

# The generalized Stokes theorem

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Abstract

When applied to a quaternionic manifold, the generalized Stokes theorem can provide an elucidating space progression model in which elementary objects float on symmetry centers that act as their parameter spaces.

## 1 Without discontinuities

Without discontinuities in the manifold  $\omega$  the generalized Stokes theorem is represented by a simple formula [1].

$$\int_{\Omega} d\omega = \int_{\partial\Omega} \omega; \left( = \oint_{\partial\Omega} \omega \right) \quad (1)$$

The manifolds  $\omega$  and  $d\omega$  represent quaternionic fields  $\mathfrak{F}$  and  $d\mathfrak{F}$ , while onside  $\partial\Omega$  the manifold  $\omega$  represents the quaternionic boundary of the quaternionic field  $\mathfrak{F}$ .

$d\omega$  is the exterior derivative of  $\omega$ .

In this way, the theorem may construct a rim  $\mathfrak{F}(\mathbf{x}, \tau)$  between the past history of the field  $[\mathfrak{F}(\mathbf{x}, t)]_{t < \tau}$  and the future  $[\mathfrak{F}(\mathbf{x}, t)]_{t > \tau}$  of that field. It means that the boundary  $\mathfrak{F}(\mathbf{x}, \tau)$  of field  $[\mathfrak{F}(\mathbf{x}, t)]_{t < \tau}$  represents a universe wide static status quo of that field.

More specifically:

$$\int_{t=0}^{\tau} \iiint_V d\mathfrak{F}(x) = \left[ \iiint_V \mathfrak{F}(x) dx \right]_{t=\tau} \quad (2)$$

Here  $[\mathfrak{F}(\mathbf{x}, t)]_{t=\tau}$  represents the static status quo of a quaternionic field at instance  $\tau$ .  $V$  represents the spatial part of the quaternionic parameter space, but it may represent only a restricted part of that parameter space. This last situation corresponds to the usual form of the divergence theorem.

### 1.1.1 Interpretation

This split of quaternionic space results in a space-progression model that is to a large extent similar to the way that physical theories describe their space time models. However, the physical theories apply a model that has a Minkowski signature. The quaternionic model is strictly Euclidean.

What happens is an ongoing process that embeds the subsequent static status quo's of the separable Hilbert space into the Gelfand triple.

The quaternionic parameter space  $\Omega$  is supposed to be defined as eigenspace of a reference operator  $\mathfrak{R}$  that resides in the Gelfand triple, which is a non-separable Hilbert space. The reverse bra-ket method relates this parameter space to an equivalent parameter space  $\mathcal{R}$ , which resides in the separable Hilbert space.

The future is kept on the outside of the discussed boundary. As a consequence, the mechanisms that generate new data, operate on the rim between past and future. These mechanisms act in a spatially stochastic fashion and will provide all generated data with a progression stamp. This progression stamp reflects the state of a universe wide clock tick. The whole universe, including its physical fields will proceed with these progression steps. However, in the Gelfand triple this progression will flow.

At the defined rim, any forecasting will be considered as mathematical cheating. Thus, at the rim, the uncertainty principle does not work for the progression part of the parameter spaces. Differential equations that offer advanced as well as retarded solutions must reinterpret the advanced solutions and turn them in retarded solutions, which in that case represent another kind of object. If the original object represents a particle, then the reversed particle is the anti-particle.

As a consequence of the construct, the history, which is stored-free from any uncertainty-in the already processed part of the eigenspaces of the physical operators, is no longer touched. Future is unknown or at least it is inaccessible.

## 2 Symmetry centers as floating parameter spaces

If we tolerate discontinuities, then these artifacts must be encapsulated by boundaries  $\partial H_n^x$  and in that way they are separated from the main parameter space  $\Omega$ .

In that case the model may apply different parameter spaces, which have their own private symmetry flavor [2]. A separable quaternionic Hilbert space can cope with coexisting parameter spaces and these spaces are served by dedicated operators. The bra-ket method relates the parameter space to a corresponding operator. For example [3]:

Let  $\{q_i\}$  be the set of **rational** quaternions in a selected quaternionic number system and let  $\{|q_i\rangle\}$  be the set of corresponding base vectors. They are eigenvectors of a normal operator  $\mathcal{R}$ .

Here we enumerate the eigenvalues and the base vectors with the same index  $i$ .

$$\mathcal{R} \equiv |q_i\rangle q_i \langle q_i| \quad (2)$$

For all bra's  $\langle x|$  and ket's  $|y\rangle$  hold:

$$\langle x|\mathcal{R}|y\rangle = \sum_i \langle x|q_i\rangle q_i \langle q_i|y\rangle \quad (3)$$

$\mathcal{R}_0 = (\mathcal{R} + \mathcal{R}^\dagger)/2$  is a self-adjoint and thus Hermitian operator. Its eigenvalues can be used to arrange the order of the eigenvectors by enumerating them with the eigenvalues. The ordered eigenvalues can be interpreted as **progression values**.

$\mathcal{R} = (\mathcal{R} - \mathcal{R}^\dagger)/2$  is the corresponding anti-Hermitian operator.

We will use the same symbol for the operator  $\mathcal{R}$ , for the eigenspace  $\{q_i\}$  and for the defined parameter space.  $\mathcal{R}$  is supposed to be ordered by using a selected Cartesian coordinate system. Eight mutually independent selections are possible. The Cartesian ordering determines the symmetry flavor of the eigenspace.

