Updating Attribute Fusion Results with Additional Evidence Using *DSmT*

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***Abstract*—This paper performs an investigation related to updating uncertain evidence with confirmed or verified information for the purpose of information fusion in C2 systems.**

The key question is to decide whether the updating of uncertain evidence with information originated from inference should apply combination rules or conditioning rules.

The author's suggestion is in the first place to investigate whether the mentioned categorisation of rules (combination and conditioning) reflects any significant difference in fusion performance to justify the category distinction in the practical term.

I. Introduction

Nowadays, modern Command & Control systems use information obtained from multiple sensors of a different kind in order to resolve problems like target tracking, attributes fusion, and textual messages processing. These tasks, however, are typically done by specified subsystems which cooperate with each other within the C2 system. The products of each subsystem perform pieces of information that, in general, are of a different degree of processing and, in the consequence, of different certainty.

In the most idealistic case these pieces of information interact effectively with each other in order to build a knowledge (so called situation awareness). Since the subsystems operate with information of different certainty, they should be equipped with adequate fusion techniques that enable to combine its product accordingly to the degree of information processing.

In the Theory of Evidence by Dezert and Smarandache (*DSmT*) [4] a process of integration of uncertain pieces of information with confirmed i.e. certain evidence is called conditioning which is complementary to operation of uncertain information integration, called combination. *DSmT* distinguishes several combination rules and over thirty different rules of conditioning. The scientific problem addressed herein is to investigate whether the updating of uncertain evidence with information originated from inference should apply combination rules or conditioning rules.

II. Combination and Conditioning Rules

Combination of information obtained from multiple sources has been an important subject of research since the Dempster's rule of combination occured. From that moment many solutions have been proposed in order to manage the evidence conflict. One of these solution is a class of *Proportional Redistriution Rules*, given by Dezert and Smarandache.

The general principle of the *Proportional Conflict Redistribution Rules* (*PCR* for short) is:

– apply the conjunctive rule
– calculate the total or partial conflicting masses
– then redistribute the conflicting mass (total or partial) proportionally on non-empty sets involved in the model according to all integrity constraints [4].

"The way the redistribution is done makes the distinction between all existing rules available in literature in the *DST* and *DSmT* frameworks (to the knowledge of the authors) and the *PCR* rules, and also the distinction among the different *PCR* versions themselves." [4]. "The *PCR* combination rules work for any degree of conflict, for any *DSm* models (Shafers model, *free DSm* model or any *hybrid DSm* model)." [4]

Another class of fusion rules is the class of conditioning rules. The justification of this class of rules was given by the authors of *DSmT*:

"While conditioning a mass $m_1(.)$, knowing (or assuming) that the truth is in *A*, means that we have an absolute (not subjective) information, i.e. the truth is in *A* has occurred (or is assumed to have occurred), thus *A* was realized (or is assumed to be realized), hence it is an absolute truth. Truth in *A* must therefore be considered as an absolute truth when conditioning, while $m_S(A) = 1$ used in *SCR*[1] does not refer to an absolute truth actually, but only to a subjective certainty in the possible occurrence of *A* given by a second source of evidence. This is the main and fundamental distinction between our approaches (*BCR*s) and Shafers (*SCR*)". [4]

III. The Research Problem

To illustrate the research problem it is suggested to consider the following fusion example referring to the threat assessment task. Assume the threat observation model defined as Figure 1. shows, where *FAKER* ($K = F \cap H$) denotes a friendly target acting as hostile for exercise purposes [5], [10], [11], and [12].

Consider a scenario where a friendly target has been observed by two imperfect sensor which provided the evidence, summarised with the basic belief assignment (*bba*) below:

[1]Shafer's Conditioning Rule

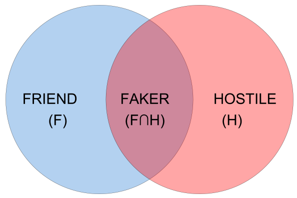

Figure 1. Venn's diagram of the observed target threat

$$m_1(F) = 0.9, \quad m_1(H) = 0.1, \quad m_1(F \cap H) = 0$$
$$m_2(F) = 0.8, \quad m_2(H) = 0.2, \quad m_2(F \cap H) = 0$$

The evidence table for such case should be defined as follows:

Table I
EVIDENCE TABLE FOR THE EXAMPLE THREAT OBSERVATION FUSION CASE

$m_2 \setminus m_1$	F [0.9]	H [0.1]	F ∩ H [0]
F [0.8]	F [0.72]	F ∩ H [0.08]	F ∩ H [0]
H [0.2]	F ∩ H [0.18]	H [0.02]	F ∩ H [0]
F ∩ H [0]	F ∩ H [0]	F ∩ H [0]	F ∩ H [0]

which leads to the following resulting *bba*:

$$m_{12}(F) = 0.72, \qquad m_{12}(H) = 0.02,$$
$$m_{12}(F \cap H) = 0.26$$

Now, imagine that due to some additional knowledge (confirmation or reasoning) one finds out that the target is really friendly.

