Multi-period medical diagnosis method using a single valued neutrosophic similarity measure based on tangent function

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Abstract

Background and Objective: Similarity measures play an important role in pattern recognition and medical diagnosis, and then existing medical diagnosis methods deal scarcely with the multi-period medical diagnosis problems with neutrosophic information. Hence, this paper proposed similarity measures between single-valued neutrosophic sets (SVNSs) based on tangent function and weighted similarity measures of SVNSs considering the importance of each element. Then, a multi-period medical diagnosis method using the proposed similarity measure was developed to solve multi-period medical diagnosis problems with single-valued neutrosophic information.

Methods: The proposed similarity measures between SVNSs were presented based on the tangent function. Then, we compared the proposed similarity measures of SVNSs with existing cosine similarity measures of SVNSs by a numerical example about pattern recognitions to indicate their effectiveness and rationality. In the multi-period medical diagnosis method, we can find a proper diagnosis for a patient by the proposed similarity measure between the symptoms and the considered

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diseases represented by SVNSs considering the weights of multi-periods. Then, a multi-period medical diagnosis example was presented to demonstrate the application and effectiveness of the proposed diagnosis method.

Results: the comparative results of pattern recognitions demonstrated the effectiveness of the proposed similarity measure of SVNSs based on the tangent function. By a multi-period medical diagnosis example, the diagnosis results show the application and effectiveness of the developed multi-period medical diagnosis method.

Conclusions: the proposed similarity measures of SVNSs based on the tangent function are effective and reasonable in pattern recognitions, and then the developed multi-period medical diagnosis method is very suitable for handling the multi-period medical diagnosis problems with single-valued neutrosophic information and demonstrates the effectiveness and rationality of the multi-period medical diagnosis method.

Keywords: Neutrosophic set; Single valued neutrosophic set; Similarity measure; Tangent function; Multi-period medical diagnosis

1. Introduction

In medical diagnosis problems, due to the complexity of various diseases and the lack of knowledge or data about the problem domain, crisp data are sometimes unavailable as medical diagnosis contains lots of uncertainties. Uncertainty is an important phenomenon of medical diagnosis problems. A symptom is an uncertain indication of a disease. Hence, the uncertainty characterizes a relation between symptoms and diseases. Thus, working with the uncertainties leads us to accurate decision making in medical diagnosis problems. In most of the medical diagnosis problems, there exist some patterns, and then experts make decision according to the similarity between an unknown sample and the basic patterns. However, the arguments introduced from an unknown sample may be vague or fuzzy in nature. To represent incomplete and uncertainty information, Zadeh [1] firstly proposed fuzzy set theory. Its characteristic is that a membership degree is assigned to each element in the set. Since then, various extensions of this concept have been introduced by many researchers. For example, Atanassov [2] extended fuzzy sets to intuitionistic fuzzy sets (IFSs). The prominent characteristic of IFS is that a membership degree and a non-membership degree are assigned to each element in the set. Since there is a number of uncertainties in medical diagnoses, they are the most interesting and fruitful areas of applications in intuitionistic fuzzy set theory. Hence, various medical diagnosis methods have been presented under the general framework of IFSs [3, 4]. Recently, Ye [5] put forward cosine similarity measures for IFSs and applied them to pattern recognition and medical diagnosis. Hung [6] introduced an intuitionistic fuzzy likelihood-based measurement and applied it to the medical diagnosis and bacteria classification problems. Tian [7] developed the cotangent similarity measure of IFSs and applied it to medical diagnosis.

However, IFSs can handle only incomplete and uncertainty information but not indeterminate and inconsistent information which exists usually in real situations. To deal with this kind of indeterminate and inconsistent information, Smarandache [8] originally proposed the concept of a neutrosophic set from philosophical point of view. A neutrosophic set A in a universal set X is characterized independently by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, such that $T_A(x)$: $X \rightarrow]^-0, 1^+[$, $I_A(x)$: $X \rightarrow]^-0, 1^+[$, and $F_A(x)$: $X \rightarrow]^-0, 1^+[$. However, the defined range of the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ in a neutrosophic set A is the non-standard unit interval $]^-0, 1^+[$, it is only used for philosophical applications and is difficult to apply in science and engineering areas. To easily use it, the defined range of functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ can be restrained to the normal standard real unit interval [0, 1]. For this purpose, Wang et al. [9] introduced the concept of a single valued neutrosophic set (SVNS), which is a subclass of the neutrosophic set and a generalization of IFS. Because of the increased volume of information available to physicians from advanced medical technologies, there is possible information of each symptom with respect to a disease including different truth-membership, indeterminacy-membership and falsity-membership degrees. Since SVNS consists of the three terms like the truth-membership, indeterminacy-membership and falsity-membership functions, it is considered to be a proper tool for representing them. However, similarity measure is a key role in the analysis and research of medical diagnosis, pattern recognition, machine learning, decision making, and clustering analysis in uncertainty environment. Therefore, some researchers have proposed various similarity measures of SVNSs and mainly applied them to decision making. For instance, Majumdar and Samanta [10] introduced several similarity measures of SVNSs based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Ye [11] further proposed the distance-based similarity measure of SVNSs and applied it to group decision making problems with single valued neutrosophic information. Furthermore, Ye [12] proposed three vector similarity measures for simplified neutrosophic sets (SNSs), including the Jaccard, Dice and cosine similarity measures for SVNSs and interval neutrosophic sets (INSs), and applied them to multicriteria decision-making problems with simplified neutrosophic information.

