14) **Magnetic moment of muon**:

$$\mu_\mu \cong \frac{e_e\hbar}{2m_\mu} \cong \left(\frac{e_e}{e_s}\right)^2\left(\frac{G_s}{G_N^{1/3}G_e^{2/3}}\right)\left(\frac{m_e}{m_\mu}\right)\left(\frac{G_s m_e e_e}{2c}\right) \quad (14)$$

15) **Magnetic moment of tau**:

$$\mu_\tau \cong \frac{e_e\hbar}{2m_\tau} \cong \left(\frac{e_e}{e_s}\right)^2\left(\frac{G_s}{G_N^{1/3}G_e^{2/3}}\right)\left(\frac{m_e}{m_\tau}\right)\left(\frac{G_s m_e e_e}{2c}\right) \quad (15)$$

16) **Magnetic moment of proton**:

$$\mu_p \cong \frac{e_s\hbar}{2m_p} \cong \left(\frac{e_e}{e_s}\right)^2\left(\frac{G_s}{G_N^{1/3}G_e^{2/3}}\right)\left(\frac{m_e}{m_p}\right)\left(\frac{G_s m_e e_s}{2c}\right) \quad (16)$$

17) **Magnetic moment of neutron**:

$$\left.\begin{array}{l}\mu_n \cong \left(\dfrac{e_s\hbar}{2m_n} - \dfrac{e_e\hbar}{2m_n}\right) \cong \dfrac{\hbar}{2m_n}(e_s - e_e) \\ \cong \left(\dfrac{e_e}{e_s}\right)^2\left(\dfrac{G_s}{G_N^{1/3}G_e^{2/3}}\right)\left(\dfrac{m_e}{m_n}\right)\left[\dfrac{G_s m_e (e_s - e_e)}{2c}\right]\end{array}\right\} \quad (17)$$

3. Understanding the mystery of quantum nature of electron in Hydrogen Atom

Considering relations (1) to (17), the authors would like to stress the following facts.

A) Along with the new strong elementary charge, within the atomic medium there exit two different gravitational constants and their existence is real, not virtual.

B) Considering $(G_s \text{ and } G_e)$ magnitudes of quantum constants like 'basic unit of angular momentum', 'basic unit of electron's distance' etc. can be fitted and understood.

C) It may be noted that, according to Bohr's theory of hydrogen atom [42,43], number of electrons that can be accommodated in any principal quantum shell is $2n^2$. Based on this idea, it is possible to assume that, probability of finding any one electron is $\left(\frac{1}{2n^2}\right)$. It can be obtained in the following way.

D) Out of $2n^2$ electrons, number of electrons that can be accommodated in *s* shell is 2. If one is willing to consider *s* shell as a basic entity in such a way that, *p* shell constitutes *3s* shells, *d* shell constitutes 5s shells, *f* shell constitutes *7s* shells etc, then, n^2 can be considered as a representation of total number of *s* shells that can be accommodated in any principal quantum shell.

E) Notation point of view, it can be assigned for *p* shell: *ps1*, *ps2*,*ps3* and for *d* shell: *ds1*,*ds2*,*ds3*, *ds4*,*ds5 etc.* Transition of electron from 2nd orbit *p* shell to 1st orbit *s* shell can be expressed as: 2*ps1* to 1*s*,2*ps2* to 1*s*, 2*ps3* to 1*s*. Thinking in this way different transition levels can be expected. With reference to *p* shell, 3 different spectral lines, with reference to *d* shell, 5 different spectral lines can be expected. Similarly with reference to *f* shell, 7 different spectral lines can be expected.