Applying any of *BCR* conditioning rules [4] for this case leads to obtaining the following updated *bba* and the corresponding belief functions:

$$m_{12}(F|F) = 0.735 \qquad Bel(F|F) = 1$$
$$m_{12}(H|F) = 0 \qquad Bel(H|F) = 0.265$$
$$m_{12}(F \cap H|F) = 0.265 \qquad Bel(F \cap H|F) = 0.265$$

Belief function error and the corresponding covariance may be calculated according to the following formulae:

$$\sigma = Bel_0(\theta_i) - Bel_m(\theta_i) \tag{1}$$
$$Cov = \sigma^T \cdot \sigma \tag{2}$$

where:

Bel_0 - belief function based on ideal *bba*,
Bel_m - belief function based on measured *bba*,
θ_i - particular hypothesis

Thus applying formula (1):

$$\sigma = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0.265 \\ 0.265 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.265 \\ -0.265 \end{bmatrix}, Cov \approx 0.14$$

On the other hand, the additional evidence may be utilized by applying combination rule e.g. *PCR5* in order to combine m_{12} with the condition m_3 which may be expressed as:

$$m_3(F) = 1, \quad m_3(H) = 0, \quad m_3(F \cap H) = 0$$

Thus finally:

$$m_{123}(F|F) = 0.72 \qquad Bel(F|F) = 1$$
$$m_{123}(H|F) = 0 \qquad Bel(H|F) = 0.28$$
$$m_{123}(F \cap H|F) = 0.28 \qquad Bel(F \cap H|F) = 0.28$$

and

$$\sigma = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0.28 \\ 0.28 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.28 \\ -0.28 \end{bmatrix}, Cov \approx 0.16$$

This simple example shows that application of two different (in terms of *DSmT*) operations for consuming the additional, relatively high quality, knowledge may return similar, however, not identical results. Should these sensors provided less specific evidence, i.e. $m_1(F) \to m_1(H)$, and $m_2(F) \to m_2(H)$, the difference between the results of the mentioned two rules would be greater. On the other hand, should these sensors provided conflicting information, e.g. $m_1(F) \to 1$, and $m_2(F) \to 0$, the difference between the results would raise even more.

According to *DSmT*, the usage of the first operation is justified if the conditioning hypothesis is true i.e. the certainty of the condition is assumed. For the second operation there is no such assumption, which makes the rule applicable for combination (fusion) of uncertain information as well. The certainty of the condition resides in the definition of its *bba*, which fosters only one hypothesis (by assigning one to it and neglects the rest of the hypotheses by assigning zeros to them).

In the real world the conditioning information may origin from external cooperating systems, data bases, inference or any other confirmed, however, imperfect sources. Keeping that in mind, a natural question is emerging, whether it is justified to use an operation that presumes the perfectness of the condition (in terms of certainty, accuracy, and precision).

Answering the question above may be very difficult due to the fact that collecting and statistical analysis of a large amount of data is required in order to make reasonable inference.

The author's suggestion is in the first place to investigate whether the mentioned categorisation of rules (combination and conditioning) reflects any significant difference in fusion performance to justify the category distinction in the practical term.

IV. RESEARCH ASSUMPTIONS

In order to make the combination/conditioning rules investigation a number of numerical experiments has been performed. The experiments were based on simulations of a manoeuvring target with preset attributes values, observations of the target characteristics by imperfect sensors, and finally revealing its characteristics by fusion subsystem. The target was described by attributes of threat, platform and activity. The simulators enabled to preset different source characteristics in order to model that the target was observed by radar, video camera and visual sightings [1], [2], [3], and [8].

It was assumed that the visual sightings provided evidence about target platform and activity, however for better clarity the particular *bba* corresponding to these two attributes had been set arbitrarily.

