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Smarandache-lattice and algorithms

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Abstract

In this paper we introduced algorithms for constructing Smarandache-lattice from the Boolean algebra through Atomic lattice, weakly atomic modular lattice, Normal ideals, Minimal subspaces, Structural matrix algebra, Residuated lattice. We also obtained algorithms for Smarandache-lattice from the Boolean algebra. For basic concept we refer to Gratzer [3].

Keywords: Smarandache-lattice, Lattice, Boolean algebra.

2010 MSC: 54A05, 54D10.

1 Introduction

In this paper we have introduced algorithms to construct Smarandache-lattice. Smarandache-lattice is one the Smarandache-2-Algebraic Structure. By [7] Smarandache n-structure on a set S means a weak structure $\{w_0\}$ on S such that there exists a chain of proper subsets $P_{n-1} < P_{n-2} < \cdots < P_2 < P_1 < S$, where '$<$' means 'included in', whose corresponding structures verify the inverse chain $\{w_{n-1}\} > \{w_{n-2}\} > \cdots > \{w_2\} > \{w_1\} > \{w_0\}$, where '$>$' signifies 'strictly stronger' (i.e., structure satisfying more axioms)By proper subset of a set S, we mean a subset P of S, different from the empty set, from the original set S, and from the idempotent elements if any. And by structure on S we mean the strongest possible structure $\{w\}$ on S under the given operation(s). As a particular case, a Smarandache 2-algebraic structure (two levels only of structures in algebra) on a set S, is a weak structure $\{w_0\}$ on S such that there exists a proper subset P of S, which is embedded with a stronger structure $\{w_1\}$.
Example: Semi lattice $<$ Lattice $<$ Boolean algebra.

2 Preliminaries

Definition 2.1. *The Lattice L is called complemented Lattice. If L has a greatest element and least element and each element has at least one complement; that is, for $b \in L$, there exists $a \in L$ such that $a \vee b = 1$, $a \wedge b = 0$.*

Definition 2.2. *The Smarandache-lattice is defined to be a lattice S, such that a proper subset of S, is a Boolean algebra (with respect to with same induced operations). By proper subset we understand a set included in S, different from the empty set, from the unit element if any, and from S.*

Definition 2.3 (Alternative Definition 2.2). *If there exists a non empty set L which is a Boolean algebra such that its Superset S of L is a Lattice with respect same induced operations. Then S is called Smarandache-lattice.*

Definition 2.4. *A Residuated lattice is an algebraic structure $(R, \wedge, \vee, \rightarrow, \otimes, \oplus, 0, 1)$ such that*

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(i) $(R, \wedge, \vee, \rightarrow, \otimes, \oplus, 1, 30)$ *is bounded lattice with least element 1 and greatest element 30.*

(ii) $(R, \otimes, 30)$ *is Commutative monoid where 30 is a unit element.*

(iii) $a * b \leq c$ *if and only if* $a \leq b \rightarrow c$.

Definition 2.5. *Let* $(L, \wedge, \vee, 0, 1)$*be a Boolean algebra. A subset* I *of* L *is called an ideal of* B *if*

(i) $0 \in I$.

(ii) $a, b \in I \Rightarrow a \vee b \in I$.

(iii) $a \in I$ *and* $b \leq a \Rightarrow b \in I$.

Definition 2.6. *Given an element* a *of a Boolean algebra (or other poset)* A*, recall that* a *is atomic in* A *if* a *is minimal among non-trivial (non-bottom) elements of* A*. That is, given any* $b \in A$ *such that* $b \leq a$*, either* $b = 0$ *or* $b = a$*. A Boolean algebra* A *is atomic if we have* $b = \bigvee_I a_i$ *for every* $b \in A$*, where* $\{a_i\}_I$ *is some set of atoms in* A*.*

Definition 2.7. *Boolean algebra is a distributive lattice which satisfies lattices whose congruences form a Boolean algebra.*

(i) *Involution:* $(a')' = a$.

