

MULTI-ULTRASONIC SENSOR FUSION BASED ON DEZERT-SMARANDACHE THEORY

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Keywords: Ultrasonic sensor, occupancy map, Dezert-Smarandache theory, fuzzy set theory.

Abstract

It is well known that ultrasonic sensor is widely used in mobile robot to get range information with low cost, and the main bottle problem of using ultrasonic sensors to build an occupancy grid map is how to deal with the uncertainty in ultrasonic sensor data. In this paper, a new kind of Multi-ultrasonic sensor fusion method is presented. The ultrasonic sensor model is built with fuzzy set theory to describe the uncertainty contained in sonar data. Environment information obtained from different measures is processed with Dezert-Smarandache (DSm) theory which is a new methodology for solving fusion problems. To get a high quality occupancy map, a paradoxical factor (PF) is introduced to help to deal with multi-reflection of ultrasonic sensor. Through simulation and experiment results, it is proved that the new algorithm behaves a good performance during mobile robot map building.

1 Introduction

Environment map obtained from sensory information is necessary for an autonomous mobile robot to accomplish a certain task, the relative researches can be found in [1,2]. Ultrasonic sensors are the most popular sensors mounted on mobile robots due to its getting range information directly [3]. Unfortunately, ultrasonic sensor data may be contaminated by the cause of the beam angle and multi-reflection problem. To get high-quality navigation map for mobile robot based on ultrasonic sensors, data fusion of multi-ultrasonic sensor is widely studied, and there are mainly three kinds of methods presented in literature: Bayesian approach [4], Dempster-Shafer theory [5] and fuzzy set theory [6]. The common ground of the applications of the three methodologies is that the work scenario of the mobile robot is described by two-dimensional discrete grids and range data are transferred into the estimation of real state of a cell. The above three types of mathematical theory all can be used to deal with uncertain information, but each has its own characteristics[7]: Bayesian approach depicted uncertainty as probability, so the state of some cell, either occupied or empty, is expressed as the probabilities of $P(\text{occupied})$ and $P(\text{empty})$ and the two probability have the relation of $P(\text{occupied}) = 1 - P(\text{empty})$. Different measures are combined by Bayesian rule. Obviously,

Bayesian approach has strong mathematical foundation and is easy to be understood. Through this theory, we can get a reasoning result of the state of certain cell either occupied or empty, and there is not other choice except for the above two. In fact, when mobile robot gets environment information from its onboard sensors, there possible is such a result that the mobile robot may have ignorance about the real world from current acquired information. It is clear that Bayesian theory has no ability to cope with this case. In contrast to the disability of Bayesian theory, Dempster-Shafer (D-S) theory admits ignorance as a reasoning result, and views uncertainty as estimation interval in which the upper bound and lower bound are degree of plausibility and degree of belief respectively. D-S theory has two strict constrains: all the elements of the frame of discernment must be exclusive and all the bodies of evidence must be assumed to be independent. When these conditions are not satisfied, the reasoning error may occur. More importantly, in practice, it may be not reasonable to assume that the two constrains must be met. Fuzzy set theory is the extension of classic Cantor theory. In this theory, uncertainty is described as the degree of membership. The advantage of fuzzy theory lies in that it accords with the model of human thinking, and it has been proved to be a good model to depict the uncertainty contained in ultrasonic sensor data [6]. Intuitively, it is a good idea to model the imprecise ultrasonic data by fuzzy set theory and combined data from different measures by the combination rule like the one in D-S theory if the frame of discernment can be extended to have the elements which are not exclusive and described by fuzzy concepts.

In this paper, we present a multi-ultrasonic sensor fusion algorithm based on DSm theory [8] and fuzzy set theory. DSm theory provides a novel mathematical frame for solving fusion problems and the application of the theory can be found in [9]. DSm theory is the extension of DS theory by freeing the two constrains mentioned above and gives strong theory foundation to our multi-ultrasonic sensor fusion algorithm. In our method, we present a paradoxical factor added to the combination rules to detect the happening of multi-reflection, and through this method, the quality of the occupancy grid map obtained from ultrasonic sensor data is improved.

2 Sensor data fusion frame work

2.1 World model

World model adopted by mobile robot is influenced by the environment it operates on, its sensory device and sensor system [10]. Occupancy grid world model [4], which views the real world as the two-dimension discrete cells, is used in this paper. It is suitable for ultrasonic sensor data fusion. The states of certain cell are depicted by fuzzy concepts Empty and Occupied, and the range information gotten from ultrasonic sensor is transferred into the degree of the membership of each fuzzy concept by sensor model introduced later. Different measures are fused by DSm theory, then the state of the cell can be estimated by the belief of occupied and empty.

