Conjecture that states that the square of any prime can be written in a certain way

Abstract. In this paper we conjecture that the square of any prime greater than or equal to 5 can be written in one of the following three ways: (i) p*q + q – p; (ii) p*q*r + p*q – r; (iii) p*q*r + p – q*r, where p, q and r are odd primes. Incidentally, verifying this conjecture, we found results that encouraged us to issue yet another conjecture, i.e. that the square of any prime of the form 11 + 30*k can be written as 3*p*q + p – 3*q, where p and q are odd primes.

Conjecture:

The square of any prime s greater than or equal to 5 can be written in one of the following three ways: (i) p*q + q – p; (ii) p*q*r + p*q – r; (iii) p*q*r + p – q*r, where p, q and r are odd primes.

Verifying the conjecture:
(up to s = 41)

: 5^2 = 25 = 3*7 + 7 - 3;

: 7^2 = 49 = 3*13 + 13 – 3; also 49 = 3*3*5 + 3*3 – 5;

: 11^2 = 121 = 3*31 + 31 - 3; also 121 = 3*3*13 + 13 – 3*3; also 121 = 3*5*7 + 3*7 – 5;

: 13^2 = 169 = 5*29 + 29 - 5; also 169 = 3*43 + 43 – 3; also 169 = 3*5*11 + 3*5 – 11;

: 17^2 = 289 = 7*37 + 37 - 7; also 289 = 3*5*19 + 19 – 3*5; also 289 = 5*5*11 + 5*5 – 11; also 289 = 5*7*7 + 7*7 – 5;

: 19^2 = 361 = 11*31 + 31 - 11; also 361 = 3*7*17 + 3*7 – 17; also 361 = 3*3*37 + 37 – 3*3;

: 23^2 = 529 = 7*67 + 67 - 7; also 529 = 5*89 + 89 – 5;

: 29^2 = 841 = 19*43 + 24; also 841 = 13*61 + 61 – 13; also 841 = 11*71 + 71 – 11;

: 31^2 = 961 = 23*41 + 18; also 961 = 3*11*29 + 3*11 – 29; also 961 = 7*7*19 + 7*7 – 19; also 961 = 3*5*61 + 61 – 3*5;

: 37^2 = 1369 = 7*11*17 + 7*11 – 17;

: 41^2 = 1681 = 23*71 + 71 - 23; also 1681 = 3*13*43 + 43 – 3*13; also 1681 = 3*19*29 + 3*19 – 29; also 1681 = 5*17*19 + 5*17 – 19.

Conjecture:

The square of any prime s of the form 11 + 30*k can be written as 3*p*q + p – 3*q, where p and q are odd primes.

Verifying the conjecture:
(up to s = 131)

: for s = 11 we have [p, q] = [13, 3] (see above);
: for s = 41 we have [p, q] = [43, 13] (see above);
: for s = 71 we have [p, q] = [73, 23];
: for s = 101 we have [p, q] = [1021, 3] and [31, 113];
: for s = 131 we have [p, q] = [331, 17], [79, 73] and [953, 7];
: for s = 191 we have [p, q] = [2281, 5], [229, 53] and [13, 1013].