A Fuzzy Neutrosophic Soft Set Model in Medical Diagnosis

Dr.I.Arockiarani

Department of Mathematics
Nirmala College for Women
Coimbatore, India
e-mail: stell11960@yahoo.co.in

Abstract— **The focus of this paper is to explore the notion of fuzzy neutrosophic sets and soft sets. A novel approach is proposed to meet the challenges in medical diagnosis. The calculation of distances, a new score function are all defined to discuss in decision making problem. A topological structure on fuzzy neutrosophic soft set is considered as a tool to derive some of their characterizations.**

Keywords—Fuzzy neutrosophic soft sets; Hammming distances;Euclidean distances; Similarity measure; Mappings of fuzzy neutrosophic soft set.

I. INTRODUCTION

[14] The soft set theory finds wide range of applications in complex medical sciences, engineering, management economics and social sciences primarily due to its flexibility without restrictions on approximate description of the situation. [10, 11]Recently many mathematicians have extended the theory by proposing the concept of fuzzy soft sets and intuitionistic fuzzy soft sets along with their properties and derived many applications in decision making.

Neutrosophic Logic has been proposed by Florentine Smarandache[19, 20] which is based on non-standard analysis that was given by Abraham Robinson in 1960s. Neutrosophic Logic was developed to represent mathematical model of uncertainty, vagueness, ambiguity, imprecision undefined, incompleteness, inconsistency, redundancy, contradiction. The neutrosophic logic is a formal frame to measure truth, indeterminacy and falsehood. In Neutrosophic set, indeterminacy is quantified explicitly whereas the truth membership, indeterminacy membership and falsity membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different sensors. And neutrosophy by its virtue of handling inconsistent and incomplete information seems to be a better choice for modeling medical knowledge base, because it is often not possible to have 100% of information at hand while making decision. From philosophical point of view, the neutrosophic set takes value from real standard or non standard subset of $]^-0,\ 1^+[$. But in real life application in scientific and Engineering problems it is difficult to use neutrosophic set with value from real standard or non standard subset of $]^-0,\ 1^+[$. Hence we consider the fuzzy neutrosophic set which takes the value from the subset of [0,1]. In this paper we define the hamming distances and Euclidean distances and similarity measures. Also we have discussed the topological structure and mappings of fuzzy neutrosophic soft set.

II. PRELIMINARIES

Definition 2.1[14]: Let U be the initial universe set and E be a set of parameters .Let P(U) denotes the power set of U. Consider a non-empty set A , $A \subset E$ .A pair (F,A) is called a soft set over U, where F is a mapping given by $F: A \rightarrow P(U)$.

Definition 2.2[1]: A Fuzzy Neutrosophic set A on the universe of discourse X is defined as

A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where

$T, I, F: X \rightarrow [0, 1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 2.3[1]: Let U be the initial universe set and E be a set of parameters. Consider a non-empty set A, $A \subset E$. Let P (U) denote the set of all fuzzy neutrosophic sets of U. The collection (F, A) is termed to be the fuzzy neutrosophic soft set over U, where F is a mapping given by

$F: A \rightarrow P(U)$.

Throughout this paper Fuzzy Neutrosophic soft set is denoted by FNS set / FNSS.

Definition 2.4[1]: A fuzzy neutrosophic soft set A is contained in another neutrosophic set B. (i.e.,) $A \subseteq B$ if $\forall$ $x \in X$, $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x) \ge F_B(x)$.

Definition 2.5[1]: The complement of a fuzzy neutrosophic soft set (F,A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, \rceil A)$ where $F^c : \rceil A \rightarrow P(U)$ is a mapping given by

$F^c(\alpha) = \langle x,\ T_{F^c}(x) = F_F(x),$

$I_{F^c}(x) = 1 - I_F(x), F_{F^c}(x) = T_F(x) \rangle$

Definition 2.6[1] : Let X be a non empty set, and

$$A = \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle, B = \left\langle x, T_B(x), I_B(x), F_B(x) \right\rangle$$

are fuzzy neutrosophic sets. Then

$$A \cup B = \left\langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \right\rangle$$

$$A \cap B = \left\langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \right\rangle$$

[Note: $\cup$ denote max or $\vee$, $\cap$ denote min or $\wedge$].

Definition 2.7[1]: A fuzzy neutrosophic soft set (F,A) over the universe U is said to be empty fuzzy

978-1-4799-4562-7/14/$31.00 ©2014 IEEE

neutrosophic soft set with respect to the parameter A if $T_{F(e)} = 0, I_{F(e)} = 0, F_{F(e)} = 1, \forall x \in U, \forall e \in A$. It is denoted by $\widetilde{0}_N$.

Definition 2.8[1]: A FNS set (F, A), over the universe U is said to be universe FNS set with respect to the parameter A if $T_{F(e)} = 1, I_{F(e)} = 1, F_{F(e)} = 0, \forall x \in U, \forall e \in A$. It is denoted by $\widetilde{1}_N$.

Definition 2.9[1]: Union of two Neutrosophic soft sets (F,A) and (G,B) over (U, E) is Fuzzy Neutrosophic soft set where $C = A \cup B$ $\forall$ e∈ C.

$$H(e) = \begin{cases} F(\mathrm{e}) & ; \text{ if } \mathrm{e} \in \mathrm{A} - \mathrm{B} \\ G(\mathrm{e}) & ; \text{ if } \mathrm{e} \in \mathrm{B} - \mathrm{A} \\ \mathrm{F(e)} \cup \mathrm{G(e)} & ; \text{ if } \mathrm{e} \in \mathrm{A} \cap \mathrm{B} \end{cases}$$ and is written as

$(F,A) \widetilde{\cup} (G,B) = (H,C)$.