F) If so, it is also possible to assume that, probability of finding any one *s* shell is $\left(\frac{1}{n^2}\right)$. Based on this proposal, from relation (4), discrete potential energy of s shell in hydrogen atom can be expressed as follows.

$$E_{pot} \cong -\left(\frac{1}{n^2}\right)\left(\frac{e_e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e_e^2 c^2}{4\pi\varepsilon_0 G_s m_p}\right) \quad (18)$$

where $\left(\frac{e_e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)$ represents a force ratio and n^2 represents the total number of s shells corresponding to n^{th} principal quantum shell. Thinking in this way, orbiting radius of n^2 number of s shells can be expressed as,

$$\left.\begin{aligned} a_n &\cong n^2\left\{\left(\frac{4\pi\varepsilon_0 G_e m_e^2}{e_e^2}\right)\left(\frac{G_s m_p}{c^2}\right)\right\} \\ &\cong n^2\left\{\left(\frac{m_p}{m_e}\right)\left(\frac{4\pi\varepsilon_0 G_s m_p^2}{e_s^2}\right)\left(\frac{G_s m_p}{c^2}\right)\right\} \end{aligned}\right] \quad (19)$$

Clearly speaking, a_n represents the orbiting radius of n^2 s shells. In this way, the long standing concept of 1:4:9:16 etc. can be understood in a more meaningful approach. Now, s shell's discrete kinetic energy can be expressed as follows.

$$\begin{aligned} E_{kin} &\cong \frac{1}{2}\left|E_{pot}\right| \\ &\cong \left(\frac{1}{2n^2}\right)\left(\frac{e_e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e_e^2 c^2}{4\pi\varepsilon_0 G_s m_p}\right) \end{aligned} \quad (20)$$

Discrete total energy of one s shell can be expressed as follows.

$$\left.\begin{aligned} E_{tot} &\cong -\left(\frac{1}{2n^2}\right)\left(\frac{e_e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e_e^2 c^2}{4\pi\varepsilon_0 G_s m_p}\right) \\ &\cong -\left(\frac{1}{n^2}\right)\left(\frac{e_e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e_e^2}{4\pi\varepsilon_0\left(2G_s m_p/c^2\right)}\right) \\ &\cong -\left(\frac{1}{n^2}\right)\left(\frac{e_e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e_e^2}{4\pi\varepsilon_0 R_0}\right) \end{aligned}\right\} (21)$$

$$\left\{\begin{array}{l}\text{where } n = 1,2,3,.. \text{ and} \\ R_0 \cong \left(2G_s m_p/c^2\right) \cong 1.239291 \text{ fm}\end{array}\right.$$

Here important points to be noted are:

1. $\left(\frac{e_e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)$ is the ratio of electromagnetic and gravitational force of electron where the operating gravitational constant is G_e not G_N.
2. $\left(\frac{e_e^2}{4\pi\varepsilon_0 R_0}\right) \approx 1.16$ MeV is the currently believed characteristic nuclear coulombic potential.
3. Ratio of total ground state energy of electron in hydrogen atom and characteristic nuclear coulombic potential is equal to ratio of electromagnetic and gravitational force of electron where the operating gravitational constant is G_e.

3.1 Understanding the origin of 'quantum of angular momentum' in hydrogen atom

Here it may be noted that, in the hydrogen atom, there exists only one electron. Hence relation (26) can be considered as a representation of the total energy of electron. Comparing this relation (21) with Bohr's theory of hydrogen atom, relation (11) can be obtained with the following relation.

$$\left(\frac{e_e^4 m_e}{32\pi^2\varepsilon_0^2 n^2\hbar^2}\right) \cong \left(\frac{1}{2n^2}\right)\left(\frac{e_e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e_e^2 c^2}{4\pi\varepsilon_0 G_s m_p}\right) \quad (22)$$

$$\hbar \cong \left(\frac{m_e}{m_p}\right)^{\frac{1}{2}}\left[\frac{\sqrt{\left(G_s m_p^2\right)\left(G_e m_e^2\right)}}{c}\right] \quad (22)$$

From relation (9),

$$\frac{G_s}{G_e} \cong \frac{e_s^2}{e_e^2} \times \frac{m_e^3}{m_p^3} \quad (23)$$

Following this relation (23), relation (22) can be written into two different forms as expressed in relation (11).

