A list of thirty-six polynomials and formulas that generate Fermat pseudoprimes

Abstract. In this paper I present a simple list of polynomials (in one or two variables) and formulas having the property that they generate Carmichael numbers or Poulet numbers, polynomials and formulas that I have discovered over time.

Polynomials that generate Carmichael numbers

1.

C = (30*n+7)*(60*n+13)*(150*n+31)

First six such Carmichael numbers: 2821, 488881, 288120421, 492559141, 776176261, 1632785701 (sequence A182085 in OEIS).

2.

C = (30*n-29)*(60*n-59)*(90*n-89)*(180*n-179)

First four such Carmichael numbers: 31146661, 2414829781, 192739365541, 197531244744661 (sequence A182088 in OEIS).

3.

C = (330*n+7)*(660*n+13)*(990*n+19)*(1980*n+37)

First two such Carmichael numbers: 63973, 461574735553 (sequence A182089 in OEIS).

4.

C = (30*n-7)*(90*n-23)*(300*n-79)

First five such Carmichael numbers: 340561, 4335241, 153927961, 542497201, 1678569121 (sequence A182132 in OEIS).

5.

C = (30*n-17)*(90*n-53)*(150*n-89)

First five such Carmichael numbers: 29341, 1152271, 34901461, 64377991, 775368901 (sequence A182133 in OEIS).

6.

C = (60*n+13)*(180*n+37)*(300*n+61)

First five such Carmichael numbers: 29341, 34901461, 775368901, 1213619761, 4562359201 (sequence A182416 in OEIS).

Polynomials that generate Poulet numbers

1.

P = 7200*n^2 + 8820*n + 2701

First eight such Poulet numbers: 2701, 18721, 49141, 93961, 226801, 314821, 534061, 665281 (sequence A214016 in OEIS).

2.

P = 144*n^2 + 222*n + 85

First eight such Poulet numbers: 1105, 2047, 3277, 6601, 13747, 16705, 19951, 31417 (sequence A214017 in OEIS).

3.

P = 3*(2*n + 1)*(18*n + 11)*(36*n + 17)

First four such Poulet numbers: 561, 62745, 656601, 11921001 (sequence A213071 in OEIS).

4.

P = (6*m - 1)*((6*m - 2)*n + 1)

First eleven such Poulet numbers: 341, 561, 645, 1105, 1905, 2047, 2465, 3277, 4369, 4371, 6601 (sequence A210993 in OEIS).

5.

P = (6*m + 1)*(6*m*n + 1)

First ten such Poulet numbers: 1105, 1387, 1729, 2701, 2821, 4033, 4681, 5461, 6601, 8911 (sequence A214607 in OEIS).

6.

P = m*n^2 + (11*m - 23)*n + 19*m - 49

First ten such Poulet numbers: 341, 645, 1105, 1387, 2047, 2465, 2821, 3277, 4033, 5461 (sequence A215326 in OEIS).

Formulas that generate Carmichael numbers

1.

C = (30*n - p)*(60*n - (2*p + 1))*(90*n - (3*p + 2)),
where p, 2*p + 1, 3*p + 2 are all three prime numbers.

First six such Carmichael numbers: 1729, 172081, 294409, 1773289, 4463641, 56052361 (sequence A182087 in OEIS).

Comment: The formula can be reduced to only two possible polynomial forms: C = (30*n - 23)*(60*n - 47)*(90*n - 71) or C = (30*n - 29)*(60*n - 59)*(90*n - 89).

2.

C = (p + 30)*(q + 60)*(p*q + 90),
where p and q are primes.

First two such Carmichael numbers: 488881, 1033669.

3.

C = (30*p + 1)*(60*p + 1)*(90*p + 1),
where p is prime.

First four such Carmichael numbers: 56052361, 216821881, 798770161, 1976295241.

4.

C = p*(2*p - 1)*(3*p - 2)*(6*p - 5),
where p is prime.

First seven such Carmichael numbers: 63973, 31146661, 703995733, 21595159873, 192739365541, 461574735553, 3976486324993 (sequence A182518 in OEIS).

5.

C = p*(2*p - 1)*(n*(2*p - 2) + p),
where p and 2*p - 1 are primes.

First ten such Carmichael numbers: 1729, 2821, 41041, 63973, 101101, 126217, 172081, 188461, 294409, 399001 (sequence 182207 in OEIS).

Comment: I conjecture that any Carmichael number C divisible by p and 2*p - 1 (where p and 2*p - 1 are primes) can be written this way.

6.

C = n*(2*n - 1)*(p*n - p + 1)*(2*p*n - 2*p + 1),
where p is odd and n natural.

Seven such Carmichael numbers: 63973, 172081, 31146661, 167979421, 277241401, 703995733, 1504651681 (sequence 212882 in OEIS).

7.

C = p*n*(3*p*n + 2)*(6*p*n - 1),
where p is prime and n natural.

