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Characterizations of normal parameter reductions of soft sets

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Abstract. In 2014, Wang et. al gave the reduct definition for fuzzy information system. We observe that the reduct definition given by Wang et. al does not retain the optimal choice of objects. In this paper, we give the drawbacks of the reduct definition of Wang et. al and give some characterizations of normal parameter reduction of soft sets. Also, we prove that the image and inverse image of a normal parameter reduction is a normal parameter reduction under consistency map.

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1. Introduction

Many practical problems involve data that contain uncertainities. These uncertainities may be dealt with existing theories such as fuzzy set theory [3, 13] and rough set theory [10]. In 1999, Molodstov [9] pointed out the difficulties of these theories and he posited the concept of soft set theory. Maji et. al [6] made a theoretical study of soft sets in 2003. In 2014, Renukadevi et. al [11] characterized some of the properties of soft sets and soft basis in terms of soft topology. Soft set theory has rich potential for applications. In [5], Maji et. al presented an application of soft sets in decision making problems. In the year 2005, Chen et. al [1] pointed out the problems in [5] and introduced a new concept called parameterization reduction. In order to overcome these problems of suboptimal choice in [1], Kong et. al [4] introduce the concept of normal parameter reduction for soft sets and fuzzy soft sets. Han et. al gave the $int^m - int^n$ decision making algorithm for soft sets in [2]. In 2012, Maji [7] gave an application of neutrosophic soft sets in decision making problem. Also, he studied the properties of neutrosophic soft sets in [8] and gave a decision making algorithm using comparision matrix. As there is no characterization for normal parameter reduction, we try to characterize normal parameter reduction of soft sets. Also, Wang et. al [12] introduced the concept of reduct in fuzzy information system. We observe that the reduct defintion given by Wang et. al did not maintain the optimal choice of objects and we prove this by giving a counter example. Also, we derive a conclusion that every fuzzy information system is a fuzzy soft set. If the parameter set is large in size, the process of finding normal parameter reduction becomes a complicated one. So we try to prove the image of a normal parameter reduction is a normal parameter redution. We introduce the concept of consistency and prove that the image and inverse image of normal parameter reduction is a normal parameter reduction. Using this characterization, instead of finding the normal parameter reduction of a soft set with large parameter set, it is enough to find the normal parameter reduction of soft set with less number of parameters whereas there is a consistency map between these soft sets.

Definition 1.1 ([9])**.** A pair $S = (F, E)$ is called a soft set over U if and only if F is a mapping of E into the set of all subsets of U. That is, $F\colon E \to P(U)$ where $P(U)$ is the power set of U. In other words, the soft set is a parameterized family of subsets of U. For every $\epsilon \in E, F(\epsilon)$ may be considered as the set of ϵ-approximate elements of the soft set (F, E).

Definition 1.2 ([7])**.** Let U be an initial universe set and E be a set of parameters (which are fuzzy words or sentences involving fuzzy words). Let $F(U)$ denotes the set of all fuzzy sets of U. Let $A \subseteq E$. A pair (F_A, A) is called a fuzzy soft set over U where F_A is a mapping given by $F_A\colon A \to F(U)$.

Suppose $U = \{u_1, u_2, ...u_n\}, E = \{e_1, e_2, ...e_m\}$ and (F, E) is a soft set (or fuzzy soft set) with tabular representation. Define $f_E(u_i) = \sum_j u_{ij}$ where u_{ij} are the entries in the soft set (or fuzzy soft set) tabular representation. For soft set (or fuzzy soft set) $(F, E), U = \{u_1, u_2, \ldots, u_n\}$, denote $c_E = \{\{u_1, u_2, \ldots, u_i\}_{f_1}, \{u_{i+1}, u_{i+2}, \ldots, u_j\}_{f_2}, \ldots, \{u_k, u_{k+1}, \ldots, u_n\}_{f_s}\}$ as a partition of objects in U which partitions according to the value of $f_E(.)$ based on indiscernibility relation and c_E is called decision partition [4] where for the subclass $\{u_v, u_{v+1}, \ldots, u_{v+w}\}_{f_i}$, $f_E(u_v) = f_E(u_{v+1}) = \cdots = f_E(u_{v+w}) = f_i$ and $f_1 \geq f_2 \geq \cdots \geq f_s, s$ is the number of subclasses. Objects with the same value of $f_E(u_i)$ are partitioned into a same subclass. If there exists a subset $A = \{e_1^{'}, e_2^{'}, \ldots, e_p^{'}\} \subset E$ satisfying $f_A(u_1) = f_A(u_2) = \cdots = f_A(u_n)$, then A is dispensable [4]. Otherwise, A is indispensable [4]. A subset A of E is a normal parameter reduction [4] of the soft set(or fuzzy soft set) (F, E) if A is indispensable and $f_{E-A}(u_1) = f_{E-A}(u_2) = \cdots = f_{E-A}(u_n)$, that is to say $E - A$ is the maximal subset of E such that the value $f_{E-A}(.)$ keeps constant. Clearly, the partitions of objects have not been changed after normal parameter reduction. If $C_{E-G} = C_E$, then $E - G$ is called pseudo parameter reduction [4] of E.