Definition 2.10[1]: The intersection (H, C) of two fuzzy neutrosophic soft sets (F,A) and (G,B) over the common universe U, denoted by (F,A) is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$, $\forall$ e ∈ C and is written as $(F,A) \widetilde{\cap} (G,B) = (H,C)$.

III. Relations On Fuzzy Neutrosophic Soft Sets

Definition 3.1: A fuzzy neutrosophic soft relation $\widetilde{R}$ between two FNSS (F,A) and (G,B) over the universe(U,E) and (U,F) respectively is defined as $\widetilde{R}$ (e, f) = F(e) $\widetilde{\cap}$ G(f), ∀e∈ E and ∀ f ∈ F, where $\widetilde{R}$: K→P(U)[P (U) denotes the set of all fuzzy neutrosophic sets of U] is an FNSS over (U,K), where K⊆E×F.

Definition 3.2: The composition ◦ of two fuzzy neutrosophic soft relations R_1 and R_2 is defined by $(R_1 \circ R_2)$ (a, c) = R_1 (a, b) $\widetilde{\cap}$ R_2 (b, c) where R_1 is a fuzzy neutrosophic soft relation form (F, A) to (G, B) over the universe (U,E)and (U,F) respectively and R_2 is a fuzzy neutrosophic soft relation from (G, B) to (H, C). over the universe (U,F) and (U,K) respectively.

Definition 3.3: The union and intersection of the relations $\widetilde{R}_1$ and $\widetilde{R}_2$ of the fuzzy neutrosophic soft set (F,A) over the universe (U,A) and the fuzzy neutrosophic soft set (G,B) over the universe (U,B) respectively is defined as follows:

$(\widetilde{R}_1 \cup \widetilde{R}_2)$ (a,b)= max {$\widetilde{R}_1$ (a,b), $\widetilde{R}_2$ (a,b)}

$(\widetilde{R}_1 \cap \widetilde{R}_2)$ (a,b) = min {$\widetilde{R}_1$ (a,b), $\widetilde{R}_2$ (a,b)}

where $\widetilde{R}_1$: A×B→P (U) and $\widetilde{R}_2$: A×B→P(U)

(P (U) denotes the set of all fuzzy neutrosophic sets of U).

Definition 3.4: Let A be a fuzzy neutrosophic set in X. Let R be the relation for X to Y. Then max-min-max composition of fuzzy neutrosophic set with A is another fuzzy neutrosophic set B of Y which is denoted by R°A. Then the membership function, indeterminate function and non-membership function of B is defined as

$T_{R \circ A}(y) = \vee_x [T_A(x) \wedge T_A(x, y)]$,

$I_{R \circ A}(y) = \vee_x [I_A(x) \wedge I_A(x, y)]$ and

$F_{R \circ A}(y) = \wedge_x [F_A(x) \vee F_A(x, y)]$ ∀ **y∈ Y**

(where ∨ = max, ∧ = min).

Definition 3.5: Let (F,A) be fuzzy neutrosophic soft set . Then the value function of (F, A) is defined as $V(F,A) = T_A + (1 - I_A) - F_A$ where T_A, I_A and F_A denotes the Truth value, indeterministic value and false value of (F,A) respectively.

Definition 3.6: Let (F,A) and (G,B) be two fuzzy neutrosophic soft set . Then the score function of (F,A)and (G,B) defined as $S_1 = V(F,A) - V(G,B)$.

Definition 3.7: Let (F,A) be fuzzy neutrosophic soft set . Then the score function of (F,A) is defined as $S_2 = T_j - I_j.F_j$.

IV. Application Of Fuzzy Neutrosophic Soft Sets In Medical Diagnosis Using Composition Function

We define mathematically; a patient is a fuzzy neutrosophic set, say P_i, on the set of symptoms S and the fuzzy neutrosophic relation from the set of symptoms S to the set of diseases D, which reveals the degree of association, indetermination and the degree of non- association between the patients and symptoms and between symptoms and diseases.

Algorithm:

Step 1: The symptoms of the patients are given in Table I i.e. the relation Q (P→S) between the patients and symptoms are noted.

Step 2: The medical knowledge relating the symptoms with the set of disease under consideration are noted in Table II i.e. R(S→D) the relation of symptoms and disease are given.

Step 3: The composition T(P→D) the relation of patients and disease are found using the definition 3.4 and noted in Table III

Step 4: Obtain the complement of Table I and is given in Table IV.

Step 5: Obtain the complement of Table II and is noted in Table V

Step 6: Applying the definition 3.4 for the values of Table IV and Table V is denoted in Table VI

*Step 7: C*alculate the value function for Table III and Table VI and is given in Table VII and Table VIII respectively.

Step 8: Find the score function using definition 3.6 for the values in Table VII and VIII and is noted in Table 1.9.

Step 9: We apply another score function for the table III using the definition 3.7 and it is given Table X

Step 10: The higher the score, higher is the possibility of .
the patient affected with the respective disease.

Table I

Q	Vomiting(S_1)	Pain in Abdomen(S_2)	Temperature(S_3)
Patient 1(P_1)	(0.7,0.4,0.1)	(0.8,0.6,0.7)	(0.4,0.8,0.5)
Patient 2(P_2)	(0.6,0.5,0.3)	(0.6,0.5,0.2)	(0.7,0.9,0.0)
Patient 3(P_3)	(0.8,0.4,0.2)	(0.5,0.1,0.5)	(1.0,0.5,1.0)
Patient 4(P_4)	(0.4,0.6,0.3)	(0.5,0.4,0.8)	(0.5,0.6,0.9)

