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Smarandache – R-Module and BF-Algebras

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Abstract. In this paper we introduced Smarandache – 2 – algebraic structure of R-Module namely Smarandache – R-Module. A Smarandache – 2 – algebraic structure on a set N means a weak algebraic structure A_0 on N such that there exist a proper subset M of N, which is embedded with a stronger algebraic structure A_1, stronger algebraic structure means satisfying more axioms, by proper subset one understands a subset different from the empty set, from the unit element if any, from the whole set. We define Smarandache-R-Module and obtain some of its characterization through S-Algebra and BF Algebras.

Keyword: R-Module, S-algebra, Smarandache – R-Module, BF-Algebras

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1. Introduction

New notions are introduced in algebra to study more about the congruence in number theory by Florentinsmarandache [2]. By <proper subset> of a set A, We consider a set P included in A and different from A, differentfrom the empty set, and from the unit element in A – if any they rank the algebraic structures using an order relationship.
The algebraic structures S1 << S2 if :both are defined on the same set :: all S1 laws are also S2 laws; all axioms of S1 law are accomplished by the corresponding S2 law; S2 law strictly accomplishes more axioms than S1 laws, or in other words S2 laws has more laws than S1.

For example : semi group <<monoid<< group << ring << field, or Semi group << commutative semi group, ring << unitary ring, etc. they define a General special structure to be a structure SM on a set A, different from a structure SN, such that a proper subset of A is an SN structure, where SM << SN.

Definition 1. Let R be a module, called R-module.If R is said to be smarandache – R – module. Then thereexist a proper subset A of R which is an S- algebra with respect to the same induced operations of R.

Definition 2. *The B-algebra* is an algebra (A;*,0) of type (2,0) (i.e., a nonempty set *A* with a binary operation * and a constant 0) satisfying the following axioms:

(Bl) x*x = 0,
(B2) $x * 0 = x$,
(B) $(x * y) * z = x * [z * (0 * y)]$.

In BH-algebras, which are a generalization of *BCK/BCI/B-algebras.* An algebra *(A;* *, 0) of type (2,0) is a*BH-algebra* if it obeys (Bl), (B2), and (BH) $x * y = 0$ and $y * x = 0$ imply $x = y$. *In* a *BG-algebra i*s an algebra (A,;*,0) of type (2,0) satisfying (Bl), (B2), and (BG) $x = (x * y) * (0 * y)$.

Definition 3. A *BF-algebra* is an algebra (A;*,0) of type (2,0) satisfying (Bl), (B2), and the following axiom:
(BF) $0 * (x * y) = y * x$.

Theorem 1. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) then the following axioms are true.
(a) $0 * (0 * x) = x$ *for all* $x \in A$;
(b) *if* $0 * x = 0 * y$, *then* $x = y$ *for any* $x, y \in A$;
(c) *if* $x * y = 0$. *then* $y * x = 0$ *for any* $x, y \in A$.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras.
Let A be a BF-algebra and $x \in A$. By (BF) and (B2)
we obtain $0 * (0 * x) = x * 0 = x$, that is, (a) holds.
Also (b) follows from (a). Let now $x, y \in A$ and $x * y = 0$. Then $0 - 0 * 0 = 0 * (x * y) -- y * x$.This gives (c).

Definition 4. A BF-algebra is called a *BF_1-algebra* (resp. a *BF2-algebra)*if it obeys (BG) (resp. (BH)).

Theorem 2. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6). Then the*algebra* A = (A;*,0) *of type* (2,0) *is a BF_1-algebra if and only if it obeys the laws* (Bl). (BF).*and*(BG).
Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras. Suppose that (Bl), (BF), and (BG) are valid in A. Let $x \in A$. Substituting $y = x$, (BG) becomes $x = (x * x) * (0 * x)$. Hence applying (Bl) and (BF) we conclude that $x = 0 * (0 * x) = x * 0$. Consequently, (B2) holds. Therefore Ais a BF_1-algebra. The converse is obvious.

Theorem 3. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6). *Then* A *is a BF2-algebra if and only if A satisfies* (B2). (BF); and the following axiom*:*
(BH') $x*y = 0 <=> x = y$.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras. Let A be a BF_2-algebra. By definition, (B2) and (BF) are valid in A. Suppose that $x * y = 0$ for x, $y \in A$. *theorem 1*(c) yields $y * x = 0$. From (BH) we see that$x = y$. If $x = y$, then $x * y = 0$ by (Bl). Thus (BH') holds in A.

Let now A satisfies (B2), (BF), and (BH'). (BH') implies (Bl) and (BH).Therefore A is BF_2algebra.

