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Particle Physics and Cosmology in the Microscopic Model

Bodo Lampe

II. Institut für theoretische Physik der Universität Hamburg

Luruper Chaussee 149, 22761 Hamburg, Germany

Abstract

This review summarizes the results of a series of recent papers[1, 2, 3, 4, 5], where a microscopic structure underlying the physics of elementary particles has been proposed. The 'tetron model' relies on the existence of an internal isospin space, in which an independent physical dynamics takes place. This idea is critically reconsidered in the present work. As becomes evident in the course of discussion, the model not only describes electroweak phenomena but also modifies our understanding of other physical topics, like gravity, the big bang cosmology and the nature of the strong interactions.

1 Basic Ideas

The Standard Model of elementary particles is very successful on the phenomenological level. The outcome of (almost) any particle physics experiment can be predicted accurately within this model, and where not, by some straightforward extension. For example, one may introduce right handed neutrinos to account for tiny neutrino masses[6].

Nevertheless, it is widely believed that the SM is only an effective low-energy theory valid below a certain energy scale, which is supposed to be larger than 1 TeV. This view is based on the fact that the SM has many unknown parameters with hitherto unexplained hierarchies. Furthermore, there is one rather mysterious component, the so-called Higgs field, needed for the spontaneous symmetry breaking (SSB) to take place in the model.

In recent papers a microscopic model has been developed[1, 2, 3, 4, 5], whose central assumption is the existence of a 3-dimensional *internal tetrahedral structure* attributed to each point of Minkowski space, in which an independent physical dynamics takes place.

Under this assumption spacetime originally is 6+1 dimensional, and at the time when the tetrahedrons are formed, it fibers into internal space and Minkowski space as $R^3 \times R^{3,1}$.

The sites $i = 1, 2, 3, 4$ of a tetrahedron in R^{6+1} are populated by spinor fields ψ, called **tetrons**. The tetron on site i will be denoted ψ_i.

The fundamental spinor representation in R^{6+1} is of dimension 8. It decomposes as[37]

$$8 \to (1, 2, 2) + (2, 1, 2) = ((1, 2) + (2, 1), 2) \tag{1}$$

under the fibration $SO(6, 1) \to SO(3, 1) \times SO(3)$[1].

[1]Here representations of $SO(3, 1) \times SO(3)$ are denoted by a set of 3 numbers (a, b, c), where (a, b) are representations of the Lorentz group and c is the dimension of a $SO(3)$-representation. For example, c=2 corresponds to a non-relativistic Pauli spinor in internal space, whose 2 spin orientations are identified with the SU(2) flavors U and D. It should be noted that (1,2,2) and (2,1,2) are complex conjugate with respect to each other, so one is the antiparticle representation of the other.

Eq. (1) means that each tetron is an isospin doublet $\psi = (U, D)$ of 3+1 dimensional Dirac fermions U and D. One may write it as a 2-index object ψ_α^a, where $\alpha = 1, 2, 3, 4$ is the Dirac index and $a = 1, 2$ the internal index. The internal spin will be called isospin.

Using the triplet $\vec{\tau}$ of internal Pauli matrices an isospin (pseudo)vector

$$\vec{Q} = \psi^\dagger \vec{\tau} \psi \tag{2}$$

may be defined for any tetron ψ. It fixes a direction in the internal space and, up to an overall constant, can be interpreted as the internal angular momentum vector of the tetron ψ.

Since the tetrons are Dirac fermions on Minkowski space, $\vec{Q}$ can be written in terms of creation and annihilation operators of a tetron ($a^\dagger$ and a) and an antitetron ($b^\dagger$ and b) as

$$\vec{Q} = \psi^\dagger \vec{\tau} \psi = a^\dagger \vec{\tau} a - b^\dagger \vec{\tau} b \tag{3}$$

For the calculation of the quark and lepton masses the chiral iso-vectors

$$\vec{S} := \vec{Q}_L = \frac{1}{2} \psi^\dagger (1 - \gamma_5) \vec{\tau} \psi \qquad\qquad \vec{T} := \vec{Q}_R = \frac{1}{2} \psi^\dagger (1 + \gamma_5) \vec{\tau} \psi \tag{4}$$

turn out to be of particular importance. For simplicity of notation they are called $\vec{S}$ and $\vec{T}$ in the following. Obviously, they fulfill $\vec{Q} = \vec{S} + \vec{T}$.

In fig. 1 the *local* ground state of the model is drawn, a configuration with 4 tetrons on the 4 sites of a tetrahedron, their isospin vectors $\vec{Q}$ pointing in radial directions away from the origin. These internal vectors fulfill the commutation relations of a system of decoupled internal angular momenta. In other words, they play the role of angular momentum observables corresponding to rotations of the internal R^3 space.

While the coordinate symmetry is S_4, the arrangement of isospin vectors in fig. 1 respects the Shubnikov symmetry[8, 10, 11]

$$G_4 := A_4 + S(S_4 - A_4) \tag{5}$$

where $A_4(S_4)$ is the (full) tetrahedral symmetry group and S the internal time reversal operation that changes the direction of internal spin vectors. This is equivalent to saying that S interchanges the role of the internal spinors in the following way

$$S : (U, D) \to (-D^*, U^*) \tag{6}$$

Note the arrangement fig. 1 does not respect S or internal parity P_{in}, but only the product SP_{in}. Furthermore it is chiral, the configuration with opposite internal chirality being given when the isospin vectors would point inwards instead of outwards. As will be shown in (2.1.19), this internal chirality is dynamically related to the $V - A$ nature of the weak interaction.

In a relativistic environment containing antiparticles the definition (5) of the Shubnikov group has to be modified to

$$G_4 := A_4 + CP_{au}T(S_4 - A_4) \quad (7)$$

This will be detailed later in (2.4.20) and figs. 4 and 5. C is the charge conjugation operator of a Dirac field and P_{au} the ('external') parity transformation in physical space. Since the elements of $S_4 - A_4$ contain an implicit factor of internal parity P_{in}, the symmetry (7) certifies CPT invariance of the local ground state in the full of R^{6+1}, cf. (2.4.20). Furthermore, the concept of an internal time S is dispensible here, so instead of S ordinary time reversal T may be used in (7).

As for the *global* ground state the set of all tetrahedrons forms a flat 3-dimensional crystal structure within the original R^6, similar to what is shown in fig. 2. This structure may be called a hyper-crystal. It is our world, the space in which all physical processes take place. Actually it will turn out to resemble an elastic or even a fluid system, so that it may as well be called a hyper-plastics or, within the Lorentz covariant cosmological framework to be developed later, the discrete micro-elastic spacetime continuum, the 'DMESC'.

Contrary to what is drawn, the tetrahedrons extend into internal space alone, not into physical space. In other words, physical space is *defined* to be the 3-dimensional subspace of R^6 orthogonal to the 3 dimensions spanned by the aligned tetrahedrons.

In addition to the coordinate alignment of tetrahedrons, there is also an alignment of isospins of neighboring tetrahedrons in fig. 2. Before the appearance of this structure the internal spins U and D, which are the building blocks of the isospin vectors, can freely rotate, and thus there is an internal spin SU(2) symmetry group. In ordinary magnetism this group is usually called Heisenberg's SU(2); in sections (2.4.16)ff it will be shown how its isospin analogon is related to the Standard Model SU(2)$_L$ gauge group.

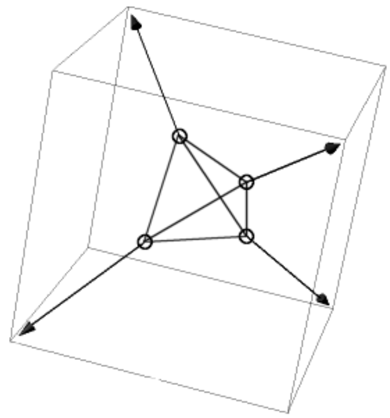

Figure 1: The local ground state of the model, living in a 3-dimensional isospin space called the 'fiber'. Shown are the tetron locations (open circles) and the 4 ground state isospin vectors $\langle\vec{Q}_i\rangle$, whose excitations will be identified with the spectrum of quarks and leptons. The origin of coordinates is taken to be the center of the tetrahedron, and is identical to the base point of the fiber in Minkowski space. The tetrahedron itself has the tetrahedral group S_4 as point group symmetry. However, due to the pseudovector property of the isospin vectors the whole system has the Shubnikov point symmetry (5). The Shubnikov group is chiral, the configuration with opposite chirality being given when the 4 isospin vectors would point inwards instead of outwards. Before the formation of the chiral tetrahedron, the internal spins U and D, which according to (2) are the building blocks of the isospin vectors, can freely rotate and thus there is an internal spin SU(2) symmetry group, which however is broken to G_4 when the chiral tetrahedron is formed. Note there are actually 2 tetrahedrons in this figure, one with respect to the internal coordinates (tetron locations) and the other one with respect to isospin vectors, and both tetrahedrons are 'aligned', in the sense that the coordinate vectors and the isospin vectors point into the same (radial) direction. This 'alignment' of coordinate and isospin vectors within one fiber has to be distinguished from the alignment of isospin vectors with respect to the isospins of neighboring tetrahedrons, as shown in fig. 2. The latter forms the basis for the electroweak phase transition, while the coordinate alignment of neighboring tetrahedrons is relevant for crystal formation at big bang temperatures.

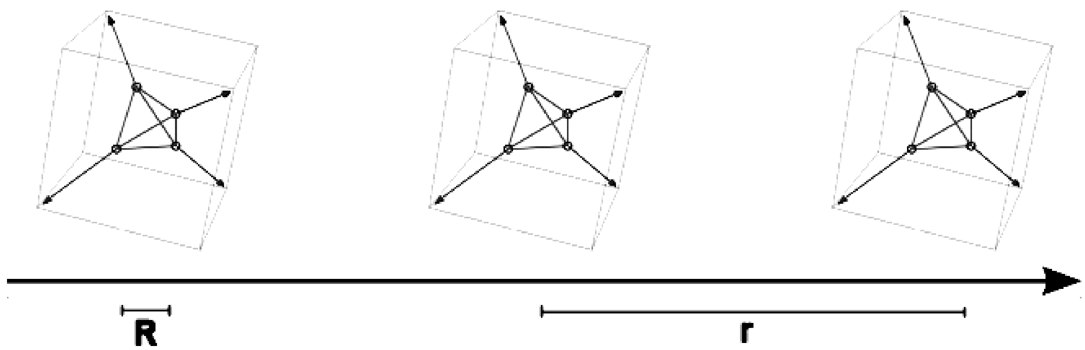

Figure 2: The global ground state of the model after the electroweak SSB consists of an aligned system of chiral tetrahedrons over physical space (the latter represented by the long arrow). R is the internal magnitude of one tetrahedron and r the distance between two of them. The figure is a bit misleading, not only because the tetrahedrons do not have an extension into physical space, but also the relative magnitudes are not correctly drawn. While r and R are tiny (of the order of the Planck length), the tetrahedrons formed by the isospin vectors are much larger, of the order of the Fermi scale (in inverse energy units). Actually there are 2 kinds of alignment in this figure: the alignment of neighboring coordinate tetrahedrons and the alignment of isospin vectors in neighboring tetrahedrons. The isospin vector alignment is associated to the electroweak symmetry breaking, because at temperatures above the Fermi scale (before the SSB) the isospins in each tetrahedron are oriented randomly (not shown) and there is a corresponding *local* $SU(2)$ symmetry which gets broken when the isospin vectors align. The figure also shows how the universe looks like in the tetron model. It is a 3-dimensional 'monolayer' of internal tetrahedrons whose average distances are given by the Planck length $\langle r \rangle = L_P$. Gravity is due to the elasticity of the coordinate bonds between neighboring tetrahedrons and corresponds to tiny deviations from this average in the vertical or horizontal direction. Finally, the coordinate alignment of the tetrahedrons is related to crystal formation at the big bang. Cosmic inflation is due to the sudden release of crystallization energy. These latter issues will be discussed in detail on pages ***15-28 and in section 2.5.

An important point in that consideration is that the SU(2) transformations are local symmetries in the sense that the isospin vectors can be rotated independently over each point of Minkowski space. The group gets broken to G_4 when the internal arrangement fig. 1 is formed. It may be given the index L, because this arrangement is chiral and because there is a dynamical relation between internal and external chirality, as explained in (2.1.19).

The mixing with the electromagnetic $U(1)$ symmetry has not been introduced at this point. This omission will be clarified later in (2.1.7) and (2.1.10), together with the tetron model interpretation of the electroweak mixing angle.

One wants to interpret the 3 generations of quarks and leptons as isospin wave excitations of the internal isospin structure. These excitations will be called **mignons**. They behave as quasi-particles while they travel through Minkowski space and can be classified according to representations of G_4, as shown below.

G_4 is a finite group which remains intact to the lowest energies. As shown in [8] it has only 1- and 3-dimensional representations. To generate all possible excitations describing the quarks and leptons one has to consider the vibrations of $\vec{S} = \vec{Q}_L$ and $\vec{T} = \vec{Q}_R$ for each of the 4 tetrons separately, cf. (2.3.1) and (2.4.16).

The isospin vibrational excitations are described by deviations δ from the ground state fig. 1, i.e.

$$\vec{S}_i = \langle \vec{S}_i \rangle + \delta \vec{S}_i \qquad \vec{T}_i = \langle \vec{T}_i \rangle + \delta \vec{T}_i \tag{8}$$

or, more precisely, by certain linear combinations of them – the eigenmodes of the isospin Hamiltonian to be discussed later in (14) and (19).

The resulting 24 mignon states can be arranged in six singlet and six triplet representations $A_{\uparrow,\downarrow}$ and $T_{\uparrow,\downarrow}$ of G_4 to yield precisely the multiplet structure of the 24 fermion states of the 3 generations, not less and not more[1]:

$$\begin{aligned} A_\uparrow(\nu_e) + A_\uparrow(\nu_\mu) + A_\uparrow(\nu_\tau) \quad &+ \quad T_\uparrow(u) + T_\uparrow(c) + T_\uparrow(t) + \\ A_\downarrow(e) + A_\downarrow(\mu) + A_\downarrow(\tau) \quad &+ \quad T_\downarrow(d) + T_\downarrow(s) + T_\downarrow(b) \end{aligned} \tag{9}$$

Details about this arrangement will be given in (2.4.18). The SM quantum numbers can be recovered from this spectrum in the following way:
–the $\uparrow$ representations can be obtained from the $\downarrow$ ones by the transformation

$\delta\vec{S} \leftrightarrow \delta\vec{T}$ for any of the tetrons, i.e. by interchanging left and right. As shown in (2.4.16) this is precisely what is needed for a weak isospin transition on the level of mignons.
–singlet and triplet Shubnikov states have a different U(1) charge. The corresponding symmetry can be interpreted as gauged tetron or $B-L$ number. Details will be given in (2.1.7) and (2.2.3). The mixture with the photon and the appearance of the Weinberg angle will be discussed in (2.1.10).
–the 3 states within each triplet T in (9) are always degenerate, because G_4 remains unbroken. The relation between those triplets and the QCD color triplets of quarks will be further discussed in (2.3.29).

Actually, to obtain the quark and lepton spectrum (9) a discrete structure is compelling only in internal space, not in physical space. Looking at fig. 2, one could try to come along with a continuous model of Minkowski space, i.e. with $r \to 0$. However, it is tempting to assume $r \neq 0$, i.e. that there is a sort of lattice underlying spacetime, with spacings so small that Lorentz symmetry is effectively maintained for all available energies.

Details of this idea will be discussed after (23) and in section 2.5, where it will be shown that the lattice must be (i) elastic and (ii) a Planck lattice, otherwise it would contradict (i) cosmological observations and (ii) Einstein's principle of equivalence[14, 15]. Due to quantum fluctuations it may be a foam[7] or a spin network – although in the tetron model there is no a priori necessity to quantize gravity, cf. (2.5.40) and (2.5.6).

Can the aligned structure fig. 2 be understood heuristically? The answer is yes, if one assumes that the arrangement of isospin vectors follows similar rules than that of spin vectors in a magnetic environment, cf. (2.2.9). What matters are value and sign of (internal) exchange integrals J of tetron wave functions as a function of the distance between 2 tetrons, because these integrals will appear as couplings in the Heisenberg isospin Hamiltonian (14).

The behavior of isospins in fig. 2 can then be understood via the so-called Bethe-Slater curve shown in fig. 3. If the tetrons are part of one tetrahedron, their distance is small $\sim R$ and according to the figure J is negative. This corresponds to antiferromagnetic behavior and leads to the formation of the frustrated structure fig. 1

with symmetry $A_4 + S(S_4 - A_4)$, because the spin vectors try to avoid each other as far as possible.

In contrast, if the internal spin vectors belong to different tetrahedrons, the distance of the corresponding tetrons is somewhat larger, of order r, and J is positive. This corresponds to ferromagnetic behavior and leads to the isospin alignment of neighboring tetrahedrons fig. 2.

Due to the tetrahedral 'star' structure fig. 1 it is appropriate to change the notion of isospin. Usually in an (anti)ferromagnetic environment, the spin vectors align into the + or - orientation of the z(=magnetization) direction, and the corresponding Pauli spinors are given by $U = (1, 0)$ and $D = (0, 1)$. In the present case the (iso)magnetic structure is defined by isospin vectors either pointing outwards or inwards in the radial direction. Correspondingly, the isospinors U and D are to be understood as 'radial' spinors[21]

$$\begin{aligned} U_\star = \sqrt{\frac{1}{3}} Y_1^0 U - \sqrt{\frac{2}{3}} Y_1^1 D &= \cos\frac{\vartheta}{2}\, U + \sin\frac{\vartheta}{2} e^{i\frac{\varphi}{2}}\, D \\ D_\star = \sqrt{\frac{2}{3}} Y_1^{-1} U - \sqrt{\frac{1}{3}} Y_1^0 D &= \sin\frac{\vartheta}{2} e^{-i\frac{\varphi}{2}}\, U - \cos\frac{\vartheta}{2}\, D \end{aligned} \tag{10}$$

where Y_l^m denote the sperical harmonics and ϑ and φ are the angles of the radial vector w.r.t. some cartesian coordinate system. These new spinors are radial in the sense that they reproduce the unit vector in polar coordinates

$$\vec{e}_r = U_\star^\dagger \vec{\tau} U_\star = -D_\star^\dagger \vec{\tau} D_\star \tag{11}$$

Furthermore they are normalized in such a way that

$$U_\star^\dagger U_\star + D_\star^\dagger D_\star = U^\dagger U + D^\dagger D \tag{12}$$

The iso-spinor state corresponding to an isospin vector pointing outwards is denoted by $U_\star$ (and similarly $D_\star$ for inwards pointing isospins). According to figs. 1 and 2, $U_\star$ is the building block of the hyper-crystal in its ground state. As shown in sections 2.1 and (2.2.6) it is unpolarized and its electric charge vanishes.

Note that this presentation is equivalent to the 'universal' z-axis approach[69, 70] used in the actual mass calculations[2]. Although according to (11) $D_\star$ has as much to do with U as it has with D, I will often leave out the star index in the following

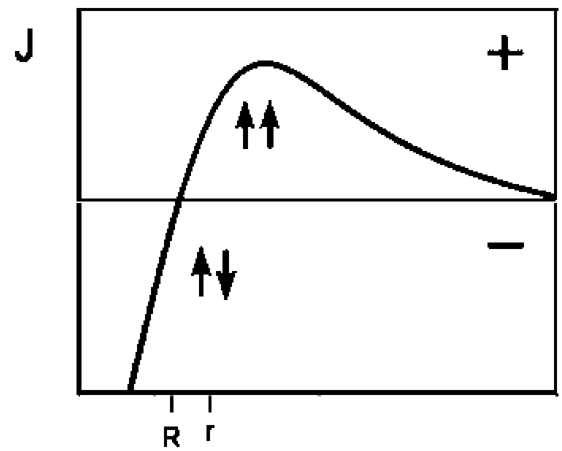

Figure 3: Qualitative behavior of the exchange integral coupling J as a function of the distance between 2 tetrons. In ordinary magnetism this is called the Bethe-Slater curve. One has anti-ferromagnetism ($J < 0$) at small, ferromagnetism ($J > 0$) at large distances.

for reasons of simplicity and understand that always $U_\star$ and $D_\star$ are meant. I will include the star only when this is needed for clarity, e.g. in (2.1.8).

In the tetron model the SM SSB arises from the 'ferromagnetic' alignment of isospin vectors in neighboring tetrahedrons. As shown in (2.1.8) and (2.3.11), the corresponding order parameter is given by a non-vanishing vacuum expectation value

$$\langle \bar{U}_\star U_\star \rangle \neq 0 \tag{13}$$

This is the way the SM Higgs mechanism is realized on the microscopic level. There is a pairing active comparable to the formation of Cooper pairs in a superconductor, and excitations of this tetron-antitetron pairing will appear as the physical Higgs field and the electroweak bosons.

In [2] the masses of the mignons (9) have been calculated, and the observed hierarchy in the quark and lepton spectrum as well as the hierarchy in the CKM and non-hierarchy in the PMNS matrix elements has been reproduced. As described above, mignon masses can be identified with the eigenfrequencies of the vibrations of the isospin vectors $\vec{S}$ and $\vec{T}$. These eigenfrequencies get contributions both from inner-and from inter-tetrahedral interactions.

Firstly, the *inner*-tetrahedral interactions are responsible for the frustrated tetrahedral configuration fig. 1, i.e. for the structure of the local vacuum. They are small distance contributions and relatively simple to treat because they can be described by an internal Heisenberg Hamiltonian for one tetrahedron alone, with corresponding internal spin vector excitations. The most general form of this Hamiltonian is

$$H_H = -J_{SS} \sum_{i \neq j=1}^{4} \vec{S}_i \vec{S}_j - J_{TT} \sum_{i \neq j=1}^{4} \vec{T}_i \vec{T}_j - J_{ST} \sum_{i,j=1}^{4} [\vec{S}_i \vec{T}_j + \vec{T}_i \vec{S}_j] - K_{ST} \sum_{i=1}^{4} \vec{S}_i \vec{T}_i \quad (14)$$

where the couplings J are internal exchange energy densities characteristic for the internal Heisenberg interactions. By introducing K_{ST}, I have allowed that the coupling $\vec{S}_i \vec{T}_j$ is different within a site ($i = j$) than outside of it ($i \neq j$).

Using (14) and (8) one is led to e.o.m. for $\delta \vec{S}_i$ and $\delta \vec{T}_i$ which can be solved in a similar way as the e.o.m. for magnons in solid state physics. On this basis the contributions from (14) to the eigenfrequencies of the 24 eigenmodes were calculated in [2].

Using $J_{SS} = J_{TT}$ (an approximation which can be justified via the generalized NJL model discussed above[2]) the following masses/eigenfrequencies are obtained

$$\pm m_\mu = 6J_{ST} + 2K_{ST} \quad (15)$$

$$\pm m_c = 4J_{SS} + 4J_{ST} \quad (16)$$

$$\pm m_s = 4J_{SS} + 2J_{ST} + 2K_{ST} \quad (17)$$

for the second family. Using the measured values of m_μ, m_s and m_c[79], the internal exchange couplings may be determined:

$$J_{ST} \approx -0.12 \text{ GeV} \qquad J_{TT} = J_{SS} \approx -0.12 \text{ GeV} \qquad K_{ST} \approx 0.32 \text{ GeV} \quad (18)$$

One concludes that one has ferromagnetic coupling $K_{ST} > 0$ of adjacent spin vectors $\vec{S}_i$ and $\vec{T}_i$, while the interactions with $i \neq j$ are anti-ferromagnetic. This is in accord with the heuristic expectations discussed before. Furthermore, it may be noted that all these values are rather small as compared to the SSB scale. A natural explanation for that is given in (2.3.21).

Secondly, the *inter*-tetrahedral interactions: these lead to the mass of the top and bottom quark and are based on the parallel (='ferromagnetic') alignment of isospins between different tetrahedrons fig. 2. Their leading effect turns out to be a contribution of order $O(\Lambda_F)$ to the top quark mass[2]. Physically speaking, this interaction handicaps the specific eigenmode describing the top quark, because this mode disturbs the SSB alignment in the strongest possible way.

Mathematically, the effect can be described by adding terms to the inner-tetrahedral Heisenberg interaction with a normal ferromagnetic plus a Dzyaloshinskii-Moriya (DM) component[23]. The sum of the 2 components will yield a quasi-democratic mass matrix[20] which in leading order only contributes a term of order Λ_F to the top-quark mass and nothing to the masses of the other quarks and leptons.

More in detail, the Hamiltonian for the SSB interactions of neighboring tetrahedrons can be derived from the W-mass term of the SM Lagrangian. By considering a $SU(2)_L$ gauge transformation, which removes the longitudinal components of the W-bosons from the Higgs part of the SM Lagrangian, one obtains

$$H_{SSB} = \ J_{inter} \ \sum_{i,j=1}^{4} [\vec{S}_i\vec{S}'_j \ + \ i\ (\vec{S}_i \times \vec{D}_{ij})\ \vec{S}'_j] \tag{19}$$

to be added to the Heisenberg Hamiltonian (14). One has

$$J_{inter} = \frac{\mu^2}{4\Lambda_F} \tag{20}$$

with μ the mass parameter of the SM Higgs potential and $\Lambda_F = \sqrt{\frac{\mu^2}{\lambda}} = 246\text{GeV}$ the Fermi scale (vacuum expectation value). Only terms involving the left handed isospin vectors $\vec{S} = \vec{Q}_L$ appear, as follows from (2.1.19) in accordance with the $V - A$ structure of the weak interactions. In (19) the factor $\vec{S}'_j$ denotes the left handed isospin vector of an adjacent tetrahedron. The precise relationship between (14) and (19) on one side and the SM Lagrangian terms on the other are worked out in (2.1.13) and [2]. The fact that the inter-tetrahedral coupling (20) is much larger than the inner-tetrahedral ones (18) will be naturally explained in (2.3.21).

Eq. (19) contains a ferromagnetic interaction plus the additional DM term which is due to the non-abelian nature of the W-bosons. The overall normalization of the DM term is dictated by $SU(2)_L$ gauge invariance, while the relative values of the DM couplings $\vec{D}_{ij}$ are fixed by the internal G_4 symmetry[2].

Quite in general, a DM component stands for a tendency to form a rotational structure (instead of the ordinary ferromagnetic alignment of neighboring tetrahedrons depicted in fig. 2) simply because the DM term tends to rotate the spin vectors instead of aligning them. In the present case it appears as a consequence of the non-abelian nature of SU(2). Therefore the DM term can be interpreted quite naturally, namely by the fact that a non-vanishing $SU(2)_L$ gauge field induces a curvature of the fiber bundle formed by the system of all tetrahedrons, and the DM term simply takes care of this curvature effect to effectively maintain the aligned structure.

This argument is supported by the fact that the gauge transformation inherent in (19) leads to the SM Lagrangian in the so-called 'unitary gauge'. This point is analyzed in detail in (2.3.15).

Using (14) and (19) one can derive the e.o.m. for the isospin vectors. With the usual ansatz $\sim \exp(i\omega t)$ one obtains a 24×24 eigenvalue problem. The eigenvalues ω correspond to the quark and lepton masses and were calculated in [2] as a function of the exchange couplings J_{SS}, J_{TT} etc. In that paper it was explicitly verified that the corresponding 24 eigenstates can be arranged into 6 singlets and 6 triplets as predicted by the Shubnikov symmetry analysis (9), i.e. as 6 leptons and 6×3 quarks. Each triplet (quark flavor) consists of 3 states with degenerate eigenvalues, because the Shubnikov symmetry G_4 is unbroken at low energies.

The dominant contribution from (19) gives the top quark a mass of the order of the Fermi scale while leaving the other quark and lepton masses unchanged. As detailed in (2.4.21), (2.4.22) and [2], b, c, s and τ get their masses mainly from (14).

In contrast, there are no contributions from (14) and (19) to u, d, e and neutrino masses. These 10 excitations remain massless on this level. To obtain their masses one has to include additional small torsional interactions[2].

The masses of the neutrinos are particularly suppressed because the 3 neutrino modes correspond to the vibrations of the 3 components of the total internal angular momentum vector

$$\vec{\Sigma} := \sum_{i=1}^{4}(\vec{S}_i + \vec{T}_i) = \sum_{i=1}^{4} \vec{Q}_i = \sum_{i=1}^{4} \psi_i^\dagger \vec{\tau} \psi_i \tag{21}$$

Whenever this quantity is conserved

$$d\vec{\Sigma}/dt = 0 \tag{22}$$

the neutrino masses will strictly vanish ($\omega = 0$). In fact, the interactions considered so far, i.e. (14) and (19), conserve total internal angular momentum. Therefore, they fulfill (22) and give no contribution to the neutrino masses. Further details can be found in (2.4.12)-(2.4.15) and in [2].

The general solution to the eigenproblem given above does not only yield the energy eigenvalues but (via the corresponding eigenvectors) can also be used to accommodate the CKM and PMNS mixing matrices[2]. The mass eigenstates are the states corresponding to the energy eigenvalues, while the interaction eigenstates naturally correspond to the original vectors $\vec{S}_i$ and $\vec{T}_i$, cf. (2.4.10).

Within this framework one can understand[2] why the CKM elements turn out to be small, whereas the PMNS matrix elements are naturally large: the lepton eigenstates (roughly given by $\vec{S} \pm \vec{T}$) are 'far away' from $\vec{S}$ and $\vec{T}$, while the up- and down-type quark eigenstates are relatively small deformations of $\vec{S}$ and $\vec{T}$, respectively. Due to the dominant contribution from (19) the top quark triplet state has the smallest mixing matrix elements with other quarks, because it corresponds to the vibration of $\vec{\Sigma}_L := \sum_i \vec{S}_i$ to an accuracy of less than 1%.

In summary, the present model describes the physical world as a huge ordered crystal of internal 'molecules', each molecule of tetrahedral form and arranged in such a way that the internal Heisenberg spin symmetry is spontaneously broken. As shown below, this approach not only provides a nice microscopic understanding of particle physics phenomena but in addition substantially supplements our understanding of the inflationary big bang cosmology. In effect, it gives the phase transitions in the early universe a microscopic meaning.