3.2 Understanding the integral nature of 'quantum of angular momentum' in hydrogen atom

If one is willing to consider the following three points, it is possible to understand the integral nature of electron's angular momentum,

1. Within the atom, electronic arrangement is 'systematic'.
2. In n^{th} principal quantum shell, there is a scope for the existence of n^2 number of (currently believed) s-shells.
3. 3 number of s-shells can be collectively called as one 'p-shell'. Similarly 5 number of s-shells can be collectively called as one 'd-shell' and so on.

Now the famous expression for integral nature of angular momentum can be expressed as:

$$\hbar_n \cong n\left\{\sqrt{\frac{m_e}{m_p}}\frac{\sqrt{\left(G_s m_p^2\right)\left(G_e m_e^2\right)}}{c}\right\} \tag{24}$$

Here, $n \cong \sqrt{n^2}$ represents the number of s shells. In hydrogen atom, as there exists only one s-shell and one electron, it appears from Bohr's theory of hydrogen atom, that – revolving electron's angular momentum is $n\hbar$ and distance is $n^2 a_0$. This is the key point to be noted here.

The emitted energy can be expressed as follows.

$$\left.\begin{aligned}
E_{emis} &\cong \left\{\left(\frac{e_e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e_e^2 c^2}{8\pi\varepsilon_0 G_s m_p}\right)\right\}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)\\
&\cong \left\{\left(\frac{e_e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e_e^2}{4\pi\varepsilon_0\left(2G_s m_p/c^2\right)}\right)\right\}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)\\
&\cong \left\{\left(\frac{e_e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e_e^2}{4\pi\varepsilon_0 R_0}\right)\right\}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)\\
&\text{where } n_2^2 > n_1^2 \text{ and } R_0 \cong \left(2G_s m_p/c^2\right) \cong 1.239291 \text{ fm}
\end{aligned}\right\} \tag{25}$$

4. To fit the Newtonian gravitational constant

It may be noted that, coupling Newtonian gravitational constant G_N with elementary physical constants is really a challenging issue and demands sound physical reasoning. In the earlier publications [44-50] and references therein, the authors proposed interesting semi empirical relations. With the proposed assumptions, it is noticed that, proton-electron mass ratio, elementary charge ratio and ratio of any two gravitational constants etc. seem to play a key role in this new approach. The authors are on the way to understand the 'back ground physics' of these relations. With further research, in near future, exact unified relations can be developed and absolute value of G_N can be estimated [51-56].

With reference to Proton-Electron mass ratio and proposed assumptions, it is noticed that,

$$\sqrt{\frac{e_s}{e_e}\left(\frac{m_p}{m_e}\right)} \cong \sqrt{\frac{G_s}{G_N^{1/3}G_e^{2/3}}} \tag{26}$$

With reference to Planck mass $M_{pl} \cong \sqrt{\hbar c/G_N}$ and proposed assumptions, it is noticed that,

$$\frac{\sqrt{M_{pl}m_e}}{m_p} \cong \left(\frac{G_e}{G_N}\right)^{\frac{1}{6}} \tag{27}$$

Interesting observation is that,

$$\sqrt{\left(G_e/G_N\right)} \approx 5.964622\times10^{23} \tag{28}$$

This number is very close to the Avogadro number [27-34]. Alternative expression can also be expressed as follows.

$$\left(\frac{\sqrt{M_{pl}m_p}}{m_e}\right)^3 \cong \sqrt{\frac{e_e}{e_s}\left(\frac{G_s}{G_N}\right)} \tag{29}$$

Thus,

$$\begin{aligned}
G_N &\cong \left(\frac{e_e}{e_e}\right)^7\left(\frac{G_s^5}{G_e^4}\right)\\
&\cong 6.679856043\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}
\end{aligned} \tag{30}$$