Recently, Ye [13] further proposed the improved cosine similarity measures of SVNSs and INSs based on the cosine function and applied them to medical diagnosis problems. Whereas, one of the medical diagnosis questions is whether only by taking single period inspection we can reach a conclusion for a particular patient with a particular decease or not. Sometimes he or she may show symptoms of different diseases also. Then, how can we reach a proper conclusion for the particular patient? One solution is to examine the patient through multi-periods and to realize comprehensive diagnosis for the patient. In this case, multi-period medical diagnosis is a more adequate method, but the medical diagnosis method using the cosine similarity measures proposed in [13] is a single period diagnosis and cannot deal with the multi-period medical diagnosis problem. Furthermore, till now existing literature deal scarcely with the multi-period medical diagnosis problems with neutrosophic information. Hence, this paper aims to propose new similarity measures of SVNSs based on tangent function and a multi-period medical diagnosis method considering the weights of multi-periods.

The rest of the article is organized as follows. Section 2 introduces some basic concepts of SVNSs and cosine similarity measures for SVNSs. Sections 3 puts forward similarity measures of SVNSs based on tangent function and weighted similarity measures of SVNSs and investigates their properties. In Section 4, we propose a multi-period medical diagnosis method based on the proposed cotangent similarity measure. In Section 5, a multi-period medical diagnosis example is provided to illustrate the application and effectiveness of the multi-period medical diagnosis method. Conclusions and further research are given in Section 6.

2. Some basic concepts of SVNSs

Smarandache [8] originally presented a neutrosophic set theory. A neutrosophic set A in a universal set X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. The three functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, such that $T_A(x)$: $X \to\ ]^-0, 1^+[$, $I_A(x)$: $X \to\ ]^-0, 1^+[$, and $F_A(x)$: $X \to\ ]^-0, 1^+[$. Hence, the sum of the three functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

However, Smarandache [8] introduced the neutrosophic set from philosophical point of view as a generalization of fuzzy set, IFS, and interval-valued IFS. But it is difficult to apply the neutrosophic set to practical problems. To easily apply in science and engineering fields, Wang et al. [9] introduced the concept of SVNS, which is a subclass of the neutrosophic set, and gave the following definition.

Definition 1 [9]. Let X be a universal set. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. Then, a SVNS A can be denoted by

$$A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle \mid x \in X \right\},$$

where $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ for each x in X. Obviously, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

For a SVNS A in X, the triplet $\langle T_A(x), I_A(x), F_A(x)\rangle$ is called single valued neutrosophic value (SVNV), which is a fundamental element in a SVNS A. For convenience, we can simply denote $a = \langle T_A, I_A, F_A\rangle$ as a SVNV in A.

Let $A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle \mid x \in X \right\}$ and $B = \left\{ \left\langle x, T_B(x), I_B(x), F_B(x) \right\rangle \mid x \in X \right\}$ be two

SVNSs in X. Then, Wang et al. [9] gave some relations:

(1) Complement: $A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle \mid x \in X\}$;

(2) Inclusion: $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$ for any x in X;

(3) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

To overcome some disadvantages of cosine similarity measures in vector space, Ye [13] introduced two improved similarity measures between SVNSs A and B based on cosine function, respectively, as follows:

$$C_1(A,B) = \frac{1}{n}\sum_{j=1}^{n}\cos\left[\pi\frac{\max\left(\left|T_A(x_j) - T_B(x_j)\right|, \left|I_A(x_j) - I_B(x_j)\right|, \left|F_A(x_j) - F_B(x_j)\right|\right)}{2}\right], \quad (1)$$

$$C_2(A,B) = \frac{1}{n}\sum_{j=1}^{n}\cos\left[\frac{\pi\left(\left|T_A(x_j) - T_B(x_j)\right| + \left|I_A(x_j) - I_B(x_j)\right| + \left|F_A(x_j) - F_B(x_j)\right|\right)}{6}\right], \quad (2)$$