To comprehend this fact, it is appropriate to redevelop the full history of the early universe within the assumptions of the tetron model: before the 'big bang' there were the free tetrons ψ floating around as a Fermi gas in R^{6+1} space at extremely high pressure and temperature. While the universe was cooling down, 3 fundamental transitions occurred:

I. the formation of tetrahedral 'molecules' from tetrons at very high tem-

perature of order Λ_R, where the scale R is roughly given by the extension of one molecule. Although this process is not a phase transition in the strict sense it has certainly released a large amount of energy which has amplified the initial temperature of the universe.
Note that with 4 molecular sites each molecule 'fills' only 3 of the 6 spatial dimensions.

II. the formation of the 'hyper-crystal' from tetrahedrons takes place at somewhat lower temperatures $T \sim \Lambda_r$, where r is the 'lattice spacing', i.e. is roughly given by the distance between 2 tetrahedrons. This alignment of all tetrahedral structures is a coordinate alignment and to be distinguished from the isospin vector alignment (item III) describing the electroweak phase transition. It puts all 3-dimensional molecular structures in parallel thus separating an internal 3-dimensional space from the rest. In other words, the crystal expands into a 3+1-dimensional subspace of R^{6+1}, while the tetrahedrons extend into what becomes the 3 internal dimensions.

Since II corresponds to the process, in which our 3+1 dimensional universe was born, it may rightfully be called the big bang. As a crystallization process it is a first order phase transition associated with the sudden release of a large amount of energy. As will be explained later, the coordinate interactions among the tetrahedrons are of elastic type. Under this condition the outcome of phase transition II is not a crystal in the strict sense, and one may *as well call it a condensation of a hyper-plastics instead of a crystallization*, cf. (2.5.9). In any case the release of crystallization/condensation energy naturally drives an inflationary expansion of the system and the corresponding metric. Therefore, within the framework of the tetron model, the big bang and the beginning of inflation are more or less identical. As argued in (2.5.9) and further below in this section, the characteristic scale Λ_r can be identified to be of the order of the Planck scale Λ_P.

III. the arrangement of isospins at temperatures of order Λ_F. Above those temperatures the isospin vectors fluctuate randomly with an associated internal 'Heisenberg' $SU(2)$ symmetry, but at Λ_F they arrange into the chiral isomagnetic structure figs. 1 and 2. At that point the so far freely rotatable internal spins get ordered and the $SU(2)$ is broken to the Shubnikov group (5). Note that the $SU(2)$

is a local symmetry group, because isospins can be rotated separately over each point of the Minkowski base space.[2]

It could happen that there is no internal coordinate order right after the crystallization, in the sense that the coordinates of tetrons in neighboring tetrahedrons are aligned.[3] In that case there is no global internal tetrahedral symmetry of the hypercrystal right after the condensation. However, since internal coordinate alignment is a prerequisite for the isomagnetic alignment (= electroweak transition) III, this has to be catched up later at the Fermi temperature, i.e. it takes place at about the same time as the isospin alignment.

This possibility will be called scenario C in later discussions, and in fact it has several benefits. For example, III automatically becomes a first order phase transition (cf. 2.3.16) with an associated second inflationary process that removes domain walls (cf. 2.3.24) from the visible parts of the universe. Furthermore, it is easier to understand that the electroweak phase transition is really spontaneous (cf. 2.3.15) and that the ground state fig. 1 is assumed by both the left- and the right-handed isospin vectors $\vec{Q}_{Li}$ and $\vec{Q}_{Ri}$ (cf. 2.4.16).

Since II happens after I, i.e. at lower temperature, one naturally expects Λ_R larger than Λ_r (i.e. $R < r$) in agreement with fig. 2 and the Bethe-Slater curve fig. 3. As argued in (2.3.17) both scales are $\geqslant \Lambda_P$ and much larger than the scale Λ_F where the isospins align. Note that while Λ_F approximately corresponds to the critical point of transition III, the values of the exchange integrals J and therefore the iso-magnetic behavior are determined at distances r and R, cf. (2.3.19).

To describe II in the framework of the Landau approach to phase transitions one should consider density fluctuations $D\exp(i\vec{p}\vec{x})$ within the gaseous assembly of tetrahedral 'molecules' and use D as the order parameter of the phase transition.

[2]The relation between the internal Heisenberg $SU(2)$ and weak isospin $SU_L(2)$ is clarified in sections (2.4.16)ff. A more detailed description of phase transition III is given in (2.3.15) and (2.3.11). The mixing with the electromagnetic $U(1)$ symmetry will be included in (2.1.7) and (2.1.10).

[3]I am not talking about a regular crystal structure in physical space, which for an elastic system is missing anyhow. Instead I am talking about the alignment of the *tetron* coordinates as depicted in fig. 2. See also the discussion at the end of (2.3.15) where the assumption of internal coordinate order is completely given up.

For an ordinary crystal these fluctuations can be identified with phonons; in the general relativistic (GR) framework of a spacetime continuum at least some of them correspond to gravitational waves. This will become clearer below in this section, where elastic deformations of the crystal will be identified with metrical changes. More information about gravitons in the microscopic model can be found in (2.5.40) and (2.5.41).

Since the density perturbation adds to the uniform density of the tetrahedron gas, there is no symmetry under changing sign of the density wave, and so the Landau free energy expansion allows for a cubic term

$$\Delta F = \alpha(T - T_c)D^2 + \beta D^3 + \gamma D^4 \tag{23}$$

where $T_c \sim \Lambda_r$ is the critical temperature for phase transition II.

Accepting the idea of an elastic spacetime continuum, the coordinate phase transition resembles a gas→liquid transition rather than a crystallization process. This even more, because the tetron particle density D is an order parameter characteristic for a condensation. While during a crystallization process the lattice symmetry plays an important role, in the hyper-crystal the bonds are elastic and there is no lattice symmetry at all in the Minkowski base space. When calling our universe a hyper-'crystal' and identifying the big bang with its 'crystallization', it should therefore be kept in mind that it shares more properties with a liquid or a deformable plastics than with a real crystal. This topic will be taken up in (2.5.10).

The appearance of the cubic term in (23) is characteristic for a first order phase transition where a second minimum, which develops in the potential when the temperature is lowered, for some time remains higher than the minimum at $D = 0$ of the gas phase, and furthermore the two minima are separated by a potential wall. When the temperature drops below the critical value, there is a discontinuity which is not present in second order transitions.

The latent heat associated with this discontinuity is released very suddenly and can be used to explain the extreme acceleration needed for cosmic inflation[12, 13]. As shown in (2.5.23) it provides all the necessary ingredients for the inflationary process to start and to eventually stop, once the condensation of tetrahedrons is completed and the energy is exhausted. Most of the initial molecular energy has then been transferred to the elastic energy of the crystal. However, some of it survives in the form of tetron-antitetron excitations (gauge bosons) and is converted into mignons (quarks and leptons), as the temperature decreases further.

In place of quarks and leptons, whose existence is tied to the isospin ordering fig. 2, shortly after crystallization other excitations are more important, like the internal coordinate vibrations discussed in (2.3.26) and [3], or excitations of the tetron-antitetron bonds discussed in (2.1.8) and (2.1.9). Most prominent among the latter are the gauge bosons and the visible and dark scalars of the 2HDM sector as described in (2.1.13) and (2.5.36).

Because of the dominance of the electroweak bosons this cosmological era is often called 'radiation dominated' or 'electroweak'. At temperatures far above the Fermi scale all these excitations are effectively massless states transforming under the local $SU(2) \times U(1)$ symmetry, and they dominate the universe all the way down to the electroweak SSB (=isospin vector alignment). More details about the tetron model view on this era can be found in (2.1.11), (2.1.12), (2.3.8) and (2.3.9).

As well known, in general relativity a non-vanishing energy momentum tensor leads to a curvature of the spacetime continuum. Many authors have interpreted this on the basis of metric elasticity[86, 92, 93, 95, 94], and some of them have speculated that gravity forces might be explainable from a microscopic structure which in some sense is analogous to the atomic structures responsible for material elasticity in low-energy physics. In such a framework, GR is equivalent to an elastic continuum and the Einstein equations are not a fundamental but merely an effective description of the microscopic dynamics, only valid at distances much larger than the Planck length.

Taking this point of view one avoids the main two problems of general relativity:
(i) the problem of quantization - as discussed in (2.5.40) and
(ii) the existence of singular solutions - because in the micro-elastic interpretation a solution to the Einstein equation makes no sense at distances of the order of one lattice spacing L_P, where the discrete nature of the hyper-crystal become apparent.

In the following I will describe some details of this approach and adapt it to the requirements of the tetron model. The fundamental dynamical quantity in general relativity is the metric which defines the distance between 2 spacetime points (or between 2 adjacent tetrahedrons). It can be calculated e.g. from the transition function between arbitrary local coordinates x_μ on the manifold and local Lorentz coordinates ξ^α of an inertial system via

$$g_{\mu\nu} = \eta_{\alpha\beta} \frac{\partial \xi^\alpha}{\partial x^\mu} \frac{\partial \xi^\alpha}{\partial x^\nu} \tag{24}$$

where $\eta_{\alpha\beta}$ is the Minkowski metric valid in the inertial frame. One may then use the resulting line element $(ds)^2 = g_{\mu\nu} dx^\mu dx^\nu$ to go further ahead and write down the curvature/field strength and the Einstein equations.

In the micro-elastic interpretation a gravitational field induces a deformation in the medium, i.e. a displacement of the internal tetrahedrons within physical space from x_μ to x'_μ. This corresponds to a modification of the metric

$$g'_{\rho\sigma} = g_{\mu\nu} \frac{\partial x^\mu}{\partial x'^\rho} \frac{\partial x^\nu}{\partial x'^\sigma} \tag{25}$$

and corresponding changes in the curvature tensor.

Since GR is locally Lorentz invariant, the reader may wonder, what the physical meaning of deformations in the time direction is. In the spatial directions it is rather clear that the distances $r \sim L_P$ between neighboring tetrahedrons get modified when a gravitational field is applied. In the time direction it is the 'hopping time', which gets modified, i.e. the time a photon or some other quasi-particle needs to travel(=be emitted, run, get absorbed) from one tetrahedron to its neighbor. This modification occurs, because the presence of massive mignons and in general of any kind of mass/energy modifies the microscopic processes behind the hopping of any 'test excitation'.

Gauge invariance, i.e. the freedom to change the local Lorentz coordinate system, mixes these concepts. It means that one can use different coordinate systems for time/positions of the tetrahedrons and implies, for example, that from inside the hyper-crystal there is no possibility to distinguish 'longitudinal' from 'transversal' curvature, cf. fig. 7. This point is further discussed in (2.5.27).

Using these ideas one can understand many features of the Einstein theory. For example, the energy released during crystallization immediately blows up the distance between the tetrahedrons thus inflating the volume of the DMESC in accordance with inflationary cosmology. The initial crystallization energy is also responsible for the subsequent FLRW expansion of the universe. More details about these issues are given in section 2.5, in particular (2.5.23) and (2.5.27). The tetron model view on inflation can be found in (2.5.23)ff, while the interpretation of dark energy is given in (2.5.37).

Another example is the Newtonian limit. For a spherically symmetric configuration the metric can be given via the line element

$$ds^2 = (1 + \frac{2\phi}{c^2})(cdt)^2 - (1 - \frac{2\phi}{c^2})(d\vec{x})^2 \tag{26}$$

and the Newtonian limit is defined by $|\phi| \ll c^2$. For a point mass M the gravitational potential far away from the source is given by

$$\phi = -\frac{GM}{|\vec{x}|} \tag{27}$$

The square root of the coefficients

$$\sqrt{g_{00}} = 1 + \frac{\phi}{c^2} \qquad \sqrt{g_{xx}} = 1 - \frac{\phi}{c^2} \tag{28}$$

give the general relativistic time dilation and length contraction, respectively. In the tetron model these effects are interpreted in the following way:
-The gravitational potential of the point source M modifies the average distance $r \sim L_P$ between 2 neighboring tetrahedrons by a factor $1 + \phi/c^2$. As a consequence, any measured length of a physical object is modified by this factor.
-The gravitational potential of the point source M modifies the average hopping time that is needed by a hyper-crystal excitation to move from one tetrahedron to the next by a factor $1 - \phi/c^2$. This applies in particular to the hopping time T_P of a photon defined in (31). As a consequence, any measured time interval between physical events is modified by this factor.

More details on the status of GR in the tetron model, as well as on FLRW, gravitational waves and the interpretation of the Newton limit will be given in section 2.5. In general one has to use the ADM formalism[45] or the approach by Carter et al.[46, 47] to describe a general relativistic elastic system which includes arbitrary transformations of the time coordinate. I have chosen to restrict myself to the special cases (26) and (103), because it makes the presentation much simpler and the arguments more transparent.

Furthermore, I have been intentionally vague about which version of GR must actually be chosen. There are generalizations like teleparallel, Poincare or Einstein-Cartan gravity where in addition to Lorentz transformations 4-dimensional translations are gauged. This leads to torsion in addition to curvature as dynamical quantity[88, 99, 100]. Due to lack of experimental information on torsion and of full knowledge of the tetron dynamics it is difficult to say whether one needs a model which describes dislocations or disclinations[94] of tetrahedrons in a flat hyper-crystal or whether 'true' curvature effects are involved, in the sense that the tetrahedrons in fig. 2 are not only shifted by tiny amounts in the horizontal but also in the vertical, i.e. internal direction.

Personally, I give preference to the latter interpretation, because it complements Einstein's original idea of a Riemannian curvature by a physically intuitive micro-picture. One simply has to assume that there are elastic inter-tetrahedral *coordinate* interactions in addition to the isomagnetism describing the phenomena of particle physics. These elastic interactions allow for a buckling and bulging of the 3-dimensional DMESC within the full R^{6+1} and can therefore be described in terms of a *non-vanishing curvature tensor*. Curvature in the time coordinate is included as described above and then patched with the spatial curvature in a Lorentz covariant way a la [46, 47] by considering a matter manifold which is orthogonal to the time slices. More details about this issue can be found in (2.5.27). Note that while the inter-tetrahedral coordinate interactions are elastic, the tetrahedral 'molecules' are rigid bodies which align in their internal spaces.

In any case, the behavior of the gravitational field is determined by the form of the gravitational action S_G. Since the equations of motion should be of second order in field derivatives, S_G must be at most quadratic in torsion and curvature

$$S_G = -\frac{c^4}{16\pi G} \int d^4x \, \sqrt{\det(g)} [\mathfrak{R} + O(\mathfrak{R}^2, \mathfrak{T}^2)] \tag{29}$$

where G is the Newton constant, $\mathfrak{T}$ the torsional and $\mathfrak{R}$ the curvature scalar. The explicit structure of S_G including $\mathfrak{R}^2$ and $\mathfrak{T}^2$ terms[61] is not given here because it is rather complicated, containing the leading term (formally identical to $\mathfrak{R}$ appearing in the Einstein-Hilbert action) plus 3 independent terms quadratic in torsion and 6 quadratic in curvature, plus possibly the cosmological constant. It can be derived from an analysis which demands consistency with the principle of equivalence and the existence of second order e.o.m. and is an example of a generalized 'f(R,T)' gravity theory[89, 91].

All in all 11 independent coupling constants[61] appear in (29). This large number of free parameters is in accord with the idea that the complete description of gravity must be quite complicated, because it is not more than an effective theory for an elastic system of microscopic entities (the internal tetrahedrons) that fill Minkowski space.

The tetron model allows to extend the view beyond this effective theory, to yield a new picture of material existence. According to this model, the world falls apart into 2 rather disparate pieces:
-firstly, *the realm of quasi-particles* like quarks, leptons, Higgs bosons and gauge fields. Since all these excitations fulfill Lorentz invariant wave equations, any phenomenon and signal propagation in this sphere is necessarily limited by the speed of light.
-secondly, *the realm of tetron matter*, i.e. of aligned tetrahedrons and of the hyper-crystal with its elastic/metrical structure.

Since the relevant scales $\Lambda_P \gg \Lambda_F$ are so vastly different, these two spheres do not have much in common. We ourselves exist in the sphere of quasi-particles and can perceive anything coming from the tetron sector only if suitable devices of ordinary matter are patched in between. Gravity, for example, which originally corresponds to a shift of tetrahedron locations on the DMESC, becomes visible in our physical world only due to the reaction with suitable conglomerations of quasi-particles. This is discussed in more detail in (2.5.41), where also the role of gravitational waves is elucidated.

Since they are an independent form of matter, tetrons and tetrahedrons can propagate with velocities larger than c. Usually, this is not relevant because they are fixed by bindings within the hyper-crystal. However, the appearance of superluminal metrical velocities shortly after the big bang can be interpreted as bound tetrahedrons moving at larger than the speed of light, cf. (2.5.42).

If the DMESC was an ordinary crystal, one could speculate about the existence of an absolute rest system. Since it is elastic, there will only be an approximate rest system at any given cosmic time, which according to the arguments in (2.5.16) can be identified with the comoving coordinates used in cosmology to describe the Hubble flow of galaxies.

At first sight the existence of such a system seems to contradict special relativity - a well established concept which I do not want to question. Indeed, in the tetron model all normal material objects are quasi-particle waves fulfilling Lorentz covariant wave equations. As such they cannot distinguish an absolute rest system, i.e. they naturally fulfill Einstein's principle of equivalence. On the other hand, the rest system fig. 2 is made of tetrons, and since it is merely the carrier of those quasi-particles, it is impossible to experimentally perceive it in Michelson Morley type of experiments. More details about this issue are given in (2.5.14)ff.

A particular consequence of tetron cosmology is that the lattice spacing r in fig. 2 is not fixed, but corresponds to an average distance between the tetrahedrons. In addition it varies with time (temperature) during cosmic expansion, simply because the properties of an elastic continuum depend on thermodynamic variables like temperature, pressure etc. The temperature dependence of c and G will be moderate, because most of it is contained in the energy momentum dependence of the Einstein equations.

For reasons explained in (2.5.6), r is to be identified with a time dependent Planck length, i.e. one has

$$L_P(t) = \langle r \rangle \tag{30}$$

with r defined in fig. 2 and 1.6×10^{-35} m being its present average value. There is clearly a relation of this quantity to the scale factor a(t) of the FLRW universe (103) because cosmic expansion is connected to a timely increase in L_P.

By definition, the Planck length is constructed from c, G and h as one of 3 dimensionful quantities which - in the absence of SM interactions - describe all the basic properties of space[m], time[s] and matter[kg]

$$L_P = \sqrt{\frac{\hbar G}{c^3}} \qquad T_P = \sqrt{\frac{\hbar G}{c^5}} \qquad M_P = \sqrt{\frac{\hbar c}{G}} \tag{31}$$

One may invert these relations to obtain

$$c = \frac{L_P}{T_P} \tag{32}$$
$$\hbar = \Lambda_P T_P \tag{33}$$
$$\kappa = \frac{L_P}{\Lambda_P} \tag{34}$$

where $\Lambda_P = M_P c^2$ is the Planck energy and $\kappa = G/c^4$ the Einstein constant.

According to (29), κ is the coupling of choice in GR. As shown below, it has a rather intuitive meaning in the micro-elastic approach, and this statement actually is true for all 3 quantities (32)-(34):
(i) since L_P is the average distance between 2 tetrahedrons, then $c = L_P/T_P$ makes T_P the 'hopping' time it takes for a photon quasi-particle to hop from one tetrahedron to the next. The question why T_P is the characteristic time for the whole physical system of quasi-particles and valid even for gravitational waves is answered in (2.5.41).
(ii) since $\Lambda_P \sim \Lambda_r$ is the binding energy of a tetrahedron in the DMESC, Planck's constant $\hbar = \Lambda_P T_P$ reflects the action of the binding energy during the characteristic time, cf. (2.5.6).
(iii) finally $\kappa = L_P/\Lambda_P$ gives the disclination of a tetrahedron in the DMESC per unit energy, i.e. applying an energy Λ_P to the tetrahedron will displace it by an amount L_P. In other words, the gravitational coupling quantifies the elasticity of the ground state tetrahedron material, i.e. its reaction to any kind of mass/energy influx, as described by the Einstein equations.

Formally, one may associate a Lame constant ζ to the tetrahedral material[62] and relate it to Einstein's constant via

$$\zeta = \frac{1}{L_P^2 \kappa} \approx 10^{112} \frac{kg}{ms^2} \tag{35}$$

The weakness of gravity (κ) thus corresponds to an extremely large stiffness (ζ) of the DMESC, which in turn is related to the high density and the rather strong coordinate forces among tetrahedrons, cf. (2.5.27) and (2.3.19). The point is that using the value of ζ it is possible to calculate the average present-day tetrahedral density in the universe

$$\rho_T = \frac{\zeta}{c^2} \approx 10^{95} \frac{kg}{m^3} \tag{36}$$

This is 121 orders of magnitude larger than the density of ordinary (=quasi-particle mignon) matter $\rho_M \approx 10^{-26} kg/m^3$. Although it is a very large value, it should not be taken as a big surprise, because after all tetrons are the omnipresent fundamental building blocks of the hyper-crystal. This issue will be further discussed in connection with (122).

It may be noted that (36) corresponds to the equation for the speed of sound in an ordinary elastic medium (see e.g. [47]) and that the tetronic c^2 according to (42) appears in the energy momentum relation for all kinds of excitations on the hyper-crystal, elastic as well as isomagnetic ones, cf. (2.5.41).

Furthermore, (35) may be re-expressed as

$$\zeta = \frac{\Lambda_P}{L_P^3} \tag{37}$$

According to this formula, the Lame parameter can be interpreted as an energy density, whose numerical value is of the order of the energy density of the vacuum arising in quantum field theories, if instead of renormalization one applies the Planck scale as a cutoff for divergent integrals[63]. This result is no accident, because the vacuum of quantum field theories is the zero point energy of all quantum oscillators, and in the framework of the tetron model is determined by the vacuum state of aligned tetrahedrons fig. 2. For details see (2.5.30).

According to the tetron model, h, c and G are derived and (moderately) temperature dependent[4] material properties of the DMESC, not valid outside of it, the only fundamental force (valid over full R^{6+1}) being the unknown interaction (2.2.10) among the tetrons. According to the above h, c and G are determined by the
-average 'lattice spacing' L_P between tetrahedrons,
-the tetrahedral density ρ_T
-and by the elastic modulus ζ.
Therefore they are in principle calculable from the fundamental force among tetrons.

The same is true for the constants of particle physics appearing in the SM lagrangian, i.e. for the Higgs parameters, the fine structure constant α (electric charge e) and

[4]It has been claimed that specifying the time evolution of these dimensional 'constants' is meaningless[52], because the standard rulers also change with time, and that the only thing that counts in the definition of worlds are the values of the dimensionless constants. This claim has been rightfully refuted for various reasons by many authors, see [53].

the weak mixing angle. While the latter will be derived in (2.1.11) and (2.1.6) from the fact that the photon is of $D_\star$-tetron content only, α can be interpreted on a similar level as h, c and G, simply because the photon - whose coupling defines α - in the tetron model is not a fundamental but a quasi-particle confined to the hyper-crystal. Finally, the Higgs parameters can be traced back to isomagnetic exchange interactions of tetrons, as described in the first half of this section.

More details of the tetron model meaning of these quantities will be given in sections (2.5.6) for h, (2.5.41) for c, (2.1.12) for the Higgs potential parameters and (2.4.10) and (2.4.21) for the Yukawa couplings. Some rudimentary ideas about the form of the fundamental tetron interaction can be found in (2.2.10).

The known particles [quarks, leptons and gauge bosons, cf. (2.5.33), (2.1.2) and (2.1.3)] are interpreted as intrinsic excitations of the hyper-crystal and as such will extend over at least one lattice spacing $\langle r\rangle = L_P$. Therefore measurements involving physical particles can never be more accurate than L_P. As proven in (2.5.6), this modifies the quantum mechanical uncertainty principle and can be used to fix the value of h as

$$\hbar = \frac{c^3}{G}\langle r\rangle^2 \tag{38}$$

Similar for the speed of light: since the photon is an excitation of the hyper-crystal, a temperature dependence as well as a dispersion of c is to be expected and calculable from crystalline parameters, like that of the speed of sound in an everyday elastic medium. A simple argument will now be given why this is not detectable in present experiments. The point is that photons on a lattice with spacing L_P have a dispersion

$$c(k) = \frac{2c(0)}{L_P k}|\sin\frac{kL_P}{2}| \approx c(0) + O(kL_P)^2 \tag{39}$$

i.e. for wavelengths $\lambda = 2\pi/k$ much larger than L_P the speed of light is constant to a very good approximation. Even with the hardest and 'oldest' cosmic gamma rays observed so far deviations from $c(0) = c$ cannot be tested.

Note that (39) relies on the existence of an equilibrium state and therefore does not control the behavior of c at the time of inflation when the hyper-crystal was formed under non-equilibrium circumstances. Furthermore, it should be mentioned that (39) is not only valid for isomagnetic excitations like the photon but also for density fluctuations of tetrons like phinons, gravitons etc, cf. (2.5.41).

Actually, it can be used as a starting point to understand the dynamical background of the special-relativistic energy-momentum relation

$$E^2 = m^2 c(0)^4 + \vec{p}^2 c(0)^2 \tag{40}$$

within the tetron model. As well known, (40) is equivalent to the Klein-Gordon equation in momentum space, and since the mignons are massive isospin waves fulfilling a d'Alembert type of wave property, they clearly must respect it. Dividing by $\hbar^2$ one obtains from (40) the dispersion relation

$$\omega(k)^2 = \omega(0)^2 + \vec{k}^2 c(0)^2 \tag{41}$$

with a low-frequency cutoff $\omega(0)^2 = m^2 c(0)^4/\hbar^2$.

Writing $\omega(k) = kc(k)$, the second term on the rhs of (41) is obtained from (39). In other words, the propagation part $\sim \vec{k}^2$ in the dispersion relation for mignons is completely fixed by the coordinate interactions of the DMESC, or more precisely by the value of $c = c(0)$, which according to (36) is determined by the stiffness and density of tetrons in the hyper-crystal.

In this respect mignons are distinguished from the magnons in ordinary magnetism whose dispersion relation $\hbar\omega = J[1 - \cos(kL_P)]$ involves the exchange coupling J even in the propagation term. In the present case, the isomagnetic exchange coupling enters the dispersion only through the mignon rest mass m which according to the calculations in [2] is proportional to J.

J is used here as a wildcard for the various internal exchange couplings introduced in (14) and (19) and in [2]. In the case of the strange quark mignon, for example, one would have $J = (4J_{SS} + 2J_{ST} + 2K_{ST})c^2$ according to (17).

In summary the relativistic energy momentum relation (40) can be rewritten in terms of tetron matter properties

$$E^2 = J^2 + \vec{k}^2 \frac{\zeta}{\rho_T} \tag{42}$$

Note that the low frequency cutoff makes sense. The tetron operators want to oscillate at their natural frequency $\omega(0) \sim J$, and cannot be compelled to oscillate any slower. Such a behavior is generic in any system that has some kind of internal oscillation. The group velocity

$$v_g = \frac{d\omega}{dk} = \frac{c(0)}{\sqrt{1 + \frac{\omega(0)^2}{c(0)^2 k^2}}} \tag{43}$$

vanishes in the long wavelength limit while at high energies, the physics of a nondispersive medium with constant group velocity $v_g = c(0)$ is recovered.

Finally, it may be noted that there is also a high frequency cutoff. This corresponds to the Planck scale and to the appearance of the sine in (39). The internal isomagnetic couplings J do not play a role in that regime. As the frequency of the wave is increased, one is probing the physics of an infinite system of tetrahedrons coupled with spring like forces. As it is further increased to still higher values ($kL_P \sim 1$) towards the Brillouin zone, the effects of the sine (the high energy cutoff) is seen. This issue is further discussed at the end of (2.5.6).

2 Questions and Answers

In this section a list of questions and answers is presented which arise in connection with the tetron model. Open problems will be specially marked and the more important ones reviewed in the summary section 3.

2.1 Questions about the Gauge Sector

According to section 1 the universe is interpreted as a discrete fiber bundle over Minkowski space R^{3+1} with fibers given by the iso-magnetic tetrahedrons fig. 1. The electroweak gauge fields are to be interpreted as connections in that fiber space, i.e. they help to define what parallel alignment in between different fibers means.

2.1.1 How can such a model lead to a local gauge theory?

The internal 3-dimensional space which hosts the tetrahedrons is naturally endowed with a $SU(2)_L \times U(1)_F$ symmetry:
–the $SU(2)$ factor arises from the rotational symmetry of the internal spin vectors before their alignment. Some details have already been explained in section 1. The

question how this symmetry is related to the weak isospin of quarks and leptons will be analyzed in (2.4.16)ff. The reason for why it receives the index L in the tetron model is explained in (2.1.19).

–the U(1) factor corresponds to tetron number conservation, which on the level of quarks and leptons translates into the $B - L$ quantum number, cf. (2.1.7).

These groups act as *local* symmetries, because their elements can be chosen different for different points of the Minkowski base space. Connections can be defined for the $SU(2)_L \times U(1)_F$ bundle, which are to be interpreted as gauge bosons. As shown in (2.1.10), there is a mixing of the $U(1)_F$ field with W_z, with mixing angle equal to the Weinberg angle. After the mixing the corresponding local gauge fields are given by the observed $W^{\pm}$, Z and γ.

2.1.2 Are the electroweak bosons and the Higgs field composite?

The answer is yes, but one should specify how this works out in detail.

–The most straightforward possibility is that they are composites of mignon antimignon pairs. However, as will be seen in (2.3.29), such a pairing is more appropriate to describe chiral symmetry breaking in QCD. Furthermore, such a construction would make the top-quark content dominate the boson sector of the SM, similar to top condensate models. Since $m_t \gg m_W$, this is usually not considered a convincing scenario.

–Secondly, one could be tempted to insist they are fundamental objects, because they are connections of the basic $SU(2) \times U(1)$ fiber bundle in the sense of differential geometry. As such they could have been induced by curvature dynamics of the full R^{6+1} geometry. I do not think this is a very attractive option, because the flat R^{6+1} knows nothing about the dynamics within the 'curved' hyper-crystal bundle. Furthermore, the Higgs field as a necessary add-on to account for the SSB does not have a simple interpretation in the pure differential geometric framework.