Theorem 4. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) then the following statements are equivalent:
(a) A *is a BF\-algebra;*
(b) $x = [x * (0 * y)] * y$ *for all* $x, y \in A$;
(c) $x = y * [(0 * x) * (0 * y)]$ *for all* $x,y \in A$.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras (a) =>(b): Let A be a BF_1-algebra and $x,y \in A$. To obtain (b), substitute$0 * y$ for y in (BG) and then use Theorem 1(a).
(b) ==> (c): We conclude from (b) that $0 * x = [(0 * x) * (0 * y)] * y$.
Hence$0 * (0 * x) = y * [(0 * x) * (0 * y)]$ by (BF). But $0 * (0 * x) = x$, and we have (c).
(c) ==> (a): Let (c) hold. (BF) clearly forces
$0 * x = [(0 * x) * (0 * y)] * y$. (1)Using (1) with $x = 0 * a$ and $y = 0 * b$
we have$0 * (0 * a) = [(0 * (0 * a)) * (0 * (0 * b))] * (0 * b)$.
Hence applying Theorem 1(a).we deduce that $a = (a * b) * (0 * b)$. Consequently, A is a BF_1algebra.

Theorem 5. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) then the following statements are true:
(a) A *is a BG-algebra;*
(b) *For* $x,y \in A$, $x * y = 0$ *implies* $x = y$;
(c) *The right cancellation law holds in* A. *i.e., If* $x*y = z*y$, *then* $x = z$ *for any* $x,y,z \in A$;
(d) *The left cancellation law holds in* A. *i.e., if* $y*x = y*z$, *then* $x = z$ *for any* $x.y,z \in A$.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras (a) is a direct consequence of the definitions.
(b): Let x, $y \in A$ and $x * y = 0$. By (BG), $x = (x * y) * (0 * y) = 0 * (0 * y)$.From Theorem 1(a) we concludethat $x = y$.
(c) is obvious, since the right cancellation law holds in every *B*G-algebra.
(d) Follows from (c) and (BF).

Definition 5. A subset *I* of *A* is called an *ideal* of A if it satisfies:
(I_1) $0 \in I$,
(I2) $x * y \in I$ and $y \in I$ imply $x \in I$ for any $x,y \in A$.

We say that an ideal I of A is *normal* if for any $x,y,z\in$ A, $x * y \in I$ implies $(z * x) * (z * y) \in I$.
An ideal I of A is said to be *proper ii* $I \neq A$.

Theorem 6. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) and *let* ***I*** *be a normal ideal of a BF-algebra* A. then the following statements are true:
(a) $x\in I \Rightarrow 0 * x \in I$,
(b) $x * y \in I \Rightarrow y * x \in I$.
Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras
(a) Let $x \in I$. Then $x = x * 0\in I$. Since I is normal, $(0 * x) * (0 * 0) \in I$.
Hence $0 * x \in I$.
(b) Let $x*y \in F$ By (a), $0*(x*y) \in$ IApplying (BF) we have $y*x \in$ **I**

Definition 6. A nonempty subset N of A is called a *subalgebra*of A ifx $* y \in$ N for any x, y $\in$ N. It is easy to see that if N is a subalgebra of A, then $0\in$ N.

Theorem 7. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) and l*et N be a subalgebra of A. If it satisfies* $x * y \in$ N, *then* $y * x \in$ N.
Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras. Let $x*y \in$ N. By (BF), $y *x = 0* (x *y)$. Since $0 \in$ N andx $* y \in N$,we see that $0 * (x * y) \in N$. Consequently, $y * x \in N$.

Theorem 8. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) *then I is a subalgebra of* A *satisfying the following condition:*

(NI) *if* $x \in A$ *and* $y \in I$, then $x * (x * y) \in I$.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras. Let $x\in A$ and $y \in I$. Theorem 3(a) shows that $0 * y\in I$. Since I is normal, we conclude that $(x*0)*(x* y) \in I$, i.e., $x *(x *y)\in I$. Thus (NI) holds. Let now $x,y\in I$. Therefore $x * (x * y) \in I$.By *Theorem* 3(b), $(x * y) * x \in I$. From the definition of ideal we have $x * y \in I$. Thus I is a subalgebra satisfying (NI).

Theorem 9. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) t*hen N is a normal subalgebra of* A *if and only if N is a normal ideal.*
Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras. Let N be a normal subalgebra of A. Clearly, $0 \in$ N. Suppose thatx $* y \in$ N and $y \in N$. Then $0 * y\in$ N. Since N is a subalgebra, we have $(x * y) * (0 * y) \in$ N. But $(x * y)$

* (0 * *y)* = x, because every B-algebra satisfies(BG). Therefore $x \in N$, and thus N is an ideal. Let now $x,y,z \in A$ and $x * y \in N$.By (NS), $(z * x) * (z * y) \in N$. Consequently, *N* is normal. The converse follows from Theorem 8. Hence the Proof.

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