This estimated value of G_N can be compared with the experimental values [51-56]. It may be noted that, as gravity is much weaker than other fundamental forces and an experimental apparatus cannot be separated from the gravitational influence of other bodies, G_N is quite difficult to measure. So far, no standard model could couple gravity with other fundamental forces and hence it does not appear possible to calculate the value of G_N directly from other (accurate) microscopic physical constants. In addition, published values of G_N have varied rather broadly, and some recent measurements of high precision are, in fact, mutually exclusive. In 2007, Fixler et al [52] described a new measurement of the gravitational constant by 'atom interferometry', reporting a value of $G_N \cong 6.693(34)\times10^{-11}$ m^{3}kg^{-1}sec^{-2}. An improved cold 'atom measurement' by Rosi et al [53] was published in 2014 and reported a value of $G_N \cong 6.67191(99)\times10^{-11}$ m^{3}kg^{-1}sec^{-2}. Most recent (CODATA: 2014) recommended value of G_N is $6.67408(31)\times10^{-11}$ m^{3}kg^{-1}sec^{-2}. In this context, the authors would like to stress the fact that, fitting the value of G_N with 'unification methodology' is quite different from the existing experimental methods of G_N and seems to be promising and versatile.

5. Discussion

5.1 The classical limits of force and power

To unify cosmology, quantum mechanics and the four observed fundamental cosmological interactions – certainly a 'unified force' is required. In this connection $\left(c^4/G_N\right) \cong 1.21\times10^{44}$ N can be considered as the classical force or astrophysical force limit. Similarly, $\left(c^5/G_N\right) \cong 3.6284\times10^{52}$ J can be considered as the classical power limit. If it is true that c and G are fundamental physical constants in physics, then $\left(c^4/G_N\right)$ and $\left(c^5/G_N\right)$ can also be considered as fundamental compound physical constants. These classical limits are more powerful than the Uncertainty limit. These two characteristic limits are for future experimental verification on Earth based man-made heavy equipments like nuclear bombs, particle accelerators, nuclear reactors and rocket propulsion centres etc. More over these two characteristic limits can be understood with future astrophysical and cosmological interpretations, observations and inferences. Without considering the current notion of black hole physics, Schwarzschild radius of black hole [57,58] can be understood with the characteristic astrophysical limiting force of magnitude $\left(c^4/G_N\right)$.

5.2 Simple applications of $\left(c^4/G_N\right)$

a) Magnitude of force of attraction or repulsion between any two charged particles never exceeds $\left(c^4/G_N\right)$.
b) Magnitude of gravitational force of attraction between any two massive bodies never exceeds $\left(c^4/G_N\right)$.
c) Magnitude of mechanical force on a revolving/rotating body never exceeds $\left(c^4/G_N\right)$.
d) Magnitude of electromagnetic force on a revolving body never exceeds $\left(c^4/G_N\right)$.

5.3 Simple applications of $\left(c^5/G_N\right)$

a) Mechanical power never exceeds $\left(c^5/G_N\right)$
b) Electromagnetic power never exceeds $\left(c^5/G_N\right)$
c) Thermal radiation power never exceeds $\left(c^5/G_N\right)$
d) Gravitational radiation power never exceeds $\left(c^5/G_N\right)$

5.4 Schwarzschild radius of a black hole

The four basic physical properties of a rotating black hole are its mass, size, angular velocity and temperature. Without going deep into the mathematics of black hole physics in this section an attempt is made to understand the Schwarzschild radius of a black hole. In all directions, if a force of magnitude $\left(c^4/G_N\right)$ acts on the mass-energy content of the assumed celestial body it approaches a minimum radius of $\left(G_N M/c^2\right)$ in the following way. Origin of the force $\left(c^4/G\right)$ may be due to self-weight or internal attraction or external compression or something else.

