Conjecture on the quadruplets of primes of the form (p, p+4k^2, p+6k^2, p+8k^2)

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Abstract. In a strict sence, the term "prime quadruplet" refers strictly to the primes (p, p + 2, p + 6, p + 8) - see Wolfram MathWorld; it is not known if there are infinitely many such prime quadruplets. In this paper I conjecture that for any k non-null positive integer there exist an infinity of quadruplets of primes of the form (p, p+2k^2, p+6k^2, p+8k^2). Finally, I define the generalized Brun's constant for prime quadruplets of the type showed and I estimate its value for the particular case k = 2 (for k = 1 the value it is known being approximately equal to 0.87).

Conjecture:

For any k non-null positive integer there exist an infinity of quadruplets of primes of the form (p, p + 2*k^2, p + 6*k^2, p + 8*k^2).

The first two quadruplets of this form for few values of k:

: for k = 1 we have (p, p + 2, p + 6, p + 8):

: (5, 7, 11, 13) and (11, 13, 17, 19).

Note that, beside the first quadruplet, the rest of them must have the form (30n+11, 30n+13, 30n+17, 30n+19).

: for k = 2 we have (p, p + 8, p + 24, p + 32):

: (5, 13, 29, 37) and (29, 37, 53, 61).

Note that, beside the first quadruplet, the rest of them must have the form (30n+29, 30n+37, 30n+53, 30n+61).

: for k = 3 we have (p, p + 18, p + 54, p + 72):

: (109, 127, 163, 181) and (139, 157, 193, 211).

Note that all of these quadruplets must have the form (30n+19, 30n+37, 30n+73, 30n+91).

: for k = 4 we have (p, p + 32, p + 96, p + 128):

: (11, 43, 107, 139) and (71, 103, 167, 199).
Note that all of these quadruplets must have the form (30n+11, 30n+43, 30n+107, 30n+139).

The first quadruplet of this form for few other values of k:

: for k = 5 we have (p, p + 50, p + 150, p + 200):

: (131, 181, 281, 331).

Note that all of these quadruplets must have the form (30n+11, 30n+61, 30n+161, 30n+211).

: for k = 6 we have (p, p + 72, p + 216, p + 288):

: (101, 173, 317, 389).

Note that all of these quadruplets must have the form (30n+11, 30n+83, 30n+227, 30n+299).

: for k = 7 we have (p, p + 98, p + 294, p + 392):

: (269, 367, 563, 661).

Note that all of these quadruplets must have the form (30n+29, 30n+127, 30n+323, 30n+421).

: for k = 8 we have (p, p + 128, p + 384, p + 512):

: (179, 307, 563, 691).

Note that all of these quadruplets must have the form (30n+29, 30n+157, 30n+413, 30n+541).

: for k = 9 we have (p, p + 162, p + 486, p + 648):

: (71, 233, 557, 719).

Note that all of these quadruplets must have the form (30n+11, 30n+173, 30n+497, 30n+659).

: for k = 10 we have (p, p + 200, p + 600, p + 800):

: (179, 307, 563, 691).

Note that these quadruplets must have one of the following four forms: (30n+11, 30n+211, 30n+611, 30n+811); (30n+17, 30n+217, 30n+617, 30n+817); (30n+23, 30n+223, 30n+623, 30n+823); (30n+29, 30n+229, 30n+629, 30n+829) .

The generalized Brun's constant for prime quadruplets

It is known that the Brun's constant for prime quadruplets represents the sum of the reciprocals of all prime quadruplets in the restricted sence that a prime quadruplet is (p, p + 2, p + 6, p + 8), that is ((1/5 + 1/7 + 1/11 + 1/13) + (1/11 + 1/13 + 1/17 + 1/19)...) and is approximately equal to 0.87. Let's see if we can find such constants for the generalized form of this prime quadruplet, i.e. the quadruplet (p, p + 2*k^2, p + 6*k^2, p + 8*k^2).

Let's take the quadruplet (p, p + 8, p + 24, p + 32) obtained from the general quadruplet for k = 2.

: (1/5 + 1/13 + 1/29 + 1/37) + (1/29 + 1/37 + 1/53 + 1/61) ≈ 0.435;

: (1/5 + 1/13 + 1/29 + 1/37) + (1/29 + 1/37 + 1/53 + 1/61) + (1/149 + 1/157 + 1/173 + 1/181) ≈ 0.459;

: (1/5 + 1/13 + 1/29 + 1/37) + (1/29 + 1/37 + 1/53 + 1/61) + (1/149 + 1/157 + 1/173 + 1/181) + (1/569 + 1/577 + 1/593 + 1/601) ≈ 0.466;

: (1/5 + 1/13 + 1/29 + 1/37) + (1/29 + 1/37 + 1/53 + 1/61) + (1/149 + 1/157 + 1/173 + 1/181) + (1/569 + 1/577 + 1/593 + 1/601) + (1/719 + 1/727 + 1/743 + 1/751) ≈ 0.471.

Finally, we conjecture that the value of *generalized Brun's constant* for prime quadruplets of the form (p, p + 2*k^2, p + 6*k^2, p + 8*k^2), for the particular case k = 2, is not greater than 0.49 and not less than 0.48.