We define a category of anti-Hermitian operators  $\{\mathfrak{S}_n^x\}$  that have no Hermitian part and that are distinguished by the way that their eigenspace is ordered by applying a polar coordinate system. We call them symmetry centers  $\mathfrak{S}_n^x$ . A polar ordering always start with a selected Cartesian ordering. The geometric center of the eigenspace of the symmetry center floats on a background parameter space of the normal reference operator  $\mathcal{R}$ , whose eigenspace defines a full quaternionic parameter space. The eigenspace of the symmetry center  $\mathfrak{S}_n^x$  acts as a three dimensional spatial parameter space. The super script  $x$  refers to the symmetry flavor of  $\mathfrak{S}_n^x$ . The subscript  $n$  enumerates the symmetry centers. Sometimes we omit the subscript.

$$\mathfrak{S}^x = |\mathfrak{s}_i^x\rangle\mathfrak{s}_i^x\langle\mathfrak{s}_i^x| \quad (4)$$

$$\mathfrak{S}^{x\dagger} = -\mathfrak{S}^x \quad (5)$$

In the companion Gelfand triple of an infinite dimensional separable Hilbert space the reverse bracket method can define continuum parameter spaces and relate them to corresponding operators. In this way the countable parameter space  $\mathcal{R}$  relates to a continuum parameter space  $\mathfrak{R}$ .

The quaternionic field  $\mathfrak{F}$  can also be represented by a dedicated operator. Here we use a parameter space  $\mathfrak{R}$  that is spanned by a full quaternionic number system.

For all bra's  $\langle x|$  and ket's  $|y\rangle$  hold:

$$\langle x|\mathfrak{R}|y\rangle = \int_q \langle x|q\rangle q \langle q|y\rangle dq \quad (6)$$

$$\langle x|\mathfrak{F}|y\rangle = \int_q \langle x|q\rangle \mathfrak{F}(q) \langle q|y\rangle dq \quad (7)$$

***Here, we use the symbol  $\mathfrak{F}$  for the field, the function and the operator. However, another parameter space  $R$  would deliver another function  $F$  for the same field  $\mathfrak{F}$ . So, what determines the field  $\mathfrak{F}$  is stored in the eigenspace  $\mathfrak{F}$  of operator  $\mathfrak{F}$  and can be coupled to different pairs of functions and parameter spaces.***

### 3 The detailed Stokes theorem

Symmetry centers represent spherically ordered parameter spaces in regions  $H_n^x$  that float on a background parameter space  $\Omega$ . These regions are platforms for local discontinuities in basic fields. The symmetry centers are encapsulated and the encapsulating boundary is part of the disconnected

boundary which encapsulates all continuous parts of physical fields that exist in the quaternionic model.

$$\int_{\Omega-H} d\omega = \int_{\partial\Omega} \omega + \sum_n \int_{\partial H_n^x} \omega \quad (1)$$

Here  $\Omega$  corresponds to the reference parameter space  $\mathcal{R}$ .

$$H = \bigcup_n H_n^x \quad (2)$$

$$\Omega \leftrightarrow \mathcal{R} \quad (3)$$

$$H_n^x \leftrightarrow \mathfrak{S}_n^x \quad (4)$$

The symmetry center  $\mathfrak{S}_n^x$  that conforms to encapsulated region  $H_n^x$ , keeps its private symmetry flavor. At the passage through the boundary the symmetry flavor of the background parameter space  $\Omega$  flips. As a consequence the symmetry related charge of the symmetry center will flip.

However, the passage of the symmetry center through the rim may also be interpreted as the annihilation of the historic symmetry center and the creation of a new symmetry center with a reverse symmetry flavor that will extend its live in the future.

The passage of the symmetry centers through the rim goes together with annihilation and creation phenomena for the objects that reside on these platforms. Thus, this passage is related to the annihilation and creation of elementary particles.

In the orthonormal base model the existence of symmetry centers is independent of progression. With other words the number of symmetry centers is a model constant. The passage through the rim does not influence this number. Only the characteristics of the combination of the symmetry center and the background parameter space are affected by the passage.

## 4 Discussion

The concept of exterior derivative is carefully crafted by skillful mathematicians, such that it becomes independent of the selection of parameter spaces. However, in a situation like this in which one parameter space floats on top of another, the selection of the parameter space does matter. The symmetry flavors of the coupled parameter spaces determine the values of the integrals that account for the contributions of the artifacts. It is represented by the symmetry related charges of these artifacts [4]. These symmetry related charges are supposed to be located at the geometric centers of the symmetry centers.

As happens so often, physical reality reveals facts (the symmetry related charges) that cannot easily be discovered by skilled mathematicians. The standard model contains a short list of electric charges

that correspond to the symmetry related charges. Also the standard model does not give an explanation for the existence of this short list. Here it becomes clear that the electric charge is a property of connected spaces and not a property of the objects that use these spaces as parameter spaces. The objects inherit the charge property from the platform on which they reside.

#### *References*

- [1] [https://en.wikipedia.org/wiki/Stokes%27\\_theorem#General\\_formulation](https://en.wikipedia.org/wiki/Stokes%27_theorem#General_formulation).
- [2] Fermion Symmetry Flavors. <http://vixra.org/abs/1512.0225>
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- [4] Foundation of a Mathematical Model of Physical Reality. <http://vixra.org/abs/1511.0074>