Then, the cosine similarity measure $C_k(A, B)$ (k =1, 2) satisfy the following properties (S1-S4) [13]:

(S1) $0 \le C_k(A, B) \le 1$;

(S2) $C_k(A, B) = 1$ if and only if $A = B$;

(S3) $C_k(A, B) = C_k(B, A)$;

(S4) If C is a SVNS in X and $A \subseteq B \subseteq C$, then $C_k(A, C) \le C_k(A, B)$ and $C_k(A, C) \le C_k(B, C)$.

3. Similarity measure of SVNSs based on tangent function

Similarity measure is usually an important mathematical tool in pattern recognition, medical diagnosis and clustering analysis. Therefore, the section proposes similarity measures between SVNSs based on the tangent function, which will be applied to multi-period medical diagnosis problems.

Definition 2. Let $A = \{\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j)\rangle \mid x_j \in X\}$ and $B = \{\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j)\rangle \mid x_j \in X\}$ be any two SVNSs in $X = \{x_1, x_2, \ldots, x_n\}$. Based on the tangent function, we define the following similarity measures between A and B:

$$T_1(A,B) = 1 - \frac{1}{n}\sum_{j=1}^{n} \tan\left[\frac{\pi}{4}\max\left(\left|T_A(x_j) - T_B(x_j)\right|, \left|I_A(x_j) - I_B(x_j)\right|, \left|F_A(x_j) - F_B(x_j)\right|\right)\right], \quad (3)$$

$$T_2(A,B) = 1 - \frac{1}{n}\sum_{j=1}^{n} \tan\left[\frac{\pi}{12}\left(\left|T_A(x_j) - T_B(x_j)\right| + \left|I_A(x_j) - I_B(x_j)\right| + \left|F_A(x_j) - F_B(x_j)\right|\right)\right]. \quad (4)$$

Then, the similarity measure $T_k(A, B)$ (k =1, 2) has the following Proposition 1:

Proposition 1. For two SVNSs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the similarity measure $T_k(A, B)$ (k =1, 2) should satisfy the following properties (S1-S4):

(S1) $0 \le T_k(A, B) \le 1$;

(S2) $T_k(A, B) = 0$ if and only if $A = B$;

(S3) $T_k(A, B) = T_k(B, A)$;

(S4) If C is a SVNS in X and $A \subseteq B \subseteq C$, then $T_k(A, C) \le T_k(A, B)$ and $T_k(A, C) \le T_k(B, C)$.

Proofs:

(S1) Since the value of the tangent function tan(x) is within [0, 1] in $x \in [0, \pi/4]$, the similarity measure value based on the tangent function also is within [0, 1]. Hence, there is $0 \le T_k(A, B) \le 1$for k =1, 2.

(S2) When $A = B$, this implies $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, $F_A(x_j) = F_B(x_j)$ for j = 1, 2, …, n and $x_j \in X$. Then $\left|T_A(x_j) - T_B(x_j)\right| = 0$, $\left|I_A(x_j) - I_B(x_j)\right| = 0$, and $\left|F_A(x_j) - F_B(x_j)\right| = 0$. Thus tan(0) = 0, Hence $T_k(A, B) = 1$ for k =1, 2.

If $T(A, B) = 1$, then it must satisfy tan(0) = 0. This implies $\left|T_A(x_j) - T_B(x_j)\right| = 0$, $\left|I_A(x_j) - I_B(x_j)\right| = 0$ and $\left|F_A(x_j) - F_B(x_j)\right| = 0$. Then, there are $T_A(x_j) = T_B(x_j)$, $I_A(x_j)$

= $I_B(x_j)$ and $F_A(x_j)$ = $F_B(x_j)$ for j = 1, 2, …, n and $x_j \in X$. Hence $A = B$.

(S3) Proof is straightforward.