(ii) *Complements:* $a \vee a' = 1$ *and* $a \wedge a' = 0$.

(iii) *Identities:* $a \wedge 1 = a$ *and* $a \vee 0 = a$, $a \vee 1 = 1$ *and* $a \wedge 0 = 0$.

(iv) *De Morgan's laws:* $(a \wedge b)' = a' \vee b'$, $(a \vee b)' = a' \wedge b'$.

3 Characterizations

3.1 Atomic lattice: Algorithm-3.1

Peter Crawly has introduced the notion, "Lattices whose congruence's form a Boolean algebra 1960. In [6] it has been proved that S is an arbitrary lattice,L is s a Boolean algebra if and only if for each proper quotient a/b of S there exists a finite chain $a = x_0 > x_1 > \cdots > x_k = b$ such that each c_{i-1}/c_i is minimal. We have proved that Boolean algebra itself is a atomic lattice ($L = A_0$), and hence every element of L is join of atoms c_{i-1}/c_i generated by minimal quotients x_i/y_j,we must have $c_{i-1}/c_i = x_i/x_j \in S$. The union of atomic lattice is called as a Lattice at the same time the intersection of atomic Lattice is non-zero unique set included in a lattice. By Gratze [3], S is a Lattice by definition S is a Smarandache-lattice. According to this hypothesis, we have to write an Algorithm for constructing a Smarandache-lattice from the Boolean algebra as follows.

Step 1: Consider a Boolean algebra L.

Step 2: Let $L = A_0$.

Step 3: Let $A_i = \theta_{c_{i-1}/c_i}$, $i = 1, 2, \ldots$ be supersets of θ_{c_0/c_1}.

Step 4: Let $S = \bigcup_{i=1}^{K} \theta_{c_{i-1}/c_i}$.

Step 5: Choose sets A_j from $A_i's$ subject to for all $a, b \in S$.
A Boolean algebra A is atomic if for every $b \in A$ such that $b = V_I a_i b \in A$, where $(a_i)_I$ is some set of atoms in A.

Step 6: Verify that $\cap A_j = \theta_{c_0/c_1} \cap \theta_{c_1/c_2} \cap \theta_{c_2/c_3} \cap \theta_{c_3/c_4} \ldots \cap \theta_{c_{k-1}/c_k} = \theta_{c_0/c_1} \neq \{0\} \subset S$.

Step 7: If Step (6) is a true, then we write S is a Smarandache-lattice.

3.2 Weakly atomic modular lattice: Algorithm-3.2

Peter Crawly has introduced the notion, "Lattices whose congruence's form a Boolean algebra 1960. In [6] it has been proved that S be a weakly atomic modular lattice. Then $\theta(L)$ is a Boolean algebra if and only if every quotient of L is finite dimensional. We have proved L be a weakly atomic modular Lattice itself Boolean algebra ($L = M_0$). The union of weakly atomic modular Lattice called as a Lattice at the same time the intersection of weakly atomic modular Lattice is non-zero unique set included in a Lattice. By Gratzer [3], S is a lattice by definition S is a Smarandache-latticeAccording to this hypothesis, we have to write an Algorithm for constructing a Smarandache-lattice from the Boolean algebra as follows.

Step 1: Consider a Boolean algebra L.

Step 2: Let $L = M_0$.

Step 3: Let $M_i,\ i = 0, 1, 2 \ldots$ be supersets of M_0.

Step 4: Let $S = \cup M_i$.

Step 5: Choose sets M_j from $M_i s$ subject to for all $a, b \in S$, $(a')' = a$, $a \vee a' = 1$ and $a \wedge a' = 0$, $a \wedge 1 = a$ and $a \vee 0 = a$, $a \vee 1 = 1$ and $a \wedge 0 = 0$, $(a \wedge b)' = a' \vee b'$, $(a \vee b)' = a' \wedge b'$.