$$R_{\min} \cong \frac{Mc^2}{\left(c^4/G_N\right)} \cong \frac{G_N M}{c^2} \tag{31}$$

If no force (of zero magnitude) acts on the mass content M of the assumed massive body, its radius becomes infinity. With reference to the average magnitude of $\left(0, \frac{c^4}{G_N}\right) \cong \frac{c^4}{2G_N}$, the presently believed Schwarzschild radius can be obtained as

$$(R)_{ave} \cong \frac{Mc^2}{\left(c^4/2G_N\right)} \cong \frac{2G_N M}{c^2} \tag{32}$$

This proposal is very simple and seems to be different from the existing concepts and may be a unified form of the Newton's law of gravity, Special theory of relativity and General theory of relativity.

5.5 Understanding the strength of any interaction

From the above relations it is reasonable to say that,

1) If it is true that c and G_N are fundamental physical constants, then $\left(c^4/G_N\right)$ can be considered as a fundamental compound constant related to a characteristic limiting force.
2) Black holes are the ultimate state of matter's geometric structure.
3) Magnitude of the operating force at the black hole surface is the order of $\left(c^4/G_N\right)$.
4) Gravitational interaction taking place at black holes can be called as 'Schwarzschild interaction'.
5) Strength of 'Schwarzschild interaction' can be assumed to be unity.
6) Strength of any other interaction can be defined as the ratio of operating force magnitude and the classical or astrophysical force magnitude $\left(c^4/G_N\right)$.
7) If one is willing to represent the magnitude of the operating force as a fraction of $\left(c^4/G_N\right)$ i.e. X times of $\left(c^4/G_N\right)$, where $X << 1$, then

$$\frac{X \text{ times of } \left(c^4/G_N\right)}{\left(c^4/G_N\right)} \cong X \to \text{Effective } G \Rightarrow \frac{G_N}{X} \qquad (33)$$

If X is very small, $\frac{1}{X}$ becomes very large. In this way, X can be called as the strength of interaction. Clearly speaking, strength of any interaction is $\frac{1}{X}$ times less than the 'Schwarzschild interaction' and effective G becomes $\frac{G}{X}$.

5.6 Are protons and electrons black holes?

1) With reference to Schwarzschild interaction, for electromagnetic interaction, $X \cong 2.811\times10^{-48}$ and for strong interaction, $X \cong 2.0\times10^{-39}$.
2) Characteristic operating force corresponding to electromagnetic interaction is $\left(c^4/G_e\right) \cong 3.4\times10^{-4}$ N and ccharacteristic operating force corresponding to strong interaction is $\left(c^4/G_s\right) \cong 242603$ N.
3) Characteristic operating power corresponding to electromagnetic interaction is $\left(c^5/G_e\right) \cong 10991$ J/sec and ccharacteristic operating power corresponding to strong interaction is $\left(c^5/G_s\right). \cong 7.273\times10^{13}$ J/sec
4) Based on these concepts, from relation (11A), can be expressed as follows.

$$\hbar c \cong \frac{\left(m_e c^2\right)^{\frac{3}{2}}\left(m_p c^2\right)^{\frac{1}{2}}}{\sqrt{\left(c^4/G_e\right)\left(c^4/G_s\right)}} \qquad (34)$$

$$\hbar \cong \frac{\left(m_e c^2\right)^{\frac{3}{2}}\left(m_p c^2\right)^{\frac{1}{2}}}{\sqrt{\left(c^5/G_e\right)\left(c^5/G_s\right)}} \qquad (35)$$

5) As $\left[\left(c^4/G_e\right),\left(c^4/G_s\right)\right] << \left(c^4/G_N\right)$ and $\left[\left(c^5/G_e\right),\left(c^5/G_s\right)\right] << \left(c^5/G_N\right)$, protons and electrons can not be considered as 'black holes'. But may be assumed to follow similar relations that black holes generally believed to follow.