(S4) If $A \subseteq B \subseteq C$, then this implies $T_A(x_j) \le T_B(x_j) \le T_C(x_j)$, $I_A(x_j) \ge I_B(x_j) \ge I_C(x_j)$ and $F_A(x_j) \ge F_B(x_j) \ge F_C(x_j)$ for j = 1, 2, …, n and $x_j \in X$. Thus, we have the following inequalities:

$$\left|T_A(x_j)-T_B(x_j)\right| \le \left|T_A(x_j)-T_C(x_j)\right|,\ \left|T_B(x_j)-T_C(x_j)\right| \le \left|T_A(x_j)-T_C(x_j)\right|,$$

$$\left|I_A(x_j)-I_B(x_j)\right| \le \left|I_A(x_j)-I_C(x_j)\right|,\ \left|I_B(x_j)-I_C(x_j)\right| \le \left|I_A(x_j)-I_C(x_j)\right|,$$

$$\left|F_A(x_j)-F_B(x_j)\right| \le \left|F_A(x_j)-F_C(x_j)\right|,\ \left|F_B(x_j)-F_C(x_j)\right| \le \left|F_A(x_j)-F_C(x_j)\right|.$$

Hence, $T_k(A, C) \le T_k(A, B)$ and $T_k(A, C) \le T_k(B, C)$ for k =1, 2 since the tangent function is an increasing function within [0, π/4].

Therefore, we complete the proofs of these properties. □

If we consider the weight of each element x_j for $x_j \in X = \{x_1, x_2, \ldots, x_n\}$ and assume that the weight of a element x_j is w_j (j = 1, 2, …, n) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then we can introduce the following weighted similarity measures between SVNSs A and B:

$$W_1(A,B) = 1 - \sum_{j=1}^{n}\left\{w_j \tan\left[\frac{\pi}{4}\max\left(\left|T_A(x_j)-T_B(x_j)\right|, \left|I_A(x_j)-I_B(x_j)\right|, \left|F_A(x_j)-F_B(x_j)\right|\right)\right]\right\}, \quad (5)$$

$$W_2(A,B) = 1 - \sum_{j=1}^{n}\left\{w_j \tan\left[\frac{\pi}{12}\left(\left|T_A(x_j)-T_B(x_j)\right| + \left|I_A(x_j)-I_B(x_j)\right| + \left|F_A(x_j)-F_B(x_j)\right|\right)\right]\right\}. \quad (6)$$

Especially, when $w_j = 1/n$ for j = 1, 2, …, n, Eqs. (5) and (6) reduce to Eqs. (3) and (4) respectively.

Obviously, the weighted similarity measures also have the following Proposition 2:

Proposition 2. For two SVNSs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, assume that the weight of a element x_j is w_j (j = 1, 2, …, n) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Then, the weighted similarity measure W_k(A,

B) for k = 1, 2 should satisfy the following properties (S1-S4):

(S1) $0 \le W_k(A, B) \le 1$;

(S2) $W_k(A, B) = 0$ if and only if $A = B$;

(S3) $W_k(A, B) = W_k(B, A)$;

(S4) If C is a SVNS in X and $A \subseteq B \subseteq C$, then $W_k(A, C) \le W_k(A, B)$ and $W_k(A, C) \le W_k(B, C)$.

By similar proof method of Proposition 1, we can give the proofs of these properties. They are not repeated here.

For the comparison of the similarity measures based on the tangent function with existing cosine similarity measures in [13] in single-valued neutrosophic setting, a numerical example is presented to demonstrate the effectiveness and rationality of the proposed similarity measures of SVNSs.

Let us consider two SVNSs A and B in $X = \{x\}$ and compare the proposed similarity measure with existing cosine similarity measures in [13] for pattern recognitions. By applying Eqs. (1)-(4), the similarity measure results for the pattern recognitions are indicated by the numerical example, as shown in Table 1.

Table 1. Similarity measure values of Eqs. (1)-(4)

	Case 1	Case 2	Case 3	Case 4	Case 5
A	$\langle x,0.2,0.3,0.4\rangle$	$\langle x,0.3,0.2,0.4\rangle$	$\langle x,1,0,0\rangle$	$\langle x,1,0,0\rangle$	$\langle x,0.4,0.2,0.6\rangle$
B	$\langle x,0.2,0.3,0.4\rangle$	$\langle x,0.4,0.2,0.3\rangle$	$\langle x,0,1,1\rangle$	$\langle x,0,0,0\rangle$	$\langle x,0.2,0.1,0.3\rangle$
$C_1(A, B)$	1	0.9877	**0**	**0**	0.8910
$C_2(A, B)$	1	0.9945	0	0.8660	0.9511
$T_1(A, B)$	1	0.9213	**0**	**0**	0.7599
$T_2(A, B)$	1	0.9476	0	0.7321	0.8416

From Table 1, we can see that the similarity measures C_1 and T_1 cannot carry out the recognition between Case 3 and Case 4, while the similarity measures C_2 and T_2 can carry out all recognitions. Hence, the similarity measures C_2 and T_2 demonstrate stronger discrimination among them and are superior to the similarity measures C_1 and T_1. In the following section, therefore, the similarity measure T_2 will be applied to multi-period medical diagnosis problems owing to its better discrimination.