–Thirdly, they could be tetron-antitetron bound states traveling freely through the hyper-crystal. However, according to the picture developed in (2.2.4) and (2.2.5), tetrons are so strongly bound within the hyper-crystal with binding energies $\sim \Lambda_P$, that they cannot be split off, not even in pairs. This requirement is also dictated by the no-dissipation concept, cf. (2.5.33). Namely, one has to take care that such

pairs do not leave the hyper-crystal and dissipate into R^{6+1}, because otherwise energy would not be conserved inside of it.
–I adhere to the idea that they must be *excitations* of tetron-antitetron bonds, i.e. arise from the tetron interactions in the crystal. While mignons are defined on one tetrahedral fiber, the gauge bosons involve a system of 2 neighboring tetrahedrons. As excitations they thus consist in a global precession of the isospin 3-bein of one fiber with respect to the 3-bein of the neighboring fiber. There are 3 types of such precessions corresponding to the 3 internal Euler angles defining the d.o.f. of the differential geometric SU(2) connection among the fibers and thus to the 3 weak gauge bosons.
As a consequence, the gauge bosons form and travel solely inside the hyper-crystal and cannot exist outside of it. 'Traveling' of such a pairing excitation is meant in the sense of a quasi-particle, i.e. the excitation hops from one tetron-antitetron pair to another, while the tetrons themselves stick to their place in the crystal. It is possible to imagine it as a density wave bilinear in $\bar{\psi}$ and ψ that travels through the crystal. This must be distinguished, however, from the density fluctuations involving $\psi^\dagger$ and ψ of a single tetrahedron which are coined phinons in (2.3.26) and (2.1.13).

2.1.3 Can a stable and massless particle like the photon be an excitation?

First of all, note that masslessness of the photon is protected by the U(1) gauge symmetry. As long as this symmetry holds, the photon remains massless, whether composite or not.
Furthermore, the masslessness of the photon implies its stability.
The answer to the given question is yes within the 'no-dissipation' hypothesis advocated in this article, cf. (2.5.33). The latter has the advantage that energy is conserved for all processes inside the crystal, so no compactification of internal spaces is needed. The only objects which are not excitations are the tetrons, the building blocks of the crystal. These, however, are bound with energies $\gg 10^{10}$ GeV.
The photon being an internal excitation cannot be scattered away from the hyper-crystal. Since according to (2.2.3) one has $Q(U) = 0$, the photon is a $\bar{D} - D$

excitation of D-tetrons, conveniently abbreviated as

$$A_\mu \sim e\, Q(D)\bar{D}\gamma_\mu D \tag{44}$$

As discussed after (11), in the SSB phase actually radial isospinors should be used

$$A_\mu \sim e\, Q(D_\star)\bar{D}_\star\gamma_\mu D_\star \tag{45}$$

Furthermore, it must be noted that the no-dissipation hypothesis (2.5.33) has rather challenging consequences. If the photon is not a fundamental particle, it is difficult to believe that the Lorentz symmetry, valid inside the hyper-crystal with the known value of c, is a fundamental property of the original full R^{6+1} spacetime. The Lorentz structure as we know it, comes into being only when the crystal is formed and holds only inside of it. This point is further elucidated in (2.5.3), (2.5.14)ff and (2.5.41).

2.1.4 What happens on the microscopic level when a mignon and an antimignon annihilate into an electroweak gauge boson?

Assume the 2 mignons are located on 2 neighboring tetrahedrons. When the gauge bosons are formed, the mignons cease to exist and are replaced by an internal excitation of a bound $\bar{\psi}$-ψ pair involving 2 tetrons from the neighboring tetrahedrons. This is in contrast to [1] where these pairs were assumed to be made from free tetrons floating around. The latter idea has been abandoned because the binding energy of a tetron in the crystal is too large, of order Λ_P, and it would furthermore allow energy in the form of $\bar{\psi}$-ψ pairs to dissipate away from the hyper-crystal.
The excitations of bound pairs of tetrons behave trivially under Shubnikov transformations (7), i.e. the information about the discrete tetrahedral structure is washed out, because mignon and antimignon compensate each other in that respect. What remains is the transformation property under $SU(2)_L \times U(1)_F$. Since $\psi = (U, D)$ is an isospin doublet, the product of ψ and $\bar{\psi}$ leads to $2 \otimes 2 = 3 + 1$, i.e. a triplet (the weak bosons) and a singlet (the B-L photon).
These serve as connections in the fiber space. As such they are useful to define, what alignment of adjacent tetrahedrons means, cf. the discussion after (19).

2.1.5 Why can mignon couplings be understood as gauge couplings?

The mignons are dynamical sections in the $SU(2)_L \times U(1)_F$ fiber bundle described above. In order to keep up gauge invariance they are naturally endowed with gauge couplings to the connections. As for the couplings of the fundamental tetron fields $\psi = (U, D)$ one may consult (2.2.3).

2.1.6 What is the meaning of the initial $U(1)_F$ symmetry?

On the tetron level F is tetron number, on the mignon level it is $B - L$.

2.1.7 How do the electric charges of mignons arise?

According to (2.1.19) parity violation of the weak interaction follows from the internal chirality of the tetrahedral 'star' configuration fig. 1. This implies that there are no separate W_R bosons and that all $V + A$ couplings to mignons necessarily vanish, cf. (2.1.17). Still it is possible to formally introduce a right handed isospin quantum number via $I_3 = I_{3R} + I_{3L}$ (with vanishing coupling g_R due to the parity violating effect).

Furthermore, $F = B - L = B + \bar{L}$ is the appropriate fermion number to choose for mignons (9), with $F(l) = -1$ for leptons and $F(q) = 1/3$ for quarks. The mixing among the neutral gauge bosons can then be described by introducing the unbroken generator Q as

$$Q = I_3 + \frac{F}{2} \tag{46}$$

so that

$$Q(u) = \frac{1}{2} + \frac{F(q)}{2} \qquad Q(d) = -\frac{1}{2} + \frac{F(q)}{2} \tag{47}$$

$$Q(\nu) = \frac{1}{2} + \frac{F(l)}{2} \qquad Q(e) = -\frac{1}{2} + \frac{F(l)}{2} \tag{48}$$

2.1.8 What is the tetron content of the Higgs field and of the SM vev?

To answer this question, the same idea is used which has led to the photon content (44), namely that all observed scalars and vector bosons arise from correlations between tetrons and antitetrons of neighboring tetrahedrons, cf. questions (2.1.2) and (2.1.4).

One of these correlations is directly related to the electroweak SSB and is called the Higgs particle. Since it is to support the radial alignment of isospinors in fig. 2 responsible for the SSB, it can be identified as

$$H \sim \bar{U}_\star U_\star \tag{49}$$

where $U_\star$ is the 'radial' iso-spinor introduced in (11) corresponding to an isospin vector $\vec{Q} = U_\star^\dagger \vec{\tau} U_\star$ pointing outward as in fig. 1. The point is that the content of the Higgs particle is in one-to-one correspondence with the vev needed to stabilize the alignment of isospins in fig. 2, and isospin vectors pointing outward correspond to radial spinors $U_\star$ while those pointing inwards correspond to $D_\star$.

According to these considerations the SM Higgs doublet Φ must be of the form

$$\Phi \sim \tau_2 (\bar{U}_{R\star} Q_{L\star})^{\dagger T} \sim \begin{pmatrix} -\bar{D}_\star (1+\gamma_5) U_\star \\ \bar{U}_\star (1+\gamma_5) U_\star \end{pmatrix} \tag{50}$$

i.e. not as in ordinary $SU(2)_L \times SU(2)_R$ symmetric Nambu-Jona-Lasinio (NJL) theories[31, 32] but formally similar to top-color models[25] – provided the use of radial isospinors is understood.

The implication of (50) on the vev and on the NJL structure inherent in the SM will be discussed in (2.1.13), (2.1.14) and (2.3.11). For use in those sections I include here the definition

$$\widetilde{\Phi} := i\tau_2 \Phi^\star \sim \bar{U}_{R\star} Q_{L\star} \tag{51}$$

2.1.9 What is the tetron content of the weak gauge bosons?

The photon is given by (44) and the $U(1)_F$ tetron number gauge boson by

$$B_\mu \sim g'\ F(\psi)\ [\ \bar{U}\gamma_\mu U + \bar{D}\gamma_\mu D\ ] \tag{52}$$

where g' is the $U(1)_F$ gauge coupling. Similar formulas hold for the $SU(2)$ gauge bosons, cf. (2.1.10).

2.1.10 γ-Z mixing and the value of the Weinberg angle in the tetron model

In (2.2.3) it is shown that $F(\psi) = -1$ and $Q(D) = -1$. Using this input one can directly infer from (44) and (52) that the weak mixing angle at the unification/crystallization point Λ_r must be 45 degrees, i.e. $\sin^2(\theta_w) = 1/2$. The form of the Z-boson is

$$Z_\mu \sim -\frac{e}{\sin(\theta_w)\cos(\theta_w)} \left[I_3(U)\bar{U}\gamma_\mu U + I_3(D)\bar{D}\gamma_\mu D + Q(D)\sin^2(\theta_w)\bar{D}\gamma_\mu D \right] \quad (53)$$

which at Λ_r reduces to $Z \sim \bar{U}\gamma_\mu U$, i.e. at the unification point the Z like the Higgs particle consists only of U-tetrons. Left- and right-handed tetrons are not distinguished in these relations, because the SU(2) gauge bosons a priori contain lefthanded as well as righthanded tetrons. It is only the internal chirality of the configuration fig. 1 that prevents the $V+A$ component to become active, cf. (2.1.16), (2.1.19), (46) and [1].

In the next subsection (2.1.11) the prediction $\sin^2(\theta_w) = 1/2$ at Λ_r will be shown to agree with the present experimental value provided one uses 3 ingredients: (i) the evolution of the SM beta function as given in [64], (ii) eq. (54) and (iii) a value of the electroweak unification scale relatively close to the Planck scale.

2.1.11 A 'unification scale' in the framework of the tetron model

In the tetron model the natural electroweak unification scale is given by the energy Λ_r at which the hyper-crystal is formed from tetrahedral 'molecules' via the phase transition II. As argued in sections 1 and (2.5.6) this scale corresponds to the average distance between 2 tetrahedrons in fig. 2 and is naturally of the order of the Planck scale. As shown in (2.1.10), at Λ_r the value of the Weinberg angle must be 45 degrees. This corresponds to a relation between the $U(1)$ and $SU(2)_L$ gauge couplings

$$g'(\Lambda_r) = g(\Lambda_r) \quad (54)$$

Note that (54) goes beyond the SM because the gauge group $SU(2)_L \times U(1)_F$, even in the form of a U(2) group, is not simply connected and therefore no relation between the values of g and g' is predicted within the SM. In contrast, in the tetron model a

prediction is possible and given by (54). This is based on the observation that the original U(1) gauge symmetry is tetron number and that the photon according to (44) should be of D-content only.
Using the SM beta funtions[64] one can extrapolate g and g' from their measured values at m_Z to ultrahigh energies in order to see for which values of Λ_r eq. (54) can be satisfied. Since there is no diminishing factor 3/5 in (54) like in typical GUT models[102], Λ_r comes out to be nearly equal to the Planck scale instead of $\Lambda_{GUT} \approx 10^{15}$ GeV. Within the present model, this is a rather convenient result, because it allows to identify the electroweak unification scale with the crystallization temperature.
It must be stressed that this is merely an order of magnitude result, because one may rightfully ask, whether the SM beta functions are really applicable up to such high energies, or whether they get appreciable corrections from other crystal excitations like the 2HDM Higgs partners discussed in (2.1.13), or from phinons and isospin density waves which appear at higher energies, cf. (2.3.26). Furthermore, as discussed in sections 1 and 2.5, the Planck scale is expected to be moderately cosmic time dependent in the tetron model.

2.1.12 Connection between the tetron model unification scale and the scales relevant for the standard cosmological model

In the Standard cosmological model the scale at which inflation ends is usually identified as the temperature below which the radiation dominated epoque starts. This era can be described as an equilibrium state of effectively massless electroweak gauge bosons.
In the tetron model, inflation is associated with the release of latent heat at crystal formation time. The end of inflation is the time when crystallization(=the inflation period) has finished, and the unification of the electromagnetic and the weak interactions is naturally interpreted as happening at this point. It is the time at which our 3+1 dimensional universe started to exist. According to the analysis in (2.1.11) and (2.5.6) this roughly corresponds to Planck scale energies, and therefore in the present model the electroweak era starts already at the Planck scale.

2.1.13 Is there a relation between the isospin interactions (14)+(19) of the tetron model and the SM Lagrangian

Yes, there is. To explain this in detail, one first has to notice that while the mignon vibrators are supposed to 'live' within one tetrahedron, the Higgs excitations according to the philosophy discussed in (2.1.2) extend over two of them.

In accordance with this observation, two types of internal vectors should be distinguished:

(i) isospin vectors $\vec{Q}$ of type (2) and (4) which are the carriers of the isospin waves (mignons). They correspond to the internal angular momentum of tetrons *within one tetrahedron* and are the 'charges' of the internal Noether currents. Their excitation spectrum leads to quark and lepton color and flavor. The smallness of neutrino masses is associated to the conservation of these currents, cf. the discussion after (21) and in (2.4.12)-(2.4.15).

(ii) fields like the gauge bosons or the $\vec{\pi}$ component of the Higgs doublet, which involve $\bar{\psi}$ instead of $\psi^\dagger$. Together with the vev $\langle\bar{\psi}\psi\rangle \neq 0$ and the Higgs particle (49) they are important for the pairing process between tetrons and antitetrons of neighboring tetrahedrons which in the tetron model is responsible for the electroweak SSB.

To understand this in more detail consider the SM Higgs potential with one doublet

$$V_{SM}(\Phi) = -\mu^2\Phi^+\Phi + \lambda(\Phi^+\Phi)^2 = -\frac{1}{2}\mu^2(\sigma^2 + \vec{\pi}^2) + \frac{1}{4}\lambda(\sigma^2 + \vec{\pi}^2)^2 \qquad (55)$$

where $\sigma = \Lambda_F + H$. This potential naturally describes the alignment of neighboring tetrahedrons and anti-tetrahedrons in fig. 2, although in that figure not the $\vec{\pi}_i$ are drawn but the $\vec{Q}_i$. The point to note is that two of the $\vec{\pi}_i$ are in parallel iff all the corresponding $\vec{Q}_i$ are. Therefore the pairing force $\sim -\mu^2\vec{\pi}_i\vec{\pi}_j$ implied by (55) exactly corresponds to a 'ferromagnetic exchange coupling' of strength μ^2 in the SSB interaction (19).

There is one drawback in this argument, and this concerns the number of d.o.f. While the SM Higgs doublet only has 4 real d.o.f., the isospin vibrators in the form of $\vec{Q}_L$ and $\vec{Q}_R$ contain 8. According to (4) these can be given as

$$\psi^\dagger\psi \qquad \psi^\dagger i\gamma_5\vec{\tau}\psi \qquad \psi^\dagger i\gamma_5\psi \qquad \psi^\dagger\vec{\tau}\psi \qquad (56)$$

I have included $\psi^\dagger\psi$ and $\psi^\dagger i\gamma_5\psi$ in this list albeit their vibrations do not correspond to mignons, but to phinons, cf. (2.3.26).
The expressions (56) are adapted to the $SU(2)_L \times SU(2)_R$ symmetric limit. In more general cases, when this symmetry does not hold, the listing reads

$$U^\dagger U \quad D^\dagger D \quad U^\dagger D \quad D^\dagger U \quad U^\dagger\gamma_5 U \quad D^\dagger\gamma_5 D \quad U^\dagger\gamma_5 D \quad D^\dagger\gamma_5 U \tag{57}$$

By comparing with the Higgs doublet (50) one sees that half of the d.o.f. are missing in (50). To account for the other half one should add a second scalar doublet to the dynamics, e.g. in the form[5]

$$\Phi' \sim \bar{D}_R Q_L \sim \begin{pmatrix} \bar{D}(1-\gamma_5)U \\ \bar{D}(1-\gamma_5)D \end{pmatrix} \tag{58}$$

Together, Φ and Φ' form the basis for an extended SM with 2 Higgs doublets. Such models are usually abbreviated as 2HDM, and have been extensively discussed in the literature[66, 67, 68].
In the $SU(2)_L \times SU(2)_R$ symmetric limit this corresponds to adding a pseudo-scalar iso-scalar particle η and a scalar iso-vector triplet $\vec{v}$ to the theory, cf. (2.1.15).
The argument about the 'ferromagnetic' alignment induced by the negative mass term $-\mu^2\vec{\pi}\vec{\pi}$ in the potential can be extended to the 2HDM model where the potential contains a term $\sim \vec{v}\vec{v}$ in addition. This term, however, must not give an appreciable contribution to the SSB interaction (19), because otherwise the b-quark mass would come out to be of order Λ_F. In other words, the second Higgs doublet Φ' must not take part in the SSB; its 'mass term' has to have a positive coefficient and correspondingly

$$\langle\Phi'\rangle = 0 \tag{59}$$

i.e. there are no quark/lepton mass contributions from Φ'.
Eq. (59) is further supported by the fact that according to (58) it implies $\langle\bar{D}D\rangle = 0$ or, more precisely, $\langle\bar{D}_\star D_\star\rangle = 0$, i.e. no alignment of isospins in the inward direction - as should be according to fig. 1.
Finally, (59) smartly agrees with the property of the inert version[67, 68] of the 2HDM model. It is interesting to note that in that model the η or the v_z (depending on which mass is smaller) is a serious dark matter candidate. For further details see (2.1.14) and (2.5.36).

[5] The radial star indices (11) in these expressions are left out for simplicity.

2.1.14 What precisely is the argument in favor of the inert version of the 2HDM model?

The most general quark Yukawa Lagrangian in a 2HDM model is given by

$$-L_Y = \bar{q}_L(\Gamma\Phi + \Gamma'\Phi')d_R + \bar{q}_L(\Delta\widetilde{\Phi} + \Delta'\widetilde{\Phi}')u_R + c.c. \tag{60}$$

where 3x3 matrices of Yukawa couplings Γ, Γ', Δ and Δ' in family space have been introduced. The resulting quark mass matrices are then given by

$$M_d = \Gamma\langle\Phi\rangle + \Gamma'\langle\Phi'\rangle \qquad\qquad M_u = \Delta\langle\Phi\rangle^\star + \Delta'\langle\Phi'\rangle^\star \tag{61}$$

Unfortunately, the diagonalization of M_u and M_d does not simultaneously diagonalize the quark-Higgs Yukawa interactions implied by (60), and this leads to the problem that unwanted FCNCs are present in the most general 2HDM model[66]. This is usually handled by the ad hoc introduction of an additional Z_2 symmetry. For example, one may demand the 2HDM Lagrangian to be invariant under the transformation

$$\Phi' \to -\Phi' \tag{62}$$

In this case all Yukawa couplings involving Φ' drop out, and all quarks and leptons couple solely to Φ. Furthermore, symmetry under (62) forbids mixing terms $\sim \Phi^\dagger\Phi' + c.c.$ in the 2HDM Higgs potential so that the Higgs field with vanishing vev can be unambiguously taken to be Φ' in accordance with the representation (58).
In the tetron model one has explicit representations (50) and (58) for Φ and Φ' and may therefore ask whether the physical origin of the Z_2 symmetry can be understood. It is easily seen from (50) and (58) that (62) corresponds to the transformation

$$D_R \to -D_R \tag{63}$$

among tetrons D_R. Indeed, this kind of symmetry naturally arises in the tetron model, because mignon interactions with the D_R field in the effective tetron Lagrangian do not appear. The point is that according to fig. 1 the system's ground state is composed of tetrons U alone, and not of D. Since quarks and leptons are excitations of the ground state, it is thus understandable that their couplings to D are strongly suppressed. This reasoning applies only to D_R and not to D_L, because

parity is broken in the tetron model and D_L couplings are present because D_L appears in an isodoublet with U_L.
Note in this and the previous question one is always talking about the radial isospinors $U_\star$ and $D_\star$ instead of U and D.

2.1.15 What is the tetron content of the additional scalar particles in the 2HDM?

This question is most easily answered in the $SU(2)_L \times SU(2)_R$ symmetric limit of the model. There are 5 observable Higgs scalars in the 2HDM model which are then given by

$$H \sim \bar{\psi}\psi \qquad \eta \sim \bar{\psi}i\gamma_5\psi \qquad \vec{v} \sim \bar{\psi}\vec{\tau}\psi \tag{64}$$

and appear in the quadratic part of the potential[32]

$$-\mu^2\Phi^\dagger\Phi - \mu'^2\Phi'^\dagger\Phi' \sim G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] + G'[(\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}\vec{\tau}\psi)^2] \tag{65}$$

where the terms with coupling G correspond to the μ^2-term in the SM Higgs potential (55) and the terms with coupling G' to an analogous quadratic term for the second Higgs doublet.
Note that although the vibrators $\vec{Q}_{L,R}$ in (4) are chosen in a $SU(2)_L \times SU(2)_R$ symmetric manner in accordance with the discussion in (2.4.7), for many other considerations it is better to use the representations (50) and (58) of the doublets Φ and Φ'.

2.1.16 How can the chiral nature of the weak bosons be ensured?

The iso-magnetic tetrahedral structure in fig. 1 violates internal parity, the state with opposite internal parity being given by a system where the 4 internal spin vectors show inwards instead of outwards. In (2.1.19) it will be proven that this internal parity violation triggers the violation of external parity as required for the V-A nature of the weak interactions, provided the interaction among tetrons stems from a common interaction in the full R^{6+1} space, cf. (66) and (67).

2.1.17 Are there SU(2)$_R$ gauge fields $\vec{W}_R$ in addition to SU(2)$_L$?

No. Z and W are originally connections of an SU(2) bundle. According to the discussion in (2.1.19) it is only the formation of the chiral structure fig. 1 together with the R^{6+1} origin of the interaction which forces them to couple to left-handed mignons only. Without the internal chirality fig. 1 the weak interactions would be vectorlike.

2.1.18 If there is no $\vec{W}_R$, why is there η in (64)

The 2HDM model (2.1.13) naturally accompanies the vibrations of $\vec{Q}_L$ and $\vec{Q}_R$. 2HDM models do not need a $\vec{W}_R$-field[66].

2.1.19 Is there a tetron interaction which gives rise to such iso-magnetic structures? Can the parity violation of the weak interactions be explained from first principles?

Before I start to discuss this question, note it is not about the fundamental coordinate forces which are responsible for the formation of (tetrahedral molecules and) the hyper-crystal at scale Λ_r, but only about the isomagnetic forces relevant for the isospin vector alignment at the Fermi scale. Of course, there is a connection between the 2 issues - to be explained in (2.2.10) and (2.2.9).

The model advocated in this paper contains a 6+1 dimensional spinor fields ψ ('tetrons'), which form tetrahedral structures and a hyper-crystal a la fig. 2. In this world quarks and leptons propagate through spacetime as quasi-particles made of isospin precessions. The Higgs field and the observed vector bosons are excitations of tetron-antitetron bonds, and the system as a whole gives rise to the $SU(2)_L \times U(1)_F$ gauge symmetric SM.

The isomagnetic tetron interactions are claimed to derive from the octonion structure which is naturally inherent in a 6+1 dimensional space. The octonions form the unique non-associative, non-commutative and normed division algebra in 8 dimensions, and their imaginary units provide for 7 of the 21 generators of SO(6,1). They are closely related to the Dirac matrices Γ_μ in 6+1 dimensions[102]. A 6+1 dimensional vector current $\bar{\psi}\Gamma_\mu\psi$ arises more or less directly from the product of

two octonions corresponding to the spinors $\bar{\psi}$ and ψ.
When the hyper-crystal is formed and R^{6+1} decomposes into Minkowski and internal space, the Γ_μ split into SO(3,1) Dirac matrices γ_μ and a remainder according to[102, 39]

$$\Gamma_{0-6} = (\gamma_{0-3}, \gamma_5 \tau_{x,y,z}) \tag{66}$$

where x, y and z denote the internal coordinates. This splitting has its physical origin in the coordinate interactions of the tetrons, which lead to the formation of the hyper-crystal, and mathematically it parallels the splitting of an octonion into 2 quaternions.
Starting from (66) one can try to derive the parity violation of the weak interaction. The important point to note is the appearance of the product $\gamma_5 \vec{\tau}$ in the internal part of (66). In principle, the presence of such a coupling corresponds already to a parity violating behavior, both in internal and Minkowski space, because γ_5 signals axial behavior in Minkowski space and $\vec{\tau}$ does the same job for the non-relativistic internal fiber.
According to (66), any 6+1 dimensional vector coupling $\bar{\psi}\Gamma_\mu\psi$ reduced to internal space will induce such a term. However, for this to actually become perceivable, an additional appropriate 'chiral situation' has to be provided, again both in internal and Minkowski space. In Minkowski space this can be achieved, for example, by using polarized beams or if there is a second vertex with a γ_5-coupling in the Feynman diagram of the process.
An analogous requirement must be met in the internal space. In other words, a configuration with a handedness must be present, in order to pick up a non-vanishing contribution from the axial coupling, and this in the case at hand is given by the local chiral ground state structure fig. 1.
As a matter of fact, the non-relativistic circumstances of the internal R^3 space make it a similar situation as one has in optical activity of molecules, where in addition to a circularly polarized photon there must be a handed molecule in order to produce a non-vanishing effect. According to (66) a 4-tetron interaction of two vector currents will induce among others a term

$$\vec{\pi}\vec{\pi} \sim [\bar{\psi}\vec{\tau}\gamma_5\psi][\bar{\psi}\vec{\tau}\gamma_5\psi] \tag{67}$$

which agrees with the quadratic term of the Higgs potential (55) responsible for the alignment of isospin vectors and on the level of isospin vectors reproduces the Heisenberg ansatz (14). As discussed in connection with fig. 3 and in (2.2.9), the sign of the coupling must be anti-ferromagnetic for inner- and ferromagnetic for inter-tetrahedral distances. The latter is in accord with the negative mass term of the Higgs potential.
An important question is whether there is a renormalizable interaction in 6+1 dimensions which can accommodate the iso-magnetic properties described here. This will partly be answered in (2.2.10).

2.2 Questions about the fundamental Fermion ψ

According to (1) the tetron matter fields ψ which form the sites of the ground state fig. 2 transform as a spinor 8 under SO(6,1) and decompose into an isospin doublet when the hyper-crystal is formed. In this section some further properties of the tetrons are elucidated.

2.2.1 What is the use of introducing an additional level of matter?

There are several good reasons to do so:
1. the existence of 3 families of quarks and leptons with altogether 24 states (plus the corresponding mass and mixing values) strongly suggests that they are not truly elementary objects.
2. a material origin for the observed internal symmetry groups is highly desirable. Traditionally, they are pasted into the theory as purely abstract groups, representing a rather static behavior of the internal spaces. This line of thinking started with Heisenberg's invention of isospin SU(2), included color SU(3) and ended with the (SUSY) GUT groups. The present model works differently, color and isospin being obtained by extending spacetime by 3 internal spatial dimensions in which an independent dynamics takes place.
3. spontaneous symmetry breaking is introduced in the SM in a more or less ad hoc way by adding a scalar field to a system which otherwise is made up solely of fermions and gauge bosons. This is similar in spirit to the Ginzburg-Landau model

for superconductors, extending it to a relativistic and local non-abelian symmetry. However, as well known from many branches of physics, a material background is required for a phase transition and SSB to occur. For example, in superconductivity the scalar field is provided by electrons bound as Cooper pairs.
In the tetron model the breaking of SU(2)$_L$ is associated to the alignment of the (material) internal spins over Minkowski space as shown in fig. 2[1]. For the interpretation of the Higgs field as a tetron-antitetron correlation see (2.1.8).

2.2.2 What is so interesting about the 8 of SO(6,1)?

This question is discussed in (2.1.19) and (2.5.11).

2.2.3 What are the couplings / charges of the tetrons?

After the hyper-crystal is formed, a tetron ψ decomposes into its isospin components U and D. This fact fixes the weak charges

$$I_3(U) = +\frac{1}{2} \qquad I_3(D) = -\frac{1}{2} \tag{68}$$

Using (46) one finds

$$Q(U) - Q(D) = 1 \qquad F(\psi) = Q(U) + Q(D) \tag{69}$$

where F is a U(1) charge and given by tetron(=fermion) number. Therefore it must be the same for both types of tetrons, i.e.

$$F(\psi) = F(U) = F(D) \tag{70}$$

Tetrons do not have a color charge because they are not involved in interactions of triplets of the Shubnikov group, cf. (2.3.29). Therefore, it is appropriate to normalize F in analogy with leptons instead of quarks, i.e. to put

$$F(\psi) = -1 \tag{71}$$

Eq. (69) then leads to

$$Q(U) = 0 \qquad Q(D) = -1 \tag{72}$$

In other words, the normalization (71) is equivalent to defining the U direction in iso-spinor space to be the one which is electrically neutral. This is in accord with the fact that there is no U component in the representation (44) of the photon and that the symmetry breaking ground state and the Higgs vev (2.3.11) are composed of U tetrons only (or actually $U_\star$).
Looking at (71) one may suspect that the result (72) is just a question of normalization and therefore cannot have much physical impact. However, there are only 2 possibilities for the tetron number of one tetrons, +1 or -1. The first choice leads to Q(D)=0, the other one to Q(U)=0. This means the only freedom one has is which tetron one wants to call U and which one is called D. In this paper the electrically neutral tetron is called U and gives the dominant contribution to the Higgs particle (49).

2.2.4 How large is the mass/binding energy of a tetrahedron within the hyper-crystal. Can it be ionized?

Not with experimental means. The binding energies are extremely large, of the order of $\Lambda_r \sim \Lambda_P$, cf. (2.5.6) and the discussion after (23).

2.2.5 How large is the mass of a single tetron?

Difficult to say. If gauge bosons and Higgs scalars would be $\bar{\psi}$-ψ bound states, the natural guess for m_ψ would be in the range of 40 to 60 GeV. However, in truth the bosons of the SM are not bound states but correlations of the $\bar{\psi}$-ψ bonds within the hyper-crystal. Note that tetrons are even more tightly bound within the tetrahedrons than the tetrahedrons are within the hyper-crystal, cf. (2.2.4).