6. Conclusion

Now a days, 'String theory' [40,41] is being believed as a promising candidate for a 'quantum theory of gravity'. It was first studied in the late 1960s as a theory of the strong nuclear force. Even though it is having a strong mathematical back ground and sound physical footing, so far, string theory could not provide any clue for understanding the observed elementary particle mass spectrum and atomic and nuclear structures in terms of gravity. In this context, qualitatively and quantitatively, by considering the proposed concepts and relations, the authors would like to stress the following points.

A) The proposed three assumptions can be given priority at fundamental level and with further research their physical existence (whether virtual or real) can be understood.
B) Characteristic quantum physical constants that are believed to be 'unique physical constants' are

compound in nature and hence can be considered as 'secondary physical constants'.

C) Discrete nature of orbiting electron can be better understood with 'systematic arrangement' of n^2 number of s-shells.

D) With further research, relations like (26) to (30) can be developed and absolute value of the Newtonian gravitational can be estimated.

E) If one is willing to explore the possibility of incorporating the proposed assumptions either in 'String theory' models or in 'Quantum gravity' models or 'Strong gravity' models, certainly, back ground physics assumed to be connected with proposed semi empirical relations can be understood and a 'practical' model of "everything" can be developed.

Acknowledgements

Author Seshavatharam U.V.S is indebted to professors K.V. Krishna Murthy, Chairman, Institute of Scientific Research in Vedas (I-SERVE), Hyderabad, India and Shri K.V.R.S. Murthy, former scientist IICT (CSIR), Govt. of India, Director, Research and Development, I-SERVE, for their valuable guidance and great support in developing this subject.

References

[1] Roberto Onofrio. On Weak Interactions as Short-Distance Manifestations of Gravity. Modern Physics Letters A, Vol. 28, No. 7 1350022 (2013)

[2] Roberto Onofrio. Proton radius puzzle and quantum gravity at the Fermi scale. EPL 104, 20002 (2013)

[3] Salam A, Sivaram C. Strong Gravity Approach to QCD and Confinement. Mod. Phys. Lett., 1993, v. A8(4), 321- 326. (1983)

[4] C. Sivaram and K. P. Sinha. Strong gravity, black holes, and hadrons. Phys. Rev. D 16 (1975).

[5] C. J. Ishame al, f-Dominance of Gravity. Phys. Rev. D 3 867. (1971)

[6] K. Tennakone. Electron, muon, proton, and strong gravity. Phys. Rev. D 10(1974) 1722

[7] C. Sivaram et al. Gravitational charges, f-gravity and hadron masses. Pramana. Vol 2, No 5, pp229-238 (1974)

[8] C. Sivaram and K. P. Sinha, Strong spin-two interaction and general relativity. Phys. Rep. 51 111(1979)

[9] C. Sivaram et al. Gravity of Accelerations on Quantum Scales. http://arxiv.org/abs/1402.5071

[10] Usha Raut and K.P. Sinha. Strong gravity and the Fine structure constant. Proc. Indian natn. Sci.Acad. 49 A, No 2, pp. 352-358 (1983)

[11] K.P.Krishna. Gauge theories of weak and strong gravity. Pramana. Vol 23, No 2, pp205-214 (1984)

[12] V. De. Sabbata and C. Sivaram. Strong Spin-Torsion Interaction between Spinning Protons. IL Nuovo Cimento Vol. 101 A, No 2, pp.273-283, (1989)

[13] Recami E. Elementary Particles as Micro-Universes, and "Strong Black-holes": A Bi-Scale Approach to Gravitational and Strong Interactions. Preprint NSF-ITP- 02-94. posted in the arXives as the e-print physics/0505149, and references therein.

[14] Abdus Salam. Strong Interactions, Gravitation and Cosmology. Publ. in: NATO Advanced Study Institute, Erice, June16-July 6, 1972 .