Step 6: Verify that for every $\cap M_j = M_0 \neq \{0\} \subset S$.

Step 7: If step (6) is a true, then we write S is a Smarandache-lattice

3.3 Normal ideals: Algorithm-3.3

In [4], it has been proved that NI is a Normal ideals itself complete semi-Lattice(Boolean algebra). The union of Normal ideals called as a Lattice at the same time the intersection of Normal ideals contained in all other nonzero normal ideals of Lattice. By Gratzer [3], S is a Lattice by definition S is a Smarandache-lattice. According to this hypothesis, we have to write an Algorithm for constructing a Smarandache-lattice from the Boolean algebra as follows.

Step 1: Consider a Boolean algebra L.

Step 2: Let $L = I_0$.

Step 3: Let $I_i,\ i = 0, 1, 2, \ldots$ be supersets of I_0.

Step 4: Let $S = \cup I_i$.

Step 5: Choose sets I_j from I_i's, subject to for all $a, b \in S$.

(i) $0 \in I$

(ii) $a, b \in I \Rightarrow$ a $\vee b \in I$

(iii) $a \in I$ and $b \leq a \Rightarrow b \in I$.

Step 6: Verify that for every$\cap I_j = I_0 \neq \{0\} \subset S$.

Step 7: If Step (6) is a true, then we write L is a Smarandache-lattice.

3.4 Minimal subspaces: Algorithem 3.4

In 2013, Emira, Barker George Philip have introduced the notion of a Lattice to be a Boolean algebra. Emira, Barker George Philip in their paper [1] have proved, if the Lattice L of subspaces of a structural algebra is complemented then the complement W is unique. Suppose $V \in S$, V is a sum of minimal subspaces, each of which is in other irreducible subspaces then V has complement in S. L is a Boolean algebra if and only if there is no chain of non zero irreducible elements. We have proved V_0 be a Minimal subspaces itself Boolean algebra. The union of Minimal subspaces called as a Lattice at the same time the intersection of Minimal subspaces is nonzero unique set included in a Lattice. By Gratzer, [3], S is a lattice by definition S is a Smarandache-lattice. According to this hypothesis, we have to write an Algorithm for constructing a Smarandache-lattice from the Boolean algebra as follows.

Step 1: Consider a Boolean algebra L.

Step 2: Let $L = V_0$.

Step 3: Let $V_i,\ i = 0, 1, 2, \ldots$ be supersets of V_0.

Step 4: Let $S = \cup V = (U_i \cap V_i)$.

Step 5: Choose sets V_j from V_i subject to for all $B_1, B_2 \in S$ such that $B_1 = B \cap V$, $B_2 = B/B_1 \Leftrightarrow$ span $B_2 \in S, V$ has a complement in S where B is a basis for S. Each U_j has a complement W_j now suppose V is the sum of minimal subspaces
$V = U_1 + U_2 + \cdots + U_S$,
$W = W_1 \cap W_2 \cap \cdots \cap W_S \in S$
$U \cap W = U_1 + \cdots + U_S \cap W \subseteq (U_1 \cap W_1) + \cdots + (U_S \cap W_S)$
$V + W = V + (W_1 \cap \cdots \cap W_S) \supseteq (U_1 + W_1) \cap \cdots \cap (U_S + W_S) = F^n$.

Step 6: $\cap V_j = V_0 \neq \{0\} \subset S$.

Step 7: If step (6) is a true, then we write S is a Smarandache-lattice.

3.5 Point lattice: Algorithm-3.5

In 2013, Emira, Barker George Philip have introduced the notion of a Lattice to be a Boolean algebra. Akkurt,Mustafa,Emira,barker George Philip in their paper [1] have proved, If the Lattice L of subspaces of a structural algebra is complemented then the complement W is unique, where $W = V_1 + V_2 + \cdots + V_k$ is the collection of the irreducible subspaces contained in W. Let $M_n(F, \rho)$be structural matrix algebra with $L = Lat(M_n(F, \rho))$ its lattice.L is Boolean algebra if and only if L is an atomic lattice. We have proved P_0 be a Point Lattice itself Boolean algebra. The union of Point Lattice is called as a Lattice at the same time the intersection of Point Lattice is nonzero unique set included in a Lattice. By Gratzer [3], S is a Lattice by definition S is a Smarandache-lattice.