2.2 Dezert-Smarandache Theory [8,9]

The inherent limitation of DS theory lies in the assumption that the elements which consist of the frame of discernment are exclusive. DSm theory extends DS theory by removing the fundamental conditions. Considering multi-ultrasonic sensor fusion problem, the discern frame is $\Theta = \{ \text{Empty, Occupied} \}$ in DS theory, where Empty and Occupied are crisp sets which are exclusive, but in DSm theory, $\Theta = \{ \text{Empty, Occupied} \}$ is called “free model” in which the limitation that Empty and Occupied are disjoint is removed. In this paper, “Occupied” and “Empty” both are fuzzy concepts which describe the possibility that there is an obstacle in some cell and not. The other cornerstone concept in DSm theory is hyper power set, which is defined as the set of all composite propositions built from elements of free model Θ with the operators of \cup and \cap [9]. Hyper power set of Θ is denoted with the sign “ D^Θ ”. It is the generalization of power set used in DS theory, and when free model degenerates into the frame of discernment, hyper power set does into power set accordingly. Considering the fusion problem in this paper again, $D^\Theta = \{ \phi, \text{Empty, Occupied, Empty} \cap \text{Occupied, Empty} \cup \text{Occupied} \}$. On the basis of the definition of hyper power, DSm theory presents the general basic belief mass (gbbm) $m(\cdot)$ which can support paradoxical information as follows:

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in D^\Theta} m(A) = 1 \quad (1)$$

In the same way as in DS theory, the belief and plausibility functions are defined as formulas (2) and (3) :

$$Bel(A) = \sum_{B \in D^\Theta, B \subseteq A} m(B) \quad (2)$$

$$Pl(A) = \sum_{B \in D^\Theta, B \cap A \neq \emptyset} m(B) \quad (3)$$

Different sources of evidence can be combined by the following formula (4):

$$m(C) = m_1 + m_2(C) = \sum_{A, B \in D^\Theta, A \cap B = C} m_1(A) m_2(B) \quad \forall C \in D^\Theta \quad (4)$$

2.3 Sensor model

In our method, we mainly reference the ultrasonic sensor model presented in [6] which is built with fuzzy set theory. It is shown in Fig.1. When a single reading r is provided by certain ultrasonic sensor, it is indicated that there is one or more obstacles (M in the figure) located somewhere along the 30° circumference of radius r . By the formulas (5)-(10), the membership functions of fuzzy concept “Empty” and “Occupied” are decided respectively :

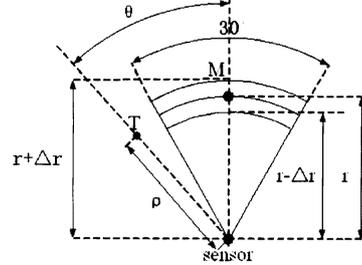


Fig. 1: Ultrasonic sensor model

$$\mu_{OCC}(\rho, r, \theta) = h_o(\rho, r) m(\theta) n(\rho) \quad (5)$$

$$\mu_{EMP}(\rho, r, \theta) = h_e(\rho, r) m(\theta) n(\rho) \quad (6)$$

$$h_o(\rho, r) = \begin{cases} 0 & 0 \leq \rho \leq r - \Delta r \\ k_o(1 - |\frac{r - \rho}{\Delta r}|)^2 & r - \Delta r \leq \rho \leq r + \Delta r \\ 0 & \text{others} \end{cases} \quad (7)$$

$$h_e(\rho, r) = \begin{cases} k_e & 0 \leq \rho \leq r - \Delta r \\ k_e(\frac{r - \rho}{\Delta r})^2 & r - \Delta r \leq \rho \leq r \\ 0 & \text{others} \end{cases} \quad (8)$$

$$m(\theta) = \begin{cases} (\frac{\theta - 15}{15})^2 & |\theta| \leq 15^\circ \\ 0 & \text{others} \end{cases} \quad (9)$$

$$n(\rho) = \begin{cases} 1 & 0 \leq \rho \leq r_o \\ 0 & \text{others} \end{cases} \quad (10)$$

where, $\mu_{EMP}(\rho, r, \theta)$ and $\mu_{OCC}(\rho, r, \theta)$ denote the membership of empty and occupied respectively, r is the reading from ultrasonic sensor, (ρ, θ) is the coordinate of a point inside sonar map, K_o, K_e are constants and Δr is range error.