2.2.6 What is the spin/helicity of a single tetron within the internal tetrahedral ground state fig. 1?

Figures 1 and 2 contain all necessary isospin information for the hyper-crystal ground state. Since the tetrons U and D are ordinary Dirac fermion in 3+1 dimensions, one may also ask what their spin direction within the hyper-crystal is.
First of all, the total spin of all tetrons within one tetrahedron should add up to

zero, because otherwise the vacuum state (i.e. the unexcited hyper-crystal) would be polarized. I do not know exactly how strong the limits are, but I am quite sure that a polarized vacuum is not a desirable option.

Now assuming the spins add up to zero, there are 2 options:

–they do so in a similar fashion as isospins do in fig. 1, i.e. because the sum of spin vectors over all tetrahedral sites vanishes.

–the spins from the left-handed and the right-handed isospin vectors $\langle\vec{Q}_L\rangle$ and $\langle\vec{Q}_R\rangle$ compensate each other on each site separately.

As a byproduct of the considerations in (2.4.16), it can be shown that the second option is fulfilled, because the spins from the tetron and the antitetron contributions in fig. 4 compensate each other.

2.2.7 Are there γ_5 anomalies in the tetron model, which could possibly make it inconsistent?

There are no anomalies in the fundamental theory of tetrons, because there are no γ_5 couplings. Such couplings arise only in the effective description (the Standard Model) due to the existence of the iso-chiral tetrahedron fig. 1, cf. (2.1.19). On the level of the effective theory the familiar anomaly cancellations among the quarks and leptons apply.

2.2.8 *How can a crystal system out of tetrons and anti-tetrons be stable?*

In other words: why is there no annihilation between its particle and antiparticle components?

One observation is that according to (44) the tetrons $U_\star$ making up the ground state of the hyper-crystal have vanishing electric charge and therefore cannot annihilate into a photon.

Actually, this is not really a fair argument, since we are talking here about tetrons and not about their excitations. Being the fundamental form of matter, tetrons anyhow are not expected to annihilate into quasi-particles like the photon or the weak gauge bosons. Writing down (44), (49) and (53) refers to *excitations* of tetron-antitetron pairs rather than the tetrons themselves.

Apart from this argument, I see two further alternatives to answer the question:

–the hyper-crystal in its ground state consists only of tetrons and not of antitetrons, i.e. the vacuum expectations values discussed in (2.1.8) and (2.3.11) do not have an antitetron contribution. Antiparticles would then only arise within the hyper-crystal on the level of excitations. Although at the moment I do not completely understand all the implications, this could modify our understanding of baryogenesis, an issue to be discussed in (2.2.17). However, because of the way weak isospin is constructed in (2.4.16), with the necessity of a tetron-antitetron pair on each tetrahedral site, I do not consider this a reasonable option.

–I adhere to the possibility that no field exists into which tetrons and antitetrons annihilate. This can happen because in the original SO(6,1) there is no notion of an antiparticle, because tetron and antitetron comprise into one single representation 8 according to (1).

Furthermore, one may argue that within the scenario developed in sections (2.5.14)ff, where the hyper-crystal including its constitutive tetron matter is a non-relativistic object with a privileged rest system, while only the quasi-particles are of relativistic nature and the familiar SO(3,1) Lorentz structure emerges but on the level of excitations, one should start with a non-relativistic spinor representation 4 of SO(6). Although antitetrons $\bar{4}$ exist in addition to this 4, annihilation processes usually do not occur in a non-relativistic environment , cf. (2.2.10) or [43, 44].

Both tetrons and antitetrons decompose into a spin and an isospin doublet according to $4 \to (2, 2)$, i.e.

$$\psi = (U_\uparrow, U_\downarrow, D_\uparrow, D_\downarrow) \tag{73}$$

when the hyper-crystal is formed. In other words, the crystallization induces a symmetry breaking

$$SU(4) \to SU(2) \times SU(2) \tag{74}$$

in 6 dimensions where SU(4) is the covering group of SO(6) and the two SU(2) factors correspond to 3-dimensional rotations in internal and physical space, respectively.

2.2.9 An intuitive understanding of the isomagnetic interactions in the hyper-crystal

In section 1/fig. 3 some heuristic arguments were given as how to understand the isospin vector configurations figs. 1 and 2 in a similar way as magnetically ordered states in solid state physics. In (2.3.2) the Pauli principle will be used to prove that the tetrahedral configuration of tetrons is extremely stable, and in (2.5.1) a simple formula is given for its energy. These arguments will be extended here in order to show that the global configuration fig. 2 is by far the lowest energy state of a many tetron system.

Much of the effect can be understood by the fermion property of tetrons alone, i.e. without considering the detailed form of the fundamental tetron interaction to be discussed in (2.2.10). One may, for example, exploit the behavior of tetrons under identical particle exchange. Since anti-tetrons are different particles, one needs to consider only half of the isospinors, let's say the ones with tetrons and without anti-tetrons. According to the discussion in (2.4.16) and figs. 4 and 5 one can therefore restrict attention to the lefthanded isovectors $\vec{Q}_L$.

The total wave function should be antisymmetric under tetron exchange. Since the ordinary spins are all 'parallel' (all left-handed), the ordinary-spin part of the wave function is symmetric. Concerning the spatial and the isospin part there are then 2 possibilities: either the isospin part is antisymmetric under tetron exchange and the spatial part is symmetric or vice versa. I want to argue that inside a tetrahedral 'molecule' the first and for the 'crystal' binding between tetrahedrons the second possibility is realized:

(i) In contrast to the inner-molecular coordinate forces, the inter-molecular coupling between 2 tetrahedrons is relatively weak, and to some extent even elastic – though with a large stiffness (35). It is an open question whether this stiffness is a many particle effect or due to an additional super-strong coordinate interaction among tetrons; see the discussion below. In any case, the isomagnetic part (19) of the inter-molecular binding corresponds to a spatial part of the wave function which is antisymmetric under tetron exchange, because under such a condition the wave function becomes small in the middle of the tetron-tetron bond, an effect which usually runs under the name 'Fermi hole' or 'exchange correlation hole'. The Pauli principle then demands that the isospin part of the wave function is symmetric, which corresponds to an aligned, i.e. 'iso-ferromagnetic' configuration. This means nothing else than aligned isospin vectors $\vec{Q}_L$ in neighboring tetrahedral molecules as shown in fig. 2 and needed for the electroweak symmetry breaking.

(ii) Inside the tetrahedrons the isospin part of the wave function is antisymmetric under tetron exchange. This corresponds to an anti-aligned, i.e. 'iso-antiferromagnetic' configuration and leads to a frustrated configuration of isospin vectors $\vec{Q}_L$ as in fig. 1. As a consequence, the spatial part of the wave function must be symmetric, and this implies that the forces inside a tetrahedron are rather strong. In the language of molecular orbital theory they are 'sigma bonds', and the strengthening effect is referred to as 'Fermi heap'.[6]

This last point, however, seems to be in contradiction to the finding that the values (18) of the internal exchange couplings are much smaller than the inter-tetrahedral coupling (20). On the basis of the preceding discussion one would expect the opposite, i.e. the anti-ferromagnetic couplings (14) should be quite large, certainly larger than the O(1 GeV) values obtained in (18). To solve this puzzle[7] one has to realize that the tetrahedral arrangement of isospins fig. 1 is not an ideal antiferromagnet, but a frustrated one. This fact severely lowers the isomagnetic energy of the state, to values of about the QCD Lambda parameter, while the ferromagnetic coupling J_{inter} in (19) which is responsible for the SM SSB remains much larger.

Note that the small values of the internal exchange couplings call for an additional super-strong coordinate interaction among tetrons, in order to maintain strong binding within one tetrahedron. This interaction would then also determine the forces of gravity.

Actually, following (35) it has been argued that the weakness of gravity corresponds to an extremely large stiffness of the coordinate forces among tetrahedrons in the hyper-crystal. In other words, the stronger a force is among tetrons, the weaker it is

[6]In a chemical bond the Fermi heap allows both electrons to be localized in the internuclear region, thus shielding the positively charged nuclei from their respective electrostatic repulsion. It is a matter of speculation whether in the present case in addition to tetrons some kind of 'nuclei' are needed to form the tetrahedral molecules.

[7]Another possibility is to give up the picture discussed in (2.2.14) and (2.3.17) and in connection with fig. 3, that the internal extension R of a tetrahedron is larger than the spatial distance $r = L_P$ between two of them.

among mignons. This rule can be applied to particle physics, too: while on the level of mignons the QCD interactions are much stronger than the electroweak ones, on the level of tetrons the internal isomagnetic stiffnesses J_{SS}, J_{ST} etc are much smaller than the inter-tetrahedral stiffness J_{inter}, the latter being inversely proportional to the Fermi coupling G_F.

One upshot of the present discussion is that the apparent strength of the nuclear forces can be traced back to the frustrated-ness of the isospins within a tetrahedral molecule. More details on QCD aspects of the tetron model can be found in (2.3.29)-(2.3.31).

2.2.10 *What is the form of the fundamental tetron interaction?*

Although I do not have a final answer to this question, there is a wealth of information (the SM Lagrangian, the family spectrum, the Shubnikov symmetry etc), which may be used to obtain some preliminary insights, and actually, important qualitative features of the tetron-tetron interaction have been derived in previous sections, eg. (2.2.9).

As argued in [1], rather weak many-particle correlations are sufficient to understand the frustrated arrangement of isospins and obtain the masses of quarks and leptons. The fact that the weak bosons $W^{\pm}$, Z and H exhibit very small lifetimes agrees with this argument and seems to indicate, that the phenomena of particle physics do not need a super-strong tetron-tetron interaction.

The formation of the hyper-crystal at big bang temperatures might also be describable by many-particle correlations, in a similar way as ordinary crystal structures are mainly due to electrostatic interactions. The extreme density (36) of tetrahedrons further strengthens the enormous stiffness (37) of the system. According to (35) it is inversely proportional to the weakness of the gravitational force, cf. (2.3.19).

Even the initial formation of the tetrahedral molecules might be understandable in terms of rather 'traditional' forces by recourse to the Pauli principle, cf. (2.2.9). The DMESC is then really similar to an ordinary crystal in that the binding occurs because there is first a separation of charges due to a tetron-antitetron pairing induced by the Pauli principle and afterwards an attraction of these charges.

It must be noted, however, that tetron coordinate interactions typically involve binding energies that are enormous (of the order of the Planck scale), and since binding energies often are a measure of the couplings involved, at this point one may face the existence of some additional super-strong binding interaction, which holds the tetrahedral crystal and molecules together.

To make the analogy with non-relativistic QED complete, in [1] the existence of an internal 'photon' as part of a 6+1 dimensional U(1) gauge theory has been assumed. It should be mentioned, however, that there are several drawbacks of the QED_{6+1} model:

-the ordinary photon interpreted as an excitation cannot be part of the fundamental 6+1 dimensional 'photon' field , cf. (2.1.2) and (2.1.3), because

-the 6+1 dimensional U(1) field must be very heavy in order that energy is not dissipated away from the hyper-crystal, cf. (2.5.33).

-Huygens principle is not valid in 6 spatial dimensions; wave fronts of d'Alembert kind of waves do not stay sharp.

For these reasons I follow here a more general approach and analyze various possible effective interactions among tetrons as to whether they yield the correct low energy phenomena.

One could consider, for example, 4-tetron contact interactions. The SM together with the tetron description of its bosonic sector [eqs. (44), (49), (50), (58) and (67)] actually set important constraints on the form of such a contact Lagrangian. These can be read off e.g. from the W-mass $m_W^2 W_\mu W^\mu$ or the $\vec{\pi}\vec{\pi}$ term in the SM Lagrangian (55) and, in a second step, must be adapted for use in a 6+1 dimensional Lagrangian framework.

In doing so, there is a scale factor to be considered when going from a 6+1 dimensional fermion Ψ to a doublet $\psi = (U, D)$ of two 3+1 dimensional spinors bound in the hyper-crystal, because Ψ has energy dimension 3 whereas U and D have energy dimension 3/2. I will write

$$\Psi = \Lambda_R^{3/2}\,\psi \tag{75}$$

where $\Lambda_R \approx \Lambda_P$ is the scale corresponding to arranging the internal dimensions to a mono-layer of tetrahedrons of thickness R, cf. fig. 2, i.e. to the extension of an internal tetrahedron.

Consider, as an example, a contact Lagrangian formed by two 7-dimensional vector currents

$$L_{6+1} = \frac{1}{\Lambda_R^5} \, [\bar{\Psi}\Gamma_\mu \Psi][\bar{\Psi}\Gamma^\mu \Psi] \tag{76}$$

In this representation Λ_R may also be interpreted as the mass of particle exchanged between the tetrons. This could for example be the above mentioned massive QED_{6+1} photon.

In any case one sees that Λ_R determines the coupling of tetrons, because the Lagrangian is to describe the formation of the internal tetrahedrons at big bang times. The appearance of the fifth power has dimensional reasons and is related to the fact that a propagator of any particle exchanged behaves differently in 6+1 dimensions than in 3+1 – namely as r^{-4} instead as r^{-1} at small distances (see below).

Γ_μ, $\mu = 0, ..., 6$ are the Dirac matrices in 6+1 dimensions. They are seven 8×8 matrices which act on the 8 components of the tetron spinor $\Psi \sim (U, D)$. When the hyper-crystal with its 3+1 dimensional 'surface' structure is formed, they break up as[102]

$$\Gamma_{0-7} = (\gamma_{0-3}, \vec{\tau}\gamma_5) \tag{77}$$

where according to the arguments in [1] and (2.1.19) the product $\vec{\tau}\gamma_5$ is one of the ingredients required to establish weak parity violation.

The 7-dimensional Lagrangian is related to a 4-dimensional one via

$$L_{3+1} = \Lambda_R^3 L_{6+1} = \frac{1}{\Lambda_R^2} \, [\bar{\psi}\vec{\tau}\gamma_5\psi][\bar{\psi}\vec{\tau}\gamma_5\psi] \tag{78}$$

This is because identifying $\int d^3x$ with the volume filled by an internal tetrahedron one has

$$S = \int d^7x L_{6+1} = \int d^4x \frac{L_{6+1}}{\Lambda_R^3} \equiv \int d^4x L_{3+1} \tag{79}$$

The result (78) formally has the same structure as (67). More precisely, the $\vec{\pi}\vec{\pi}$ term in (55) can be rewritten as

$$V_{SM} = ... + \frac{\mu^2}{\Lambda_P^4} \, [\bar{\psi}\vec{\tau}\gamma_5\psi][\bar{\psi}\vec{\tau}\gamma_5\psi] \tag{80}$$

where Λ_P is the Planck scale. To obtain (80) I have identified

$$\vec{\pi} = \frac{1}{\Lambda_P^2} \, \bar{\psi}\vec{\tau}\gamma_5\psi \tag{81}$$

In a traditional technicolor model the TC scale (=mass of the TC gauge bosons exchanged) would appear in (81). Here we identify it with the scale Λ_P set by the Planck lattice constant of the hyper-crystal , cf. the discussion in (2.3.17).

A disadvantage of using an approach with contact interactions like (80) is that it ignores the above mentioned problem with Huygens' principle as well as the Galilean nature of the hyper-crystal to be discussed in (2.5.3) and (2.5.41). According to that point of view, tetrons and tetron matter inside the hyper-crystal should not be described by d'Alembert type of wave equations, but a non-relativistic framework should better be used.

It is relatively simple to write down non-relativistic 4-tetron contact terms in 6 dimensions, e.g.

$$L_{nr} = \frac{1}{\Lambda_R^5} \, [\Psi^\dagger \lambda^a \Psi][\Psi^\dagger \lambda^a \Psi] \tag{82}$$

where λ^a, a=1,...,15 are the generators of SO(6). Ψ in this equation is formally given by (75), but instead of the tetron field ψ (= a relativistic spinor 8 of SO(6,1)) the non-relativistic 4 of SO(6) should be taken, as defined in (73).

Since tetrons remain fermions in the non relativistic framework, a quantum mechanical environment must be maintained to describe their interactions, with a Planck constant h_6 possibly different from the ordinary one. In such a framework bound states are most conveniently analyzed using a generalization of Schrödinger's equation to 6 dimensions with a potential $U(r)$ between two tetrons. Spin and isospin effects can in principle be included by extending this to a 6 dimensional equation of Pauli type.

For many aspects of the dynamics like

–the coordinate formation of tetrahedrons,

–the big bang crystallization process and

–the shifts of tetrahedrons inside the elastic hyper-crystal (giving rise to the gravitational interactions)

it is sufficient to average over spins and isospins, so that one can do without the Pauli terms and use the 6 dimensional version of the Schrödinger equation. Even the exchange couplings responsible for the iso-magnetic interactions (14) relevant for particle physics can be calculated with the scalar potential of the Schrödinger equation alone. Only when polarization effects matter, one should extend this to a 6-dimensional analog of the Pauli equation. In many respects the 6-dimensional Schrödinger equation is similar to its 3-dimensional version. First, it is invariant under 6+1 dimensional Galilean transformations, if U is. The wavefunction Φ for one internal tetrahedron transforms as

$$\Phi \to \Phi \exp[\frac{i}{\hbar_6}(Et - \vec{p}\vec{r})] \tag{83}$$

where $\vec{p} = m\vec{v}$ is the 6-dimensional momentum and $E = mv^2/2 + U$ the energy. Secondly, in the case of free tetrons ($U = 0$) there are plane wave solutions

$$\Phi \sim \exp[i(\omega t \pm \vec{k}\vec{r})] \tag{84}$$

with a quadratic dispersion $\omega = \hbar^2 k^2/2m$. Such a relation for tetrons, derived within a 6-dimensional non-relativistic quantum mechanics and holding for the highest energies, is completely different than the $\omega \sim k$ dispersion obtained from the d'Alembert/Klein-Gordon type of equation which controls mignon behavior, cf. (41).

The predominant question in the non-relativistic approach is what the r-dependence of the (iso)scalar potential U(r) appearing in the 6-dimensional Schrödinger equation is and what kind of 'charge' it contains. Since the Green function of the Laplacian in 6 dimensions is r^{-4}, an educated guess is $U(r) \sim r^{-4}$. This guarantees the validity of Gauss' law and thus of charge conservation for the new tetron interaction. Furthermore, the structure of propagators is maintained, because the Fourier transform of r^{-4} in 6 dimensions is $\sim \vec{p}^{-2}$. Nevertheless, other choices are possible, e.g. $U(r) \sim r^{-2}$ or $\sim r^{-1}$, which has been used by some authors in their studies on the stability of hydrogen like atoms in higher dimensions[71, 72, 73]. Another possibility is $U(r) \sim \exp(-\Lambda_R r)/r^4$ in case of the above mentioned massive photon model.

In order to obtain a tetrahedral bound state, a Newtonian attraction instead of a Coulomb repulsion among the tetrons has to be assumed, i.e. $U(\vec{r}_i - \vec{r}_j) < 0$. This could either be handled *en face* or as discussed above by charge separation due to the Pauli principle.

In any case, inverse power potentials in higher dimensions pose the additional problem that they usually lead to rather weakly bound states (as compared e.g. to hydrogen in 3 spatial dimensions)[71].

If one does not like any of the above alternatives, one can assume instead the existence of some kind of central potential within each tetrahedron. In other words, in addition to tetrons there are yet unknown other components stabilizing the attraction among tetrahedrons within the hyper-crystal by an additional super-strong interaction, cf. discussion and footnote at the end of (2.2.9).

2.2.11 *Why do the 'molecules' formed by tetrons have a tetrahedral coordinate structure?*

In principle this question can be answered by analyzing the fundamental tetron interaction and showing that among the 'molecules' composed of n tetrons the ones with n=4 (or n=8) are energetically most favored. Subsections (2.5.32) and (2.5.10) about the growth of the hyper-crystal and (2.5.11) about the octonion origin of the interactions as well as (2.3.2) and (2.2.9) are recommended to read in this connection.

2.2.12 Why not use an internal molecular model instead of a crystal?

According to (2.5.3) Lorentz invariance can be approximately established for small enough lattice spacings (of order L_P). Nevertheless, some readers may find it difficult to imagine the world as an irregular elastic crystal, with every point in physical space occupied by an internal tetrahedron. So why not use a model, where the quarks and leptons are excitations of isolated tetrahedrons in an otherweise empty space? The molecules would extend into internal dimensions and have a frustrated anti-ferromagnetic structure as in fig. 1. Even an explanation of the SSB as a rearrangement within the molecules happening below a certain temperature is feasible. However, with such a picture one would run into all the known problems of classic composite models[30]. The strongest counter argument certainly is, how one and the same molecule can sometimes have a mass larger than 100 GeV and sometimes be as light as neutrinos.

2.2.13 Why not use a tetrahedral lattice in ordinary space, without any internal dimensions?

Since higher dimensions have never been observed experimentally, it is important to critically scrutinize their introduction. In this subsection I follow the idea that the tetrahedrons extend into ordinary space and only *mimic* the existence of internal symmetries by forming encapsulated, neutral and ordered tetrahedral systems a la figs. 1 and 2 in which mignons can be excited just as in the model with internal dimensions. As before, the extension of the tetrahedrons would have to be tiny, of order L_P.

The spin-$\frac{1}{2}$ nature of the mignons would be ensured by the spin-$\frac{1}{2}$ nature of the tetrons just as described in (2.4.2), while the 'internal' quantum numbers arise from the relative angular momentum of the tetrons inside the tetrahedron, i.e. from the Shubnikov symmetry. The whole system including gravity would look similar to a Cosserat continuum, where in addition to the Cosserat deformations describing gravity[94] there are interactions among the encapsulated tetron spins with corresponding spin wave excitations.

There are then several advantages of this approach as compared to the model with internal d.o.f.:

–there is no problem (2.5.33) with the dissipation of energy into internal dimensions.

–the question (2.5.10) why there is no growth of the crystal into the internal dimensions, does not arise.

–parity violation of the weak interaction would work analogous to optical activity in molecules, without the necessity to recur to an octonion structure, cf. (2.1.19) and (2.5.11).

However, there is also a drawback. It is related to the fact that a rotation in physical space would not only flip the ordinary spin of a mignon, but also transform the 'internal' coordinates.

This argument seems to kill the idea. One can only come around this conclusion, if one assumes rather strange behavior of the tetrahedrons, e.g. that they are immersed into a spacetime medium in such a way that they can be rotated independently of spatial rotations, i.e. have an extremely large relaxation time against outside rotations.

It may be noted that color has only been observed in singlet states which would not be sensitive to rotations anyhow. As for internal SU(2), weak isospin partners like the electron and its neutrino after the SSB are Shubnikov singlets, too. Weak isospin transitions in the tetron model are constructed rather indirectly as transition between excitations of $\vec{Q}_L$ and $\vec{Q}_R$, cf. the discussion in (2.4.16), and this construction can in principle be taken over to the scenario discussed in this subsection.

As before, the electroweak bosons would be related to $U(1) \times SU(2)$ gauge transformations referring to tetron number and the rotations of the tetron spin vectors, respectively.

2.2.14 How large is the internal extension R of one tetrahedron? How large is the average spacing $\langle r(t) \rangle$ between 2 tetrahedrons?

In most parts of the paper the following scenario is considered: R is smaller than r, and r is of the order of the Planck length. This is a reasonable assumption because R is the scale of tetrahedral 'molecule' formation, which took place at higher temperatures than the hyper-crystallization, i.e. before our universe was born. Furthermore, the single internal tetrahedrons are rigid and strongly bound objects, while the condensed system of tetrahedrons is elastic and its 'lattice spacing' r grows with cosmic time.
Since $\langle r(t) \rangle$ was identified with the Planck length L_P in (30), and is thus related to the uncertainties of quantum theory, the introduction of a length R smaller than L_P may look problematic. However, as discussed in (2.5.6) and in connection with (38), the quantum nature of matter arose when the hyper-crystal was formed during the big bang (via crystallization), i.e. *after* the time of tetrahedral molecule formation.

2.2.15 Why should the distance between 2 adjacent tetrahedrons be identified with the Planck scale?

One may ask whether there is the possibility, that r und R are much larger than the Planck length. In that case tetrons would have nothing to do with general relativity, and one could forget all reasoning about gravity and cosmology presented in this work. Only the particle physics sections would apply, and r and R would be scales like appear in technicolor models. However, the arguments in (2.5.6) indicate that the most consistent picture is obtained by choosing $r \approx L_P$.

2.2.16 Is the quantum theory of angular momentum used in (14)-(19) applicable in case $R < L_P$?

Yes, because the isomagnetic interactions responsible for the Standard Model physics take place within the hyper-crystal and at energies much smaller than Λ_P and Λ_r.

2.2.17 *The baryon asymmetry in the light of the tetron model*

Naively one could think that baryon asymmetry is due to a statistical fluctuation shortly after the big bang which makes the observable part of the universe 'baryonic' while other parts are predominantly 'antibaryonic'. However, the experts seem to agree that such an asymmetry would have long been washed out, e.g. by sphaleron effects from the non-perturbative sector of the SM. Instead they prefer to locate baryogenesis at temperatures near Λ_F[54]. In that kind of approach, it is then noted that the SM can only partly explain baryogenesis, mainly because of lack of 'enough' CP violation in the CKM sector. Additional new physics not too far above the Fermi scale is needed to remedy the situation.

What can be learned from the tetron model about this issue? First of all, within the tetron approach the SM is an effective theory, and it is not clear whether its equations are valid beyond the perturbative regime, i.e. whether the sphaleron argument is really true. Secondly, the low energy limit of the tetron model is a 2HDM model rather than the SM. Since the amount of CP violation is generally larger in 2HDM models than in the SM, baryogenesis can in principle be explained more easily[55].

2.2.18 A simple memo to understand the role of the permutation group in the ordering of quarks and leptons

Since the 24-dimensional representation (9) of G_4 is faithful, one can assign each of the 24 quark and lepton states to an element of G_4, in a similar way as suggested in table 2 of [5].

2.3 Questions about the local tetrahedral Structure and the Nature of the SSB

To obtain the correct mass spectrum for quarks and leptons, not only the internal geometry but also other features of the model have to fixed.

2.3.1 Are the tetrahedrons formed by 4 or 8 tetrons, i.e. how many vibrators are needed on each lattice site?

A priori one may consider several options:
(i) the 4-tetron option: naively, one would think that 4 tetrons can give rise to only $4 \times 3 = 12$ excitations of their isospin vectors $\vec{Q}$. However, there are 2 independent vibrators $\delta\vec{Q}_L$ and $\delta\vec{Q}_R$ for each tetron, and for 4 tetrons this gives the desired $2 \times 4 \times 3 = 24$ states of eq. (9). For this picture to work the ground state values $\langle\vec{Q}_L\rangle$ and $\langle\vec{Q}_R\rangle$ must contain antitetron contributions. This is discussed in (2.4.16) and actually brings option (i) close to option (ii).
(ii) the 8-tetron option: here $\vec{Q}_{Li}$ and $\vec{Q}_{Ri}$ are carried by 2 different particles approximately occupying the same tetrahedral site i, as depicted in figs. 4 and 5. As discussed in (2.4.16), it is most appropriate to assume this to be a particle and an antiparticle. Since these are sitting very close together, the system keeps its Shubnikov symmetry (7), and all arguments concerning the mignon spectrum remain unchanged.
The reason why the options (i) and (ii) are equivalent, can also be seen in the following way: according to (1) a tetron which transforms as 8 under SO(6,1) decomposes into a particle (1,2) and an antiparticle (2,1) under the Lorentz group SO(3,1), and these are just the d.o.f. required in fig. 4.
When assuming a hyper-crystal with an originally Galilean structure as in (2.5.14)ff, the tetron and antitetron correspond to a 4 and $\bar{4}$ of SU(4), cf. (73). Within the above philosophy this is merely restating the fact that one may consider the particle and antiparticle contributions to the isospin vectors separately.
(iii) in case of 8 tetrons there is another option which however will be abandoned for reasons discussed below: namely one could consider unpolarized isospin vectors $\vec{Q}_{1-8}$, which appear in pairs $\vec{Q}_i$ and $\vec{Q}_{i+1}$ on each tetrahedral site i=1-4, again to

be interpreted as tetrons i and i+1 to be very close to each other (with a tiny but non-vanishing internal distance d_8), i.e. more tightly bound than to the others. Mathematically it corresponds to a coordinate ground state symmetry $A_4 \times Z_2$ instead of S_4. Assuming ground state isospins $\langle \vec{Q}_i \rangle = \langle \vec{Q}_{i+1} \rangle$ on each site to be parallel, the isomagnetic ground state will again be symmetric under the Shubnikov group $A_4 + S(S_4 - A_4)$. It is then straightforward to see that a spectrum of the same form (9) as before is obtained from the vibrations of the $\vec{Q}_{1-8}$.

This scenario is clearly distinguished from those (i and ii) with chiral isospin vectors. First of all, the origin of isospin of mignons (quarks and leptons) is different in the 2 cases. According to (2.4.16) the transition $L \leftrightarrow R$ can be chosen to accompany an isospin transformation. In contrast in the case with $\vec{Q}_{1-8}$, the Z_2 exchange $(i \leftrightarrow i+1)$ has to provide for an isospin transition.

In addition, there is another, stronger disadvantage of using the approach with $\vec{Q}_{1-8}$, because the intimate connection between left-handed vibrators and the top-quark gets lost, cf. (19) and (2.4.21), i.e. the understanding why m_t is of the order of the SSB scale while all other quarks and leptons, in particular the b-quark, have much smaller masses, cf. (2.4.22).

2.3.2 Why is the tetrahedral 'molecule' so stable?

The shortcut answer is that the tetrahedron is the 'helium' of the tetron model.

One can use the Pauli principle to understand this point, cf. (2.2.9). While in the case of helium, two SU(2) spinors arrange antiparallel to form the most stable and abundant element in the universe, for tetrons the spinor representation 8 of SO(6,1) is relevant. The most stable configuration corresponds to a 'shell' filled with 8 tetrons all with *different* SO(6,1) quantum numbers, as depicted in fig. 5.