According to this hypothesis, we have to write an Algorithm for constructing a Smarandache-lattice from the Boolean algebra as follows.

Step 1: Consider a Boolean algebra L.

Step 2: Let $L = P_0$ point lattice.

Step 3: Let $P_i,\ i = 0, 1, 2, \ldots$ be super sets of P_0.

Step 4: Let $S = \cup P_i$.

Step 5: Choose sets P_j from P_i subject to for all $P_1, P_2 \in L$ such that $P_1 = B \cap P$, $P_2 = B/B_1 \Leftrightarrow$ span $B_2 \in S, V$ has a complement in L, where S is a basis for L each U_j has a complement W_j now suppose V is the sum of minimal.

Step 6: $W = \cap P_j = P_0 \neq \{0\} \subset S$.

Step 7: If step (6) is a true, then we write S is a Smarandache-lattice.

3.6 Residuated lattice: Algorithm-3.6

$L = \{1, 30\}$ is a Boolean algebra with respect to $(L, \vee, \wedge, 1, 30)$ [5]. We have proved that all axioms are satisfied for Boolean algebra and this Boolean algebra itself is a Residuated Lattice. The union of Residuated Lattice is called as a Lattice at the same time the intersection of Residuated lattices is a unique nonzero set included in Lattice. By Gratzer [3], S is a Lattice by definition S is a Smarandache-lattice. According to this hypothesis, we have to write an Algorithm for constructing a Smarandache-lattice from the Boolean algebra as follows.

Step 1: Consider a nonempty Set $L = \{1, 30\}$.

Step 2: Verify that $L = \{1, 30\}$ is a Boolean algebra with respect to $\wedge, \vee$.
For, check the following conditions

(i) Associative Law: For any $a, b, c \in L$, $a \vee (b \vee c) = (a \vee b) \vee c \vee$ is defined as follows:

$$1 \vee (1 \vee 1) = 1 \vee 1 = 1 \in L$$
$$(1 \vee 1) \vee 1 = 1 \vee 1 = 1 \in L$$
$$1 \vee (1 \vee 1) = (1 \vee 1) \vee 1$$
$$30 \vee (30 \vee 30) = 30 \vee 30 = 30 \in L$$
$$(30 \vee 30) \vee 30 = 30 \vee 30 = 30 \in L$$
$$30 \vee (30 \vee 30) = (30 \vee 30) \vee 30$$
$$1 \vee (30 \vee 30) = 1 \vee 30 = 30 \in L$$
$$(1 \vee 30) \vee 30 = 30 \vee 30 = 30 \in L$$
$$1 \vee (30 \vee 30) = (1 \vee 30) \vee 30$$
$$1 \vee (30 \vee 1) = 1 \vee 30 = 30 \in L$$
$$(1 \vee 30) \vee 1 = 30 \vee 1 = 30 \in L$$
$$1 \vee (30 \vee 1) = (1 \vee 30) \vee 1$$
$$1 \wedge (1 \wedge 1) = 1 \wedge 1 = 1 \in L$$
$$(1 \wedge 1) \wedge 1 = 1 \wedge 1 = 1 \in L$$
$$1 \wedge (1 \wedge 1) = (1 \wedge 1) \wedge 1$$
$$30 \wedge (30 \wedge 30) = 30 \wedge 30 = 30 \in L$$
$$(30 \wedge 30) \wedge 30 = 30 \wedge 30 = 30 \in L$$
$$30 \wedge (30 \wedge 30) = (30 \wedge 30) \wedge 30$$