4. Multi-period medical diagnosis method

Due to more and more complexity of real medical diagnoses, a lot of information available to physicians from modern medical technologies is often incomplete, indeterminate and inconsistent information. However, by only taking one period inspection, one is difficult to reach a conclusion from a particular patient with a particular decease. Sometimes he/she may also show the symptoms of different diseases. Then, how can we give a proper conclusion? One solution is to examine the patient across different periods and to realize comprehensive diagnoses for the patient. To do so, we present a multi-period medical diagnosis method as a better tool for reasoning such a situation.

Let $S = \{S_1, S_2, \ldots, S_\mathrm{m}\}$ be a set of symptoms, $T = \{t_1, t_2, \ldots, t_q\}$ be a set of periods, and $D = \{D_1, D_2, \ldots, D_n\}$ be a set of considered diseases. For a patient P with various symptoms, we can indicate characteristic values between a patient and symptoms in multi-period medical diagnosis problems, which are shown in Table 2. Then, Table 3 shows characteristic values between symptoms and the considered diseases.

Table 2. Characteristic values between a patient and symptoms

	t_k	S_1	S_2	…	S_m
P	t_1	$C_1(t_1)$	$C_2(t_1)$	…	$C_\mathrm{m}(t_1)$
	t_2	$C_1(t_2)$	$C_2(t_2)$		$C_m(t_2)$
	…	…	…		…
	t_q	$C_1(t_q)$	$C_2(t_q)$		$C_m(t_q)$

In Table 2, $C_j(t_k)$ denotes the characteristic value between a patient P and the jth symptom S_j (j =

1, 2, …, m) in the kth period t_k for k = 1, 2, …, q. Obviously, if q = 1, the medical diagnosis problem is actually a single period medical diagnosis problem [13].

Table 3. Characteristic values between symptoms and the considered diseases

	S_1	S_2	...	S_m
D_1	C_{11}	C_{12}	…	C_{1m}
D_2	C_{21}	C_{22}	…	C_{2m}
…	…	…	…	…
D_n	C_{n1}	C_{n2}	…	C_{nm}

In Table 3, C_{ij} denotes the characteristic value between the jth symptom S_j (j = 1, 2, …, m) and the ith considered disease D_i (i = 1, 2, …, n).

For a multi-period medical diagnosis problem with single-valued neutrosophic information, the characteristic values between a patient and symptoms are denoted by the form of a SVNV $C_j(t_k) = \langle T_j(t_k), I_j(t_k), F_j(t_k)\rangle$ and the characteristic values between symptoms and the considered diseases are denoted by the form of a SVNV $C_{ij} = \langle T_{ij}, I_{ij}, F_{ij}\rangle$ for convenience. If we consider that the weight vector of the symptoms is $\boldsymbol{w} = (w_1, w_2, …, w_m)^T$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{m} w_j = 1$ and the weight vector of the periods is $\boldsymbol{\omega} = (\omega(t_1), \omega(t_2), …, \omega(t_q))^T$ with $\omega(t_k) \in [0, 1]$ and $\sum_{k=1}^{q} \omega(t_k) = 1$.

Since the similarity measure of Eq. (6) demonstrates stronger discrimination mentioned above, we apply it to multi-period medical diagnosis problems. Thus, the diagnosis steps are given as follows:

Step 1: Calculate the similarity measure between a patient P and the considered disease D_i (i = 1, 2, …, n) in each period t_k (k = 1, 2, …, q) by the following formula:

$$W_i(t_k) = 1 - \sum_{j=1}^{m} \left\{ w_j \tan\left[\frac{\pi}{12} \left(\left|T_j(t_k) - T_{ij}\right| + \left|I_j(t_k) - I_{ij}\right| + \left|F_j(t_k) - F_{ij}\right| \right) \right] \right\}. \quad (7)$$

Step 2: Obtain the weighted measure M_i for i = 1, 2, …, n by the following formula:

$$M_i = \sum_{k=1}^{q} W_i(t_k)\omega(t_k). \quad (8)$$

Step 3: Rank all the weighted measures of M_i for P with respect to D_i (i = 1, 2, …, n) in a descending order.

Step 4: Give a proper diagnosis according to the maximum weighted measure value.

Step 5: End.

5. Multi-period medical diagnosis example

Since physicians can obtain a lot of information from modem medical technologies, medical diagnosis contains a lot of incomplete, uncertainty and inconsistent information. Then, SVNS is a very suitable tool for expressing and handling it. In this section, we provide a medical diagnosis example to demonstrate the application and effectiveness of the proposed multi-period medical diagnosis method.