Table II

R	Intestinal Obstruction (D_1)	Inguinal Hernia (D_2)	Appendicitis (D_3)	Ureteric Colic (D_4)
Vomiting(S_1)	(0.9,0.6,0.7)	(0.9,1.0,0.5)	(0.9,0.2,0.8)	(0.6.0.2,0.3)
Pain in Abdomen(S_2)	(0.5,0.3,0.3)	(0.4,0.6,0.6)	(0.4,0.5,0.3)	(0.9,0.5,0.8)
Temperature(S_3)	(0.8,0.8,0.9)	(0.7,0.8,0.3)	(0.8,0.1,0.8)	(0.3,0.4,0.5)

Table III

T	Intestinal Obstruction (D_1)	Inguinal Hernia (D_2)	Appendicitis (D_3)	Ureteric Colic (D_4)
Patient 1(P_1)	(0.7,0.8,0.7)	(0.7,0.8,0.5)	(0.7,0.5,0.7)	(0.8.0.5,0.3)
Patient 2(P_2)	(0.7,0.8,0.3)	(0.7,0.8,0.3)	(0.7,0.5,0.3)	(0.6,0.5,0.3)
Patient 3(P_3)	(0.8,0.5,0.5)	(0.8,0.5,0.5)	(0.8,0.1,0.5)	(0.5,0.4,0.3)
Patient 4(P_4)	(0.5,0.6,0.7)	(0.5,0.6,0.5)	(0.5,0.4,0.8)	(0.5,0.4,0.3)

Table IV

Q’	Vomiting(S_1)	Pain in Abdomen(S_2)	Temperature(S_3)
Patient 1(P_1)	(0.1,0.6,0.7)	(0.7,0.4,0.8)	(0.5,0.2,0.4)
Patient 2(P_2)	(0.3,0.5,0.6)	(0.2,0.5,0.6)	(0.0,0.1,0.7)
Patient 3(P_3)	(0.2,0.6,0.8)	(0.5,0.9,0.5)	(1.0,0.5,1.0)
Patient 4(P_4)	(0.3,0.4,0.4)	(0.8,0.6,0.5)	(0.9,0.4,0.5)

Table V

R’	Intestinal Obstruction (D_1)	Inguinal Hernia (D_2)	Appendicitis (D_3)	Ureteric Colic (D_4)
Vomiting(S_1)	(0.7,0.4,0.9)	(0.5,0.0,0.9)	(0.8,0.8,0.9)	(0.3.0.8,0.6)
Pain in Abdomen(S_2)	(0.3,0.7,0.5)	(0.6,0.4,0.4)	(0.3,0.5,0.4)	(0.8,0.5,0.9)
Temperature(S_3)	(0.9,0.2,0.8)	(0.3,0.2,0.7)	(0.8,0.9,0.8)	(0.5,0.6,0.3)

Table VI

T’	Intestinal Obstruction (D_1)	Inguinal Hernia (D_2)	Appendicitis (D_3)	Ureteric Colic (D_4)
Patient 1(P_1)	(0.5,0.4,0.8)	(0.6,0.4,0.7)	(0.5,0.6,0.8)	(0.7.0.6,0.4)
Patient 2(P_2)	(0.3,0.5,0.6)	(0.3,0.4,0.6)	(0.3,0.5,0.6)	(0.3,0.5,0.6)
Patient 3(P_3)	(0.9,0.7,0.5)	(0.5,0.5,0.5)	(0.8,0.6,0.5)	(0.5,0.6,0.8)
Patient 4(P_4)	(0.9,0.6,0.5)	(0.6,0.4,0.5)	(0.9,0.5,0.5)	(0.8,0.5,0.5)

Table VII

Value function	D_1	D_2	D_3	D_4
Patient 1(P_1)	0.2	0.4	0.5	1
Patient 2(P_2)	0.6	0.6	0.9	0.8
Patient 3(P_3)	0.8	0.8	1.2	0.6
Patient 4(P_4)	0.2	0.4	0.3	0.8

Table VIII

Value function	D_1	D_2	D_3	D_4
Patient 1(P_1)	0.3	0.5	0.1	0.7
Patient 2(P_2)	0.2	0.3	0.2	0.2
Patient 3(P_3)	0.7	0.5	0.7	0.1
Patient 4(P_4)	0.8	0.7	0.7	0.8

Table IX

Score function	D_1	D_2	D_3	D_4
Patient 1(P_1)	-.01	-0.1	-.04	**0.3**
Patient 2(P_2)	0.4	0.3	**0.7**	0.6
Patient 3(P_3)	0.1	0.3	**0.5**	**0.5**
Patient 4(P_4)	-0.7	-.03	-0.4	**0**

Table X

Score function	D_1	D_2	D_3	D_4
Patient 1(P_1)	0.14	0.3	0.35	**0.65**
Patient 2(P_2)	0.46	0.46	**0.55**	0.45
Patient 3(P_3)	0.55	0.55	**0.75**	0.38
Patient 4(P_4)	0.08	0.2	0.18	**0.38**

Therefore from Table IX and Table X we conclude that P_1 and P_4 are suffering from D_4 and P_2 and P_3 are suffering from D_3.

V. HAMMING DISTANCES AND SIMILARITY MEASURES OF FUZZY NEUTROSOPHIC SOFT SETS

Definition 5.1: Let U = $\{u_1, u_2, \ldots\ldots u_n\}$be a universe, E=$\{e_1, e_2, \ldots. e_m\}$ be a set of parameters. A, B $\subseteq$E and F_A and G_B be two FNSS on U with their fuzzy neutrosophic soft function $\mu_A(e_i) = \{u, T_A(u), I_A(u), F_A(u) : u\in U\}$ and $\lambda_B(e_i) = \{u, T_B(u), I_B(u), F_B(u) : u\in U\}$ respectively . Then the distances of F_A and G_B are defined as follows.