[15] P. Caldirola, M. Pavsic and Recami E. Explaining the Large Numbers by a Hierarchy of Universes: A Unified Theory of Strong and Gravitational Interactions. IL Nuovo Cimento Vol. 48 B, No. 2, 11 (1978)

[16] Recami E and V. T. Zanchin. The strong coupling constant: its theoretical derivation from a geometric approach to hadron structure. Foundations of Physics letters, vol-7, no.1, pp. 85-93.(1994).

[17] V. T. Zanchin and Recami E. Regge like relations for stable (non-evaporating) black holes. Foundations of Physics letters, vol-7, no.2, pp.167-179 (1994).

[18] S. I. Fisenko, M. M. Beilinson and B. G. Umanov. Some notes on the concept of strong gravitation and possibilities of its experimental investigation. Physics Letters A, Vol-148, Issues 8-9, 3 Sep 1990, pp 405-407.

[19] Fedosin S.G. Model of Gravitational Interaction in the Concept of Gravitons. Journal of Vectorial Relativity, Vol. 4, No. 1, pp.1-24. (2009)

[20] M. Kumar and S. Sahoo. Elementary Particles as Black Holes and Their Binding Energies. The African Review of Physics 8:0025. pp. 165-168(2013)

[21] David Gross, Einstein and the search for Unification. Current science, Vol. 89, No. 12, 25 (2005).

[22] P. A. M. Dirac, The cosmological constants. Nature, 139, 323, 9. (1937)

[23] P. A. M. Dirac, A new basis for cosmology. Proc. Roy. Soc. A 165, 199, (1938).

[24] Abdus Salam. Einstein's Last Dream: The Space - Time Unification of Fundamental Forces, Physics News, Vol.12, No.2, p.36. (1981)

[25] U. V. S. Seshavatharam, Lakshminarayana S. Lakshminarayana. To Validate the Role of Electromagnetic and Strong Gravitational Constants via the Strong Elementary Charge. Universal Journal of Physics and Application 9(5): 210-219 (2015)

[26] U. V. S. Seshavatharam et al. On Fundamental Nuclear Physics & Quantum Physics in Light of a Plausible Final Unification. Prespacetime Journal, Vol 6, Issue 12, pp. 1451-1468 (2015)

[27] U. V. S. Seshavatharam, Lakshminarayana S. Understanding nuclear binding energy with low energy elementary charge and root mean square radius of proton. Journal of Applied Physical Science International. Vol 6, Issue 1,pp.1-13, (2016)

[28] U. V. S. Seshavatharam, Lakshminarayana S. To confirm the existence of nuclear gravitational constant, Open Science Journal of Modern Physics. 2(5): 89-102 (2015)

[29] U. V. S. Seshavatharam, Lakshminarayana S. Final unification with Schwarzschild's Interaction. Journal of Applied Physical Science International 3(1): 12-22 (2015).

[30] Seshavatharam, U. V. S. et al. Fermi's weak coupling constant and Newtonian gravitational constant in the light of final unification. To be appeared in Prespacetime journal (December 2015)

[31] P.J. Mohr, B.N. Taylor, and D.B. Newell CODATA Recommended Values of the Fundamental Physical Constants:2010, by in Rev. Mod. Phys. 84, 1527 (2012)

[32] K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014)

[33] Brower, Richard C. et al. Glueball Spectrum for QCD from AdS Supergravity Duality. Nuclear Physics B 587: 249–276. (2000)

[34] L.D.Landau. The theory of fermi liquid. Soviet physics. JETP. Vol 3, No 6, pp. 920-925 (1957)

[35] L.P. Kadanoff and G. Baym, Quantum Statistical Mechanics W.A. Benjamin, New York (1962)

[36] W. Kohn and L. Sham, Phys. Self-Consistent Equations Including Exchange and Correlation Effects. Rev. 140, A1133 (1965)

[37] B. D. Day. Elements of the Brueckner-Goldstone Theory of Nuclear Matter. Rev. Mod. Phys. 39, 719 (1967)

[38] A.L. Fetter and J.D. Walecka. Quantum Theory of Many-Particle Systems. McGraw-Hill, San Francisco, (1971).