2.3.3 Can there be a tetron and an antitetron on one and the same tetrahedral site?

If we accept the idea that there are 2 independent fermions on each tetrahedral site (a tetron and an antitetron, cf. (2.3.1) and figs. 4 and 5), then strictly speaking they cannot exist on exactly the same spot. In other words, there must be a tiny nonvanishing distance d between them. For obvious reasons one should have $d \leq r, R$ where r and R were defined in fig.2 and identified as the Planck scale in sections 1 and (2.5.6).

2.3.4 Is the binding which makes up the Higgs field due to isospin or is it due to tetron coordinate interactions?

It is due to both. The Higgs particle (49) relies on the alignment of $U_\star$ isospinors and is therefore a natural part of the iso-magnetic interactions. On the other hand, the Higgs (as well as all other scalar and vector fields) is an excitation of the tetron-antitetron bonds in the crystal, and therefore controlled by the coordinate interactions.

2.3.5 What are the unbroken symmetries of the model?

The Shubnikov group G_4 and the electromagnetic $U(1)_Q$. The unbroken Shubnikov group has only singlet and triplet representations and leads exactly to the observed color and flavor spectrum of 3 families of quarks and leptons (9).

2.3.6 Can one calculate the Fermi scale, the Weinberg angle and W/Z and Higgs mass from first principles?

The origin of the Weinberg angle was discussed in (2.1.10). The Fermi scale and the Higgs mass arise from iso-magnetic exchange and pairing interactions, as discussed in (2.1.8) and (2.2.9). Therefore if one would know the exact form of the fundamental tetron interaction (2.2.10), these quantities would be calculable from 6-dimensional exchange integrals.

2.3.7 Did gauge bosons exist at temperatures above the Fermi scale?

Yes, they did. In the tetron model gauge bosons are particle-antiparticle correlations of crystally bound tetrons. They came into being shortly after the crystal was formed at temperature $\Lambda_r \approx \Lambda_P$ and made up for the bulk of particles in the 'radiation dominated epoque', cf. (2.1.12).

2.3.8 Did quarks and leptons exist at temperatures above the Fermi scale?

No, because from their very nature they require the existence of the iso-magnetically ordered state fig. 2. When the tetron gas cooled down and the hyper-crystal began to form, there was only the coordinate alignment of tetrahedrons but no alignment of isospin vectors. Therefore, at that stage, at temperatures above the Fermi scale, quarks and leptons did not exist, because they could not travel as quasi-particles through the isomagnetically disordered hyper-crystal. Only gravitons, phinons (2.3.26) and scalar and vector bosons (as excitations of the tetron-antitetron bonds) were present. This era is usually called radiation dominated, cf. (2.1.12).

2.3.9 Then why can quarks and leptons be produced at energies above the Fermi scale?

In collider experiments they can exist at energies much larger than 1 TeV because the alignment of isospins is stable much beyond Λ_F in the fully ordered hyper-crystal, i.e. in our universe. This is due to a collective hysteresis effect in which the crystal stabilizes itself by the concerted action of all aligned tetrahedrons to maintain the isomagnetic ordering. As a result, quarks and leptons can be produced and propagate normally, even in cases where energies locally exceed the critical temperature Λ_F.

2.3.10 Why is $\bar{\psi}$ involved in the order parameter and not $\psi^\dagger$, whereas the total 'iso-magnetization' $\vec{\Sigma}$ eq. (21) is defined just like in ordinary magnetic models?

Short answer to a long question: the ground state value of the total internal angular momentum vector $\vec{\Sigma} = \sum_{i=1}^{4} \psi_i^\dagger \vec{\tau} \psi_i$ vanishes due to the tetrahedral arrangement of isospin vectors within any internal tetrahedron fig. 1, and is therefore not useful as an order parameter in the present case.

Furthermore, the definition of $\vec{\Sigma}$ involves only tetrons of one single tetrahedron, whereas the particle physics SSB consists in the isospin alignment of two *neighboring* tetrahedrons.

2.3.11 Is the vev $\langle \bar{U}U + \bar{D}D \rangle$ or $\langle \bar{U}U \rangle$ or what?

The vev is given by $\langle \bar{U}_\star U_\star \rangle$ in accordance with (49) where $U_\star$ is the 'radial' iso-spinor introduced in (11) corresponding to an isospin vector pointing outward as in fig. 1. Such a vev is precisely what is needed to stabilize the alignment of isospins in fig. 2.

2.3.12 Why not use some other order parameter which is closer related to the isomagnetic alignment than the Higgs vev?

First of all it must be noted that according to (2.3.11) the Higgs vev in the tetron model has a lot to do with the isomagnetic alignment.
Secondly, it is true that in general for a given phase transition different order parameters are possible. In the present case one may for example consider 2 neighboring tetrahedrons A and B with tetrons ψ_{Ai} and $\bar{\psi}_{Bi}$, and $i = 1, 2, 3, 4$ counting the tetrahedral sites. Unfortunately, the aligned tetrahedral 'star' configuration of these 2 tetrahedrons not only implies $\langle \vec{\Sigma}_{A,B} \rangle = 0$ for each tetrahedron separately, but also $\langle \vec{\Xi} \rangle = 0$, where $\vec{\Xi}$ is defined as $\vec{\Xi} = \sum_i \bar{\psi}_{Bi} \vec{\tau} \psi_{Ai}$. This is because one can show that all vectors $\bar{\psi}_{Bi} \vec{\tau} \psi_{Ai}$ of 2 adjacent tetrahedrons are parallel, iff the corresponding isospin vectors $\vec{Q}_{Ai,Bi}$ are. Thus $\vec{\Xi}$ is not a useful order parameter either.

2.3.13 So what is the microscopic interpretation of the Higgs particle?

As discussed in (2.1.2) the Higgs is neither fundamental nor a bound state of mignons, but an excitation of tetron-antitetron pairs which are themselves bound within the hyper-crystal.

2.3.14 Should one consider separate tetrahedrons for anti-tetrons, with isospin vectors pointing inward?

This question may seem justified, because anti-fermions usually react to magnetic forces with an opposite sign. However, using isospin vectors (2) one is treating the problem in a covariant way. As can be seen in (3), the isospin vectors contain particle as well as antiparticle contributions, and the antiparticle contributions have a negative sign. More details are given in (2.4.16), where a $\langle\vec{Q}_R\rangle$ pointing inwards will be defined in terms of a charge conjugate tetron field. Sections (2.3.3) and (2.2.8) may also be consulted in connection with this question.

2.3.15 Is there a difference between the SM SSB and a ferromagnet, apart from the fact that the SM SSB takes place in internal space? Is the symmetry breaking in the tetron model really spontaneous?

Both cases (ferromagnet and tetron structure) are similar in that at high energies / temperatures the directions of (iso)spins are oriented randomly with an associated SU(2) Heisenberg symmetry, and this defines the symmetric state.
In an uni-axial ferromagnet an accidental magnetization axis usually appears spontaneously, based on a thermodynamic potential

$$V_{FM}(\vec{M}) = -a\vec{M}^2 + b\vec{M}^4 \tag{85}$$

where $\vec{M}$ is the total magnetization and the minimum of the potential is at $\langle\vec{M}^2\rangle = a/2b$.
In the case at hand the crystallization process at scale Λ_r is accompanied by a coordinate alignment of all tetrahedrons, i.e. there is a spontaneous selection of one global internal coordinate system for all tetrahedrons. This coordinate alignment, however, happens prior to the alignment of isospins and has not much to do with it. When the temperature decreases towards Λ_F, the anti-ferromagnetic tetrahedral 'star' configurations fig. 1 appear where the isospin vectors within one tetrahedron avoid each other as far as possible. Note there is an infinite SU(2) symmetric set of such 'star' configurations just as in a ferromagnet there is an infinite set of possible magnetized states corresponding to all possible magnetization directions in R^3. The difference as compared to a ferromagnet is that not only the stars over one tetrahedron have to be included but also those over all the other tetrahedrons over Minkowski space, with their independent SU(2) degeneracies, and this makes the problem a local gauge symmetric one.
The SSB consists in the simultaneous selection of one among all the possible star configurations over all Minkowski base points – namely the one with $\langle\Phi\rangle \sim (0, 1)$.

According to (50) this corresponds to a vev for the $U_\star$ isospinor component, i.e. the one with an isospin vector pointing outwards in the radial direction. The choice of $(0,1)$ - and of U - is notational convention and, in the framework of the gauge theory, corresponds to choosing a certain gauge (the so called unitary gauge). There is again a similarity to the situation in a ferromagnet where some axis is selected by the spontaneous magnetization, and the coordinate system is then 'gauged' in such a way that this axis is called the z-axis $\sim (0,0,1)$.

One may ask what role the coordinate alignment of tetrahedrons at the crystallization point $\Lambda_r \sim \Lambda_P$ plays in this game, because it seems plausible that the state, where the tetrahedrons of coordinate and isospin both point in the same radial directions, is energetically preferred. (This geometry is in fact depicted in figures 1 and 2, and the conditions, under which this happens, will be called scenario Q.)

A similar situation is sometimes encountered in ordinary uni-axial ferromagnets in cases when the coordinate backbone of the crystal prefers one specific magnetization direction, so that the ferromagnetic phase transition is not really spontaneous. This effect can be modeled by adding a tiny explicit symmetry breaking contribution to the potential (85) by hand. At high temperatures due to thermal fluctuations this structural / coordinate effect is not important. But it becomes relevant near the Curie temperature where it fixes the magnetization direction.

In the present case, however, this possibility needs no consideration. The reason is that Λ_P is so large as compared to Λ_F, that the granular internal coordinate structure is not noticed by the isospin vectors (nor by any human experiment). From the perspective of the isospin vectors it looks as if they are sitting on an internal coordinate structure which is rotationally invariant. They only feel the anti-ferromagnetic aversion towards their 3 fellows within one tetrahedron.

As a consequence of these considerations all 'star' configurations are energetically equivalent, and the symmetry breaking is spontaneous.

One can even go as far to say that there could be no coordinate alignment among the tetrahedrons at all. This would be in accord with the idea that the inter-tetrahedral coordinate (=gravitational) interactions are elastic and therefore can give rise to any relative coordinate orientation between neighboring tetrahedrons. The alignment of isospins could live with this option, because the only thing which matters for the SM SSB is that the 4 isospin vectors point in radial direction and build up the isomagnetically aligned multi-tetrahedral configuration, irrespective of what the coordinates of the tetrons are.

Further it may be noted that the question whether the SB is really spontaneous, is much easier to answer in what was called scenario C in section 1. In that case the electroweak phase transition III consists in a *simultaneous* alignment of coordinate and isospin vectors as shown in fig. 2. In other words the tetrahedral star configuration consists in a coordinate and an isospin star where the coordinate and the associated isospin vector always point into the same radial directions. The transition to the ordered state is then necessarily spontaneous because all rotated (coordinate + isospins) tetrahedral star configurations are energetically equivalent.

2.3.16 Is the electroweak phase transition first or second order?

Lattice calculations in the SM with one Higgs doublet give no definite answer to this question. The transition seems to be second order for $m_H \lesssim 120$GeV, while for $m_H \gtrsim 130$GeV one obtains a first order transition[48]. In the intermediate region it may be a cross-over[49]. By contrast, in 2HDM models the situation is clearer, because the electroweak phase transition turns out to be first order[50] and terms of order $\sim \Phi^3 T$ arise in the temperature dependent Higgs potential.

Since the 2HDM model (2.1.15) arises as the low energy approximation of the tetron model, one may be content with this result, in particular because a first order transition is preferable for phenomenological reasons, cf. (2.3.24) and [49]. However, it should also be possible to directly determine the nature of the phase transition in the tetron model without recurring to an effective theory. To achieve this aim, a calculation in the framework specified in (2.3.15) and by figs. 1 and 2 should be performed. If one looks at fig. 2, such an isomagnetic alignment is normally expected to be of second order. However, first order magnetic transitions are also known, in particular in connection with deformable structures[51].

2.3.17 What are the relevant scales in the model?

Naively, there are only 2 scales: the Fermi scale Λ_F and the Planck scale Λ_P. The binding and crystallization process with coordinate alignment of tetrahedrons but erratic directions of isospin vectors corresponds to scales Λ_r and Λ_R both or order Λ_P, while Λ_F is the scale where the isospin vectors align.

On a more sophisticated level some other scales might seem reasonable:

–the formation of tetrahedral 'molecules' from a tetron gas in 6 dimensions and of the hyper-crystal from the 'molecules' may happen at different scales Λ_R and $\Lambda_r(t=0)$ where $t=0$ corresponds to big bang time when the hyper-crystal was formed. Furthermore, due to the elasticity of the DMESC, the crystal binding energy at that point was different than it is now: $\Lambda_r(0) \gg \Lambda_r(now)$. This corresponds to the fact that the average distance between neighboring tetrahedrons in the hyper-crystal has grown since the big bang: $\langle r(0)\rangle \ll \langle r(now)\rangle$ where according to (30) we should identify $\langle r(now)\rangle$ with the present value of the Planck length. In other words one has $\Lambda_r(now) = \Lambda_P$ and thus a hierarchy of scales

$$\Lambda_R \geq \Lambda_r(0) \gg \Lambda_r(now) \approx \Lambda_P \gg \Lambda_F \tag{86}$$

–it is conceivable that the crystallization and the coordinate alignment of tetrahedrons do not happen at the same temperature, i.e. there is Λ_P for the crystallization and another scale Λ_A for the coordinate alignment fulfilling $\Lambda_P > \Lambda_A > \Lambda_F$. However, this would imply another phase transition in the early universe for which there is no indication. As explained in section 1 and (2.3.15) it is best to assume that either $\Lambda_A \approx \Lambda_P$ or $\Lambda_A \approx \Lambda_F$ or that there is actually no coordinate alignment at all, only isospin alignment.

–there may be a separate scale Λ_d for the pairing interaction of 2 tetrons on one tetrahedral site, as discussed in (2.3.3), (2.4.16) and (2.3.1).

2.3.18 What is the geometrical meaning of these scales?

According to fig. 2, r and R can be interpreted as lengths of certain tetron bonds within the discrete structure fig. 2. Namely, R is the fixed bond length of 2 tetrons within a tetrahedron, while r is the variable (elastic) bond length of 2 tetrahedrons in the hyper-crystal. By contrast, the Fermi scale measures the 'length' of isospin vectors of the ground state fig. 1.

2.3.19 Why are the scales Λ_R and Λ_r so much larger than the quark and lepton masses? Why is gravity so weak as compared to the isomagnetic interactions? Why is the Planck scale so large in comparison to the Fermi scale?

These questions are variation of why gravity is so weak as compared to QED and the other particle physics interactions.

An important point to note is that the weakness of the gravitational forces and the strong coordinate forces among tetrons and tetrahedrons are related. The latter express themselves in the close packing and the associated extreme stiffness of the tetrahedrons in the hyper-crystal, which was deducted from the smallness of Newton's constant in the discussion after (35). In other words, it is the strong binding of tetrons together with the close-meshed packing of tetrahedrons which makes the hyper-crystal react to mignon excitations with only tiny distortions. In the Newtonian approximation, for example, writing g_{00} in (28) as $g_{00} = -1 - 2\phi$, the gravitational potential is $\phi \approx 10^{-39}$ at the surface of a proton and not larger than 10^{-6} at the surface of the sun.

Not knowing the precise nature (2.2.10) of the fundamental interaction among tetrons, one can only say that gravity is some kind of remnant elastic interaction among the mignons, while the tetrons themselves are strongly bound to the hyper-crystal structure.

On the other hand, the energies $\sim \Lambda_F$ involved in the isospin alignment are much smaller than the energies $\Lambda_r \approx \Lambda_P$ needed for the coordinate formation of the crystal. R and r are the length scales at which the iso-magnetic exchange integrals J have to be calculated, i.e. they are on the *abscissa* of the Bethe-Slater curve fig. 3, while the values of J are drawn on the *ordinate* of fig. 3 and always $\leq \Lambda_F$. The reason why one can have $J \ll \Lambda_P$ is explained in (2.3.20).

2.3.20 Why are the exchange energies $J \sim O(GeV)$ in (14) so much smaller than the tetron binding energies $\sim O(E_P)$?

Exchange couplings/integrals within a tetrahedron generically are of the form

$$J = \int d^3y_1 d^3y_2 f_1(\vec{y}_1) f_2(\vec{y}_2) V(\vec{y}_1 - \vec{y}_2) f_1(\vec{y}_2) f_2(\vec{y}_1) \tag{87}$$

where y_1 and y_2 denote internal coordinates and f_i the corresponding tetron wave functions. The value of J is the smaller, the smaller the overlap of the wave functions at different sites is. If the wave functions are very strongly concentrated, it is no problem to have $J \ll E_P$. Furthermore, as discussed in (2.2.10), there may be super-strong coordinate forces among tetrons which constitute E_P but do not touch the isomagnetic interactions $\sim J$ among isospin vectors.

2.3.21 Why is the inter-tetrahedral exchange coupling so much larger than the inner-tetrahedral one?

In order to establish the SM SSB, the inter-tetrahedral exchange coupling J_{inter} must be chosen to be as in (20) corresponding to a numerical value $J_{inter} \approx 7.8$ GeV. In order to understand why the inner-tetrahedral couplings (18) come out much smaller than this value, one has to note that J_{inter} as defined in (19) contains an implicit factor which counts the number of nearest neighbors of a given tetrahedron. Assuming that on the average within the elastic hyper-crystal there are always at least 6 neighboring tetrahedrons (and the next-to-nearest neighbors probably also count), this will naturally enlarge J_{inter} by an order of magnitude. In other words, the inter-tetrahedral isomagnetic forces are not per se unnaturally large as compared to the 'inner' ones. Instead, originally, all interactions among 2 isospin vectors are the same generic order of magnitude of about 1 GeV.

2.3.22 Do GUT theories play any role?

No. It is difficult to imagine why there should be other SSBs in the tetron model besides those described in section 1 and (2.1.11)-(2.1.12). I see no reason for the proliferated Higgs sector characteristic for most GUT models, cf. (2.5.34) and (2.5.35).

2.3.23 Is there a unification of electroweak and strong couplings?

No. In the tetron model the QCD forces are on a less fundamental footing than the electroweak interactions. For further details see (2.5.34) and (2.3.29)-(2.3.31). The existence of a unification scale for electromagnetism and the weak interactions has been discussed in (2.1.11).

2.3.24 What about domain walls?

Phase transitions in physics are usually associated with the formation of domains. However, domain structures have never been observed in cosmology.

In the tetron model, cosmological domains either appear as separate universes, which according to (2.5.38) were created in R^6 via a condensation process similar to the one that has led to our own universe, or, if they are part of our own universe, they have long disappeared beyond our event horizon. To understand this in detail, one should first realize that one has to distinguish (i) domains arising at crystallization time (coordinate alignment of tetrahedrons) from (ii) those arising at the electroweak phase transition (tetrahedral alignment of isospins).

(i) In an ordinary crystal, one expects the appearance of domains with different values of the order parameter arising from concurrent nucleations of crystal germs in different points of space. In principle, this is also true for the R^{6+1} space under consideration, and one would expect domains, where the ordered coordinate tetrahedrons are rotated by some angle as compared to those shown in fig. 2. Tetron model adapted cosmic inflation (2.5.23) gives an argument, why the corresponding domain walls have moved so far away from us as not to be observable. Another, more absolute reason, why domains do not arise in the case at hand, has to do with the fact that the hyper-crystal grows into and occupies only a quasi 3-dimensional subspace of R^6. Therefore it intersects with other hyper-crystals from concurrent nucleation points, which grow into other 3-dimensional subspaces of R^6, in at most 1 point (because the intersection of 2 almost flat 3-dimensional submanifolds in R^6 in general is just 1 point). This means the result of the other nucleations will be different hyper-crystals, i.e. they correspond to different worlds whose intersection with our universe consists of at most one point. At this point there will be a defect within the hyper-crystal structure, cf. (2.5.38).

Another possibility to avoid domain walls at the crystallization temperature is scenario C as described in section 1 where one can do without internal coordinate order.

(ii) In an ordinary ferromagnet, one expects the appearance of 'Weiss domains' with different ordering directions of spin vectors. In the case of isospin vectors such domains can in principle exist, too, and would differ by a global rotation of the isomagnetic tetrahedral 'star' configuration figs. 1 and 2. However, as discussed in

(2.3.16), the phase transition is first order, i.e. associated with a sudden release of latent heat, which blows up the micro-elastic continuum, i.e. triggers an inflationary process which in turn shifts domain walls outside the visible part of the cosmos.
Note that models with inflation near the electroweak scale have been discussed extensively in the literature[74, 75, 76, 77].
Note added: in (2.3.15) arguments have been given to show that the electroweak symmetry breaking (alignment of isospins) is truely spontaneous. It should be noted, however, that the non-existence of domain walls is much easier to understand in what was called scenario Q in (2.3.15). The point is that in scenario Q the electroweak domains always coincide with the coordinate domains. There is then only one electroweak domain in the whole hyper-crystal because according to the discussion under item (i) different coordinate domains correspond to different universes (hyper-crystals).

2.3.25 Why are quarks and leptons the same everywhere in the universe? Why does one electron look exactly like the other?

A question which has bothered me already before I invented the tetron model. The answer: because the hyper-crystal is the same everywhere. It is built from tetrons in the same way and according to the same laws everywhere, and its mignon excitations are therefore the same everywhere in the universe.

2.3.26 Are there excitations of the hyper-crystal besides the known quarks, leptons and scalar and vector bosons?

Yes. An incomplete listing includes:
–phinons. They are the analogs of phonons in a solid and have been described under more general circumstances in [3]. In the case at hand there are 12 phinon states, that can be classified according to representations of the permutation group S_4. They travel as quasi-particles through the hyper-crystal in the same way as mignons do. Phinon masses are expected to be much larger than mignon (quark/lepton) masses. While the mignon spectrum is lying at and below Λ_F, the phinon spectrum is concentrated towards the crystallization energy Λ_P. Note, this is not a very

accurate characterization, in view of the fact that neutrino masses are so tiny with respect to the Fermi scale. Note further, phinons are internal coordinate vibrations, and thus have to be distinguished from gravitational waves. An interesting question is whether mignon-phinon scattering is possible.
–isospin density waves: they are to be distinguished from phinons and from mignons. The vibrators in this case are similar to the isospin vectors (4), however without the factor of $\vec{\tau}$, i.e. given by $\psi^\dagger(1 \pm \gamma_5)\psi$. As far as I can judge these excitations correspond to a fourth family of fermions, i.e. a lepton-like and a quark-like isospin doublet, probably higher in mass, because they are not related to the other families by the Z_3 family quantum number inherent in the Shubnikov group $A_4 + S(S_4 - A_4)$. In particular, the fourth 'neutrino' is expected to be much heavier than the known neutrinos, because its mass is not suppressed by internal angular momentum conservation, cf. (2.4.12).
–one should mention scalar fields other than the standard Higgs boson. They are the components of the second Higgs doublet (58), the lightest among them being the most promising candidate for dark matter, cf. (2.5.36).

2.3.27 What about vibrations of $\bar{\psi}\gamma_\mu\vec{\tau}\psi$, $\bar{\psi}\sigma_{\mu\nu}\vec{\tau}\psi$ etc?

These are other examples of higher mass excitations of the hyper-crystal.

2.3.28 Could quark and leptons be phinons?

or in other words: what is the advantage of using mignons with Shubnikov symmetry $A_4+S(S_4-A_4)$ over phinon excitations with symmetry group $A_4 \times Z_2$ as advocated in [3]?
The answer is that many of the attractive features of mignons are absent, like the explanation of the Higgs mechanism, of why $m_t \gg m_b$, of tiny neutrino masses etc.

2.3.29 *Is it possible to understand the dynamics of the strong interactions from within the tetron approach?*

I don't have a final answer to this question. From its very construction the tetron model is concerned mainly with the symmetries and interactions of electroweak physics. The colors of quarks arise merely as a byproduct, because they are interpreted as the 3 d.o.f. of a Shubnikov triplet representation. It is therefore clear that QCD with a color gauge group and SU(3) color triplets does not directly arise in the tetron model.

In section 1 it was argued that the phase transitions of a 6+1 dimensional space-time filled with a condensing tetron gas supplies all relevant physics for the early universe and it may even account to understand the forces of gravity. As for the latter the suggestion is that it arises from elastic forces between tetrahedrons which are the remnants of the fundamental 6-dimensional tetron-tetron coordinate interactions (2.2.10) and induce curvature and/or torsion effects on Minkowski space. One would like to interpret the strong interactions in a similar spirit, namely starting with the paradigm that there are no other interactions in the universe besides the ones among tetrons.

Gluons cannot be part of the isomagnetic geometry, because the SU(2)×U(1) bundle connection allows only for the 4 electroweak gauge bosons. One may note, however, that a Shubnikov invariant mignon-mignon interaction is to be expected among the triplets, transforming as

$$T \times T = A + A' + A'' + 2T \tag{88}$$

As shown in [5] this structure can be embedded into a $\mathrm{SU}_c(3)$ algebra where the color indices correspond to the 3 d.o.f. of the triplet representations in (9). Although as yet there is no proof that gluons and QCD gauge interactions can really arise from this line of reasoning, some hints will be given in (2.3.31).

2.3.30 Is color a part of isospin?

In the tetron model the color triplets of quarks transform according 3-dimensional representations of the Shubnikov group (5), and this symmetry group is defined in terms of transformations within the same internal space as weak isospin. This could lead one to suspect that color in some sense is part of isospin. However, this is not the case. While the Shubnikov group is unbroken to the lowest energies, isospin symmetry corresponds to the free rotations of isovectors on each tetron site separately and is completely broken in the ordered state.

2.3.31 Is there a connection between the QCD vacuum and the electroweak condensate?

Chiral symmetry breaking of the strong interactions is the appearance of a non-vanishing quark condensate

$$\langle \bar{u}u \rangle \approx \langle \bar{d}d \rangle \approx \langle \bar{s}s \rangle \approx -(0.25GeV)^3 \tag{89}$$

which breaks $SU(2)_L \times SU(2)_R$ to the diagonal isospin $SU(2)_V$ group.
One difference as compared to the electroweak case is that these groups are global, not local symmetries. Furthermore, according to (49) the Higgs condensate is related to $U_\star$, i.e. the tetrahedral star configuration pointing outwards as in fig. 1. In contrast, the quark condensates are singlets w.r.t. the relevant quantum number (in this case color, while for the Higgs condensate it is isospin). In other words

$$\langle \bar{q}q \rangle = \langle \bar{q}_1 q_1 + \bar{q}_2 q_2 + \bar{q}_3 q_3 \rangle \tag{90}$$

does not correspond to a preferred direction or orientation in color space.
The quark condensates essentially are a measure of the nucleon masses, and in the QCD framework this role is taken over by the QCD Lambda parameter. Thus it appears that the nucleons get their mass not from the Higgs mechanism (mignon oscillations) but from the condensates (89). However, as discussed below, in the tetron model there may be a loose connection between the 2 mechanisms.
It is interesting to note that the strange condensate has about the same magnitude as up and down condensates, although its mignon mass (17) is much larger. This is an indication that chiral symmetry breaking is due to a *mignon triplet condensation* at temperatures of about 1 GeV.
This phase transition is also responsible for the confinement of quarks[8]. The point is that each (Shubnikov) triplet state by definition defines a direction in internal space. This is nothing else than the oscillating isospin eigenvector which according to (14) interacts via a Heisenberg Hamiltonian with other isospin vectors. Therefore, it is no accident that Λ_{QCD} and the values of the inner-tetrahedral isospin exchange couplings (15)-(17) [or the masses of the second family mignons] are of the same

[8] I have played with the idea that confinement arises from the stiffness of the DMESC but did not really find a good argument for that.

order of magnitude!

If one continues on this road, one arrives at a picture which leads to a proof of quark confinement. However, this picture differs somewhat from the usual QCD folklore. Namely, one can interpret the (Shubnikov) triplet state describing a quark as a vector in internal space, which tends to destroy the frustrated local isomagnetic configuration, for example by inverting one of the vectors in fig. 1.

Let me call this destroyed frustrated configuration a DFC. Since the neighbouring frustrated tetrahedrons are still aligned to each other as drawn in fig. 2, this will cost the system a large amount of energy of order Λ_F. If in addition an anti-quark is put on the hyper-crystal at some distance to the quark, this will again cost energy of order Λ_F.

In such a situation the system will prefer a state where all the tetrahedrons on the line between the quark and the anti-quark are in the DFC configuration, because in this case the alignment along the line is preserved, i.e. energy of order Λ_F saved.

If now one tries to increase the distance between the quark and the anti-quark, an amount of energy $\Lambda_{QCD} \ll \Lambda_F$ has to be spend for each additional tetrahedron between the quark and the anti-quark put in DFC configuration. This increase in energy for each step of distance-increase corresponds to a linear, i.e. confining potential.

2.4 Questions about the Quark and Lepton Mass Spectrum and the CKM and PMNS mixing Matrices

According to the discussion in section 1, quark and lepton masses can be calculated as excitation frequencies of mignons. The frequencies are obtained quite naturally by considering isomagnetic interactions among the tetrons of one or two tetrahedrons only. Analytic expressions for masses in terms of internal Heisenberg and Dzyaloshinskii-Moriya exchange couplings have been derived in [2] and reviewed in section 1.

2.4.1 Using exchange couplings instead of Yukawas – isn't one replacing one set of unknown parameters by another set and one effective theory (the Standard Model) by another one (the internal Heisenberg model)?

No, because the internal couplings in (14) and (19) can in principle be calculated from first principles as exchange integrals over internal space, just as in ordinary magnetism the exchange couplings of the Heisenberg model are in principle calculable from exchange integrals of electronic wave functions over physical space. What one needs to know is the underlying 6+1 dimensional dynamics of tetron interactions, cf. (2.1.19), (2.2.10) and (2.2.9).

2.4.2 Why are mignons spin-$\frac{1}{2}$ particles?

Since they are constructed from excitations of 'bosonic' isospin operators (2), one could be led to believe that they are bosons, just like magnons in ordinary ferromagnets are bosonic quasi-particles.