2.4 Data fusion

The main advantage of DSm theory lies in its ability to cope with paradoxical information, and the uncertain information contained in certain sonar response can cause paradox due to the beam angle of sonar cone, specular reflections and environment variation. In the process of ultrasonic sensor data fusion with DSm theory, the universal set should be defined firstly as a crisp set that consists of all the cells involved in mobile working space, and denoted by $W = \{ C_{ij}, i \in Z, j \in Z \}$. As mentioned above, the free model is $\Theta = \{ \text{Empty, Occupied} \}$, the hyper power set $D^\Theta = \{ \phi, \text{Empty, Occupied, Empty} \cap \text{Occupied, Empty} \cup \text{Occupied} \}$. In D^Θ ,

Empty, Occupied are fuzzy concept as the literal meaning which are used to describe the state information of certain cell in W . $Empty \cap Occupied$ represents the paradox in the evidence, and $Empty \cup Occupied$ describes ignorance. When a sonar response is gotten, it can be viewed as a information source, and for each cell C in W , the gbbm $m_c(\cdot)$ should be computed and used to revise the global map. Respectively, $m_c(\phi) = 0$, $m_c(Empty) = \mu EMP(\rho, r, \theta)$ and $m_c(Occupied) = \mu OCC(\rho, r, \theta)$ which can be computed by formula (5) - (10). $m_c(Empty \cap Occupied)$ and $m_c(Empty \cup Occupied)$ can be computed as followed formulas(11)-(12) :

$$m_c(Empty \cap Occupied) = \min(m_c(Empty), m_c(Occupied)) \quad (11)$$

$$m_c(Empty \cup Occupied) = 1 - m_c(Empty) - m_c(Occupied) - m_c(Empty \cap Occupied) \quad (12)$$

When range data is gotten from some sonar, it is converted into gbbm of each cell according to formulas (5)-(12). With combination rule which is presented with formula (3), global map is updated by the new sonar reading. Concretely, the process of map updating can be depicted by formulas (13)-(17)

$$m_c^g(\phi) = 0 \quad (13)$$

$$m_c^g(Occupied) = m_c^o(Occupied) \times m_c^n(Occupied) + m_c^o(Occupied) \times m_c^n(Empty \cup Occupied) + m_c^o(Occupied) \times m_c^n(Empty \cup Occupied) \quad (14)$$

$$m_c^g(Empty) = m_c^o(Empty) \times m_c^n(Empty) + m_c^o(Empty) \times m_c^n(Empty \cup Occupied) + m_c^o(Empty) \times m_c^n(Empty \cup Occupied) \quad (15)$$

$$m_c^g(Empty \cap Occupied) = m_c^o(Empty) \times m_c^n(Occupied) + m_c^o(Occupied) \times m_c^n(Empty) + m_c^o(Empty \cap Occupied) \times m_c^n(Empty \cup Occupied) + m_c^o(Empty \cup Occupied) \times m_c^n(Empty \cap Occupied) \quad (16)$$

$$m_c^g(Empty \cup Occupied) = m_c^o(Empty \cup Occupied) \times m_c^n(Empty \cup Occupied) \quad (17)$$

where $m_c^g(\cdot)$ is the gbbm after being updated for certain cell c in the working space, $m_c^o(\cdot)$ is the gbbm before being updated for some cell c in the working space, $m_c^n(\cdot)$ is the gbbm obtained from certain sonar reading. According to formula (2), the belief functions of "Empty" and "Occupied" are:

$$Bel_c(Empty) = m_c^g(Empty) + m_c^g(Empty \cap Occupied) \quad (18)$$

$$Bel_c(Occupied) = m_c^g(Occupied) + m_c^g(Empty \cap Occupied) \quad (19)$$

The main idea of introducing DSsm theory in sonar data fusion lies in its ability to deal with paradoxical information contained in sonar data. We have noticed the theory is suit for cope with multi-ultrasonic sensor fusion problem in dynamic environment, and the combination rules can be modified to be helpful to treat with sonar specular problem. The argument can be illustrated in the simulation process: supposing the ultrasonic sensor is fixed and gets two serials of reading $\{5, 5, 5, 5, 10, 10, 10, 10, 10, 10\}$ and $\{5, 5, 5, 5, 10, 5, 5, 5, 5, 5\}$. We can view the first situation as that there is an obstacle at a distance

of 5 unit originally and then the obstacle is moved to the place where is 10 unit far away from the sonar. The situation depicted with the second serial of readings can be regarded as that multi-reflection has occurred on the surface of the obstacle which is 5 unit away from the sonar. The idea of the above simulation process is firstly presented in [11]. The fusion results of the probability of occupancy based on DSsm theory and fuzzy set theory are shown in Fig. 2 and Fig. 3 respectively:

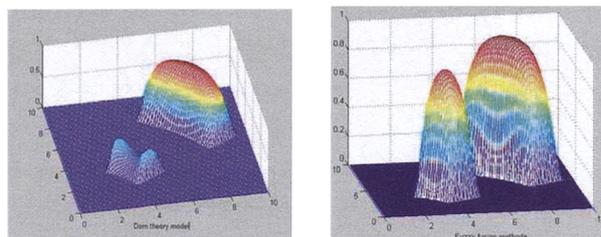


Fig. 2 DSsm theory(left) and fuzzy set theory(right) fusion results for the dynamic environment

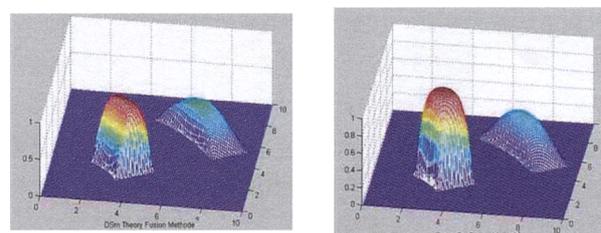


Fig.3 DSsm theory(left) and fuzzy set theory(right) fusion results for the multi-reflection environment

The same sensor model is adopted in the simulation process, but the sensor data coming from different information sources are combined according to corresponding combination rules respectively. The world model of occupancy grid is applied with a side of 0.01 unit. From Fig.2, DSsm combination rule assign relatively low estimation value of being occupied to the cells which is about 5 unit away from the sensor in the sonar radiation cone in comparison with fuzzy set method. So it is concluded that DSsm theory behaves a better performance than the fuzzy set fusion method in variable environment. When multi-reflection occurs, there is no obvious difference between the two methods according to Fig.3. It is known that when multi-reflection occurs, the error reading is often larger than the right reading. In the sensor model, formula (10) acts as filter to deal with multi-reflection. Concretely, when reading larger than r_0 , it will be viewed as an error reading caused by multi-reflection. r_0 is set according to the environment which the mobile robot works in [6]. But if the reading caused by multi-reflection is less than r_0 , formula (10) is of no effect, and the error reading must bring about the degeneration of sonar map quality. To cope with this problem, the paradoxical factor (PF) is presented and applied in the combination rules of DSsm theory. Paradoxical factor can be depicted with formulas (20)-(21):

$$PF = \frac{1}{1 + \exp\left[\frac{(\kappa - \kappa_0)}{t_0}\right]} \quad (20)$$

$$\begin{aligned} \kappa = & m_c^o(Empty) \times m_c^n(Occupied) + \\ & m_c^o(Occupied) \times m_c^n(Empty) + \\ & m_c^o(Empty \cap Occupied) \times m_c^n(Empty \cup Occupied) + \\ & m_c^o(Empty \cup Occupied) \times m_c^n(Empty \cap Occupied) \end{aligned} \quad (21)$$

DSm theory combination rules are modified as formula (22)-(26) with application of PF:

$$m_c^s(\phi) = 0 \quad (22)$$

$$\begin{aligned} m_c^s(Occupied) = & PF \times [m_c^o(Occupied) \times m_c^n(Occupied) + \\ & m_c^o(Occupied) \times m_c^n(Empty \cup Occupied) + \\ & m_c^o(Occupied) \times m_c^n(Empty \cup Occupied)] \end{aligned} \quad (23)$$

$$\begin{aligned} m_c^s(Empty) = & PF \times [m_c^o(Empty) \times m_c^n(Empty) + \\ & m_c^o(Empty) \times m_c^n(Empty \cup Occupied) + \\ & m_c^o(Empty) \times m_c^n(Empty \cup Occupied)] \end{aligned} \quad (24)$$

$$\begin{aligned} m_c^s(Empty \cap Occupied) = & PF \times [m_c^o(Empty) \times m_c^n(Occupied) + \\ & m_c^o(Occupied) \times m_c^n(Empty) + \\ & m_c^o(Empty \cap Occupied) \times m_c^n(Empty \cup Occupied) + \\ & m_c^o(Empty \cup Occupied) \times m_c^n(Empty \cap Occupied)] \end{aligned} \quad (25)$$

$$m_c^s(Empty \cup Occupied) = 1 - m_c^s(Occupied) - m_c^s(Empty) - m_c^s(Empty \cap Occupied) \quad (26)$$

where κ_0 and t_0 are constants, other signs have the same meanings as in formulas (13)-(17). In formula (20), κ serves as an indicator to detect the occurrence of specular reflection.