1. *Hamming Distance*:

$$d^S_{FNS}(F_A, G_B) = \frac{1}{3m}\left\{\begin{array}{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left|T_A(e_i)(u_j) - T_B(e_i)(u_j)\right| \\ +\left|I_A(e_i)(u_j) - I_B(e_i)(u_j)\right| \\ +\left|F_A(e_i)(u_j) - F_B(e_i)(u_j)\right|\end{array}\right\}$$

2. *Normalized Hamming Distance:*

$$l^S_{FNS}(F_A, G_B) = \frac{1}{3mn}\left\{\begin{array}{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left|T_A(e_i)(u_j) - T_B(e_i)(u_j)\right| \\ +\left|I_A(e_i)(u_j) - I_B(e_i)(u_j)\right| \\ +\left|F_A(e_i)(u_j) - F_B(e_i)(u_j)\right|\end{array}\right\}$$

3. Euclidean *Distance* :

$$e^S_{FNS}(F_A, G_B) = \left(\frac{1}{3m}\left\{\begin{array}{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(T_A(e_i)(u_j) - T_B(e_i)(u_j)\right)^2 \\ +\left(I_A(e_i)(u_j) - I_B(e_i)(u_j)\right)^2 \\ +\left(F_A(e_i)(u_j) - F_B(e_i)(u_j)\right)^2\end{array}\right\}\right)^{\frac{1}{2}}$$

4. *Normalized Euclidean Distance* :

$$q^S_{FNS}(F_A, G_B) = \left(\frac{1}{3mn}\left\{\begin{array}{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(T_A(e_i)(u_j) - T_B(e_i)(u_j)\right)^2 \\ +\left(I_A(e_i)(u_j) - I_B(e_i)(u_j)\right)^2 \\ +\left(F_A(e_i)(u_j) - F_B(e_i)(u_j)\right)^2\end{array}\right\}\right)^{\frac{1}{2}}$$

Example 5.2:

Let U = {u_1, u_2, u_3, u_4} is a universal set E = {e_1,e_2,e_3,e_4} is a set of parameters. A = {e_1, e_2, e_3}; B = {e_1,e_2,e_3} are subsets of E. If two FNSS F_A and G_B over U are connected as follows.

F_A = {(e_1 , {(u_1,0.5,0.5,0.5) , (u_2,0.4,0.6,0.5) (u_3,0.7,0.4,0.2) (u_4,0.8,0.7,0.1)})
(e_2 , {(u_1,0.4,0.5,0.6) , (u_2,0.2,0.5,0.7) (u_3,0.2,0.7,0.8) (u_4,0.2,0.8,0.2)})
(e_3 , {(u_1,0.2,0.4,0.7) , (u_2,0.1,0.7,0.9) (u_3,0.5,0.6,0.4) (u_4,0.7,0.5,0.2)}) }

G_B= { (e_1 , {(u_1,0.2,0.4,0.7) , (u_2,0.1,0.7,0.9) (u_3,0.5,0.6,0.4) (u_4,0.4,0.5,0.4)})
(e_2 {(u_1,0.5,0.6,0.5) , (u_2,0.4,0.6,0.5) (u_3,0.3,0.7,0.6) (u_4,0.4,0.6,0.5)})
. (e_3 , {(u_1,0.4,0.5,0.6) , (u_2,0.2,0.6,0.7) (u_3,0.2,0.7,0.8) (u_4,0.2,0.7,0.5)}) }

(i) $d^S_{FNS}(F_A,G_B) = 0.811$ (ii) $l^S_{FNS}(F_A,G_B) = 0.2027$

(iii) $e^S_{FNS}(F_A,G_B) = 0.4606$ (iv) $q^S_{FNS}(F_A,G_B) = 0.2303$.

Theorem 5.3: Let FNSS(U) be a set of all fuzzy Neutrosophic soft sets over U. Then the distances $d^S_{FNS}(F_A,G_B)$, $l^S_{FNS}(F_A,G_B)$, $e^S_{FNS}(F_A,G_B)$ and $q^S_{FNS}(F_A,G_B)$, defined from FNSS(U) to the non-negative real number R^+ are metric.

Definition 5.4: Let F_A and G_B be two FNSS over U. Then by using the hamming distance, similarity measure of F_A and G_B is defined as $S_{FNS}(F_A,G_B) = \dfrac{1}{1+d^S_{FNS}(F_A,G_B)}$.

Definition 5.5: : Let F_A and G_B be two FNSS over U. Then F_A and G_B are said to be α- similar, denoted as $F_A \approx^{\alpha} G_B$ if and only if $S_{FNS}(F_A,G_B) \geq \alpha$ for $\alpha \in (0,1)$.

Theorem 5.6: Let E be a parameter set, A, B ⊆ E and F_A and G_B be two FNSS over U. Then the following hold.

(i) $0 \leq S_{FNS}(F_A,G_B) \leq 1$

(ii) $S_{FNS}(F_A,G_B) = S_{FNS}(G_B,F_A)$.

(iii) $S_{FNS}(F_A,G_B) = 1$ iff $F_A = G_B$

HAMMING DISTANCES AND SIMILARITY MEASURES IN DECISION MAKING

We construct a decision making that is based on the similarity measure of two FNSS. The algorithm is given below.

Step 1: Construct a FNSS F_A over U based on an expert

Step 2: Construct a FNSS G_B over U based on a responsible person for the problem.

Step 3: Construct a distance of F_A and G_B

Step 4: Calculate the similarity measure of F_A and G_B.

Step 5: Estimate the result by using the similarity.

Application:

The hamming distance and similarity measure of two FNSS can be applied to detect whether an ill person is suffering from a certain disease or not

Assume that the universal set contains only two elements Brain tumor and not brain tumor i.e. U = {u_1, u_2}. The set of Certain visible symptoms E = { x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8} where x_1 = headache, x_2 = vomiting , x_3 = personality or mood changes, x_4 = seizures, x_5 = cognitive decline, x_6 = vision and hearing problems, x_7 = physical changes, x_8 = speech changes.