Another assumption was made regarding the origin of the processed information. Namely, deduced values of the attributes were taken into account as well as the observed values. Particular frames of discernment referring to observed and deduced values of the same attributes had been assumed not to be identical [1], [2], [3], and [9].

Additionally, the following assumptions had also been made:.

– The hybrid *DSmT* model is applied:
  - Hypotheses are not disjoint:
  - Hypotheses are defined according to *JC3* model [5]
– Frame of discernment for platform attribute is defined as follows:

$$\Theta_{VC} = \{MHC,\ MHI,\ MHO,\ MSC,\ MSO,\ D\} \tag{3}$$

  where:
  – *MHC* - mine-hunter, coastal [5];
  – *MHI* - mine-hunter, inshore [5];
  – *MHO* - mine-hunter, inshore [5];
  – *MSC* - mine-hunter, coastal [5];
  – *MSO* - mine-hunter, inshore [5];
  – *D* - mine-hunter, inshore [5];
  – *VC* index - obtained from video camera
– For platform attribute, with $\cup$ and $\cap$ operators secondary hypotheses may be created [2] and they have their interpretation as in *JC3* model (surface-vessel-type-category code) [5], namely:
  - $MHC \cup D = MHCD$ (coastal mine-hunter equipped with drones)
  - $MHI \cup MHO \cup MHC \cup D = MH$ (mine-hunter, general)
  - $MHO \cap MSO = MHSO$ (mine-hunter or mine sweeper)
  - $(MHC \cap MSC) \cup D = MHSD$ (mine-hunter or mine sweeper equipped with drones)
  - $(MHO \cap MSO) \cup (MHC \cap MSC) \cup D = MHS$ (coastal mine-hunter or mine sweeper general)

Another assumption was made regarding a structure of the condition, when combination rules were applied for the purpose of conditioning. Application of any combination rule for the reason of conditioning may be performed in two ways:

The first way is to combine the base *bba* with 'degraded' *bba* of the condition, consisting of one hypothesis only, with mass of one.

The second way is to combine the base *bba* with *bba* of the condition, which encompasses multiple hypotheses, named in the condition, with mass distributed equally. This is according to Laplace's principle of insufficient reason. In the author's opinion this technique is preferable for two reasons:

– It is much easier to find interpretations of compound (secondary) hypotheses if they encompass lower number of primary hypotheses, which happens when the condition is split for multiple hypotheses;
– Such solution enables to make advantage of the potential of the particular combination rule due to the fact it involves multiple compound hypotheses.

[2]For some readers the convention of naming secondary hypotheses may seem to be inconsistent due to the fact that adding a particular letter sometimes stands for intersection and sometimes for union. Nevertheless this is not the convention of the author, who only adopted it from *JC3* lexicons.

V. Simulated Scenario

During numerical experimentation the following fusion scenario was considered:

A friendly target follows a randomly generated trajectory, as Figure 2 shows. The aim of the research is to reveal the value of the target threat attribute. Observations are taken from three sources of evidence: visual sightings, video camera and radar (equipped with IFF).

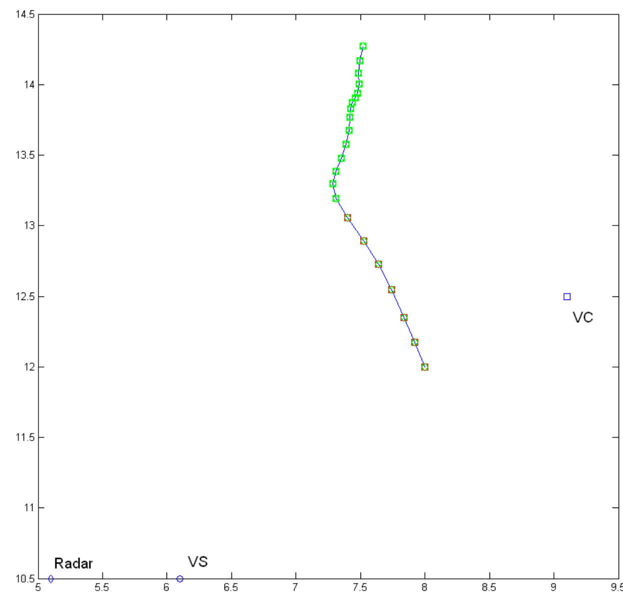

Figure 2. Simulation of randomly-generated target trajectory with threat evaluation based on radar, visual sightings, and video camera observations