In the following example, we shall discuss the medical diagnosis problem adapted from [4, 5, 13].

Let $D = \{D_1, D_2, D_3, D_4, D_5\}$ = {Viral fever, Malaria, Typhoid, Gastritis, Stenocardia} be a set of diseases and $S = \{S_1, S_2, S_3, S_4, S_5\}$ = {Temperature, Headache, Stomach pain, Cough, Chest pain} be a set of symptoms. Then characteristic values between symptoms and the considered diseases are represented by the form of SVNVs, which are shown in Table 4 [13].

Table 4. Characteristic values between symptoms and the considered diseases represented by SVNVs

	S_1 (Temperature)	S_2 (Headache)	S_3 (Stomach pain)	S_4 (Cough)	S_5 (Chest pain)
D_1 (Viral fever)	$\langle 0.4, 0.6, 0.0\rangle$	$\langle 0.3, 0.2, 0.5\rangle$	$\langle 0.1, 0.3, 0.7\rangle$	$\langle 0.4, 0.3, 0.3\rangle$	$\langle 0.1, 0.2, 0.7\rangle$
D_2 (Malaria)	$\langle 0.7, 0.3, 0.0\rangle$	$\langle 0.2, 0.2, 0.6\rangle$	$\langle 0.0, 0.1, 0.9\rangle$	$\langle 0.7, 0.3, 0.0\rangle$	$\langle 0.1, 0.1, 0.8\rangle$

D_3 (Typhoid)	⟨0.3, 0.4, 0.3⟩	⟨0.6, 0.3, 0.1⟩	⟨0.2, 0.1, 0.7⟩	⟨0.2, 0.2, 0.6⟩	⟨0.1, 0.0, 0.9⟩
D_4 (Gastritis)	⟨0.1, 0.2, 0.7⟩	⟨0.2, 0.4, 0.4⟩	⟨0.8, 0.2, 0.0⟩	⟨0.2, 0.1, 0.7⟩	⟨0.2, 0.1, 0.7⟩
D_5 (Stenocardia)	⟨0.1, 0.1, 0.8⟩	⟨0.0, 0.2, 0.8⟩	⟨0.2, 0.0, 0.8⟩	⟨0.2, 0.0, 0.8⟩	⟨0.8, 0.1, 0.1⟩

In the medical diagnosis, assume that we have to take three different samples from a patient in three periods. For example, the SVNV of a symptom S_1 for a patient P is given as $C_1(t_1)$ = ⟨0.8, 0.6, 0.5⟩ in the first period t_1, which indicates that the characteristic value $C_1(t_1)$ between the patient P and the symptom S_1 is the truth degree 0.8, falsity degree 0.5 and indeterminacy degree 0.6. Thus, the characteristic values between the patient and symptoms are represented by SVNVs, which can be constructed as Table 5.

Table 5. Characteristic values between a patient and symptoms represented by SVNVs

	t_k	S_1 (Temperature)	S_2 (Headache)	S_3 (Stomach pain)	S_4 (Cough)	S_5 (Chest pain)
	t_1	⟨0.8, 0.6, 0.5⟩	⟨ 0.5, 0.4, 0.3⟩	⟨0.2, 0.1, 0.3⟩	⟨0.7, 0.6, 0.3⟩	⟨0.4, 0.3, 0.2⟩
P	t_2	⟨0.7, 0.3, 0.2⟩	⟨0.6, 0.3, 0.2⟩	⟨0.3, 0.2, 0.4⟩	⟨0.6, 0.5, 0.2⟩	⟨0.6, 0.5, 0.3⟩
	t_3	⟨0.5, 0.2, 0.4⟩	⟨0.6, 0.3, 0.4⟩	⟨0.3, 0.3, 0.5⟩	⟨0.4, 0.3, 0.2⟩	⟨0.6, 0.4, 0.4⟩

If we consider that the weight vector of the five symptoms is $\boldsymbol{w} = (1/5, 1/5, 1/5, 1/5, 1/5)^T$ and the weight vector of the three periods is $\boldsymbol{\omega} = (0.25, 0.35, 0.4)^T$, then the diagnosis steps are given as follows:

Step 1: Calculate the similarity measures between the patient P and the considered disease D_i in each period t_k for i = 1, 2, 3, 4, 5 and k = 1, 2, 3 by Eq. (7), as shown in Table 6.