Construct a FNSS F_A over U for brain tumors and this can be prepared with help of a medical person.

F_A = {(x_1 , {(u_1,0.5,0.5,0.5) , (u_2,0.4,0.6,0.5)})
(x_2 , {(u_1,0.7,0.3,0.2) , (u_2,0.8,0.4,0.1)})
(x_3 , {(u_1,0.4,0.5,0.6) , (u_2,0.2,0.3,0.7)})
(x_4 , {(u_1,0.2,0.3,0.8) , (u_2,0.2,0.4,0.2)})
(x_5 , {(u_1,0.2,0.5,0.7) , (u_2,0.1,0.6,0.9)})
(x_6 , {(u_1,0.5,0.3,0.4) , (u_2,0.7,0.6,0.2)})
(x_7 , {(u_1,0.3,0.5,0.7) , (u_2,0.4,0.5,0.4)})
(x_8 , {(u_1,0.5,0.4,0.2) , (u_2,0.7,0.6,0.1)})}.

Construct a FNSS G_B over U based on the data of ill person.

G_B = {(x_1 , {(u_1,0.9,0.6,0.1) , (u_2,0.9,0.4,0.0)})
(x_2 , {(u_1,0.1,0.4,0.9) , (u_2,0.1,0.4,0.8)})
(x_3 , {(u_1,0.7,0.4,0.1) , (u_2,0.9,0.6,0.9)})
(x_4 , {(u_1,0.9,0.6,0.1) , (u_2,0.9,0.6,0.8)})
(x_5 , {(u_1,0.9,0.4,0.1) , (u_2,0.9,0.4,0.2)})
(x_6 , {(u_1,0.1,0.6,0.9) , (u_2,0.1,0.4,0.8)})
(x_7 , {(u_1,0.9,0.6,0.1) , (u_2,0.7,0.4,0.9)})
(x_8 , {(u_1,0.9,0.1,0.0) , (u_2,0.0,0.0,1.0)})}.

$d^S_{FNS}(F_A,G_B) = 0.8833$ & $S_{FNS}(F_A,G_B) = 0.5309$

Construct a FNSS H_C over U based on the data of another ill person.

H_C = {(x_1 , {(u_1,0.5,0.3,0.4) , (u_2,0.4,0.4,0.4)})
(x_2 , {(u_1,0.7,0.6,0.1) , (u_2,0.8,0.4,0.1)})
(x_3 , {(u_1,0.4,0.5,0.5) , (u_2,0.2,0.4,0.6)})
(x_4 , {(u_1,0.2,0.3,0.7) , (u_2,0.2,0.4,0.1)})
(x_5 , {(u_1,0.2,0.5,0.6) , (u_2,0.1,0.4,0.8)})
(x_6 , {(u_1,0.5,0.2,0.3) , (u_2,0.7,0.3,0.1)})
(x_7 , {(u_1,0.2,0.4,0.6) , (u_2,0.1,0.7,0.8)})
(x_8 , {(u_1,0.5,0.4,0.3) , (u_2,0.7,0.4,0.1)})}.

$$d^S_{FNS}(F_A,H_C) = 0.166.$$

$$S_{FNS}(F_A,H_C) = 0.857.$$

Suppose we take α = 0.6

From F_A and G_B we have $S_{FNS}(F_A,G_B) = 0.53 < \alpha$.

Therefore we conclude that the person is not possibly suffering from brain tumor.

From F_A and H_C we have $S_{FNS}(F_A, H_C) = 0.85 > \alpha$.

Therefore we conclude that the person is possibly suffering from brain tumor.

VI. MAPPINGS OF FUZZY NEUTROSOPHIC SOFT SETS

Definition 6.1: Let (X, E) and (Y, E') be fuzzy neutrosophic soft sets over X and Y with parameters set E and E' respectively. Let $u:X\to Y$ and $p:E\to E'$ be mappings. Then mapping $f:(u,p): (X,E)\to(Y,E')$ is defined as follows:

For a FNSS (F,A) in (X,E), f(F,E) is a FNSS in (Y,E') obtained as follows for $\beta\in p(E)\subseteq E'$ and $y\in Y$,

$$f(F,A)(\beta)(y) = \begin{cases} \bigvee_{x\in u^{-1}(y)} \left(\bigvee_{\alpha\in p^{-1}(\beta)\cap A} F(\alpha) \right)(x), \\ \quad If\ u^{-1}(y) \neq \phi,\ p^{-1}(\beta)\cap A \neq \phi \\ \tilde{0}_N, \quad otherwise \end{cases}$$

f(F,A) is called a fuzzy neutrosophic soft image of FNSS (F,A) . [$\vee$ - max and $\wedge$ - min]

Definition 6.2: Let $u:X\to Y$ and $p:E\to E'$ be mapping. Let $f:(X,E)\to(Y,E')$ be a mapping and (G,B) a FNSS in (Y,E') where $B\subseteq E'$, then $f^{-1}(G,B)$ is a FNSS in (X,E), defined as follows for $\alpha\in p^{-1}(B)\subseteq E$ and $x\in X$,

$$f^{-1}(G,B)(\alpha)(x) = \begin{cases} G(p(\alpha))u(x)), & for\ \ p(\alpha)\in B \\ \tilde{0}_N, & otherwise \end{cases}$$

$f^{-1}(G,B)$ is called a fuzzy neutrosophic soft inverse image of (G,B).

