Notable observation on a property of Carmichael numbers

Marius Coman
email: mariuscoman13@gmail.com

Abstract. In this paper I conjecture that for any Carmichael number C is true one of the following two statements: (i) there exist at least one prime q, q lesser than Sqr (C), such that p = (C – q)/(q – 1) is prime, power of prime or semiprime m*n, n > m, with the property that n – m + 1 is prime or power of prime or n + m – 1 is prime or power of prime; (ii) there exist at least one prime q, q lesser than Sqr (C), such that p = (C – q)/((q – 1)*2^n) is prime or power of prime. In two previous papers I made similar assumptions on the squares of primes of the form 10k + 1 respectively 10k + 9 and I always believed that Fermat pseudoprimes behave in several times like squares of primes.

Conjecture:

For any Carmichael number C is true one of the following two statements:

(i) there exist at least one prime q, q lesser than Sqr (C), such that p = (C – q)/(q – 1) is prime, power of prime or semiprime m*n, n > m, with the property that n – m + 1 is prime or power of prime or n + m – 1 is prime or power of prime;

(ii) there exist at least one prime q, q lesser than Sqr (C), such that p = (C – q)/((q – 1)*2^n) is prime or power of prime.

Verifying the conjecture:
(for the first ten Carmichael numbers)

: C = 561 and (C – 5)/4 = 139, prime; also (C – 11)/10 = 5*11, semiprime such that 11 – 5 + 1 = 7, prime; also (C – 17)/(16*2) = 17, prime;

: C = 1105 and (C – 7)/6 = 3*61, semiprime such that 61 – 3 + 1 = 59, prime; also (C – 13)/12 = 7*13, semiprime such that 13 – 7 + 1 = 7, prime and 13 + 7 – 1 = 19, prime; also (C – 17)/(16*2^2) = 17, prime;

: C = 1729 and (C – 5)/4 = 431, prime; also (C – 17)/16 = 107, prime; also (C – 37)/36 = 47, prime;

: C = 2465 and (C – 29)/28 = 3*29, semiprime such that 29 – 3 + 1 = 27 = 3^3, power of prime and 29 + 3 – 1 = 31, prime;

: C = 2821 and (C – 7)/6 = 7*67, semiprime such that 67 – 7 + 1 = 61, prime and 67 + 7 – 1 = 73, prime; also (C – 11)/10 = 281, prime; also (C – 31)/30 = 3*31, semiprime such that 31 – 3 + 1 = 29, prime;

: C = 6601 and (C – 5)/4 = 17*97, semiprime such that 97 – 17 + 1 = 81 = 3^4, power of prime and 97 + 17 – 1 = 113, prime; also (C – 7)/6 = 7*157, semiprime such that 157 – 7 + 1 = 151, prime and 157 + 7 – 1 = 163, prime; also (C – 11)/10 = 659, prime; also (C – 23)/22 = 13*23, semiprime such that 23 – 13 + 1 = 11, prime; also (C – 31)/30 = 3*73, semiprime such that 73 – 3 + 1 = 71, prime; also (C – 61)/60 = 109, prime;

: C = 8911 and (C – 23)/(22*2^2) = 101, prime; also (C – 31)/(30*2^3) = 37, prime; also (C – 67)/(66*2) = 67, prime;

: C = 10585 and (C – 7)/6 = 41*43, semiprime such that 43 – 41 + 1 = 3, prime and 43 + 41 – 1 = 83, prime; also (C – 13)/12 = 881, prime; also (C – 19)/18 = 587, prime; also (C – 29)/28 = 13*29, semiprime such that 29 – 13 + 1 = 17, prime and 13 + 29 – 1 = 41, prime; also (C – 37)/36 = 293, prime; also (C – 43)/42 = 251, prime; also (C – 73)/(73*2) = 73, prime;

: C = 5841 and (C – 5)/4 = 37*107, semiprime such that 107 – 37 + 1 = 71, prime; also (C – 11)/10 = 1583, prime; also (C – 13)/12 = 1319, prime; also (C – 61)/60 = 263, prime; also (C – 67)/66 = 239, prime; also (C – 73)/72 = 3*73, semiprime such that 73 – 3 + 1 = 71, prime; also (C – 89)/88 = 179, prime; also (C – 97)/(96*2^2) = 41, prime;

: C = 29341 and (C – 7)/6 = 4889, prime; also (C – 31)/30 = 977, prime; also (C – 61)/(60*2^3) = 61, prime.