Table 6. Similarity measure values of $W_i(t_k)$

	t_1	t_2	t_3
$W_1(t_k)$	0.8035	0.7974	0.8458
$W_2(t_k)$	0.7760	0.7904	0.7865
$W_3(t_k)$	0.7741	0.7885	0.8178
$W_4(t_k)$	0.7298	0.7141	0.7758
$W_5(t_k)$	0.6944	0.6900	0.7539

Step 2: Obtain the weighted measure M_i for i = 1, 2, 3, 4, 5 by applying Eq. (8):

M_1 = **0.8183**, M_2 = 0.7852, M_3 = 0.7966, M_4 = 0.7427, and M_5 = 0.7167.

Step 3: The ranking order is $M_1 > M_3 > M_2 > M_4 > M_5$.

Step 4: The patient P suffers from viral fever according to the maximum weighted measure value M_1.

Compared with the medical diagnosis methods in [4, 5, 13], the multi-period medical diagnosis in this paper is a comprehensive medical diagnosis method with examining a patient through multi-periods, which obtains the diagnosis conclusion by using the tangent similarity measure of SVNSs. However, the medical diagnosis methods in [4, 5, 13] cannot handle the multi-period medical diagnosis problem with neutrosophic information. Since the single period medical diagnosis problem is a special case of the multi-period medical diagnosis problem, the proposed multi-period medical diagnosis method can deal with the medical diagnosis problems with intuitionistic fuzzy information and single valued neutrosophic information in [4, 5, 13]. Furthermore, the multi-period medical diagnosis method presented in this paper is superior to the single period medical diagnosis methods proposed in [4, 5, 13] because the later is difficult to give a proper diagnosis for a particular patient with a particular decease in some situations and the former has to examine the patient through multi-periods and to consider the weights of multi-periods in order to reach a proper conclusion for the patient. From multi-period medical diagnosis point of view, the proposed multi-period medical diagnosis method is more suitable and more reasonable to find a proper disease diagnosis than the existing medical diagnosis methods.

6. Conclusion

This paper proposed the similarity measures for SVNSs based on the tangent function and the weighted similarity measure of SVNSs introduced by considering the importance of each element, and then investigated their properties. Further, we developed a multi-period medical diagnosis method based on the proposed similarity measure considering the weight of periods. Finally, a medical diagnosis example with single valued neutrosophic information was provided to demonstrate the applications and effectiveness of the developed multi-period medical diagnosis method.

In the further work, it is necessary to apply the similarity measure of SVNSs to other areas such as pattern recognition, image processing, and clustering analysis. We also shall extend the proposed multi-period medical diagnosis method to multi-period decision making problems with neutrosophic information.

Acknowledgment

This paper was supported by the National Natural Science Foundation of China (No. 71471172).

References

[1] L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

[2] K.T. Atanassov, Intuitionistic Fuzzy sets, Fuzzy sets and Systems 20 (1986) 87–96.

[3] S.K. De, R. Biswas, A.R. Roy, An application of intuitionistic fuzzy sets in medical diagnosis, Fuzzy Sets and Systems 117(2) (2001) 209–213.

[4] I.K. Vlachos, G.D. Sergiadis, Intuitionistic fuzzy information — Applications to pattern recognition, Pattern Recognition Letters 28 (2007) 197-206.

[5] J. Ye, Cosine similarity measures for intuitionistic fuzzy sets and their applications, Mathematical and Computer Modelling 53(1-2) (2011) 91-97.

[6] K.C. Hung, Applications of medical information: Using an enhanced likelihood measured

approach based on intuitionistic fuzzy sets, IIE Transactions on Healthcare Systems Engineering 2(3) (2012) 224-231.

[7] M.Y. Tian, A new fuzzy similarity based on cotangent function for medical diagnosis, Advanced Modeling and Optimization 15(2) (2013) 151-156.

[8] F. Smarandache, A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic. Rehoboth: American Research Press, 1999.

[9] H. Wang, F. Smarandache, Y.Q. Zhang, R. Sunderraman, Single valued neutrosophic sets, Multispace and Multistructure 4 (2010) 410-413.

[10] P. Majumdar, S.K. Samanta, On similarity and entropy of neutrosophic sets, Journal of Intelligent and Fuzzy Systems 26(3) (2014) 1245-1252.

[11] J. Ye, Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making, International Journal of Fuzzy Systems 16(2) (2014) 204-211.

[12] J. Ye, Multiple attribute group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment, Journal of Intelligent and Fuzzy Systems 27(12) (2014) 2927–2935.

[13] J. Ye, Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses, Artificial Intelligence in Medicine 63(3) (2015) 171–179.