Example 6.3: Let X = {a,b,c} ;Y = {x,y,z} E ={e_1, e_2, e_3, e_4}, E'={e_1',e_2', e_3'} and (X,E) and (Y,E') are FNSSs:
Let $u : X\to Y$ and
$p:E\to E'$ be mappings defined as u(a) = z; u(b) = y; u(c) = y,
p(e_1)= e_1' , p(e_2)= e_1', p(e_3)= e_3' and p(e_4)= e_2'.A= { e_1, e_2, e_4}.
Choose two FNSSs in (X,E) and (Y,E') respectively as
(F,A) ={{(e_1 ,{(a,0.5,0.6,0.5) , (b,0.0,0.1,1.0) ,(c,0.8,0.6,0.2)})
{(e_2 ,{(a,0.1,0.5,0.9) , (b,0.9,0.6,0.1) ,(c,0.5,0.6,0.5)})
{(e_4 ,{(a,0.4,0.5,0.6), (b,0.3,0.6,0.7),(c,0.6,0.6,0.4)})}.
(G,B) ={{(e_1',{(x,0.3,0.5,0.7) , (y,0.5,0.6,0.5) , (z,0.1,0.4,0.9)})
{(e_2',{(x,0.9,0.6,0.1) , (y,0.1,0.8,0.9) , (z,0.5,0.7,0.5)})
{(e_3',{(x,0.7,0.6,0.3), (y,0.5,0.8,0.5),(z,0.6,0.7,0.4)})}.
Then the fuzzy neutrosophic soft image of (F,A) under f: (X, E)$\to$(Y,E') is obtained as
f (F, A) ={(e_1',{(x, 0,0,1),(y,0.9,0.6,0.1),(z,0.5,0.6,0.5)})
(e_2',{(x, 0,0,1),(y,0.6,0.6,0.4),(z,0.4,0.5,0.6)})
(e_3',{(x, 0,0,1),(y, 0,0,1),(z, 0,0,1)})}
f^{-1}(G,B) ={(e_1,{(a, 0.1,0.4,0.9),(b,0.5,0.6,0.5),(c,0.5,0.6,0.5)})
(e_2,{(a, 0.1,0.4,0.9),(b,0.5,0.6,0.5),(c,0.5,0.6,0.5)})
(e_3,{(a, 0.6,0.7,0.4),(b, 0.5,0.8,0.5),(c, 0.5,0.8,0.5)})
(e_4,{(a, 0.5,0.7,0.5),(b, 0.1,0.8,0.9),(c, 0.1,0.8,0.9)})

Definition 6.4: Let f: $(X,E)\to(Y,E')$ be mapping and (F,A) and (G,B) be FNSS in (X,E). Then for $\beta\in E'$, $y\in Y$, then fuzzy neutrosophic soft union and intersection of fuzzy neutrosophic soft images f(F,A) and f(G,B) in (Y,E') are defined as follows:

$$(f(F,A)\,\tilde{\cup}\, f(G,B))(\beta)(y) = f(F,A)(\beta)(y)\,\tilde{\cup}\, f(G,B)(\beta)(y)$$

$$(f(F,A)\,\tilde{\cap}\, f(G,B))(\beta)(y) = f(F,A)(\beta)(y)\,\tilde{\cap}\, f(G,B)(\beta)(y).$$

Definition 6.5: Let f: $(X,E)\to(Y,E')$ be mapping and (F,A) and (G,B) be FNSS in (Y,E'). Then for $\alpha\in E$, $x\in X$, then fuzzy neutrosophic soft union and intersection of fuzzy neutrosophic soft inverse images f^{1}(F,A) and f^{1}(G,B) in (Y,E') are defined as follows:

$$(f^{1}(F,A)\,\tilde{\cup}\, f^{-1}(G,B))(\alpha)(x) = f^{-1}(F,A)(\alpha)(x)\,\tilde{\cup}\, f^{-1}(G,B)(\alpha)(x)$$

$$(f^{1}(F,A)\,\tilde{\cap}\, f^{1}(G,B))(\alpha)(x) = f^{1}(F,A)(\alpha)(x)\,\tilde{\cap}\, f^{-1}(G,B)(\alpha)(x)$$

Theorem 6.6: Let f: $(X,E)\to(Y,E')$ be a mapping, then for FNSS (F,A) and (G,B) in (X,E) we have

(i) $f(\phi) = \phi$

(ii) $f(X,E)\subseteq(Y,E')$

(iii) $f((F,A)\,\tilde{\cup}\,(G,B)) = f(F,A)\,\tilde{\cup}\, f(G,B)$

(iv) $f((F,A)\,\tilde{\cap}\,(G,B)) \subseteq f(F,A)\,\tilde{\cap}\, f(G,B)$

(v) If $(F,A) \subseteq (G,B)$, then $f(F,A) \subseteq f(G,B)$.

Proof:

(i) and (ii) are obvious.

(iii) for $\beta\in E'$ and $y\in Y$, we have to prove

$f((F,A)\,\tilde{\cup}\,(G,B))(\beta)(y) = f(F,A)(\beta)(y).\ \tilde{\cup}\ f(G,B)(\beta)(y)$

Consider $f((F,A)\,\tilde{\cup}\,(G,B))(\beta)(y) = f(H,A\cup B)(\beta)(y)$(say)

$$= \begin{cases} \bigvee_{x\in u^{-1}(y)} \left(\bigvee_{\alpha\in p^{-1}(\beta)\cap(A\cup B)} H(\alpha) \right)(x), \\ \quad If\ u^{-1}(y) \neq \phi,\ p^{-1}(\beta)\cap(A\cup B) \neq \phi \\ \tilde{0}_N, \quad otherwise \end{cases}$$

where

$$H(\alpha) = \begin{cases} F(\alpha) & ;\ \text{if}\ \alpha\in A - B\cup p^{-1}(\beta) \\ G(\alpha) & ;\ \text{if}\ \alpha\in B - A\cup p^{-1}(\beta) \\ F(\alpha)\cup G(\alpha) & ;\ \text{if}\ \alpha\in (A\cap B)\cap p^{-1}(\beta) \end{cases}$$

for $\alpha\in(A\cup B)\cap p^{-1}(\beta)$.