However, it is important to distinguish the behavior in internal space from that in Minkowski space. While mignons transform as Shubnikov singlets A and triplets T (i.e. *not* as projective representations) w.r.t. internal space, it is not hard to see that they are Dirac fermions w.r.t. Minkowski space.

The point to note is that mignons are not bound states of tetrons but eigenmodes of their excitations. As such they are not tensor products but linear combinations of (fluctuations of) tetron fields ψ_a^α (where $a = 1, 2$ is the internal and $\alpha = 0, 1, 2, 3$ the Dirac index of the tetron). Since each mignon is a vibration of *one* isospin eigenmode, one concludes that it must be a Dirac particle w.r.t. the Minkowski base space. Using the 'bosonic' isospin vectors $\vec{Q} = \psi^\dagger \vec{\tau} \psi$ is merely a tool to separate the isospin triplet vibrations from singlet density fluctuations of $\psi^\dagger \psi$, cf. (2.3.26). Looked at from 'below', i.e. from the Minkowski base space, the tetron excitations are Dirac fermions. If such an excitation travels through the crystal as a quasi-particle, it can be either L or R, particle or anti-particle.

2.4.3 Why are there exactly the quark and lepton states of the 3 generations?

The unbrocken (Shubnikov) symmetry group has only singlet and triplet representations, and the 8 independent isospin vectors on a tetrahedron lead exactly to the flavor spectrum of 24 quarks and leptons as given in (9).

2.4.4 Are there other ground states than the tetrahedral one, which yield the appropriate quark and lepton spectrum (9)?

No. I scanned other geometries with 8 iso-magnetic vibrators and found that for most systems mignons appear in 2-dimensional representations[8, 9], not useful for the q/l spectrum of particle physics. This applies in particular to the configuration (m2) described in (2.5.1).

2.4.5 Should one really use covariant isospin vectors (3) containing both particle and anti-particle contributions as vibrators?

Yes, one should. It is important for the vanishing electric charge of the hyper-crystal, for the formation of gauge bosons out of tetron-antitetron pairs, and in general to maintain relativistic covariance as seen in (7) and figs. 4 and 5. Technically it is important for the mass calculations corresponding to mignon mass terms $\langle 0|T(\bar{q}q)|0\rangle$.

2.4.6 Why not use only $\vec{Q} = \vec{Q}_L + \vec{Q}_R$ as vibrators instead of $\vec{Q}_L$ and $\vec{Q}_R$ separately?

In order to obtain the 3 families of quarks and leptons one would have to consider systems with 8 instead of 4 tetrons. This possibility has been discussed in (2.3.1). The main counterargument is that the $SU(2)_L$ vibrators $\vec{Q}_L$ are needed to account for the observed parity violation of the weak interaction.

2.4.7 Is it okay to start the mass calculations using chiral $SU(2)_L \times SU(2)_R$ symmetry quantities $\vec{S} = \vec{Q}_L$ and $\vec{T} = \vec{Q}_R$?

Yes. This is just the way the SM works. Non-zero fermion masses are developed via SSB starting from a massless, i.e. $SU(2)_L \times SU(2)_R$ chirally symmetric theory. The role of custodial SU(2) in the tetron model is discussed in (2.4.17).

2.4.8 Can the simple Heisenberg interaction (14) really explain the full q/l mass spectrum with its extreme hierarchies?

No. As shown in [2] the masses of some of the fermions get contributions from other physical sources, namely

–the top mass is dominated by a contribution of order Λ_F which stems from the symmetry breaking *inter*-tetrahedral interactions (19). Physically it arises because the top quark corresponds to the 3 eigenmodes which 'disturb' the global ground state in the strongest possible way. This disturbance is also responsible for the hierarchy observed in the CKM matrix elements.

–only strange-, charm- and muon-mass are dominated by anti-ferromagnetic exchange couplings within one tetrahedron, and thus can be obtained from the *inner*-tetrahedral exchange couplings (14) alone.

–down-quark, up-quark and electron are left massless by the Heisenberg and DM interactions (14) and (19). They get their relatively small masses from energetically favored torsion contributions[2].

–neutrino masses are protected by internal angular momentum conservation, i.e. by the internal rotational symmetry, cf. (2.4.12). The way how they acquire their tiny mass values is described in [2].

2.4.9 Is there a mismatch between the internal Heisenberg interaction (14) for a single tetrahedron and the DM interaction (19) involving neighboring tetrahedrons?

This is a technical question concerning the calculation of quark and lepton masses presented in [2], and the answer is that technically one can treat the isospin vectors from the neighbors as if they were vectors of the original tetrahedron.

Denoting 2 neighboring tetrahedrons as primed and unprimed and starting with interactions of the form $\sim \vec{S}_i \vec{S}'_j$, one would expect the doubling of the number of eigenmodes. However, due to the symmetry between the 2 tetrahedons, i.e. between the $\vec{S}$ and $\vec{S}'$, the modes arrange in pairs of identical energy. In other words, the doubling of modes is a trivial one.

To understand this in more detail consider the first term (Heisenberg contribution) in the inter-tetrahedral interaction (19) and assume that the inner-tetrahedral distances are much smaller than the inter-tetrahedral ones. This is in accord with arguments given in (2.3.17), that after the long time of cosmic expansion one expects $r \gg R$ in fig. 2. This assumption implies that the couplings J_{inter} of an $\vec{S}_i$ to all isospin vectors in the neighboring tetrahedron are identical (as anticipated in (19)) and one obtains

$$\frac{d\vec{S}_i}{dt} = J_{inter} \, [\vec{S}_i^0 \times \sum_{j=1}^{4} \vec{S}'_j] \tag{91}$$

where the superscript 0 denotes ground state values. A second set of equations is obtained for $d\vec{S}'_i/dt$ by exchanging the role of the 2 tetrahedrons, i.e. the primed and unprimed quantities. An obvious set of solutions to these equations fulfills $\vec{S}_i = \vec{S}'_i$, i.e. the vibrations are completely in step, and one obtains the trivial doubling of modes mentioned above.

As for the DM part (second term) in (19) a similar argument can be given. Following the results in [2] this leads to the conclusion that the top quark receives the overwhelming mass contribution from the inter-tetrahedral interaction (19).

It is interesting to note that in a cosmological scenario where one would have $r \ll R$ the mignon mass spectrum would turn out quite different because in that case it is natural to assume that the coupling of $\vec{S}_i$ to $\vec{S}'_i$ is much larger than to the other $\vec{S}'_j$.

2.4.10 How are the CKM and PMNS matrix elements obtained in the tetron model?

In the SM the possibility of inter-family mixing arises because interaction eigenstates are distinguished from mass eigenstates. In the tetron model there are the original modes which correspond to small vibrations of the isospin vectors, and on the other hand the eigenmodes of the vibrations whose eigenfrequencies correspond to the

quark and lepton masses. The original modes $\delta\vec{Q}_{iL/R}$ are directly related to the SM interaction eigenstates because $\vec{Q}_L$ according to (4) defines the isospin 'charge' of the left handed SU(2) current appearing in the Glashow theory.
Therefore, in order to obtain the CKM and PMNS matrices in the tetron model, one can make use of the results of the diagonalization procedure from the $\delta\vec{Q}_{iL/R}$ to the mass/frequency eigenmodes. Since the mixing in the right-handed sector is not observable, only projections onto the $\delta\vec{Q}_{iL}$ need to be treated.

2.4.11 Why are interfamily interactions suppressed in the tetron model?

It is a prevalent problem in many composite models, to explain why - apart from CKM mixing effects - transitions between fermions of different families do not exist, e.g. why $\mu \to e\gamma$ is forbidden.
In the tetron model this fact can be understood from the symmetry of states. Since the gauge bosons W, Z and γ (2.1.4) behave trivially under the Shubnikov group, the same must be true for the fermion-antifermion conglomerates, which get pair produced from them.
Analyzing products of representations of (94), it turns out that all inter-family conglomerates behave non-trivially under Shubnikov transformations. For example, a combination of a μ^-- and an e^+-mignon transforms as a non-trivial singlet and therefore cannot become a photon.
In order to actually carry out this analysis, the exposition in (2.4.18) is useful. It may then be noted that the antiparticle of A'_{1+is} transforms as A''_{1-is}.

2.4.12 How can the smallness of neutrino masses be understood?

Neutrinos are interpreted as internal Goldstone particles of the breaking of the tetronic isospin by the formation of the discrete structure fig. 1. The associated conserved Noether charge is given by the total internal angular momentum $\vec{\Sigma}$ defined in (21) which implies the existence of 3 zero-frequency modes. This is analogous to how magnons are interpreted as Goldstone modes in ordinary magnetism, except that here one is considering the physics of spin waves in the internal spaces.
While in ordinary ferromagnets after magnetization a U(1) symmetry about the z-

axis survives, in the given frustrated configuration fig. 1 all three SO(3) generators give rise to Goldstone bosons, to be identified as the internal magnons corresponding to the 3 neutrino species.

2.4.13 Neutrinos are fermions. How can they be Goldstone modes?

One has to distinguish the dynamics in internal from that in physical space. In physical space the neutrinos are fermions, but they are (Goldstone) bosons w.r.t. the dynamics in internal space, because in internal space they are described by (bosonic) excitations of the total internal angular momentum $\vec{\Sigma}$ defined in (21) which is the conserved quantity associated with the internal rotational symmetry. As discussed before, none of the representations in (9) are projective representations of the Shubnikov group, so all quarks and leptons are 'bosonic' w.r.t. the internal dynamics, cf. (2.4.2) and (2.4.18).

2.4.14 How do neutrinos obtain their tiny non-zero masses?

As Goldstone modes neutrinos are strongly protected to getting masses. However, as proven in [2] the observed non-zero neutrino masses can be generated on the phenomenological level by tiny torsional interactions which violate (22). These can also be used to accommodate appropriate PMNS mixing values. Physically the existence of such interactions is a signal for the activity in isospin space of small anisotropic forces. This is discussed in more detail in (2.5.21).

2.4.15 Are neutrinos Dirac or Majorana particles?

They are Dirac fermions. Like all other quark and lepton flavors they inherit this property from the tetrons, cf. (2.4.2).

2.4.16 How is weak isospin realized on the mignon level?

The first fact to note is that the weak isospin of the SM cannot be directly identified with the tetronic isospin SU(2). As discussed below in connection with (93), the construction of mignon isospin relies on exchanging the roles of $\vec{S}$ and $\vec{T}$, i.e. of the

$SU(2)_L$ and the $SU(2)_R$ sector.

The most appropriate way to construct quark and lepton isospin is to assume the existence of a tetron ψ_L and an antitetron $(\chi_R)^c = (\chi^c)_L$ on each tetrahedral site i=1,2,3,4, as depicted in figs. 4 and 5. Here the charge conjugation operator is defined as usual for a Dirac spinor F, i.e. $F^c = C\gamma_0 F^*$.

One can either interpret fig. 4 in such a way that there are 2 particles (a tetron and an antitetron) on each site, or by saying that one is considering the tetron and the antitetron contribution of a single field to the isospin vectors $\vec{Q}_L$ and $\vec{Q}_R$, cf. the discussion in (2.3.1).

Although $(\chi_R)^c$ is left-handed, it transforms under SU(2)$_R$ and thus can be used to build a $\vec{Q}_R$ vibrator. For the question why tetrons and antitetrons do not annihilate inside the hyper-crystal see (2.2.8).

I also tried an approach without antiparticles, i.e. with χ_R instead of $(\chi_R)^c$, but have abandoned this for the following reasons:

–since tetrons are fermions, the wave function for each pair would have to respect the Pauli principle. While the spatial part must be symmetric (the argument is similar as for in the helium atom), the spin and the isospin part are both expected to be anti-symmetric singlets, and this would violate the Pauli principle. If one of the tetrons is an antiparticle, one does not run into this problem of two identical fermions.

–$A(\nu_e)$ and $A(e)$, $T(u)$ and $T(d)$ etc would not be true isospin partners but components of Kramers doublets. The exchange $\vec{Q}_L \leftrightarrow \vec{Q}_R$ induces an isospin exchange $U \leftrightarrow D$ only if one antiparticle is involved. The reason is that charge conjugation compensates for the internal time reversal active in a Kramers doublet. This will be explained in more detail in (2.4.19).

–finally, since the Higgs field and the gauge bosons have been interpreted as tetron-antitetron excitations, it seems a good idea to have antitetrons already appearing in the ground state.

The isospin vectors, whose vevs are depicted in fig. 4, are given by

$$\vec{Q}_L = \frac{1}{2}\,\psi_L^\dagger \vec{\tau}\psi_L \qquad\qquad \vec{Q}_R = \frac{1}{2}\,(\chi_R)^{c\dagger}\vec{\tau}(\chi_R)^c \tag{92}$$

A transition between them is then necessarily accompanied by an exchange of isospins U and D via[87]

$$(U_L, D_L) \rightarrow (-D_R^c, U_R^c) \tag{93}$$

If one strips off the Dirac structure, (93) is identical to an internal time reversal (6), which was considered in connection with the Shubnikov transformations in section 1. One concludes that the Shubnikov symmetry (5) can be defined by using charge conjugation and without introducing the concept of an internal time, as anticipated in the discussion after (7). For more details on the action of discrete symmetries like C, P and T on the tetron ground state see (2.4.20). It is interesting to note that the internal reflection operators which exchange the elements of A_4 and $S_4 - A_4$ comprise isospin transformations, charge conjugation as well as transitions between left and right.

To summarize, it has been shown that the transition between weak isospin partners like $A_1(e)$ and $A_1(\nu_e)$ can be obtained by exchanging $\vec{Q}_{Li}$ and $\vec{Q}_{Ri}$ on the tetrahedral sites i. In the actual calculation of mignon eigenstates, it turns out that the top-quark is predominantly an $\vec{S} = \vec{Q}_L$ excitation while the b-quark is $\vec{T} = \vec{Q}_R$. On the other hand the neutrinos are given by vibrations of the conserved quantity $\sum_i(\vec{S}_i + \vec{T}_i)$, while charged leptons are approximately excitations of the $\sum_i(\vec{S}_i - \vec{T}_i)$ combination.

The connection between the attributes 'left-handed' and 'up-type' (and similarly 'right-handed' and 'down-type') is of fundamental importance in the tetron model. It relies on the chiral property of the internal ground state fig. 5 and the octonion induced form of the tetron interaction (77) and (2.1.19). Furthermore, it is at the heart of the tetron model explanation of weak parity violation (2.1.19) and of the large value of the top quark mass from (19) as compared to the other q/l masses, cf. (2.4.21).

2.4.17 The difference between tetron isospin and 'custodial' SU(2)

In the bosonic sector of the SM (i.e. without quarks and leptons) there is a global vectorlike SU(2) symmetry which remains intact after the SSB. This is usually called 'custodial SU(2)', and it should not be mixed up with the isospin SU(2) of the 3-dimensional internal space considered in the tetron model. While custodial SU(2)

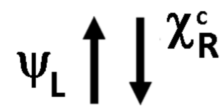

Figure 4: Ground state configuration of the tetron-antitetron pair on a single tetrahedral site. The vertical axis corresponds to the radial direction in the sense of (11), i.e. isospin pointing outwards=up means U, inwards=down means D. Drawn are the ground state values of isospin vectors $\vec{Q}_L$ and $\vec{Q}_R$ which in (92) are defined in terms of ψ_L and $(\chi_R)^c$, respectively. Considering the tetrahedron as a whole, the configuration is shown in fig. 5 and is chiral, both in internal and physical space. The internal chirality is flipped by $U \leftrightarrow D$, the external by $L \leftrightarrow R$. Note that $(\chi_R)^c$ is left-handed, but transforms under $SU(2)_R$. Therefore and since the configuration shown is preferred over that of opposite chiralities through the sign of the interaction (78), $(\chi_R)^c$ does not take part in the symmetry-breaking *inter*-tetrahedral interactions (19).

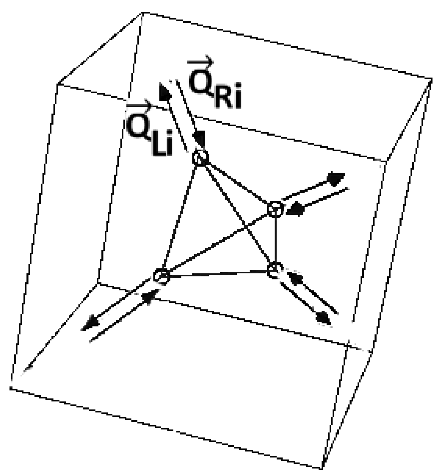

Figure 5: The local ground state of the tetron model with 8 internal spin vectors $\vec{Q}_{Li}$ (pointing outwards) and $\vec{Q}_{Ri}$ (pointing inwards), i=1,2,3,4, accounts for 3×8 d.o.f. corresponding to 24 quarks and leptons according to (9). Due to the anti-particle nature of the isospin vectors pointing inwards, the depicted configuration is chiral and internal parity maximally violated.

arises on the level of mignons, the tetron model SU(2) exists on the level of tetrons and comprises not only electroweak but also color and family symmetries. While the dominant corrections from custodial SU(2) breaking are $\sim m_t - m_b$, tetron SU(2) breaking terms generate the neutrino masses, cf. (2.4.12)ff. More details on the relation between tetron and electroweak(=mignon) SU(2) can be found in (2.3.30) and (2.4.16).

2.4.18 Do pairs of ordinary and true Shubnikov representations form isospin doublets?

No. To explain this point, I start with the remark, that 'true' Shubnikov representations have nothing to do with 'projective' representations mentioned above. While the latter are representations of the covering group, a true Shubnikov representation is a representation of $G_4 = A_4 + S(S_4 - A_4)$ which is not a representation of A_4. True Shubnikov representations are labeled by an index s in the following. The Shubnikov group G_4 under consideration has the property that for each ordinary representation D of G_4 there is exactly one true Shubnikov representation D_s. If one puts an isospin along the z-direction, true Shubnikov representations are related to the excitations of the y-components of the isospin vectors, while Q_x and Q_z correspond to the ordinary representations.

Analyzing the representations appearing in (9) one finds that A and A_s, T and T_s arise in the combinations

$$\begin{aligned} A(\nu_e) + A'_{1+is}(\nu_\mu) + A''_{1-is}(\nu_\tau) \quad &+ \quad T(u) + T_{1+is}(c) + T_{1-is}(t) + \\ A(e) + A'_{1+is}(\mu) + A''_{1-is}(\tau) \quad &+ \quad T(d) + T_{1+is}(s) + T_{1-is}(b) \end{aligned} \tag{94}$$

where the notation $1 \pm is$ means that the second and third family are generated by excitations of $Q_x \pm iQ_y$, while the first family corresponds to excitations of Q_z. According to this result an isospin doublet is not given by a pair (A, A_s) or (T, T_s) of an ordinary representation and a Shubnikov representation. Instead, the rows A, A_{1+is}, A_{1-is} and T, T_{1+is}, T_{1-is} correspond to particles of the 1., 2. and 3. family. Finally nota bene that the classification (94) can be used to show that interfamily transitions are suppressed in the tetron model, as explained in (2.4.11).

2.4.19 A simple mathematical understanding of all the 24 Shubnikov states in (94)

First of all note that, although mignons are actually precessions of internal angular momentum vectors, I sometimes call them 'vibrations' or simply 'excitations' in this paper.

In this section the 24 mignon precessions are derived successively starting with the 3 mignons arising from a single tetron, then going over to 2 tetrons and ending with the 8 tetron configuration fig. 5. The results of the discussion will amount to the following statements:

–the existence of 3 families reflects the 3 dimensions of the internal space, as already claimed in the first paper on the subject[85].

–weak isospin of quarks and leptons corresponds to isospin transformations of tetron and anti-tetron within one tetrahedral site i=1,2,3 or 4.

–the color d.o.f. arise from the 4-fold structure of the tetrahedrons.

By slightly abusing notation, mignon excitations of a single tetron can be described as isospinors of the form

$$\psi = (\langle U\rangle + \delta U, \langle D\rangle + \delta D) = (1 + \delta U, \delta D) \tag{95}$$

$\langle U\rangle$ and $\langle D\rangle$ are the isospinor values in the ground state. For the convenience of the following discussion, $\langle U\rangle$ can be put to 1 and $\langle D\rangle$ assumed to vanish. δU and δD contain a time dependence $\sim \exp(i\omega t)$ where ω is the mass/energy/frequency of the precession. The corresponding fluctuation of the isospin vector $\vec{Q} = \psi^\dagger \vec{\tau}\psi$ is given by

$$\delta\vec{Q} = \vec{Q} - \langle\vec{Q}\rangle = \begin{pmatrix} \mathrm{Re}\,\delta D \\ \mathrm{Im}\,\delta D \\ \mathrm{Re}\,\delta U \end{pmatrix} + O(\delta^2) \tag{96}$$

where $\langle\vec{Q}\rangle = (0, 0, 1)$.

It may be noted, that on the tetrahedron one has non-vanishing vevs $\langle\vec{Q}\rangle \neq 0$ only for the single isospin vectors, whereas for the tetrahedron as a whole the total internal angular momentum vanishes in the ground state, i.e.

$$\sum_{i=1}^{4} \langle\vec{Q}_i\rangle = 0 \tag{97}$$

and the only relevant vev becomes the Higgs vev as described in (2.3.10)-(2.3.11).
Coming back to the single tetron states (96), the 3 vibrations $\mathrm{Re}\,\delta D$, $\mathrm{Im}\,\delta D$ and $\mathrm{Re}\,\delta U$ in the x, y and z-direction essentially correspond to the third, second and first family of quarks and leptons. The y-vibration is a true Shubnikov excitation in the sense of (2.4.18).

The color quantum number arises as soon as all four tetrahedral sites are taken into account. In place of one vibrator (95) one then has four, and arrives at one singlet A (the lepton) and one triplet T (the quark) for each family and for each isospin value $\pm 1/2$. More details about the origin of color in the tetron model can be found in (2.3.29) and (2.3.30).

So far, we have ignored the fact that there are 2 vectors $\vec{Q}_L$ and $\vec{Q}_R$ that vibrate on each tetrahedral site instead of one. These vectors are given in (92), and the doubling of mignon states which they induce gives rise to weak isospin of quarks and leptons, i.e. to partners like $A(\nu_e)$ and $A(e)$ in (94).

In explicit terms the mignons from the 2 tetrons fig. 4 on a tetrahedral site are given by isospinors of the form

$$\psi_L = (U_L, D_L) = (1 + \delta U_L, \delta D_L) \qquad \chi_R^c = (-D_R^c, U_R^c) = (-\delta D_R^c, -1 + \delta U_R^c) \quad (98)$$

and the corresponding isospin vectors are

$$\vec{Q}_L - (0,0,\frac{1}{2}) = \begin{pmatrix} \mathrm{Re}\,\delta D_L \\ \mathrm{Im}\,\delta D_L \\ \mathrm{Re}\,\delta U_L \end{pmatrix} \qquad \vec{Q}_R - (0,0,-\frac{1}{2}) = \begin{pmatrix} \mathrm{Re}\,\delta U_R^c \\ \mathrm{Im}\,\delta U_R^c \\ \mathrm{Re}\,\delta D_R^c \end{pmatrix} \quad (99)$$

which makes the weak isospin transformation between the 2 kinds of mignons evident.

Actually, for a more precise analysis, terms of order δ^2 or, equivalently, fluctuations of $\mathrm{Im}\,\delta U$ have to be included in the analysis. For example, they enter in

$$\delta Q_x = \mathrm{Re}\,\delta D + \frac{1}{2}[\delta U^\dagger \delta D + c.c.] \quad (100)$$

or in Q_z, because $Q_z = \psi^\dagger\psi/2 - |\delta D|^2$ with the tetron density given by

$$\psi^\dagger\psi = |1 + \delta U|^2 + |\delta D|^2 \quad (101)$$

This makes the analysis of the weak isospin transition somewhat more complicated than appears in (99).

The complete set of the 24 quark and lepton eigenstates is worked out in [2].

2.4.20 The role of CPT symmetry in the tetron model

In this section I want to show that CPT plays an eminent role in the relativistic generalization (7) of the Shubnikov group (5). The point is that in the presence of relativistic particles, CPT supersedes the PT symmetry inherent in non-relativistic (iso)-magnetic phenomena.
Actually for this argument to work, the parity transformation P is to be interpreted as the product $P_{in}P_{au}$ of internal and external parity, where P_{in} reverses the sign of the internal and P_{au} that of the physical coordinates. In other words, CPT is the CPT transformation not in Minkowski space (this would be CP_{au}T) but in the underlying 6+1 dimensional spacetime. Therefore this invariance will sometimes be called CPT6 to distinguish it from ordinary CPT in Minkowski space.
We start our line of reasoning by summarizing what we already know about C, P and T in the tetron model:
-in (2.1.19) the (maximal external) parity violation observed in the weak interactions was related to the maximal violation of internal parity P_{in} induced by the ground state fig. 1. This relation arises via the appearance of products $\gamma_5\vec{\tau}$ in the tetron dynamics.
-in (2.4.16) charge conjugation was discussed as a necessary ingredient, if one is working in a relativistic environment and includes antitetrons in the ground state as done in figs. 4 and 5. Due to the antiparticle nature of the isospin vectors pointing inwards, internal parity in fig. 5 is maximally violated just as it is in fig. 1.
-it is well known that time reversal is an essential ingredient in any kind of magnetic phenomena. One may even go as far to say that magnetism is the physics of the breaking of T symmetry. For example, T appears in the nonrelativistic version (5) of our Shubnikov symmetry. T itself is broken as in any magnetic alignment, but the product TP_{in} is an implicit part of the symmetry (5).
The action of CPT6 on an isospin vector $\vec{Q}_L$ pointing up(=outwards) as in figs. 4 and 5 is given by

$$\vec{Q}_L \xrightarrow{C} -\vec{Q}_L \xrightarrow{P_{au}} -\vec{Q}_R \xrightarrow{P_{in}} \vec{Q}_R \xrightarrow{T} -\vec{Q}_R \tag{102}$$

As discussed in section 1, T and P_{in} reverse the orientation of isospin vectors, while P_{au} exchanges L and R. C also reverses the orientation of an isospin vector, because antiparticle contributions enter in isospin vectors $\vec{Q}$ with an opposite sign, cf. (3).

Eq. (102) shows that the ground state fig. 5 consisting of 4 tetrons and 4 anti-tetrons fulfilling $\langle\vec{Q}_{Li}\rangle = -\langle\vec{Q}_{Ri}\rangle$ on each tetrahedral site i=1,2,3,4 is CPT6 invariant. In contrast, if the local ground state would be $\langle\vec{Q}_{Li}\rangle = \langle\vec{Q}_{Ri}\rangle$, i.e. if the isospin vectors would have the same orientation, one would have PT or CP symmetry, but CPT6 would be violated.
Note, this discussion of the ground state's discrete symmetries has to be distinguished from the CP violation in the CKM matrix. As discussed in [2], the latter arises from the complex nature of the exchange couplings and the resulting phases in the mixing of mignon states.

2.4.21 Why is top so heavy, why not bottom?

Up to tiny CKM and V+A mixing effects, the top quark corresponds to vibrations of $\vec{\Sigma}_L = \sum_i \vec{Q}_{Li}$. This vector plays a special role in the tetron model, which has to do with the chiral nature of the SSB and with the relation between internal and external parity violation[1] induced by (78), and because nature has chosen to break internal parity, i.e. prefers the state fig. 1 with isospin vectors pointing outwards over the one with those pointing inwards. The top-mignon is defined as that internal precession, where all isospin vibrations act against the SSB alignment in the strongest possible way.

2.4.22 Why is m_b much smaller than m_t and from where does it get its predominant contributions?

In contrast to the top quark, the b-quark is mostly an excitation of the $\sum_i \vec{Q}_{Ri}$, and therefore does not get a contribution from the $SU(2)_L$ Dzyaloshinskii-Moriya interaction (19).
I have considered several sources for an m_b value of order 5 GeV, the most straightforward being the existence of a tiny right handed (=V+A) current. Actually, in the calculation[2] it is more difficult to accommodate the bottom mass than the values of the lighter quark masses. This is because contributions to m_b are naturally associated to the right handed version of (19) where $\vec{T}$ instead of $\vec{S}$ appears, and this V+A contribution must be radically suppressed according to the argument

that weak parity violation is due to the internal chirality of the tetrahedrons, as discussed in (2.1.19). Thus from a general point of view the introduction of a small V+A current does not serve as a fully satisfactory explanation for a noteworthy bottom mass.

On the other hand one is talking here about a m_b/m_t =3% effect, and a 3% V+A correction to the leading V-A coupling is compatible with present experimental bounds[6]. Such a correction can then be used to account for the observed value of m_b[2].

2.4.23 If up type quarks arise mainly from vibrations of $\vec{Q}_L$, how can their right-handed version be produced?

As shown in [2], lepton states originate dominantly from vibrations of the form $\vec{S}\pm\vec{T}$, while up and down quark states are related to vibrations of $\vec{S}$ and $\vec{T}$, respectively, where $\vec{S} = \vec{Q}_L$ and $\vec{T} = \vec{Q}_R$. Therefore one might suspect that the helicities of quarks and leptons generated in this way are restricted, too. However, it must be noted that an excitation δ of a left-handed isospin vector can in principle vibrate into any chiral direction. The same is true for right handed vectors $\vec{Q}_R$.