Table(1)

Table 1. Similarity measure values of Eqs. (1)-(4)

	Case 1	Case 2	Case 3	Case 4	Case 5
A	$\langle x,0.2,0.3,0.4\rangle$	$\langle x,0.3,0.2,0.4\rangle$	$\langle x,1,0,0\rangle$	$\langle x,1,0,0\rangle$	$\langle x,0.4,0.2,0.6\rangle$
B	$\langle x,0.2,0.3,0.4\rangle$	$\langle x,0.4,0.2,0.3\rangle$	$\langle x,0,1,1\rangle$	$\langle x,0,0,0\rangle$	$\langle x,0.2,0.1,0.3\rangle$
$C_1(A, B)$	1	0.9877	**0**	**0**	0.8910
$C_2(A, B)$	1	0.9945	0	0.8660	0.9511
$T_1(A, B)$	1	0.9213	**0**	**0**	0.7599
$T_2(A, B)$	1	0.9476	0	0.7321	0.8416

Table(2)

Table 2. Characteristic values between a patient and symptoms

	t_k	S_1	S_2	…	S_m
	t_1	$C_1(t_1)$	$C_2(t_1)$		$C_m(t_1)$
P	t_2	$C_1(t_2)$	$C_2(t_2)$	…	$C_m(t_2)$
	…	…	…		…
	t_q	$C_1(t_q)$	$C_2(t_q)$		$C_m(t_q)$

Table(3)

Table 3. Characteristic values between symptoms and the considered diseases

	S_1	S_2	...	S_m
D_1	C_{11}	C_{12}	…	C_{1m}
D_2	C_{21}	C_{22}	…	C_{2m}
…	…	…	…	…
D_n	C_{n1}	C_{n2}	…	C_{nm}

Table(4)

Table 4. Characteristic values between symptoms and the considered diseases represented by SVNVs

	S_1 (Temperature)	S_2 (Headache)	S_3 (Stomach pain)	S_4 (Cough)	S_5 (Chest pain)
D_1 (Viral fever)	⟨0.4, 0.6, 0.0⟩	⟨0.3, 0.2, 0.5⟩	⟨0.1, 0.3, 0.7⟩	⟨0.4, 0.3, 0.3⟩	⟨0.1, 0.2, 0.7⟩
D_2 (Malaria)	⟨0.7, 0.3, 0.0⟩	⟨0.2, 0.2, 0.6⟩	⟨0.0, 0.1, 0.9⟩	⟨0.7, 0.3, 0.0⟩	⟨0.1, 0.1, 0.8⟩
D_3 (Typhoid)	⟨0.3, 0.4, 0.3⟩	⟨0.6, 0.3, 0.1⟩	⟨0.2, 0.1, 0.7⟩	⟨0.2, 0.2, 0.6⟩	⟨0.1, 0.0, 0.9⟩
D_4 (Gastritis)	⟨0.1, 0.2, 0.7⟩	⟨0.2, 0.4, 0.4⟩	⟨0.8, 0.2, 0.0⟩	⟨0.2, 0.1, 0.7⟩	⟨0.2, 0.1, 0.7⟩
D_5 (Stenocardia)	⟨0.1, 0.1, 0.8⟩	⟨0.0, 0.2, 0.8⟩	⟨0.2, 0.0, 0.8⟩	⟨0.2, 0.0, 0.8⟩	⟨0.8, 0.1, 0.1⟩

Table(5)

Table 5. Characteristic values between a patient and symptoms represented by SVNVs

	t_k	S_1 (Temperature)	S_2 (Headache)	S_3 (Stomach pain)	S_4 (Cough)	S_5 (Chest pain)
	t_1	$\langle 0.8, 0.6, 0.5\rangle$	$\langle 0.5, 0.4, 0.3\rangle$	$\langle 0.2, 0.1, 0.3\rangle$	$\langle 0.7, 0.6, 0.3\rangle$	$\langle 0.4, 0.3, 0.2\rangle$
P	t_2	$\langle 0.7, 0.3, 0.2\rangle$	$\langle 0.6, 0.3, 0.2\rangle$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.6, 0.5, 0.2\rangle$	$\langle 0.6, 0.5, 0.3\rangle$
	t_3	$\langle 0.5, 0.2, 0.4\rangle$	$\langle 0.6, 0.3, 0.4\rangle$	$\langle 0.3, 0.3, 0.5\rangle$	$\langle 0.4, 0.3, 0.2\rangle$	$\langle 0.6, 0.4, 0.4\rangle$

Table(6)

Table 6. Similarity measure values of $W_i(t_k)$

	t_1	t_2	t_3
$W_1(t_k)$	0.8035	0.7974	0.8458
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$W_4(t_k)$	0.7298	0.7141	0.7758
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