$f((F,A)\,\tilde{\cup}\,(G,B))(\beta)(y) =$

$$\underset{x \in u^{-1}(y)}{\vee}\left\{ \vee \begin{cases} F(\alpha) & ;\ \text{if}\ \alpha \in \mathrm{A} - \mathrm{B} \cup \mathrm{p}^{-1}(\beta) \\ G(\alpha) & ;\ \text{if}\ \alpha \in \mathrm{B} - \mathrm{A} \cup \mathrm{p}^{-1}(\beta) \\ \mathrm{F}(\alpha) \cup \mathrm{G}(\alpha) & ;\ \text{if}\ \alpha \in (\mathrm{A} \cap \mathrm{B}) \cap \mathrm{p}^{-1}(\beta) \end{cases} \right. \tag{1}$$

By definition

$(f(F,A) \tilde{\cup} f(G,B))(\beta)(y) = f(F,A)(\beta)(y) \tilde{\cup} f(G,B)(\beta)(y)$

$$= \underset{x\in u^{-1}(y)}{\vee}\left(\underset{\alpha\in p^{-1}(\beta)\cap(A\cup B)}{\vee} F(\alpha) \right)(x) \tilde{\cup}$$

$$\underset{x\in u^{-1}(y)}{\vee}\left(\underset{\alpha\in p^{-1}(\beta)\cap(A\cup B)}{\vee} G(\alpha) \right)(x)$$

$$\underset{x \in u^{-1}(y)}{\vee}\left\{ \vee \begin{cases} F(\alpha) & ;\ \text{if}\ \alpha \in \mathrm{A} - \mathrm{B} \cup \mathrm{p}^{-1}(\beta) \\ G(\alpha) & ;\ \text{if}\ \alpha \in \mathrm{B} - \mathrm{A} \cup \mathrm{p}^{-1}(\beta \\ \mathrm{F}(\alpha) \cup \mathrm{G}(\alpha) & ;\ \text{if}\ \alpha \in (\mathrm{A} \cap \mathrm{B}) \cap \mathrm{p}^{-1}(\beta) \end{cases} \right. \tag{2}$$

From (1) and (2) the result follows.

(iv) For $\beta\in E'$ and $y\in Y$ and using definition we have

$f((F,A) \tilde{\cap} (G,B))(\beta)(y) = f(H,A\cap B)(\beta)(y).$

$$=\left\{ \underset{x\in u^{-1}(y)}{\vee}\left(\underset{\alpha\in p^{-1}(\beta)\cap(A\cap B)}{\vee} H(\alpha) \right)(x), \right.$$

$$=\left\{ \underset{x\in u^{-1}(y)}{\vee}\left(\underset{\alpha\in p^{-1}(\beta)\cap(A\cap B)}{\vee} [F(\alpha)\cap G(\alpha)] \right)(x), \right.$$

$$=\left\{ \underset{x\in u^{-1}(y)}{\vee}\left(\underset{\alpha\in p^{-1}(\beta)\cap(A\cap B)}{\vee} [F(\alpha)(x)\cap G(\alpha)(x)] \right) \right.$$

$$\subseteq\left[\left\{ \underset{x\in u^{-1}(y)}{\vee}\left(\underset{\alpha\in p^{-1}(\beta)\cap(A\cap B)}{\vee} F(\alpha) \right)(x), \right.\right] \tilde{\cap}$$

$$\left[\left\{ \underset{x\in u^{-1}(y)}{\vee}\left(\underset{\alpha\in p^{-1}(\beta)\cap(A\cap B)}{\vee} G(\alpha) \right)(x), \right.\right]$$

$= f(F,A)(\beta)(y) \tilde{\cap} f(G,B)(\beta)(y) = (f(F,A) \tilde{\cap} f(G,B))(\beta)(y).$

(v) for $\beta\in E'$ and $y\in Y$, $(F,A) \subseteq (G,B)$, we have

$$f(F,A)(\beta)(y) = \underset{x\in u^{-1}(y)}{\vee}\left(\underset{\alpha\in p^{-1}(\beta)\cap A}{\vee} F(\alpha) \right)(x)$$

$$= \underset{x\in u^{-1}(y)}{\vee}\left(\underset{\alpha\in p^{-1}(\beta)\cap A}{\vee} F(\alpha)(x) \right)$$

$$\subseteq \underset{x\in u^{-1}(y)}{\vee}\left(\underset{\alpha\in p^{-1}(\beta)\cap B}{\vee} G(\alpha)(x) \right)$$

$$= f(G,B)(\beta)(y)$$

Hence (v) is proved.

Remark 6.7:

In the above theorem (ii) , (iv) and (v) the reverse implication need not be true.

Definition 6.8[1]:

Let (F_A ,E) be FNS set on (U,E) and τ be a collection of Fuzzy Neutrosophic soft subsets of (F_A , E). (F_A ,E) is called Fuzzy neutrosophic soft topology(FNST) if the following conditions hold.

(i) $\phi_E ,U_E \in \tau$

(ii) $F_E ,G_E \in \tau$ implies $F_E \tilde{\cap} G_E \in \tau$

(iii) $\{(F_\alpha)_E : \alpha\in\Gamma\} \subseteq \tau$ implies $\tilde{\cup}\{(F_\alpha)_E : \alpha\in\Gamma\} \in \tau$

The triplet (U, τ ,E) is called an Fuzzy Neutrosophic soft topological space (FNSTS)over U.

Every member of τ is called an Fuzzy Neutrosophic soft open set in U.