The green colour of the trajectory denotes measurements that have been successfully taken for each of the simulated sensors. For example, considering the last sample, the collected *bba*s are as follows:

Combination of evidence obtained from IFF [10] and visual sightings [10] has led to acceptation of FRIEND hypothesis [5].

$$Threat_{VC} \oplus Threat_{IFF} = FRIEND \tag{4}$$

From the visual sightings it is also acquired that the target activity is mine-hunting (*MINE HUNTING MARITIME*). Thus, the combination of threat and the activity attributes results in selection of the target platform, related to searching for mines.

$$(FRIEND,\ MINE\ HUNTING\ MARITIME) \rightarrow Platform \tag{5}$$

Table II
EVIDENCE TABLE FOR THE EXAMPLE THREAT OBSERVATION FUSION CASE

Threat	Visual sightings	Video camera	Radar/IFF
HOSTILE	0.0024	0.0004	0.0008
UNKNOWN	0.0060	0.0012	-
NEUTRAL	0.0068	0.0015	-
JOKER	0.0109	-	-
FRIEND	0.2400	0.4369	0.8774
FAKER	0.0292	0.0049	0.0119
SUSPECT	0.0032	0.0005	0.0011
ASSUMED FRIEND	0.0215	0.0046	0.0088
PENING	0.0068	0.5500	0.1000

that is:

(Threat, Activity) $\rightarrow$ *Platform:*
(FRIEND, MINE HUNTING MARITIME) $\rightarrow$

- *MINEHUNTER COASTAL (MHC)*
- *MINEHUNTER COASTAL WITH DRONE (MHCD)*
- *MINEHUNTER GENERAL (MH)*
- *MINEHUNTER INSHORE (MHI)*
- *MINEHUNTER OCEAN (MHO)*
- *MINEHUNTER/SWEEPER COASTAL (MHSC)*
- *MINEHUNTER/SWEEPER GENERAL (MHS)*
- *MINEHUNTER/SWEEPER OCEAN (MHSO)*
- *MINEHUNTER/SWEEPER W/DRONE (MHSD)*

Taking into account the assumptions of 2 and 3 *bba*, based on video camera observation, is assessed as follows:

$$\begin{array}{ll} m_{VC}(MHC) = 0.1, & m_{VC}(MHCD) = 0.1, \\ m_{VC}(MSC) = 0.2 & m_{VC}(MHI) = 0.3, \\ m_{VC}(MHO) = 0.2, & m_{VC}(MSO) = 0.1 \end{array}$$

Taking into account the implication (5) the above *bba* may be updated according to any of the conditioning rules with the following condition (Cond) [4]:

$$Cond : Truth = MHC \cup MHO \cup MHI \quad (6)$$

In information systems, where concept lexicons are ordered hierarchically, this condition could be expressed in a simpler way, e.g. *MH* (mine-hunter, general). However, due to the given frame of discernment (3) it has to be defined as an alternative of the subsequent mine-hunter types: coastal, oceanic and inshore.

An effective utilisation of the condition (*Cond*) performed the subject of the comparative analysis and the key point of the investigation. The next steps were to repeat the operation with different rules of conditioning or combination.

Updating bba according to *BCR1* led to the subsequent results, which were the basis of the comparison:

$$\begin{array}{l} m_{BCR1}(MHC|Cond) = 0.1429, \\ m_{BCR1}(MHI|Cond) = 0.4286, \\ m_{BCR1}(MHCD|Cond) = 0.1429, \\ m_{BCR1}(MHO|Cond) = 0.2856, \end{array}$$

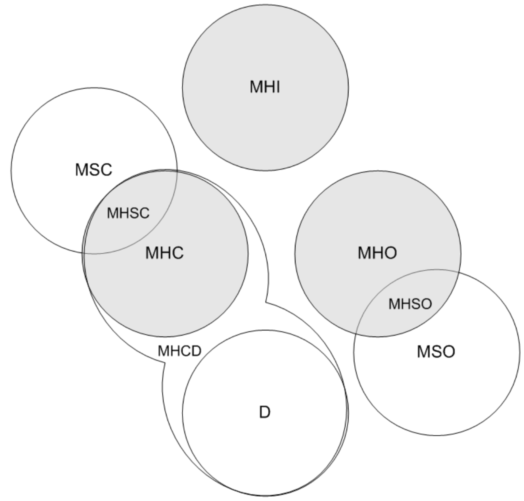

Figure 3. Venn's diagram for target *platform* attribute. The condition is indicated with grey

