Notable observation on the squares of primes of the form 10k+1

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Abstract. In this paper I conjecture that for any square of prime of the form p^2 = 10k + 1, p greater than or equal to 11, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime and, in case that this conjecture turns out not to be true, I considered three related "weaker" conjectures.

Conjecture:

For any square of prime of the form p^2 = 10k + 1, p greater than or equal to 11, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime.

Verifying the conjecture:
(for the first ten primes p with the property mentioned)

: p = 11 and p^2 = 121; (p^2 – 5)/4 = 29, prime;
: p = 19 and p^2 = 361; (p^2 – 5)/4 = 89, prime; also (p^2 – 7)/6 = 59, prime; also (p^2 – 13)/12 = 29, prime;
: p = 29 and p^2 = 841; (p^2 – 7)/6 = 139, prime; also (p^2 – 11)/10 = 83, prime;
: p = 31 and p^2 = 961; (p^2 – 5)/4 = 239, prime; also (p^2 – 13)/12 = 79, prime; also (p^2 – 17)/16 = 59, prime;
: p = 41 and p^2 = 1681; (p^2 – 5)/4 = 419, prime; also (p^2 – 11)/10 = 167, prime; also (p^2 – 13)/12 = 139, prime; also (p^2 – 29)/28 = 59, prime;
: p = 59 and p^2 = 3481; (p^2 – 11)/10 = 347, prime;
: p = 61 and p^2 = 3721; (p^2 – 5)/4 = 929, prime; also (p^2 – 7)/6 = 619, prime;
: p = 71 and p^2 = 3721; (p^2 – 5)/4 = 1259, prime; also (p^2 – 7)/6 = 839, prime; also (p^2 – 11)/10 = 503, prime; also (p^2 – 13)/12 = 419, prime; also (p^2 – 29)/28 = 179, prime; also (p^2 – 31)/30 = 167, prime; also (p^2 – 37)/36 = 139, prime; also (p^2 – 61)/60 = 83, prime;
: p = 79 and p^2 = 6241; (p^2 – 5)/4 = 1559, prime; also (p^2 – 7)/6 = 1039, prime; also (p^2 – 17)/16 = 389, prime; also (p^2 – 61)/60 = 103, prime;

: p = 89 and p^2 = 7921; (p^2 – 5)/4 = 1979, prime; also (p^2 – 7)/6 = 1319, prime; also (p^2 – 13)/12 = 659, prime; also (p^2 – 19)/18 = 439, prime; also (p^2 – 23)/22 = 359, prime; also (p^2 – 31)/30 = 263, prime; also (p^2 – 41)/40 = 197, prime; also (p^2 – 61)/60 = 131, prime; also (p^2 – 73)/72 = 109, prime.

Note:

In case that the conjecture above turns out not to be true there are three "weaker" conjectures that may be considered:

(i) For any square of prime of the form p^2 = 10k + 1, p greater than or equal to 11, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime or a power of prime.

Example:

: p = 59, p^2 = 3481, (p^2 – 13)/12 = 17^2, square of prime.

(ii) For any square of prime of the form p^2 = 10k + 1, p greater than or equal to 11, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime or semiprime m*n, n > m, with the property that n – m + 1 is prime or power of prime or n + m – 1 is prime or power of prime.

Examples:

: p = 61, p^2 = 3721, (p^2 – 11)/10 = 7*53 and 53 – 7 + 1 = 47, prime; also 53 + 7 – 1 = 59, prime;
: p = 71, p^2 = 5041, (p^2 – 43)/42 = 7*17 and 17 – 7 + 1 = 11, prime; also 17 + 7 – 1 = 23, prime.

(iii) For any square of prime of the form p^2 = 10k + 1, p greater than or equal to 11, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/((q – 1)*2^n)) is prime.

Examples:

: p = 61, p^2 = 3721, (p^2 – 41)/(40*2^2) = 23, prime;
: p = 71, p^2 = 5041, (p^2 – 17)/(16*2) = 157, prime.