F_E is called an Fuzzy Neutrosophic soft closed set in U if $F_E\in \tau^c$, where $\tau^c = \{F_E^C : F_E \in \tau\}$

Example 6.9[1]:

Let U = $\{b_1 ,b_2 ,b_3\}$ and E=$\{e_1 ,e_2\}$. Let F_E, G_E, H_E, L_E be Neutrosophic soft set where

$H(e_1)$={< b_1,0.8,0.4,0.5> ,< b_2, 0.7,0.7,0.3> ,< b_3,0.7,0.5,0.4> }

$H(e_2)$={< b_1,0.9,0.5,0.6>,< b_2, 0.8,0.8,0.4> ,< b_3, 0.8,0.6,0.5> }

$F(e_1)$={< b_1,0.5,0.4,0.5>,< b_2, 0.6,0.7,0.3> , < b_3, 0.6,0.5,0.4> }

$F(e_2)$={< b_1,0.6,0.5,0.6>,< b_2, 0.7,0.8,0.4> , < b_3, 0.7,0.6,0.5> }

$G(e_1)$={< b_1,0.8,0.4,0.7>,< b_2, 0.7,0.3,0.4>, < b_3, 0.7,0.5,0.6> }

$G(e_2)$={< b_1,0.9,0.5,0.8>,< b_2, 0.8,0.4,0.5>, < b_3, 0.8,0.6,0.7> }

$L(e_1)$={< b_1,0.5,0.4,0.7>,< b_2, 0.6,0.3,0.4>, < b_3, 0.6,0.5,0.6> }

$L(e_2)$={< b_1,0.6,0.5,0.8> ,< b_2, 0.7,0.4,0.5>,< b_3, 0.7,0.6,0.7> }

τ = {F_E, G_E, H_E, L_E, ϕ_E , U_E } is an Fuzzy Neutrosophic soft topology on U.

VII. CONCLUSION

This paper introduces the notion of fuzzy neutrosophic soft relations and the novel score function with a view to evolve an expert system for the diagnosis of patients. We have proposed some new techniques and measures namely hamming distances and similarity measures and derived some of their properties. A decision making method based on the similarity measure is constructed. The concept of similarity measures could be further extended to have natural applications in the field of pattern recognition, feature extraction, region extraction, image processing, coding theory etc. Finally the mappings on fuzzy neutrosophic soft set are defined and their properties are discussed and it can be developed in topological structures of fuzzy neutrosophic soft sets.

REFERENCES

[1] I.Arockiarani, I.R.Sumathi,J.Martina Jency, "Fuzzy Neutrosophic Soft Topological Spaces,"IJMA-4(10), 2013., 225-238.

[2] K.Attanassov, "Intuitionistic fuzzy sets," Fuzzy Sets and Systems, 20(1986) 87-96.

[3] K.Attanassov, "Intuitionistic fuzzy sets Theory and Applications," Physica-Verlag, Wyrzburg,1999.

[4] N.Cagman, S.Karatas and S.Enginogly, "Soft Topology," Computers and Mathematics with Applications, 62(2011)351-358.

[5] S.K.De, R.Biswas and A.R.Roy, "An application of intuitionistic fuzzy sets in medical diagnosis," Fuzzy Sets and Systems, 117(2001),209-213.

[6] Y.Guo and H.D.Cheng, "A new neutrosophic approach to image segmentation, Pattern Recognition, 42(5) (2009), 587-595.

[7] A.Kharal and B.Ahmad, "Mappings on Fuzzy Soft Classes,"Advances in Fuzzy Systems, Volume 2009 (2009), Article ID 407890, 6 pages.

[8] A.Kharal, "Distance and similarity measures for soft sets," New Math. And Nat. Computation, 06,321(2010) DOI:10.1142/S1793005710001724.

[9] A.Kharal, "Automated Medical Diagnosis: A method Using Neutrosophic Sets,"The journal of Fuzzy Mathematics, Vol.229, No. 1. (2014).

[10] P.K.Maji, A.R.Roy, R.Biswas, Fuzzy Soft Sets. The Journal of Fuzzy Mathematics, 9(3)(2001)589-602.

[11] P.K.Maji, A.R.Roy, R.Biswas, "Intuitionistic Fuzzy Soft Sets," The Journal of Fuzzy Mathematics, 9 (3)(2001)677-692.

[12] P.K.Maji, A.R.Roy, R.Biswas, "On Intuitionistic Fuzzy Soft Sets;," The Journal of Fuzzy Mathematics 12(3) (2004) 669-683.

[13] P.Majumdar, S.K.Samanta, "Similarity measure of soft Sets," New Mathematics and Natural Computation, vol.4,no.1,pp.1-12,2008.

[14] D.Molodtsov,"Soft set theory – First results," Computer and Mathematics with Applications, vol.37,no.4-5, pp.19-31, Feb-Mar 1999.

[15] A.Mukherjee, S.B.Chakraborty, "On intuitionistic fuzzy soft relations," Buletin of Kerala Mathematics Association, 5(1)(2008) 35-42.

[16] Naim Cagman, Irfan Deli , "Similarity measures of intuitionistic fuzzy soft sets and their decision making ," arXiv:1301.0456v3 [math.LO] 16 Dec 2013.

[17] E.Sanchez, "Resolution of composition of fuzzy relation equations," Information and control, 30(1976),38-48.

[18] E.Sanchez, "Solutions in composite fuzzy relation equation, Application to Medical diagnosis in Brouwerian Logic, in :M.M.Gupta, G.N.Saridis, B.R.Gaines(Eds.), Fuzzy automata and decision process,"Elsevier, North – Holland (1977).

[19] F.Smarandache, "Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics," University of New Mexico, Gallup, NM 87301, USA (2002).

[20] F.Smarandache, "Neutrosophic set, a generalization of the intuitionistic fuzzy sets," Inter.J.Pure Appl.Math.,24 (2005),287 – 297.

[21] L.A.Zadeh, "Fuzzy Sets," Inform and Control 8(1965) 338 – 353.