The following step was to update bba with *BCR12*, which resulted in the subsequent *bba*:

$$\begin{array}{l} m_{BCR12}(MHC|Cond) = 0.2, \\ m_{BCR12}(MHSC|Cond) = 0.2, \\ m_{BCR12}(MHI|Cond) = 0.3, \\ m_{BCR12}(MHO|Cond) = 0.2, \\ m_{BCR12}(MHSO|Cond) = 0.1, \end{array}$$

In case of application of *BCR12* it is worth of notice that calculation of the respective belief and plausibility functions leads to acceptation of *MHC* hypothesis as the resulting decision. Before updating, the belief function for *MHC* was of the least value since:

$$Bel_{VC}(MHC) = m_{VC}(MHC) = 0.1 \quad (7)$$

After updating, due to the fact that $m_{VC}(MHSC)$ supports the belief in *MHC* hypothesis, this hypothesis becomes the most credible since:

$$\begin{aligned} Bel_{BCR12}(MHC) &= m_{BCR12}(MHC|Cond) \\ &+ m_{BCR12}(MHSC|Cond) = 0.4 \quad (8) \end{aligned}$$

The experiment procedure was repeated once again with *BCR17* conditioning rule, which led to the following *bba*:

$$\begin{array}{l} m_{BCR17}(MHC|Cond) = 0.4, \\ m_{BCR17}(MHI|Cond) = 0.3, \\ m_{BCR17}(MH)|Cond) = 0.3, \end{array}$$

Figure 4 performs a summary comparison of the *bba*s resulted by application of *BCR1, BCR12* and *BCR17*. Application of *BCR12* leads to relatively equal distribution of mass among the platform hypotheses, which is typical since as a strongly pessimistic *BCR* rule (see [2]) constrains the condition influence in the highest degree. Application of *BCR1* leads to slightly more diversified distribution, according to which except of mine-hunters, named in the condition (i.e. *MHO, MHC*, and *MHI)* a drone equipped mine-hunter *MHCD* hypothesis is supported. *BCR17* has proven to be the most optimistic conditioning rule of the three. As Figure 4. shows, in the considered fusion case *BCR17* has filtered all the hypotheses to ones named in the condition.

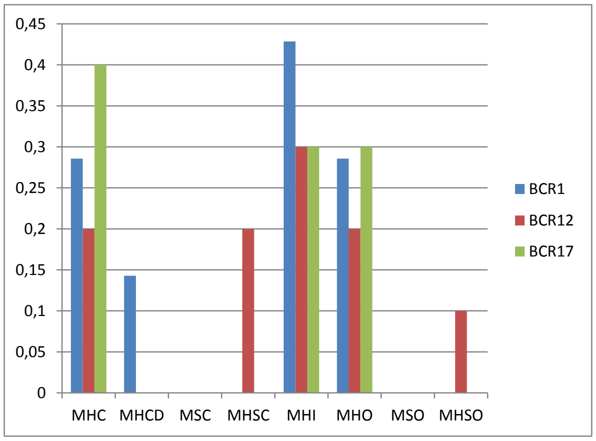

Figure 4. Basic belief assignment updated with: *BCR1*, *BCR12*, and *BCR17*

The last step of the numerical experiments was application of proportional conflict redistribution rule no. 5 (*PCR5*) for the conditioning operation. As it was mentioned in section III application of any combination rule for the reason of conditioning may be performed in to ways. In the considered case application of combination of base *bba* with an unstructured condition i.e. $m_{Cond}(MHC \cup MHO \cup MHI) = 1$ would bring exactly the same results as for conditioning with *BCR12*. On the other hand, application of combination of base *bba* with a structured condition i.e.

$$m_{Cond}(MHC) = 1/3, \quad m_{Cond}(MHI) = 1/3, \quad m_{Cond}(MHO) = 1/3$$

has brought the following results:

$$m_{PCR}(MHC) = [m_R \oplus m_{Cond}](MHC) = 0.202$$
$$m_{PCR}(MHCD) = [m_R \oplus m_{Cond}](MHCD) = 0.0154$$
$$m_{PCR}(MHI) = [m_R \oplus m_{Cond}](MHI) = 0.355,$$
$$m_{PCR}(MHO) = [m_R \oplus m_{Cond}](MHO) = 0.2622,$$
$$m_{PCR}(MHSC) = [m_R \oplus m_{Cond}](MHSC) = 0.0667,$$
$$m_{PCR}(MHSO) = [m_R \oplus m_{Cond}](MHSO) = 0.0333,$$
$$m_{PCR}(MSC) = [m_R \oplus m_{Cond}](MSC) = 0.05,$$
$$m_{PCR}(MSO) = [m_R \oplus m_{Cond}](MSO) = 0.0154,$$