[39] Serot, B.D. and Walecka, J.D. Advances in Nuclear Physics, Vol. 16. Negele, J.W. and Vogt, E., Eds., Plenum, New York. (1986)

[40] Juan M. Maldacena. Gravity, Particle Physics and Their Unification. Int.J.Mod.Phys. A15S1 840-852 (2000)

[41] Sen, Ashoke. Strong-weak coupling duality in four-dimensional string theory. International Journal of Modern Physics A 9 (21): 3707–3750 (1994)

[42] Niels Bohr. On the Constitution of Atoms and Molecules, Part I . Philosophical Magazine 26 (151): pp. 1–24. (1913).

[43] Niels Bohr. On the Constitution of Atoms and Molecules, Part II Systems Containing Only a Single Nucleus. Philosophical Magazine 26 (153): pp.476–502. (1913)

[44] U. V. S. Seshavatharam and S. Lakshminarayana. Analytical estimation of the gravitational constant with atomic and nuclear physical constants. Proceedings of the DAE-BRNS Symp. on Nucl. Phys. 60 (2015).

[45] Seshavatharam, U. V. S. & Lakshminarayana, S., On the Plausibility of Final Unification with Avogadro Number. Prespacetime Journal. Vol 5, Issue 10, pp. 1028-1041(2014).

[46] U. V. S. Seshavatharam and S. Lakshminarayana. Nucleus in Strong nuclear gravity. Proceedings of the DAE Symp. On Nucl. Phys. 56: 302 (2011)

[47] U. V. S. Seshavatharam and S. Lakshminarayana, To confirm the existence of atomic gravitational constant. Hadronic journal, Vol-34, No 4, p.379.(Aug 2011)

[48] U. V. S. Seshavatharam and S. Lakshminarayana. Logic Behind the Squared Avogadro Number and SUSY. International Journal of Applied and Natural Sciences. Vol. 2, Issue 2, p.23-40 (2013).

[49] U. V. S. Seshavatharam, S. Lakshminarayana. Past, Present and Future of the Avogadro Number. Global Journal of Science Frontier Research Volume 12, Issue 7, p. 27-37 (2012)

[50] U. V. S. Seshavatharam, S. Lakshminarayana. Molar Electron Mass and the Basics of TOE, Journal of Nuclear and Particle Physics, Vol. 2 No. 6, p. 132-141 (2012)

[51] S. Schlamminger and R.D. Newman. Recent measurements of the gravitational constant as a function of time. Phys. Rev. D 91, 121101 (2015)

[52] J. B. Fixler et al. Atom Interferometer Measurement of the Newtonian Constant of Gravity, Science 315 (5808): 74–77 (2007)

[53] G. Rosi, et al. Precision measurement of the Newtonian gravitational constant using cold atoms. Nature 510, 518-521. (2014)

[54] Terry Quinn, Harold Parks, Clive Speake and Richard Davis. An uncertain big G..Phys.Rev. Lett. 112.068103. (2013)

[55] George T Gillies. The Newtonian gravitational constant: recent measurements and related studies. Rep. Prog. Phys. 60 151, (1997)

[56] J Stuhler, M Fattori, T Petelski and G M Tino. MAGIA using atom interferometry to determine the Newtonian gravitational constant. J. Opt. B: Quantum Semiclass. Opt. 5 S75–S81 (2003)

[57] Roger Penrose. Chandrasekhar, Black Holes, and Singularities. J. Astrophys. Astr. (1996) 17, 213-231

[58] Subrahmanyan Chandrasekhar. On Stars, Their Evolution and Their Stability. Nobel Prize lecture, December 8 (1983).