Ten such Carmichael numbers: 2465, 62745, 11119105, 3249390145 (obtained for p = 5); 6601 (obtained for p = 7); 656601 (obtained for p = 11); 41041, 271794601 (obtained for p = 13); 11119105, 2159003281 (obtained for p = 17) (sequence 212882 in OEIS).

Formulas that generate Poulet numbers

1.

P = (2^(3*k + 1) - 1)/3,
where k natural.

First three such Poulet numbers: 341, 1398101, 5726623061.

Comment: The formula can be generalized as (n^(n*k + k + n - 1) – 1)/(n^2 – 1), formula which generates, I conjecture, an infinity of Fermat pseudoprimes to base n for any integer n, n > 1 (for n = 3 the formula becomes (3^(4*k + 2) - 1)/8 and generates Fermat pseudoprimes to base 3 for 14 values of k from 1 to 20).

2.

P = q*((n + 1)*q - n*q)*((m + 1)*q - m*q),
where q prime and m, n natural.

Five such Poulet numbers: 10585, 13741, 13981, 29341, 137149.

3.

P = q*((n*q - (n + 1)*q)*(m*q - (m + 1)*q),
where q prime and m, n natural.

Such Poulet number: 6601.

4.

P = q*(q + 2*n)*(q + 2^2*n - 2),
where q prime, n natural:

Two such Poulet numbers: 561, 1105.

5.

P = q*(q + 2*n)*(q + 2^k*n),
where q prime and n, k natural.

Four such Poulet numbers: 1729, 2465, 2821, 29341.

6.

P = (1 + 2^k*m)*(1 + 2^k*n)*(1 + 2^k*(m + n)),
where k, m, n natural.

Two such Poulet numbers: 13981, 252601.

7.

P = 3*(3 + 2^k)*(3 + q*2^h),
where q prime and k, h natural.

Three such Poulet numbers: 645, 1905, 8481.

8.

P = q^2 + 81*q + 3*q*r,
where q, r primes.

Four such Poulet numbers: 2821, 6601, 14491, 19951.

Comment: Note that the numbers (2821, 6601) and (14491, 19951) are "pairs" because 2821 = 13^2 + 81*13 + 3*13*41 while 6601 = 41^2 + 81*41 + 3*13*41 and also the values of the [q, r] for 14491 and 19951 are [43, 71] respectively [71, 43].

9.

P = r*q*(n*(q – 1) + r),
where r, q primes and n natural.

Six such Poulet numbers: 137149, 340561, 852841, 950797, 1052503, 1357621.

Comment: I conjecture that any Poulet number having as prime factors both the numbers 23 and 67 can be written this way, also any Poulet number having as prime factors both the numbers 11 and 61.

10.

P = 3*q^3*(3*n + 1) – q^2*(15*n + 2) + 6*q*n,
where q prime and n natural.

Six such Poulet numbers: 4335241, 13421773, 17316001, 17098369, 93869665, 170640961.

Comment: I conjecture that any Poulet number having as prime factors both a number of the form 30*k + 23 and a number of the form 90*k + 67 can be written this way.

11.

P = 6*q^3*(6*n + 1) – q^2*(66*n + 5) + 30*q*n,
where q prime and n natural.

Six such Poulet numbers: 5148001, 7519441, 9890881, 12262321.

Comment: I conjecture that any Poulet number having as prime factors both a number of the form 30*k + 11 and a number of the form 180*k + 61 can be written this way.

12.

P = ((2^n)^k)*((2^n)^(k + 1) + 2^n + 1) + 1,
where k, n natural.

Ten such Poulet numbers: 561, 33153 (obtained for n = 1); 1105, 16705 (obtained for n = 2); 4369, 1052929, 268505089 (obtained for n = 4), 266305 (obtained for n = 6); 2113665 (obtained for n = 7); 16843009 (obtained for n = 8).

13.

P = 2*q^2 - q,
where q is also a Poulet number.

First six such Poulet numbers: 831405, 5977153, 15913261, 21474181, 38171953, 126619741 (sequence A215343 in OEIS).

14.

P = (q^2 + 2*q)/3,
where q is also a Poulet number.

First six such Poulet numbers: 997633, 1398101, 3581761, 26474581, 37354465, 63002501 (sequence A216276 in OEIS).

15.

P = q^2*n - q*n + p,
where q is also a Poulet number and n natural.

First six such Poulet numbers: 348161, 831405, 1246785, 1275681, 2077545, 2513841 (sequence A217835 in OEIS).

16.

P = (n^m + n*m)/(m + 1),
where m, n natural.

Ten such Poulet numbers: 341, 645, 2465, 2821, 4033 (obtained for m = 2); 341, 1729, 188461, 228241, 1082809 (obtained for m = 3) (sequence A216170 in OEIS).

17.

P = (6*k - 1)*((6*k - 2)*n + 1),
where k, n natural.

First eleven such Poulet numbers: 341, 561, 645, 1105, 1905, 2047, 2465, 3277, 4369, 4371, 6601 (sequence A210993 in OEIS).