The following short paper was written in 2001 (together with more material), but is never was published.

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Birthday Present for the Chudnovskys

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Abstract

A formula for $\frac{1}{7}!^{42} = \Gamma(8/7)^{42}$ is given.

I. Introduction

I really don't know much about the Chudnovsky brothers - in fact less more than the astonishing story told in /1/. But they surely have a birthday or maybe even two. So it is time to send them an appropriate present: a formula for pi.

II. Here it is

$$\pi = \left[\frac{1}{7}!^{42} \cdot \frac{7^{29}}{2^{31}} \cdot \sin^2 \frac{2\pi}{7} \cdot \left(\frac{1}{2} + \frac{1}{8^2} + \frac{6^2}{8^2 \cdot 15^2} + \frac{6^2 \cdot 13^2}{8^2 \cdot 15^2 \cdot 22^2} + \frac{6^2 \cdot 13^2 \cdot 20^2}{8^2 \cdot 15^2 \cdot 22^2 \cdot 29^2} + \ldots \right)^{-9} \right.$$
$$\cdot \left(\ldots + \frac{4^2 \cdot 11^2}{6^2 \cdot 13^2 \cdot 20^2} + \frac{4^2}{6^2 \cdot 13^2} + \frac{1}{6^2} + \frac{1}{3^2} + \frac{1}{3^2 \cdot 10^2} + \frac{8^2}{3^2 \cdot 10^2 \cdot 17^2} + \ldots \right)^3 \tag{1}$$
$$\left. \cdot \left(\ldots + \frac{2^2 \cdot 9^2}{4^2 \cdot 11^2 \cdot 18^2} + \frac{2^2}{4^2 \cdot 11^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{3^2}{5^2 \cdot 12^2} + \frac{3^2 \cdot 10^2}{5^2 \cdot 12^2 \cdot 19^2} + \ldots \right)^4 \right]^{1/20}$$

I am sorry for the series expansions. Indeed they don't converge very quickly.

III. The main result

Of course there is a strategy you need to reach this formula. Mainly one should look about series expansions for different values of the gamma function. For example this one:

$$\frac{1}{7}!^{42} = \frac{2^{31} \cdot \pi^{20}}{7^{29} \cdot \sin^2 \frac{2\pi}{7}} \cdot \frac{\left(\frac{1}{2} + \frac{1}{8^2} + \frac{6^2}{8^2 \cdot 15^2} + \ldots \right)^9}{\left(\ldots + \frac{1}{6^2} + \frac{1}{3^2} + \frac{1}{3^2 \cdot 10^2} + \ldots \right)^3 \cdot \left(\ldots + \frac{1}{4^2} + \frac{1}{5^2} + \frac{3^2}{5^2 \cdot 12^2} + \ldots \right)^4} \tag{2}$$

Then (1) follows straightforward.

IV. Happy Birthday

Proofing formula (2) right should be shivering easy. And proofing this formula wrong could make you a great mathematician because (2) for sure is correct. Being a physicist giving a crude and somehow strange *Herleitung* is no problem. But this is another story. It's now up to you, mathematicians of the world, to total up a proof in these stormy times.

Literature

/1/ David Blatner: π – Magie einer Zahl,
Rowohlt Verlag, Reinbek bei Hamburg 2000.