$\wedge$ is defined as follows:

$$1 \wedge (30 \wedge 30) = 1 \wedge 30 = 1 \in L$$
$$(1 \wedge 30) \wedge 30 = 1 \wedge 30 = 1 \in L$$
$$1 \wedge (30 \wedge 30) = (1 \wedge 30) \wedge 30$$
$$1 \wedge (30 \wedge 1) = 1 \wedge 1 = 1 \in L$$
$$(1 \wedge 30) \wedge 1 = 1 \wedge 1 = 1 \in L$$
$$1 \wedge (30 \wedge 1) = (1 \wedge 30) \wedge 1$$

(ii) Commutative law: For any a, b ∈L, $(a \vee b) = (b \vee a)$

$$1 \vee 1 = 1 \vee 1 = 1 \in L$$
$$30 \vee 30 = 30 \vee 30 = 30 \in L$$
$$1 \vee 30 = 30 \vee 1 = 30 \in L$$

(iii) Distributive law: For all

$$a, b, c \in L \; a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$
$$a, b, c \in L \; a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$
$$1 \vee (30 \wedge 30) = (1 \vee 30) \wedge (1 \vee 30) = 30 \in L$$
$$1 \vee (1 \wedge 1) = (1 \vee 1) \wedge (1 \vee 1) = 1 \in L$$
$$30 \vee (30 \wedge 30) = (30 \vee 30) \wedge (30 \vee 30) = 30 \in L$$
$$30 \vee (1 \wedge 30) = (30 \vee 1) \wedge (30 \vee 30) = 30 \in L$$
$$1 \wedge (30 \vee 30) = (1 \wedge 30) \vee (1 \wedge 30) = 1 \in L$$
$$1 \wedge (1 \vee 1) = (1 \wedge 1) \vee (1 \wedge 1) = 1 \in L$$
$$30 \wedge (30 \vee 30) = (30 \wedge 30) \vee (30 \wedge 30) = 30 \in L$$
$$30 \wedge (1 \vee 30) = (30 \wedge 1) \vee (30 \wedge 30) = 30 \in L.$$

(iv) Identity element there exists identity 1 (‘0’element) for $\vee$ and 30(‘1’ element) for $\wedge$
For any $a \in L(a \vee 1) = a,\ a \wedge 30 = a$
For $1 \in L(1 \vee 1) = 1,\ 1 \wedge 30 = 1 \in L$
For $30 \in L(1 \vee 30) = 30,\ 30 \wedge 30 = 30 \in L$

(v) Complement every element of L has a complement with in L there exists a′ is the complement of a then $a \in L,\ a \vee a' = 30,\ a \wedge a' = 1$, $1' = 30$ $30' = 1$.

(vi) Idempotent Laws: For any $a \in L$, $a \vee a = a$, $a \wedge a = a$, $1 \vee 1 = 1$, $1 \wedge 1 = 1$, $30 \vee 30 = 30$, $30 \wedge 30 = 30$

(vii) Null Law: For any $a \in L$, $a \vee 1 = 1$, $a \wedge 0 = 0$, 0 element is 1 and 1 element is 30.
$30 \vee 1 = 30$, $30 \vee 30 = 30$, $1 \wedge 1 = 1$, $30 \wedge 1 = 1$.

(viii) Absorption Law: For any $a, b \in$L, $a \wedge (a \vee b) = a, a \vee (a \wedge b) = a$

$$30 \wedge (30 \vee 1) = 30, 30 \vee (30 \wedge 1) = 30$$
$$1 \wedge (1 \vee 30) = 1, 1 \vee (1 \wedge 30) = 1$$

(ix) De-Morgan’s Law: For any $a, b \in L$, $(a \vee b)' = a' \wedge b'$, $(a \wedge b)' = a' \vee b'$.