VI. Discussion

In Figure 4 one sees that updating *bba* with *BCR1*, *BCR12*, and *BCR17* rules provides significantly different resulting basic belief assignments. Rules of *BCR12* and *BCR17* are specifically recommended by the authors of *DSmT* as the tools for fusing uncertain pieces of information with confirmed information. Nevertheless, it is important to notice that in the result of a particular application of *BCR17* the hypothesis of *MHC* was assigned a mass twice as big as for *BCR12*.

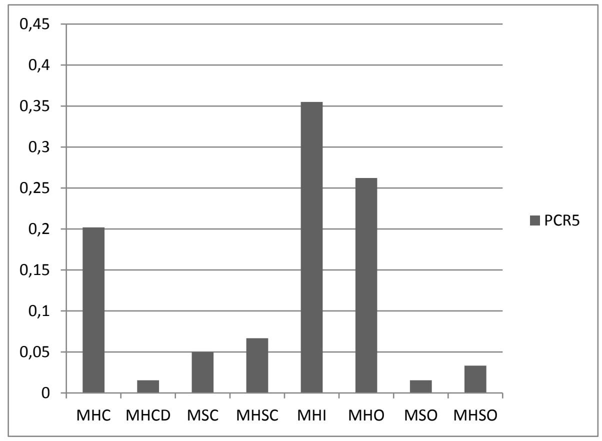

Figure 5. Basic belief assignment updated with: *PCR5*

Observation of the difference mentioned above has pushed the author into consideration of using *PCR5* rule in order to update the evidence in a similar manner as it was presented in section III. As Figure 5. shows, the resulting *bba* is closer to one obtained after performing *BCR12*. The difference resides mainly in fact that *PCR5* assigns the mass to all hypotheses, however the *PCR5*-enhanced distribution is much more selective than one obtained with *BCR12*. The only problem in application of *PCR5* with the structured condition *bba* could be the mass distribution, when hypotheses, which make the condition, are not disjoint. However, that does not happen in practice due to the fact it is a general requirement for the conditioning hypotheses to encompass only disjoint hypotheses.

It is worth of notice that the differences in results of application of the considered combination and conditioning rules become less observable after calculating the respective belief functions, according to the hypotheses relations [6], and [9], contained in the attribute model. This is due to the fact that belief functions may have nonzero values for the hypotheses unrelated to the condition. It was intentionally, by the author, not to introduce the calculation of the subsequent belief functions and provide the analysis on *bba* level in order to enhance the important differences among the diverse fusion rules.

VII. Conclusion

Application of Dezert-Smarandache Theory enables to unify information obtained from miscellaneous (in terms of information processing level) sources like sensor data and deductive reasoning. In this paper it was presented how conditioning information (obtained from inference or confirmed by another highly reliable source) may be used in order to update the incomplete and uncertain evidence based on sensor data.

The fundamental question that forced to start the research works was deciding whether the updating of uncertain evidence with information originated from inference should apply combination rules or conditioning rules. *DSmT* indicates the conditioning rules as ones that should be used to bind uncertain information with confirmed events. However, in real C2 systems information originated from inference is never entirely sure. On the other hand it was desirable to check whether in the considered case of attribute fusion it is important to distinguish the particular class of rules.

The results of the research works have proven that there are significant differences among the distributions obtained for diverse conditioning rules. On the other hand application of combination rules, even if observable in *bba* provided results comparable to pessimistic rule of conditioning *BCR12*. That brings in the following conclusion:

The hypothesis that updating attribute fusion results with additional evidence in *DSmT* framework should be done using condtioning rules only was not confirmed. Depending on the particular fusion problem the choice of the particular rule should follow the comparative analysis in order to select the most adequate one. The particular ordination of the rule (whether it belongs to class of combination or conditioning) is much less important and in the author's opinion both classes should be equally taken into account.

Since there is no significant difference in perfomance between these two classes of rules applied for C2 systems purposes the minimum implementation of *DSmT* framework may be constrained to application of one class of rules - the rules of combination.

ACKNOWLEDGMENT

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