2.5 Connections to Gravitation and Cosmology

This section relies on the interpretation of the world as an elastic system of internal tetrahedrons. According to section 1 the tetrahedrons form an extremely dense monolayer mesh which can buckle and bulge into the surrounding higher dimensional space thus inducing the forces of gravity. The following list of questions and answers shows how this picture may be incorporated within a larger, cosmological framework. Throughout it will be assumed that after crystallization spacetime is homogeneous and isotropic, i.e. it can be described by a Friedmann / Robertson-Walker (FLRW) metric

$$ds^2 - c^2dt^2 = -a^2(t)[\frac{(d\vec{x})^2}{1-k\vec{x}^2} + \vec{x}^2 d\Omega^2] \qquad (103)$$

where $a(t)$ is the dimensionless 'scale factor' and k is the Gaussian curvature of the space (not of spacetime) at the time when a(t)=1. k has units of length^{-2} and is positive, zero or negative for an elliptical, Euclidean or hyperbolic universe, respec-

tively. The time where a(t)=1 is often chosen to be the presence; however, in (2.5.27) it will be chosen to be the time when the universe has reached its equilibrium.
Note, the x^i in (103) are 'comoving' coordinates, from which the physical coordinates are obtained as $a(t)x^i$. Accordingly, the physical velocity of an object can be decomposed as

$$v^i = a(t)\frac{dx^i}{dt} + \frac{da}{dt}x^i = w^i + Hax^i \tag{104}$$

where the second term introducing the Hubble parameter $H = \dot{a}/a$ is called the 'Hubble flow' and the first term w^i defines the 'peculiar velocity' of an object, i.e. its velocity relative to the Hubble flow.
Note that the FLRW metric fulfills $g_{00} = 1$ and $g_{0i} = 0$, i.e. it corresponds to a metric in a 'synchronous gauge'[96]. Such a metric has a particular simple interpretation in the tetron model, because the only non-trivial elements are the spatial g_{ij}. These can be identified with variable 'longitudinal' distances among the tetrahedrons in the 3-dimensional elastic system which is our physical universe. This point will be further discussed in connection with (2.5.27) and fig. 7a.

2.5.1 Is the tetrahedral ground state stable or metastable?

In other words, are there isomagnetic states X with a lower energy? If yes, this might threaten our world when such a state would be produced in high energy collisions. More precisely, what could happen is that at collider energies of order Λ_F the ordered state fig. 2 is locally destroyed and in the process of re-ordering of isospin vectors a germ of the state X appears. Energy would then be released, which would destroy the ordering in the neighborhood of the original collision, so that a chain reaction would start at the end of which the metastable ground state would be completely replaced by the stable one.
In the microscopic model it is possible to compare the energies of all isomagnetic configurations with 4 isovectors to the energy of fig. 1. Essentially, one deals with a system of 4 isospin vectors interacting via an 'anti-ferromagnetic' coupling $J_A > 0$. The local energy of any such state X is given by

$$E(X) = J_A\,[\,\vec{Q}_1\vec{Q}_2 + \vec{Q}_1\vec{Q}_3 + \vec{Q}_1\vec{Q}_4 + \vec{Q}_2\vec{Q}_3 + \vec{Q}_2\vec{Q}_4 + \vec{Q}_3\vec{Q}_4\,] \tag{105}$$

With this input one may run over all possible ground state configurations of isospin vectors. As a result, one finds 2 minima with exactly the same energy. One minimum (m1) corresponds to fig. 1 while the other (called m2) is characterized by the 4 isospins arranged into 2 pairs of opposite orientation as depicted in fig. 6 of [97]. This conclusion remains unchanged, if one considers separate left- and right-handed isospin vectors on each tetrahedral site as in (14) and fig. 5. However, it may get changed, if the *inter*-tetrahedral energies are different. In general, one expects roughly identical inter-tetrahedral energies, because in both cases (m1) and (m2) the same number of isospin pairs are aligned in a ferromagnetic way, with identical exchange couplings J_{inter}. However, in reality there may be small differences between the J_{inter} values for the (m1) and the (m2) configuration due to the different geometries of the two ground states.

2.5.2 Is physical space discrete, i.e. is there a granular structure of physical space in addition to the discrete tetrahedral structure in internal space?

Most probably yes. Although the discrete structure of physical space is not compelling and the distance r between two tetrahedrons could be identically zero, the discussions in section 1 suggest that r has a tiny non-vanishing value of the order of the Planck length. Note, due to the elasticity of the system only an average $\langle r \rangle$ can be given, in accord with (30).

2.5.3 How can such a granular structure be compatible with Lorentz invariance?

In other words: how can the elastic continuum of tetrahedrons be Lorentz invariant? The point to note is that in the tetron model all physical objects that we know are superpositions of excitations which travel as quasi-particles through the hypercrystal. This holds true even for photons and also for ourselves as well as for all experiments and 'reference frames' we can preparate. In a quantum mechanical framework such excitations always have a wave nature and are to be described by (generalizations of) the d'Alembert wave equation. This fact alone fixes the system of waves, which constitutes our physical environment, to be Lorentzian, because the d'Alembert operator

$$\Box = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \tag{106}$$

leaves the squared 4-momentum p^2 invariant. In particular, if one wave packet is emitted from another one, their velocities add up according to the rules of Lorentz transformations.

For massive particles which move at velocities $< c$, the operator is modified to $\Box - m^2c^2/\hbar^2$ corresponding to a 'dispersion relation' $p^2 = m^2c^2/\hbar^2$. It is important for this argument (and in general to retain the Lorentz structure) that in all those wave equations there is a universal maximum speed c that fixes the relation between space and time. This is defined to be the speed of the massless excitations (2.1.3) and according to (32) is given by the ratio of 'lattice constant' L_P and 'hopping time' T_P, cf. the discussion after (41).

The value of c is universal for all SM particles because all of them arise from the same isomagnetic interactions introduced in section 1. This is analyzed in more detail in (2.5.13), and the question of (metrical) velocities larger than c will be discussed in (2.5.42). The question why gravitational waves propagate at c will be answered in (2.5.41).

It should be stressed, that in particle physics interactions usually only the excitations move. The tetrahedrons stay fixed at their location in the hyper-crystal. They only move in connection with metrical changes induced by gravity, for example when the universe expands after the crystallization or on a much tinier level in any kind of gravitational interaction, cf. the discussion after (28) and in (2.5.42).

2.5.4 Is time discrete?

Time is an averaged construct induced by the superposition of many elementary thermodynamics processes. In the tetron model it can be interpreted to have a granularity of order T_P – at least from the standpoint of the quasi-particles which constitute our physical environment, because T_P is the minimum time it takes a quasi-particle to jump from one tetrahedron to the other. In our world of quasi-particles the duration of any observable physical process can never fall below this value.

2.5.5 Does quantum mechanics arise from material properties of the hyper crystal?

My claim is yes. The proof will be given in the following section (2.5.6). It relies on the fact that ordinary matter (including the photon) consists of internal excitations traveling as quasi-particles on a discrete structure with Planck length lattice spacing.

2.5.6 Why is the Planck length the natural lattice spacing for the hyper-crystal?

The short answer: the Planck length L_P arises as a lower limit on Δx in the 'generalized' Heisenberg uncertainty relation (108), which includes the effects of gravity. On the other hand in the tetron model *all* known particles including the photon are interpreted as excitations with an extension of at least one lattice spacing $\langle r \rangle$, i.e. they are quasi-particle waves with wavelength $\lambda > L_P$. Since every physical experiment necessarily makes use solely of these quasi-particles, its resolution cannot be better than r. This strongly suggests $r \approx L_P$, because if r would be smaller than L_P, r instead of L_P would determine the uncertainties of quantum mechanics.
Due to the extremely small lattice spacings, spacetime as we perceive it effectively looks like a continuum. This guarantees local rotational symmetry. The item of invariance under Lorentz boosts is discussed in (2.5.3).
Extended answer: in ordinary quantum mechanics there is a fixed dimensionful quantity h which relates the canonical Fourier variables of frequency to energy and inverse distance to momentum, i.e. it transforms the spacetime quantities x and t into physically 'active' quantities

$$\vec{p} = \hbar \vec{k} \qquad\qquad E = \hbar\omega \tag{107}$$

These relations are the reason why h appears in the uncertainty principle $\Delta x \Delta p > \hbar$ which otherwise would just be the Cauchy-Schwarz inequality known from the Fourier analysis of waves, i.e. $\Delta x \Delta k > 1$ with no dimension on the r.h.s. I consider them as further evidence that h is a material property of the hyper-crystal not valid outside of it, cf. the discussion later in this section and at the end of section 1.
The 'generalized' uncertainty relation includes gravitational effects of the photon on a test particle[58]. These occur because the general relativistic effect from the

photon adds to the uncertainty about the test particle. The Heisenberg relation is then modified to

$$\Delta x \geq \frac{\hbar}{\Delta p} + L_P^2 \frac{\Delta p}{\hbar} = \lambda[1 + (\frac{L_P}{\lambda})^2] \tag{108}$$

where λ is the photon wavelength to be identified with the limit of resolution.
Eq. (108) can be derived e.g. by extending the 'Heisenberg microscope' thought experiment (which imagines a photon to measure x and p of a probe particle) to include gravitational effects of the photon[58]. Without gravity, the position of the particle can be determined to an accuracy of about the wavelength of the light used, and this determines the ordinary uncertainty relation. If one includes gravity, there is a tiny gravitational force from the photon acting on the particle, because the photon has an effective mass

$$m_\gamma = \frac{h\nu}{c^2} = \frac{h}{c\lambda} \tag{109}$$

and this force will accelerate the particle, making the already uncertain position of the particle a bit more uncertain. The gravitational uncertainty can be estimated as follows: using Newton's law the additional acceleration due to gravity is given by $a = Gm_\gamma/r^2$, so that the relevant length can be estimated to be $aT_P^2 = L_P^2/\lambda$. This is precisely the second term in (108).
This result has quite a natural interpretation, because the gravitational effect of the photon modifies the average spacing $\langle r \rangle = L_P$ between the tetrahedrons involved in the interaction and thus the resolution $\Delta p = \lambda$ of the photon. The modification factor can be derived from the general relativistic length contraction according to (28) by inserting the photon's gravitational mass (109) and amounts to

$$1 - \frac{\phi}{c^2} = 1 + \frac{Gh}{|\vec{x}|\lambda c^3} \tag{110}$$

Since the relevant regime of discussion is distances $|\vec{x}| \approx \lambda$, the factor (110) agrees with the factor in (108).
In any case eq. (108) implies that Δx considered as a function of λ takes on a minimum value

$$\Delta x \geq L_P \tag{111}$$

corresponding to a photon with wavelength L_P. In the usual folklore, this is interpreted in such a way that at distances/wavelengths smaller than L_P all matter dissolves into quantum fluctuations and the laws of physics do not have a meaning any more.

In the tetron model, however, the interpretation is a little different. First of all, one should be reluctant in just adding up uncertainties and 'deriving' from that a minimum position uncertainty. Rather, the condition $\Delta x > L_P$ arises in the tetron model from the limitation of wavelengths of quasi-particles which holds true inside the hyper-crystal. In particular, since the photon is such a quasi-particle (a correlation among 2 tetrahedrons), the minimal wave length of photons, that can be produced and used in experiments, roughly corresponds to one lattice spacing L_P, and it is this minimal wavelength which absolutely restricts the precision of any experiment.

It may be noted that this limitation is valid only in the realm of quasi-particles (in which we ourselves exist), not of tetrons. On the other hand, this remark does not preclude, that there are analogous restrictions for tetrons, with maybe a different Planck constant, cf. the discussion in (2.2.10). In this connection it is worth mentioning that on the hyper-crystal the value of h is given by (33) where the Planck energy Λ_P corresponds to the binding energy of the tetrahedrons and T_P is the hopping time needed to absorb and re-emit the photon. Obviously, such a value of h can only be valid within the hyper-crystal and does not at all apply to, let's say, tetrons in a tetron gas before the crystallization.

Actually, within the microscopic model one can do better than just augmenting arguments in favor of (111). Starting from the dispersion (39) of a quasi-particle photon one can extent the Fourier analysis on the Planck lattice to derive a modified uncertainty relation

$$\Delta x \Delta k \geq |\langle \cos(k L_P) \rangle| \tag{112}$$

Eq. (112) may be evaluated in the limit of long wavelengths to obtain the analogon of (108)

$$\Delta x \geq \frac{\hbar}{\Delta p} - L_P^2 \frac{\Delta p}{2\hbar} \tag{113}$$

No need to worry about the factor 1/2, because the second term in (108) is only an estimate anyhow. More important seems the change in sign of the second term. However, as explained above, in the tetron model one is not reliant on the minimum argument based on (108).

Note further, that for wave vectors near the border of the first Brillouin zone, i.e. for wavelengths $\lambda \sim L_P$ the r.h.s. of (112) vanishes. Therefore it seems that lattice quantum mechanics at Planck scale energies can exhibit classical, non-quantum mechanical behavior. This, however, will happen only for extremely high photon energies, in a region where photons cease to exist because their quasi-particle nature forbids wavelengths $\leq L_P$.

2.5.7 How is physical space defined within the 6+1 dimensional world? How is it distinguished from the internal dimensions?

The aligned tetrahedrons define a 3-dimensional subspace of R^6. Everything orthogonal gives physical space.

2.5.8 What is the exact lattice structure of the hyper-crystal?

This question is obsolete because of the elastic nature of inter-tetrahedral coordinate bonds. It is an irregular lattice of tetrahedrons in the base 3+1 dimensional spacetime without a discrete symmetry. As discussed in various places - e.g. after (23) in section 1, in (2.5.9) and (2.5.29) - there are more similarities to a fluid than to a crystal.

2.5.9 Then why is this structure called a 'crystal'?

Because there is a rigid tetrahedral structure in the *internal* directions. Concerning physical space it is a disordered system which resembles a plastics or a fluid. However, the quasi-particles in this space follow an approximate Lorentz symmetry at low energies, cf. (2.5.3).

2.5.10 Why is there no growth of the crystal into the internal directions?

This has to do with the form of the fundamental tetron interaction, cf. (2.2.10) and (2.2.11), and its preference to form the spiky tetrahedral 'star' system fig. 1, which do not allow tetrahedrons to be stacked on top of each other in fig. 2. It is the main reason why internal dimensions need not be compactified, cf. (2.5.32).
In the following I want to make the argument more explicit and start by considering the 1+1 dimensional configuration in fig. 6, i.e. one internal and one physical

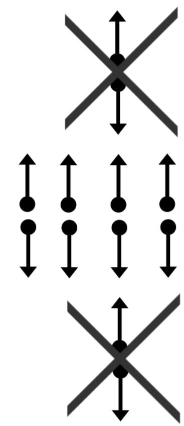

Figure 6: This picture illustrates the impossibility of the crystal to grow into the internal dimension for the 1+1 dimensional case. It is based on the assumption that the interactions among isospins of different internal molecules are ferromagnetic, so that adjacent molecules repel each other. The argument can be extended to the realistic 3+3 dimensional case. Note the definition of 'internal' as given in (2.5.7).

dimension with 'di-atomic' molecules consisting of 2 tetrons instead of 4. While the isospin interactions within one molecule are anti-ferromagnetic, the interactions among isospins of *different* tetrahedrons are assumed to be ferromagnetic. (Remember that the tetron model understanding of the SM symmetry breaking in section 1 was based on this assumption.) Therefore the crystal structure depicted in fig. 6 can only grow in the horizontal(=physical) but not in the vertical(=internal) direction. Actually, the essence of this argument can be transferred to the 3+3 dimensional case of tetrahedrons, because if there is to be a ferromagnetic arrangement, then this first of all builds up the physical base space. If one further tries to arrange in parallel e.g. isospins $\vec{S}_1$ and $\vec{S}'_1$ of 2 tetrahedrons t and t' in the vertical(=internal) direction, the ferromagnetic interaction $J\vec{S}_1\vec{S}'_1$ will not be effectual because between $\vec{S}_1$ and $\vec{S}'_1$ there will always be the vector $\vec{S}_2 + \vec{S}_3 + \vec{S}_4 \approx -\vec{S}_1$.

2.5.11 *Where do the 6 spatial dimensions come from?*

I have only a partial answer to this question. A tetron spinor $\psi = (U, D)$ transforming as 8 under SO(6,1) can be interpreted as an octonion field living in 6+1 dimensions. Octonions form a rather unique mathematical structure[40, 41, 42]. They are the next thing to consider when complex numbers (used for amplitudes in quantum mechanics) and quaternions (used for rotations and spinors in physical space) are not ample enough to describe physical phenomena. The octonion nature of tetrons has been used in (2.1.19) to determine the form of the isomagnetic tetron interactions. The splitting of R^6 into an internal space and physical space corresponds to a splitting of an octonion into two quaternions.

Note that if octonion multiplication should really be relevant for the behavior of tetrons, it seems somewhat more natural to have 7 instead of 6 spatial dimensions. This, however, is pure speculation. It is the product $\vec{\tau}\gamma_5$ appearing in (78), which provides the link between internal and physical space, necessary to explain particle physics phenomena such as parity violation of the weak interaction.

2.5.12 How does the value for c come about? Why is it finite?

In the tetron model light is interpreted as a (massless) excitation of tetron-antitetron pairs, cf. (2.1.2) and (2.1.3). While the pairs themselves are bound over 2 tetrahedrons with fixed positions within the hyper-crystal, the excitations propagate through physical space as quasi-particles. In contrast to massive excitations, whose propagation was considered in (41) and (42), photons cannot cling to one fixed internal tetrahedron, but are constantly moving at the speed c of light. Nevertheless, c is not infinite, because even a massless excitation needs a certain 'hopping time' T_P to jump from one tetrahedron to another. Since the tetrahedrons are distributed over the hyper-crystal with average distance L_P, this corresponds to $c = L_P/T_P$ in agreement with (31).

2.5.13 Why is c the universal limiting speed for all particles?

In the tetron model all SM particles are internal excitations whose interactions have the same type of isomagnetism as universal origin. Therefore it is of no surprise that a common maximum speed exists, to be identified with the ratio L_P/T_P, as explained in (2.5.12). Furthermore, it was demonstrated at the end of section 1, that while the mignon masses are determined by isomagnetic interactions, their

propagation proceeds via the elastic/spacetime properties of the DMESC.
For the interpretation of (metrical) velocities larger than c see (2.5.42).

2.5.14 Is there a rest system of the hyper-crystal?

At first sight the existence of such a system seems to contradict Einstein's principle of equivalence and special relativity - well established concepts which I do not want to question. Still I think the answer to the question is affirmative.
The point is that in the tetron model all material objects and all normal matter and gauge bosons including the photons from their very nature are quasi-particle waves, i.e. entities fulfilling Lorentz covariant Klein-Gordon equations, as discussed in (2.5.3). As such they cannot distinguish an absolute rest system, i.e. they naturally fulfill Einstein's principle of equivalence. By contrast, the original tetron matter, which forms the fixed hyper-crystal ground state, is merely the carrier of those quasi-particles, and mostly invisible for human experiments.
This point of view is close to the Fitzgerald interpretation of special relativity[103] and allows for a rest system of the hyper-crystal. More details are given in the next answers.

2.5.15 Why has a rest system never been observed in Michelson Morley type of experiments?

The essence of the answer to this question has already been given at the end of (2.5.14). An extended version can be found in [103]. In that interpretation of special relativity a Galilean ground state like fig. 2 is not a contradiction to special relativity but rather supplements it. It supplies the 6-dimensional framework which leads to the observed spectrum of physical particles.

2.5.16 Can the hyper-crystal's rest system ever be identified?

It follows from (2.5.14) that it is difficult to experimentally perceive the ground state fig. 2 which forms the rest system of the hyper-crystal - the reason being that our reality (ourselves, our detectors as well as all test particles) consists of quasi particle waves and not of tetrons themselves.

The puzzle cannot be resolved on the level of excitations. Even gravitational waves propagate at the speed of light, cf. (2.5.41). However, it is well known that the metrical expansion of the universe (and metrical changes in general) can proceed at superluminal velocities, cf. (2.5.26) and (2.5.41).

In section 1 it was suggested that metrical changes correspond to displacements of the internal tetrahedrons within the elastic hyper-crystal, cf. the discussion after (24). Applied to the standard model of cosmic expansion, this leads to the idea that *the hyper-crystal's rest system is given by the spatial coordinates of the FLRW metric* (103). Of course one must be aware that due to the expansion factor a(t) this is an elastic 'rest system' permanently changing with cosmic time, but at least momentarily it can be considered to be at rest. The time dependence of a(t) defines the expansion of the elastic hyper-crystal, and what comes nearest to the notion of 'matter at rest' are the galaxies on the Hubble flow. The peculiar velocity (104) gives their 'true' nontrivial motion with respect to the elastic rest system.

The underlying idea is that to a good approximation mignon-matter forming the galaxies was originally produced 'at rest' (at least on the average) during and shortly after the big bang / crystallization and has since been moving together with the expanding elastic hyper-crystal. This is in accord with the finding that most large lumps of matter, such as galaxy clusters, are nearly comoving with the Hubble flow.

2.5.17 Is there an internal time different from the ordinary time variable?

In magnetism, time reversal is a far-spread concept because it allows to reverse the orientation of the magnetization / of a spin vector. This way it enters the definition of magnetic point groups, for example the Shubnikov group (6).

However, as shown in (2.4.16) and (2.4.20), for the case of iso-magnetism one may replace the role of internal time reversal by charge conjugation, so no separate time is needed for the internal dimensions.

2.5.18 Is there an absolute time in the hyper crystal?

Yes, it is given by the comoving Hubble time coordinate, i.e. the elapsed time since the big bang according to a clock of a comoving observer.

2.5.19 What is the status of the Copernican and of the Cosmological Principle in the tetron model?

The Copernican Principle states that no place in the universe is 'special' or preferred, while the Cosmological Principle demands that the universe looks the same in all directions (is isotropic) and contains everywhere roughly the same amount and mixture of material (is homogeneous). These principles are not questioned by the microscopic model, assuming a suitable uniformity of the hyper-crystal built from tetrons. However they will *not* be fulfilled at the edges of the DMESC where there may be steadily new accretions of tetrahedrons to the crystal. Those edges are therefore expected to be both 'special' and anisotropic places.

2.5.20 Is the original $R^{6,1}$ Lorentzean oder Galilean?

It was argued in section 1, that the speed of light, which is at the heart of Lorentz symmetry, is an intrinsic property of the hyper-crystal, not valid outside of it. Furthermore, the fact that metric expansion of our universe can proceed with velocities larger than c gives support for Galilean (or, in case of a SO(6,1) Lorentz type symmetry, with a limiting speed much larger than the speed of light).

2.5.21 Do we know anything about the position of our universe in full R^{6+1}?

To show up in experiments such an information requires *anisotropic* interactions, which would get their anisotropy from 6-dimensional structures which go beyond the hyper-crystal. It cannot be obtained from the dominant isospin interactions (14) and (19), because these are rotationally invariant, i.e. the same mass spectrum is obtained after a global rotation of the isospin axes.

A simple anisotropic Hamiltonian of isospin vectors $\vec{Q}_i$ would look like

$$H_a = -J_z \sum_{i \neq j}^{4} Q_{iz} Q_{jz} - J_{xy} \sum_{i \neq j}^{4} (Q_{ix} Q_{jx} + Q_{iy} Q_{jy}) \tag{114}$$

with $J_z \neq J_{xy}$.

Since they explicitly break internal rotational symmetry, anisotropic interactions like (114) violate internal angular momentum conservation (22). As discussed in (2.4.12), neutrinos are the Goldstone modes corresponding to that symmetry. More precisely, the 3 neutrino species were identified as the 3 vibrating components of total internal angular momentum. Therefore, the anisotropic Hamiltonian (114) contributes to the neutrino masses, and measuring the neutrino mass matrix has the potential to answer the present question.

2.5.22 Is there a contribution from the tetron ground state configuration to the mass/energy density and to the expansion rate of the universe?

According to general relativity the mass/energy density of the universe determines its curvature, its expansion rate and its future. So one might expect that the energy of the ground state fig. 2 should also contribute. However, as argued in (2.5.30), this energy does not appear in energy balance equations of ordinary matter.

The main effect of the enormous tetron ground state energy is to initialize big bang inflation by releasing a vast amount of crystallization energy.

2.5.23 How does all of this fit into inflationary cosmology?

As discussed in (2.1.11) and after (23), inflation in the microscopic model is associated to the crystallization process of tetrahedrons with an accelerated expansion due to the elastic nature of the bindings and the sudden release of crystallization energy.

The major signature of inflation is the exponential increase of the scale parameter a(t) in (103). According to the Einstein equations a(t) has to fulfill

$$\ddot{a} = -\frac{4\pi}{3} G(\rho + 3p/c^2) a \tag{115}$$

where ρ is the mass density and p the pressure, and the combination $\rho + 3p/c^2$ corresponds to the trace of the energy momentum tensor. Eq. (115) makes it clear that the exponential increase needed for inflation can be obtained for constant and negative $\rho + 3p/c^2$.

What kind of matter fulfills such a condition? The immediate answer: matter undergoing a phase transitions. Indeed, it is normally assumed in models of inflation that a false vacuum decays in the framework of some abstract phase transition being active in the very early universe. However, the physical background of this phase transition is never specified.

In the tetron model the understanding is clearer, because the phase transition relies on the Landau free energy ΔF in (23). The order parameter is defined in terms of density fluctuations D of tetrahedrons and is thus a material quantity. Inflation(=crystallization) starts almost immediately after the big bang, at the time when the germ of the hyper-crystal comes into being. This point corresponds to the maximum value of the free energy curve (23) from where the system rolls down to its non-trivial minimum – the moment when the latent heat is released.

A word of warning: although the qualitative features of an exponential expansion are well described by (115), one should be aware that the Einstein equations are not valid close to the crystallization point. According to the discussions after (23) and after (37), the least one must expect is a temperature dependence of c and G which will modify the details of the description. Since I have not quantitatively estimated the amount of energy released in the crystallization, it is even possible, that the dominant part of the growth of the hyper-crystal arises from simultaneous accretion of tetrahedrons and only a minor part is due to subsequent expansion, i.e. to inflation in the proper sense. In that case the accretion would almost instantaneously produce a huge hyper-crystal, and expansion literally would start only at tetrahedral distances of let's say $r \geq 0.1\, L_P$. I will call this possibility scenario X.

2.5.24 What are the tetron model answers to the flatness and the horizon problem?

If one rejects scenario X, they are similar to those in ordinary inflationary models:
–flatness problem (the question why the universe is almost flat everywhere): due to

the exponential expansion, triggered by the initial release of crystallization energy, the universe is much larger than anticipated.
–horizon problem (the question why the universe looks almost the same everywhere): all parts of the universe were causally connected at the time when the hyper-crstal was born. Due to the subsequent exponential expansion they have lost their causal contact.
On the other hand, within scenario X, the universe is extremely large right from the beginning, and also flat because the interactions of tetrahedrons are such that they form a flat hyper-crystal. Concerning the horizon problem: what appears to be a causal contact shortly after the big bang, would be due to the fact that the physics and initial conditions are everywhere the same at the point of accretion.

2.5.25 Is there an inflaton field?

Inflation was explained in section 1 and (2.5.23) as arising from the latent heat released in the crystallization of the hyper-plastics. Therefore in the tetron model, the inflaton can be interpreted as the energy/density wave that carries the initial crystallization energy.

2.5.26 How can the release of crystallization energy lead to metric velocities much larger than the speed of light?

It is well known that metrical changes proceeding faster than c do not contradict Einstein's equations, because in GR there are rules about matter moving through space, but there is no rule about space expanding faster than light. They necessarily occur in the standard cosmological model at times shortly after the big bang.
In the microscopic model metric velocities have a special interpretation due to the fact that an expanding space corresponds to elastic motion of tetrahedrons in the hyper-plastics, cf. (2.5.42) and section 1. Since tetron velocities are not bounded by c, the release of a large amount of crystallization energy at the big bang can push this motion to faster than c. It is only excitations and quasi-particles whose propagation on the hyper-crystal is bounded by c, cf. (2.5.3), (2.5.12), (2.5.13), (36) and (42).

2.5.27 Nature of the forces among tetrahedrons

The elastic forces among the tetrahedrons are responsible for the curvature of spacetime, i.e. for the gravitational interactions. In principle, they should be calculable from the fundamental tetron interaction considered in (2.2.10). Because of the many particles involved, in practice this is not an easy task.

On a heuristic level the forces can be understood as depicted in fig. 7 for a linear chain of tetrahedrons. Longitudinal displacements/accelerations are felt as local or global contraction or expansion of flat physical space and can be used to understand the physics of the FLRW metric. Transverse displacements go into one of the 3 internal dimensions thus inducing genuine extrinsic spatial curvature.

Neither of the two can explain the appearance of a gravitational field in the Newtonian limit, i.e. the potential (27) appearing in the g_{00} component of the metric (26). To account for that, a 'timely' curvature, i.e. variations of the hopping time T_P are needed, i.e. the time a quasi-particle needs to travel(=be emitted, run, get absorbed) from one tetrahedron to its neighbor. These variations happen, because the presence of mass/energy modifies the microscopic processes behind the hopping of any 'test excitation'. Note that the Newton effect has an extremely simple form - it is proportional to M and inversely proportional to r, and actually arises from the elastic nature of the tetronic environment in which the excitation propagates, cf. (42).

In more formal terms the FLRW cosmology relies on a longitudinal spatial expansion $r \to a(t)r$ in the sense of fig. 7a, while the Newtonian limit (26) corresponds to a change of the local time and space variables according to

$$t \;\to\; t(1+\frac{GM}{c^2r}) \tag{116}$$

$$r \;\to\; r - r_p + \frac{GM}{c^2}\ln\frac{r}{r_p} \tag{117}$$

in the sense of (24). r_p arises from an integration constant and can be chosen to be the position of the local observer.

According to (117), switching on a Newtonian gravitational field, not only the hopping time is modified as discussed above, but also the tetrahedrons are concentrating near the source. For a comoving observer it appears that they have moved away from him, their distance increased by a factor $1+\frac{GM}{c^2r_p}$.

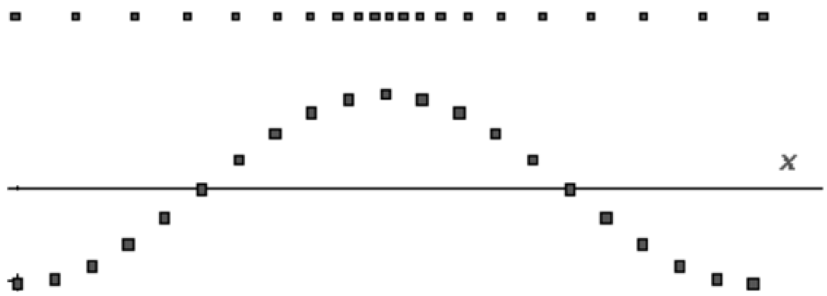

Figure 7: This picture illustrates 'longitudinal' vs 'transverse' curvature for a 1-dimensional chain of tetrahedrons. The tetrahedrons are drawn as tiny black squares. Physical space is represented by the x-axis, similar to fig. 2. Longitudinal displacements/accelerations (fig. 7a) are felt as local or global contraction or expansion of physical space like in the FLRW metric. Transverse displacements (fig. 7b) go into the internal dimension. Modifications of the hopping time (relevant for the Newtonian approximation) are not drawn in the figure.