$$\begin{aligned}
(1 \vee 30)' &= 30' = 1 \\
1' \wedge 30' &= 30 \wedge 1 = 1 \\
(1 \vee 30)' &= 1' \wedge 30' = 30 \\
(1 \wedge 30)' &= 1' = 30 \\
1' \vee 30' &= 30 \vee 1 = 30 \\
(1 \wedge 30)' &= 1' \vee 30'
\end{aligned}$$

(x) Involution Law: $a \in L$, $(a')' = a$, $(1')' = 30' = 1$ and $(30')' = 1' = 30$.
$L = \{1, 30\}$ satisfies all the conditions of Boolean algebra.
Hence $L = (L, \wedge, \vee, 1, 30')$ is a Boolean algebra.

Step 3: Let $L = R_0$ be a Residuated lattice. Let $R_0 = L = \{1, 30\}$.

Step 4: Consider super sets $R_i; i = 01, 2, 3$ of R_0.

$$\begin{aligned}
R_0 &= \{1, 30\}, \\
R_1 &= \{1, 2, 15, 30\}, \\
R_2 &= \{1, 2, 3, 10, 15, 30\}, \\
R_3 &= \{1, 2, 3, 5, 6, 10, 15, 30\}.
\end{aligned}$$

Step 5:

$$\begin{aligned}
&S = \bigcup_{i=0}^{3} R_i. \\
&S = R_0 \cup R_1 \cup R_2 \cup R_3 \\
&S = \{1, 30\}, \cup \{1, 2, 15, 30\} \cup \{1, 2, 3, 10, 15, 30\} \cup \{1, 2, 3, 5, 6, 10, 15, 30\} \\
&\quad = \{1, 2, 3, 5, 6, 10, 15, 30\} \supseteq L
\end{aligned}$$

Step 6: A Residuated lattice is an algebraic structure $(R, \wedge, \vee, \rightarrow, \otimes, \oplus, 0, 1)$ such that

(i) $(R, \wedge, \vee, \rightarrow, \otimes, \oplus, 1, 30)$ is bounded lattice with least element 1 and greatest element 30.

(ii) $(R, \otimes, 30)$ is Commutative monoid where 30 is a unit element.

(iii) $a * b \leq c$ if and only if $a \leq b \rightarrow c$ for all $a, b, c \in R$.

Step 7: $(R, \oplus, \otimes, \rightarrow, 1, 30)$ is a Residuated Lattice. $\oplus, \otimes, \rightarrow$ is defined as follows.

(i) $a \otimes b = GLB\{a, b\}$.
$1 \otimes 1 = 1, 1 \otimes 30 = 1, 30 \otimes 1 = 1$ and $30 \otimes 30 = 30$.

(ii) $a \oplus b = LUB\{a, b\}$.
$1 \oplus 1 = 1, 1 \oplus 30 = 30, 30 \oplus 1 = 30$ and $30 \oplus 30 = 30$.

(iii) $a \rightarrow b = a' \oplus b$.
$1 \rightarrow 1 = 30, 1 \rightarrow 30 = 30, 30 \rightarrow 1 = 1, 30 \rightarrow 30 = 30$.
Hence R_0 satisfies required condition. We observe that $a^*b \leq c$ if and only if $a \leq b \rightarrow c$ for all $a, b, c \in R$.
Hence for $R_1 = \{1, 2, 15, 30\}$ and $S = \{1, 2, 3, 5, 6, 10, 15, 30\}$.

Step 8: Verify $\cap R_j = \{1, 30\} \cap \{1, 2, 15, 30\} \cap \{1, 2, 3, 10, 15, 30\} \cap \{1, 2, 3, 5, 6, 10, 15, 30\} = R_0 \subseteq S$.

Step 9: If the Step 8 is true, then write S is Smarandache-lattice

4 Conclusion

In this paper we have to study Algorithm for construct a Smarandache-lattice from the Boolean algebra by an algorithmic approach through its substructures and smarandache lattice has been introduced in some applications.

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