# Two conjectures on Super-Poulet numbers with two respectively three prime factors

Marius Coman
email: mariuscoman13@gmail.com

**Abstract.** In this paper I make two conjectures on Super-Poulet numbers with two, respectively three prime factors.

**Definition:**

Super-Poulet numbers are the Poulet numbers whose divisors d all satisfy the relation d divides 2^d – 2 (see the sequence A050217 in OEIS for the list of Super-Poulet numbers).

**Note:**

Every 2-Poulet number (Poulet number with only two prime factors) is also a Super-Poulet number (see the sequence A214305 for the list of 2-Poulet numbers).

**Conjecture 1:**

For any 2-Poulet number q*r (obviously q and r primes, distinct (q < r) beside the case of the two 2-Poulet numbers which are the squares of the two known Wieferich primes) is true one of the following two statements:

i) there exist n positive integer such that r = n*q – n + 1;
ii) there exist p prime, p greater than 7, also n and m positive integers, such that q = n*p – n + 1 and r = m*p – m + 1.

**Verifying the conjecture:**
(For the first twenty-two 2-Poulet numbers)

: 341 = 11*31 and 31 = 11*3 – 2;
: 1387 = 19*73 and 73 = 19*4 – 3;
: 2047 = 23*89 and 89 = 23*4 – 3;
: 2701 = 37*73 and 73 = 37*2 – 1;
: 3277 = 29*113 and 113 = 29*4 – 3;
: 4033 = 37*109 and 109 = 37*3 – 2;
: 4369 = 17*257 and 257 = 17*6 – 5;
: 4681 = 31*151 and 151 = 31*5 – 4;
: 5461 = 43*127 and 127 = 43*3 – 2;

: 7957 = 73*109 and 73 = 6*13 - 5 while 109 = 9*13 - 8;
: 8321 = 53*157 and 157 = 53*3 - 2;
: 10261 = 31*331 and 331 = 31*11 - 10;
: 13747 = 59*233 and 233 = 59*4 - 3;
: 14491 = 43*337 and 337 = 43*8 - 7;
: 15709 = 23*683 and 683 = 23*31 - 30;
: 18721 = 97*193 and 193 = 97*2 - 1;
: 19951 = 71*281 and 281 = 71*4 - 3;
: 23377 = 97*241 and 97 = 6*17 - 5 while 241 = 15*17 - 14;
: 31417 = 89*353 and 353 = 89*4 - 3;
: 31609 = 73*433 and 433 = 73*6 - 5;
: 31621 = 103*307 and 307 = 103*3 - 2;
: 35333 = 89*397 and 89 = 4*23 - 3 while 397 = 18*23 - 17.

Note that the conjecture is obviously true for the case of the two 2-Poulet numbers which are the squares of the two known Wieferich primes, i.e. 1194649 = 1093^2 and 12327121 = 3511^2. For instance, the prime 1093 can be written in seven distinct ways like n*p - p + 1, where p prime: 1093 = 2*547 - 1 = 7*157 - 6 = 14*79 - 13 = 21*53 - 20 = 26*43 - 25 = 39*29 - 38 = 197*7 - 6 (and, of course, 1093 = 1093*1 - 0).

**Conjecture 2:**

For any Super-Poulet number with three prime factors p*q*r (obviously p, q and r primes, p < q < r) is true one of the following two statements:

iii) there exist n and m positive integers such that q = n*p - n + 1 and r = m*p - m + 1;

iv) there exist s prime, s greater than 7, also a, b and c positive integers, such that p = a*s - a + 1, q = b*s - b + 1 and r = c*s - c + 1.

**Verifying the conjecture:**
(For the first 18 such Super-Poulet numbers)

: 294409 = 37*73*109 and 73 = 37*2 - 1 while 109 = 37*3 - 2;
: 1398101 = 23*89*683 and 89 = 23*4 - 3 while 683 = 23*31 - 30;
: 1549411 = 31*151*331 and 151 = 31*5 - 4 while 331 = 31*11 - 10;
: 1840357 = 43*127*337 and 127 = 43*3 - 2 while 337 = 43*8 - 7;
: 12599233 = 97*193*673 and 193 = 97*2 - 1 while 673 = 97*7 - 6;

: 13421773 = 53*157*1613 and 157 = 53*3 - 2 while 1613 = 53*31 - 30;

: 15162941 = 59*233*1103 and 233 = 59*4 - 3 while 1103 = 59*19 - 18;

: 15732721 = 97*241*673 and 97 = 17*6 - 5 while 241 = 17*15 - 14 also 673 = 17*42 - 41;

: 28717483 = 59*233*2089 and 233 = 59*4 - 3 while 2089 = 59*36 - 35;

: 29593159 = 43*127*5419 and 127 = 43*3 - 2 while 5419 = 43*129 - 128;

: 61377109 = 157*313*1249 and 313 = 157*2 - 1 while 1249 = 157*8 - 7;

: 66384121 = 89*353*2113 and 353 = 89*4 - 3 while 2113 = 89*24 - 23;

: 67763803 = 103*307*2143 and 307 = 103*3 - 2 while 2143 = 103*21 - 20;

: 74658629 = 89*397*2113 and 89 = 23*4 - 3 while 397 = 23*18 - 17 while 2113 = 23*96 - 95;

: 78526729 = 43*337*5419 and 337 = 43*8 - 7 while 5419 = 43*129 - 128;

: 90341197 = 103*307*2857 and 307 = 103*3 - 2 while 2857 = 103*28 - 27;

: 96916279 = 167*499*1163 and 499 = 499*3 - 2 while 1163 = 167*7 - 6;

: 109322501 = 101*601*1801 and 601 = 101*6 - 5 while 1801 = 101*18 - 17.