Normally these effects are tiny, however for $r \to 0$ or $GM \sim c^2$ the Newton approximation breaks down, and the Schwarzschild metric

$$ds^2 = (1 - \frac{2GM}{c^2 r})(cdt)^2 - (1 - \frac{2GM}{c^2 r})^{-1}(dr)^2 \tag{118}$$

should better be used. Based on Kruskal's coordinates, a similar analysis as in the Newtonian case can be carried out in order to obtain the rearrangement of tetrahedrons near a black hole. Details on the interpretation of black holes in the tetron model will be given in (2.5.39).

I will now end the discussion of the effects of point masses and return to the question to what extent the standard FLRW cosmology is modified by the tetron model. As described in section 1, the underlying idea is the existence of an elastic *and curved* 3-dimensional space embedded in R^6. The possibility to have curvature is important here, otherwise one would have a flat elastic system which in the Einstein framework would correspond to 'pure gauge' configurations (137), which do not affect ordinary mignon matter. Note that curvature in the time direction corresponds to accelerations of tetrahedrons within the elastic medium. It can be combined with spatial curvature in the way described in section 1 [45, 46].

According to the analysis in (2.2.10) the force among the tetrahedrons may be of such a form that these are eventually driven towards an equilibrium distance r_0 corresponding to a minimum of the inter-tetrahedral potential. In that case the force can be expanded in powers of $r - r_0$, and in the leading harmonic approximation it is of Hooke's form

$$m_T \ddot{r} = -d_T (r - r_0) \tag{119}$$

where m_T is the tetrahedron mass and

$$\omega_T^2 = d_T / m_T \tag{120}$$

the frequency characteristic for the interaction.
In an elastic medium of many tetrahedrons this interaction corresponds to

$$\rho_T \ddot{u} = \zeta u'' \tag{121}$$

where ζ is the Lame parameter introduced in (35), u the deformation vector and ρ_T the mass density (36) of tetrahedrons. Note that eq. (36) actually corresponds to a tetrahedron mass

$$m_T = \rho_T L_P^3 = M_P \tag{122}$$

In other words, the tetron masses and binding energies within one tetrahedron plus the binding energy of one tetrahedron within the hyper-crystal sum up to define the Planck mass M_P.
Since ζ can be traced back to Hooke's constant via

$$d_T = \zeta \, r_0 \tag{123}$$

it can be considered as Hooke's constant per unit of length.
According to (121) one expects elastic waves at speed

$$c^2 = \zeta / \rho_T \tag{124}$$

in agreement with (36). It must be noted, however, that this kind of elastic wave can only be perceived by mignon matter, i.e. by human observers, if it is associated to a non-vanishing curvature, i.e. is not just a pure gauge transformation (137). In that case it constitutes a gravitational wave.

There is another caveat in these considerations, and this concerns the use of the non-equilibrium quantities ζ, ρ_T etc in the above equations. These were introduced in section 1 to describe the properties of the universe today. However, since cosmic expansion has not reached the equilibrium r_0 but currently corresponds to an average distance $r = L_P < r_0$ between tetrahedrons, quantities ζ and ρ_T in (123) and (124) differ from the values (36) and (37) which they take at present. It is true that there will always be a minimum in the inter-tetrahedral potential, so that a harmonic approximation is reasonable; however, the values of the potential parameters certainly depend on cosmic time.

For reasons of simplicity I will ignore this point in the following; in other words, I will assume that the time dependence of these parameters can be neglected. Under this assumption, one obtains

$$\omega_T \equiv \sqrt{\frac{d_T}{m_T}} = \frac{1}{T_P} \tag{125}$$

This is an extremely high frequency, which evidently does not play any role in the universe's expansion. The point to note is that in the presence of the densely packed many other tetrahedrons, a given tetrahedron is not able to immediately attain the relaxed equilibrium distance r_0 to its neighbors. Instead, due to the large stiffness (35) of the DMESC, the characteristic time for this to happen is much larger. In other words, there exists another characteristic frequency $\omega_M \ll \omega_T$ contributing to the enhanced expansion of the hyper-crystal, so that (115) should be replaced by

$$\ddot{a} = -\frac{4\pi}{3} G(\rho + 3p/c^2)a - \omega_M^2(a - a_0) \quad [+\Lambda_C c^2 a] \tag{126}$$

where ρ and p are the (mignon) matter density and pressure, respectively, and in a matter dominated universe one has $\rho \sim 1/a^3$ as usual.

For reasons of comparison I have also added a cosmological constant term. It may be noted that apart from the constant $\omega_M^2 a_0$ it is of the same form as the harmonic term. Actually, all of the terms in (126) can be interpreted to arise from the total energy-momentum of the tetrahedral system, an issue which is further discussed in (2.5.30).

It is important to note that for a universe expanding towards equilibrium a_0 one has $a < a_0$ so that the sign of the harmonic term is the same as that of a Λ_C term with a positive cosmological constant. If one analyzes this more closely, it

turns out that it is possible to accommodate the observed dark energy effect to a harmonic contribution $\omega_M \neq 0$ instead of a cosmological constant provided one chooses $\omega_M \approx 10^{-18}$Hz (very small as compared to $\omega_T \approx 10^{43}$Hz).
The 2 terms on the rhs of (126) are then of the same order. The first one leads to the well-known $t^{2/3}$ behavior of a(t) for a dust dominated universe. For larger values of a and t the second term becomes relevant, with $a \sim a_0(1-\cos(\omega_M t))$. We live at $0 \ll a \ll a_0$. This condition corresponds to times where the universe has started to re-accelerate but does not yet feel the equilibrium.
Note that for a proper treatment of (126) the temperature dependence of a_0 must be taken into account

$$a_0 \sim (T_c - T)^{-\frac{1}{6}} \tag{127}$$

where $T < T_c$ and T_c was introduced in (23).
Finally, it should be stressed that the two characteristic frequencies ω_T and ω_M introduced above are related to Newton's constant and to the cosmological constant, respectively. While the former describes the reaction of the tetrahedral coordinates to isomagnetic and other excitations, the latter is a collective effect of the densely packed hyper-crystal.

2.5.28 How is the curved hyper-crystal embedded in R^6?

In this section I will show that it is not very difficult to embed the standard solutions of the Einstein equations like the FLRW cosmology and the Schwarzschild black hole into the flat 6(+1) dimensional space, which according to section 1 represents the basis of the tetron model. As will be seen, it is actually enough to have 1 additional spatial dimension instead of 3, i.e. an R^4 instead of an R^6; and as a matter of fact it remains unclear at this moment, whether this one additional dimension should be chosen to be one of the isospin directions or as the seventh spatial dimension on which I speculated in (2.5.11).
Consider, for example, the case $k = 1$ of a closed FLRW universe (103), where the spatial part of the FLRW metric describes a 3-sphere S^3. In terms of R^4 coordinates

this is given by

$$u_1 = a\cos\phi \tag{128}$$

$$u_2 = a\sin\phi\cos\theta_1 \tag{129}$$

$$u_3 = a\sin\phi\sin\theta_1\cos\theta_2 \tag{130}$$

$$u_4 = a\sin\phi\sin\theta_1\sin\theta_2 \tag{131}$$

and the well known S^3 line element

$$ds^2 = a^2[d\phi^2 + \sin^2\phi(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2)] \tag{132}$$

can be identified as the spatial part of the FLRW metric via the identification $\vec{x}^2 = \sin^2\phi$ because of $d\phi^2 = d\vec{x}^2/(1-\vec{x}^2)$.
One is thus led to the conclusion, that such a structure - as well as small disturbances of it - can always be embedded into R^4 and therefore also in R^6 or R^7.
The situation is similar for the Schwarzschild geometry (118). To understand that point, one may consider the Schwarzschild singularity at r=0 in the rest frame of the hyper-crystal. While the curvature in the time direction is due to a modification of the hopping time as explained in section 1, the spatial component of (118) is obtained from local R^4 coordinates u_1, u_2, u_3 and

$$u_4 = 2\sqrt{r_b(r - r_b)} \tag{133}$$

where $r^2 = u_1^2 + u_2^2 + u_3^3$ and $r_b = \frac{2GM}{c^2}$ is the Schwarzschild radius.

2.5.29 Is there a similarity to the behavior of superfluids?

There is a certain similarity, but there are also appreciable differences.
The growth and expansion of the elastic hyper-crystal along a lower (in this case 3) dimensional structure is reminiscent of the behavior of superfluids, which can creep along arbitrary surfaces. There are 2 prototypes of superfluids whose representatives are He-3 and He-4. While in He-3 fermion condensates populate the macroscopic quantum state, in He-4 superfluidity is a consequence of a normal Bose-Einstein condensation of the He-4 bosons. There are actually speculations that associate gravity to a He-3 type of superfluid[90]. In the tetron model, however, it is the tetrahedrons and not the tetrons themselves which are involved in the gravitational

interactions. The tetrahedrons are bosons, so the ordinary Bose-Einstein approach to superfluidity could be useful to describe the growth and expansion of the hyper-crystal. More precisely, the Bose-Einstein condensate of tetrahedrons within the hyper-crystal would obey a Gross-Pitaevskii equation in 3 dimensions[101]

$$i\hbar\frac{\partial}{\partial t}D(\vec{r},t) = [-\frac{\hbar^2}{2m}\Delta + \alpha|D(\vec{r},t)|^2]D(\vec{r},t) \tag{134}$$

Although it fulfills a (nonlinear) Schrödinger equation and is sometimes called the 'wave function' of the condensate, D is a classical field and can be identified with the order parameter of the system, i.e. with the density fluctuations of tetrahedrons introduced in (23).

One should, however, be hesitant to accept the idea of a superfluid, for the following reasons:

–quite in general superfluidity is a low temperature effect. There is no indication of an additional cosmic phase transition in the low energy regime.

–in the last 10 billion years the universe has expanded rather slowly. This is not what one would expect from a *super*fluid.

–in a superfluid there is a macroscopic quantum state governed by a Planck constant h, as appears in (134). This picture is different from the interpretation (2.5.6) of quantum theory on the hyper-crystal and the model with elastic Hooke's inter-tetrahedral forces advocated in sections 1 and (2.5.27).

As a result I prefer to consider the DMESC in its ground state to be a perfect fluid (not a superfluid), and by definition without any stresses. Only if mignons and other excitations are put into the system, this will produce stress and strain, thus modifying metric and curvature in the way discussed in section 1 and giving rise to the effects of gravity.

2.5.30 Is energy conserved in the tetron dynamics?

Yes. This is in contrast to general relativity, where energy is only 'covariantly' conserved, i.e. the energy momentum tensor fulfils $D^\mu T_{\mu\nu} = 0$ where D^μ is the covariant derivative involving the Christoffel symbols. This point can be understood by considering the Einstein equation $G_{\mu\nu} = \kappa T_{\mu\nu}$, which relates the energy-momentum $T_{\mu\nu}$ of matter(=mignons) to the intrinsic curvature equivalent to displacements of tetrahedrons, in the following way: the Einstein equation can be re-interpreted by saying that the total energy momentum of the universe with contributions $G_{\mu\nu}$ (from the tetrahedrons) and $T_{\mu\nu}$ (from the mignons) is not only constant but actually zero, and that energy can be shifted from one side of the equation to the other, i.e. from mignons to tetrahedrons and vice versa. A famous example is the red shift of photons in the expanding universe where the photons lose energy to the FLRM metric. In the tetron model, this is not the whole story. As discussed after (37), the vacuum energy density of the universe does not only consist in the (small) contribution from the cosmological constant responsible for dark energy, cf. (2.5.37), and the (somewhat larger) contribution from the QCD and electroweak symmetry breaking vacua, but there is also a contribution

$$V_T \sim \Lambda_P^4 \tag{135}$$

to the vacuum energy from the tetronic ground state energy fig. 2. This contribution arises as the product of the energy of a bound tetrahedron ($\sim \Lambda_P$) and its inverse volume ($\sim \Lambda_P^3$) and is very large, because the tetrahedrons are closely packed and strongly bound within the hyper-crystal.

In the framework of quantum field theories the vacuum energy density arises as the sum of all zero point oscillations. For example, the vacuum energy for a free field is the sum of the ground state energies $\omega_k \sim \sqrt{k^2 + m^2}$ of all oscillator modes k. For a cubic box of side-length L_P, one has to sum over integers $n = kL_P/2\pi$, and in the continuum limit this turns into an integral

$$\int \sqrt{k^2 + m^2} d^3k \sim \Lambda_P^4 \tag{136}$$

Using the Planck scale as a natural cutoff, this is again much larger than the cosmological constant and the energies of the QCD and electroweak symmetry breaking vacua. In field theory[63] it is usually renormalized away, so that it does not appear in any particle or gravitational energy balance consideration. By contrast, in the tetron model it must be included and added to the ground state energy of the hyper-crystal, which according to (135) is of the same order of magnitude.

There is also a third contribution to be included in tetron energy considerations, and that is elastic energy $\sim (\partial_\mu u_\nu + \partial_\nu u_\mu)^2$ from displacements of tetrahedrons with vanishing curvature. It is true that these contributions do not matter in the energy balance of ordinary matter, because mignons do not respond to these 'flat' elastic deformations (pure gauges)

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu u_\nu + \partial_\nu u_\mu \tag{137}$$

For the tetron dynamics, however, they are relevant.

2.5.31 We are living in a rather cold universe. Why don't we see a transition from the elastic hyper-crystal to a rigid crystalline structure at zero temperature?

One could speculate that the present temperatures of the universe are not small enough for this phase transition to occur, or that pressure is required like for the solidification of He-3.
In such a rigid crystal no shifts of the tetrahedrons would be allowed, and therefore one expects gravitation to completely disappear.
On the other hand, this remark is only true in the absence of mignons. When a mignon excitation is present, its mass/energy induces a stress in the hyper-crystal which at least locally re-liquidizes the system. In other words, the mignon's mass/energy is automatically accompanied by curvature, i.e. a by non-vanishing gravitational potential.

2.5.32 Are the internal spaces compact or infinite?

They are infinite, no assumption about compactification of internal spaces needs to be made. The reason why we cannot step into the internal dimensions is because we are built from quasi-particles and thus cannot leave the mono-layered hyper-crystal(=our universe), which is so to say a flat 'carpet' embedded in R^6.
According to fig. 2, the hyper-crystal is restricted to a 3+1 dimensional 'surface' in R^{6+1}. Going away from this 'surface', internal space is empty, because according to (2.5.33) and (2.5.10) ordinary matter cannot dissipate into the internal dimensions. Exception: other hyper-crystals may have condensed at big bang times. In general these will lie skewed with respect to the one we live in and form separate 'universes', cf. (2.3.24) and (2.5.38).

2.5.33 How can one avoid dissipation of energy into the internal dimensions?

Since internal space is infinite, matter and energy could in principle disappear into it. This could happen in the form of particles which move in the direction orthogonal to the hyper-crystal. However, in (2.1.2) and (2.1.3) I have taken the viewpoint that all observed particles including the photon are excitations of the crystal and as such cannot exist away from it. Furthermore, the tetrons, from which the crystal is made, are assumed to be so strongly bound, that they can be split off the hyper-crystal only by supply of Planck scale energies.

2.5.34 Was there a GUT era in the early universe where electroweak and strong couplings were unified?

No, because there is no GUT – cf. (2.3.22). In the tetron model the strong force has a different origin than the electroweak one (cf. 2.3.29), and thus GUT unification seems unlikely.

The proper history of the universe starts with the end of the crystallization process (=the inflation era), at which point electromagnetism and weak forces are unified, cf. (2.1.11). In the standard terminology this is the starting point of the radiation dominated era, with photons, effectively massless W/Z and dark matter as the dominant excitations. At the end of the electroweak era at temperatures of order Λ_F there is the alignment of isospin vectors corresponding to the electroweak phase transition which gives masses to q/l and W/Z.

2.5.35 Is there a unification of the SM and gravitational forces at the Planck scale?

Not in the sense of supergravity and related models. Even at big bang temperatures gravity and SM forces have a very different nature. Although it is true that everything observed can eventually be derived from the fundamental forces among tetrons, the SM interactions trace back to interactions between isospin vectors of tetrons, whereas gravity is an elastic force between tetrahedrons which stems from remnant tetron coordinate interactions.

The question, why the forces of gravity appear to be much weaker than particle physics effects, has been treated in (2.3.19).

2.5.36 Are there dark matter candidates in the model?

Yes, there are several possibilities:
–further internal excitations of the crystal, like phinons, cf. (2.3.26) and [3]. Their interactions with mignons(=ordinary matter) is tiny, because they are not involved in the isomagnetic correlations giving rise to the SM.
–the pseudoscalar η arising in the 2HDM ansatz (2.1.13). Such a possibility is widely discussed in the literature [67, 68] provided the η is inert, i.e. does not interact with quarks and leptons. This condition can be fulfilled in the present model essentially because of (59). Note that right after inflation there is the radiation dominated era where the inert scalar is copiously produced together with a soup of many other tetron-antitetron bound state excitations (photon, W/Z, Φ and Φ').

2.5.37 Is there an explanation for dark energy?

Observations indicate that the universe's expansion rate was decelerating until about 5 billion years ago, after which time the expansion began re-accelerating. Phenomenologically, this can be explained by including a cosmological constant Λ_C in the theory (this amounts to saying that a volume in space has some intrinsic fundamental vacuum energy creating a pressure which makes the universe expand) or by a weakly fluctuating scalar 'quintessence' field (this does a similar job).
Furthermore, such a type of energy that is not matter or dark matter is also needed to explain the apparent flatness of the universe (absence of any detectable global curvature). According to that argument the contribution of dark energy should be more than twice as large as that of matter and dark matter together.
A third explanation of the dark energy effect is offered within the f(R,T) models (29) discussed in section 1 by suitable accommodation of the 11 phenomenological coupling constants[60].
To understand dark energy within the tetron model one should remember, that the micro-elastic forces that have initially induced cosmic expansion during the big

bang crystallization process are still at work today. For example, there may still be accretions to the hyper-crystal at its edges which are setting free large amounts of crystallization energy. The energy is then transferred to the other tetrahedrons of the crystal in the form of weakly fluctuating energy/density waves traveling through the universe. This picture is well along the line of the quintessence idea mentioned above.

Another possibility is that the increased acceleration arises, because the average distance L_P between tetrahedrons has not yet reached its equilibrium value r_0. This line of argument was followed in (2.5.27) and gives rise to a self-contained explanation of the dark energy effect.

2.5.38 Are there other universes?

Probably yes. One may consider the original $R^{(6,1)}$ spacetime as a container of universes. When the tetron gas cooled down and temperatures reached the crystallization temperature (Planck energy), germs of 3+1-dimensional hyper-crystals came into being in various places of $R^{(6,1)}$, cf. (2.3.24). These crystals then grew in their respective 3+1 dimensionsal subspaces, each of them making up for a separate $R^{(3,1)}$ spacetime. Since they are of low dimension as compared to the whole $R^{(6,1)}$, they hardly interfere with one another. If at all, they intersect in isolated (1-dimensional) points. At those points a defect in the isomagnetic and/or coordinate crystal structure will show up, because the isospin and/or coordinate vectors of the tetrons do not know how to orient themselves.

2.5.39 On the interpretation of black holes in the tetron model

Before discussing the fate of a black hole in the tetron model, some general wisdom on black holes in Einstein's gravity will be listed:

(i) When a mass M is squeezed into a roughly spherical volume of size smaller than $2GM/c^2$, it collapses into a black hole. The gravitational forces at the boundary are so strong that photons or other quasi-particle excitations cannot leave the black hole to reach an outside observer.

(ii) The boundary of a black hole at $r = 2GM/c^2$ is nothing material, but just a

boundary to an external observer for not seeing light from the black hole.
(iii) the energies at each point inside a stellar or galactic black hole are relatively small and not large enough to restore the electroweak isospin symmetry.
(iv) When inside a black hole, any mass will be further accelerated towards its center. Due to this fact a singularity at $r = 0$ will arise. Near the singular point the Einstein equations loose their validity.
Now let's have a look on black holes from the tetron model point of view. At first sight one may think that the accretion of matter at $r = 0$ will eventually lead to a dissolution of the hyper-crystal and a corresponding evaporation of tetronic material into the full R^6 space.
However, this would only be true for unrealistically small black holes of Planck size L_P. In general, the energies at single sites are simply not large enough, even at $r = 0$. The point to be noted is that according to section 1 in the tetron model the Einstein equations arise from the effective action (29) for the DMESC, but are not applicable at arbitrary small distances / high energies, where the discrete structure becomes perceptible. In particular there will be no mathematical singularity at $r = 0$ in case when two mignon excitations approach each other at a distance r. Instead, what happens is that as soon as they become direct neighbours on the hyper-crystal lattice, they cannot come any closer. There is still a mutual attraction which deforms the hyper-crystal a bit, but the 2 mignons cannot come closer than roughly L_P. This is the reason why there is no singularity in tetron model gravitation.
If one assumes maximum mignon masses to be of order Λ_F, i.e. between 1 GeV and 1 TeV, one can predict the volume of the core of a black hole of mass M to be $L_P^3 Mc^2/\Lambda_F$:
-for a stellar black hole with mass 10^{31} kg and a Schwarzschild radius ~ 10 km one obtains a core radius of $\sim 10^{-13}$ meters
-for a galactic black hole with mass 10^{45} kg and a Schwarzschild radius of ~ 0.01 light years one obtains a core radius of $\sim 10^{-10}$ meters
These values are tiny but still much larger than the Planck length. They do not suffice to dissolve the hyper-crystal, simply because the maximum excitation energy carried by an excitation on a crystal site is Λ_F, which is much smaller than the value Λ_P needed to dissolve the crystal at that site.

2.5.40 Should gravity be quantized?

As emphasized in (2.5.6), the quantum behavior of nature is closely related to the granularity of physical space. Therefore it seems natural to believe that in circumstances where this discrete structure becomes relevant, gravitational effects should be treated in a quantum theoretical manner.
However, gravity in the tetron model is an effective interaction of (internal) tetrahedrons in an elastic/plastics system. Its description by the Einstein-Hilbert action or its generalization (29) is valid only at distances $\gg L_P$, i.e. looses its validity when probed at distances where the discrete structure becomes apparent, cf. the discussion in (2.5.39). Instead of 'quantizing' gravity one should quantize the fundamental interaction among tetrons.

2.5.41 Why is the speed of gravitational waves equal to the speed of light? Why is there a universal maximum speed for all the objects in the universe?

This question has already been answered in (2.5.12), (2.5.13) and at the end of section 1. Summarizing those arguments one may say that the propagation part in the dispersion relation for mignons is determined by the same elastic properties as that of gravitational waves. In this connection it is interesting to note that gravitational waves are not deformations into the 3 internal dimensions, but they deform the 2 transverse directions in ordinary space, i.e. the plane perpendicular to the propagation direction. For a plus-polarized plane gravitational wave, for example, with polarization tensor $\epsilon_{\mu\nu} \sim$ diag $(0, 1, -1, 0)$ the line element is $ds^2 = (cdt)^2 - \exp(i\omega(t-z))(dx^2 - dy^2)$ and the curvature is non-vanishing due to the exponential factor.
In the remainder of this section I want to add some more comprehensive remarks about the topic of gravitational waves: in section 1 general relativity has been interpreted as an effective theory for an elastic system of internal tetrahedrons. Gravitational waves exist in this theory on the classical level, as solutions to the Einstein equations. They are metrical waves, whose velocity is forced to be equal to the speed of light by the condition of local Lorentz invariance. In the tetron model

they can be associated to some of the density fluctuations of tetrahedrons[9] discussed in connection with (23).

In the tetron theory elastic waves propagate at the speed (124) of a typical excitation in an elastic medium, and thus at the same speed as photons and massless mignons, cf. (42). At first sight this looks like a rather amazing feature, because it was argued in (2.5.35) that the isomagnetic particle physics interactions of the photon and the elastic gravitational interactions do not have much in common.

In order to get a clearer understanding one should realize that the world according to the microscopic model falls apart into 2 rather disparate pieces:

-the realm of what philosophers would call emergent or appearance phenomena, i.e. isomagnetic quasi-particles like quarks, leptons, Higgs and gauge fields. Since all these excitations fulfill Lorentz invariant wave equations, any phenomenon and signal propagation in this sphere is necessarily limited by the speed of light.

-the realm of what could be called 'true' or tetron matter, consisting of tetrons, of aligned tetrahedrons and of the DMESC with its elastic/metric structure. This may rightfully be called $'\upsilon\lambda\eta\ \ \pi\rho\omega\tau\eta$. However, while for Aristotle this was more an idea than a concrete material, here it can be understood in a real, *materialistic* sense. Just for joke one could call it 'tatter' to distinguish it from the ordinary mignon material which remains m-atter.

Since the relevant scales $\Lambda_P \gg \Lambda_F$ are so vastly different, these two spheres do not have much in common. We ourselves live in the sphere of appearances and can perceive anything coming from the tatter sector only if suitable devices of mignon matter are patched in between. Gravity, for example, which originally corresponds to a shift of tetrahedron locations on the DMESC, becomes visible in our physical world only due to the small and stiff reaction to suitable conglomerations of m-atter. In particular, the physical effects of a gravitational wave can only be seen by plugging appearances in between, i.e. m-atter which 'rides' the gravitational waves. The principle of relativity states, that m-atter must respect local Lorentz invariance. Therefore, although the fundamental interactions among tetrons may proceed at larger velocities than c, the gravitational interactions between m-atter particles always appear to proceed at c.

[9] In distinction the *internal* translational excitations were coined phinons in [3], while the internal rotational excitations are mignons, i.e. quarks and leptons.

Let me repeat that these arguments are supported by considering the dispersion relation for mignons (41) and (42). As shown at the end of section 1, mignon propagation can be completely described by c, which itself according to (36) is given in terms of tetron variables as $c^2 = \zeta/\rho_T$.

2.5.42 How can metric velocities larger than c be interpreted in the tetron model?

This question is related to the discussion in (2.5.14),(2.5.15), (2.5.16), (2.5.26) and (2.5.41). According to (40)-(42), c is the maximum speed for all the isomagnetic quasi-particles that build our known universe. However, this limit does not apply to tetrons nor to the bound tetrahedrons which make up the hyper-crystal and are the carriers of the quasi-particles. As evident from (24), in the tetron model metrical changes are associated to displacements of tetrahedrons. The corresponding velocities of the tetrahedrons have been particularly large ($> c$) in the inflationary period shortly after the big bang (crystallization) where a lot of crystallization energy had been released.
According to (104) one can roughly identify the metric velocity in an FLRW universe with the Hubble flow Hd. In the tetron model this can be interpreted as the relative velocity of 2 tetrahedrons at distance d.

3 Conclusions

The present review is devoted to a model which tries to give a microscopic meaning to physical phenomena usually described by the Standard Model of elementary particles. By introducing an additional level of matter one is able to understand and to calculate known particle properties (like the quark and lepton masses and mixings) from first principles and furthermore to make predictions for future experiments. Most prominent among the latter are:
–the existence of a second Higgs doublet similar as in inert 2HDM models[67, 68].
–the existence of a fourth family of quarks and leptons. This family, however, is distinct from the other three, not only because it has a very massive neutrino but also because its couplings are not given by the SM. The point is that the 8 Dirac

particles of this family do not arise from vibrations of iso-'magnetizations' $\psi^\dagger\vec{\tau}\psi$ but of internal 'densities' $\psi^\dagger\psi$, and therefore they do not obtain their masses via the Higgs mechanism and the SSB fig.2.

After discussing particle physics properties, implications of the tetron model on the big bang and on phase transitions in the early universe have been elucidated. This has led to the idea that besides the
–isomagnetic interactions among aligned isospin vectors which are relevant for particle physics
one should consider 2 other forces:
–strong rigid coordinate forces among tetrons fixing the form and extension of the (internal) tetrahedrons.
–weak elastic forces between these tetrahedrons, which are the basis of the gravitational interactions.
All 3 types of forces are assumed to derive from one universal interaction among the tetrons, and it was suggested that one should use octonion multiplication and perhaps supersysmmetry as a guideline which eventually will lead to the correct renormalizable theory in 6+1 dimensions.

The various viewpoints on the model presented in the preceding sections have supplied a set of important requirements as to the nature of tetron interaction. We already know that
–tetrons have a tendency to form excited pairs with antitetrons of neighboring tetrahedrons in the crystal with similarity to Cooper pairs or Frenkel excitons of solid state physics.
–tetron bonds are extremely short, of the order of the Planck length.
–they get saturated in tetrahedral configurations.
–these configurations form 3-dimensional monolayer crystal structures, i.e. there is no stacking of tetrahedrons on top of each another, no growth of the hyper-crystal into internal directions.
–isospins within the quartets of tetrons are maximally frustrated (fig. 1).
–once tetrons are in such a saturated hyper-crystal configuration, there is a left-over elastic force among the internal tetrahedrons, which gives rise to the gravitational interactions.

In summary a new picture of the physical world was presented. It was shown that
–quarks and leptons can be interpreted as internal magnons of a discrete iso-magnetic structure and that their spectrum is due to a tetrahedral symmetry group which remains unbroken down to the lowest energies
–the SU(2)×U(1) gauge group of the SM is related to an iso-magnetic 'Heisenberg' SU(2) and the SM SSB can be obtained from a global ordering of internal magnets
–the big bang is due to a crystallization process which has released a large amount of latent energy and has led to the formation of a rapidly expanding 3+1 dimensional universe
–the physical world including the gauge bosons consists of excitations which travel as quasi-particles through the hyper-crystal. Since they are described by relativistic wave equations, this leads to the appearance of Minkowski space (and the associated Lorentz structure).
–an 'elastic' rest system of the hyper-crystal exists which is identical to the cosmological comoving coordinates. However, due to the quasi-particle nature of our physical world this cannot be observed in Micholsen-Morley type of experiments.

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