3 Experiments and discussions

The fusion method is tested on self-developed mobile robot THUNDER which works in the corridor of our laboratory. THUNDER has a column appearance with the diameter of 50cm, and the area of the corridor is $240 \times 600 \text{ cm}^2$. The overview of both the mobile robot THUNDER and its working space are shown in Fig.4 :



Fig.4 Overview of robot THUNDER and the testing environment.

In [12], it is summarized that robot's misunderstanding of real world according to sensor reading caused mainly by one or more of four factors, which are flawed world model, inaccurate sensor readings, poor self-location and over

simplified sensor model. In other words, the four kinds of problems should be overcome to get a high-quality sonar map. To focus on the performance of the fusion method in dealing with inaccurate sonar readings, the robot THUNDER is remotely operated in the testing experiment, so self-location problem is avoided. When the robot is moved to different locations, raw sensor data are gathered by a sonar ring of 8 ultrasonic sensors which is mounted on THUNDER. To get a changing environment, the passage was blocked with obstacles originally and the obstacles are removed after the robot is controlled to move forward several times later. The raw sensor data are processed with DSm theory, then the occupancy map is obtained. For a certain cell c , the states of being occupied and empty are estimated with $Bel_c(Occupied)$ and $Bel_c(Empty)$ respectively. When $Bel_c(Occupied) > 0.5$, the cell c is figured out with black rectangle otherwise with white one, and the result is shown in Fig. 5. The same raw sensor data are fused with the method based on fuzzy set theory depicted in [6] and the result is shown in Fig. 6. If any cell's probability of being occupied is higher than 0.5, it is shown in the same way as in Fig.5. When the obstacles that block the corridor originally have been removed, it is reflected in the Fig.5, but the cells where the obstacles situating on still have a high probability of being occupied in Fig.6. To clearly illustrate the performance of DSm theory in multi-ultrasonic sensor fusion, sonar data in static environment are also collected and processed. The sensor data are fused with DSm combination rules firstly, then PF is added to the combination rules to process the same data. To make a clear comparison, the data are also fused with fuzzy set theory just like previous experiment. The three fusion results can be found in Fig.7, from which it can be conclude that the result map applied PF reflects the real world better than the one without applying it. It is obvious that the map obtained by the method based on fuzzy set theory in Fig.7 is more conservative than the other two maps because there are more cells are estimated with high probability of being occupied. To be more persuasive, a quantitative map quality metric which is presented in [12] is used to measure the three maps. The core idea of the metric is to calculate the error according to the difference between the ground truth map of experiment scenario and each map obtained from corresponding fusion method. The more the difference is, the worse the fusion methodology performs in the sonar data fusion.



Fig.5. Occupancy map obtained from DSm theory combination rules.



Fig.6. Occupancy map obtained from fuzzy set combination rules.



Fig.7. Maps obtained based on DS theory (left), DS theory with PF applied (middle) and fuzzy set theory(right).

The error can be computed by formulas (27)-(29) [12]:

$$Error = \sum_{x=0}^w \sum_{y=0}^h MAX(error_o, error_e) \quad (27)$$

$$error_o = |grid_o(x, y) - truth_o(x, y)| \quad (28)$$

$$error_e = |grid_e(x, y) - truth_e(x, y)| \quad (29)$$

Where, w and h together determine the size of both occupancy and ground truth maps, $grid_o(x,y)$ and $grid_e(x,y)$ denote both the occupancy and empty values according to the occupancy map, $truth_o(x,y)$ and $truth_e(x,y)$, which have the value of either 1.0 or 0, denote the real state of position (x, y) in the real world. Error of the three occupancy maps in fig.7 is measured with the value of 193.55, 190.36, and 210.90 respectively. Obviously, it can be concluded that the map based on DS theory has better quality than the one based on fuzzy set combination rules, and when PF is applied, the quality of occupancy map is also improved.

3 Conclusions

In this paper, the multi-ultrasonic sensor fusion strategy based on DS theory is presented. Through simulations and testing experiments it is proved that the method proposed in this paper behaves a good performance in changing environment. Paradoxical factor is put forward to modify the combination rules. The new rules are helpful to deal with multi-reflection, and the quality of occupancy map is also improved. With the quantitative map quality metric, the effectiveness of PF has been verified. However, so far our experiment environment is relatively simple and there are no moving obstacles in it, so the further research will be focused on extending the method to adapt more complex environment.

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