Definition 1.3 ([12])**.** Let A, B be two universes and $i\colon A \to B$ a mapping from A to B. For each $x \in A$, the equivalence class is defined by $[x]_i = \{y \in A | i(x) = i(y)\}$. Then $\{[x]_i | x \in A\}$ is a partition on A with respect to i.

Lemma 1.4 ([12], Theorem 4.4)**.** *Let (U, C) be a fuzzy information system, $(V, f(C))$ an $f-$ induced fuzzy information system, f a consistency-based homomorphism from*

(U, C) to $(V, f(C))$ and $P \subseteq C$. Then P is a reduct of C if and only if $f(P)$ is a reduct of $f(C)$.

2. Normal parameter reduction of soft sets

The normal parameter reduction plays an important role in attribute reduction. In 2008, Kong et. al [4] introduced the concept of pseudo parameter reduction and normal parameter reduction. But they did not find any relationship between these two parameter reductions. As there is no characterization for normal parameter reduction, we try to give some characterizations of normal parameter reduction for soft sets. In this section, we prove that every normal parameter reduction is a pseudo parameter reduction but not conversely. Also, we characterize the normal parameter reduction of soft sets using equivalent decision partition, N_A and V_A where A is the set of parameters.

Proposition 2.1. *Normal parameter reduction of (F, E) is a pseudo parameter reduction of (F, E).*

Proof. Suppose A is a normal parameter reduction of E. Then $E - A$ is a maximal subset of E such that $f_A(u_i) \neq f_A(u_j)$ for some i, j and $f_{E-A}(u_i) = f_{E-A}(u_j)$ for all i, j. Since $f_{E-A}(u_i) = f_{E-A}(u_j)$ for all i, j, after removing $E - A$, the decision partition value of C_E will be changed but the decision partition of C_E will not be changed. Therefore, $C_E = C_A$. Hence A is a pseudo parameter reduction of (F, E). □

The converse of the above Proposition 2.1 need not be true as shown by the following Example 2.2.

Example 2.2. Consider the universe $U = \{u_1, u_2, u_3, u_4, u_5\}$ is the set of houses and the parameter set for these houses is $E = \{e_1 =$ beautiful, $e_2 =$ cheap, $e_3 =$ green surroundings, $e_4 =$ good water supply, $e_5 =$ wooden$\}$. The soft set of (F, E) is given by $(F, E) = \{(e_1, \{u_1, u_2, u_3\}), (e_2, \{u_2, u_4, u_5\}), (e_3, \{u_1\}), (e_4, \{u_1, u_2, u_4\}), (e_5, \{u_2, u_3\}\}$. The tabular representation for this soft set is given below.

U	e_1	e_2	e_3	e_4	e_5	$f_E(.)$
u_1	1	0	1	1	0	3
u_2	1	1	0	1	1	4
u_3	1	0	0	0	1	2
u_4	0	1	0	1	0	2
u_5	0	1	0	0	0	1

Table 1

The decision partition of E is $C_E = \{\{u_2\}_4, \{u_1\}_3, \{u_3, u_4\}_2, \{u_5\}_1\}$. Let $A = \{e_1, e_3, e_4\}$. Then the decision partition of A is $C_A = \{\{u_2\}_2, \{u_1\}_3, \{u_3, u_4\}_1, \{u_5\}_0\}$. Therefore, $C_E = C_A$. Hence A is a pseudo parameter reduction. But $f_{E-A}(u_1) = 0 \neq 2 = f_{E-A}(u_2)$. Therefore, A is not a normal parameter reduction of (F, E).

Remark 2.3. Consider the soft set given in Example 2.2. For the parameter set E, the maximum of $f_E(.)$ is $f_E(u_2) = 4$. Therefore, the optimal choice with respect to E is u_2. But for the parameter set A, the maximum of $f_A(.)$ is $f_A(u_1) = 3$. Therefore, the optimal choice with respect to A is u_1. Thus, A does not maintain the optimal choice object of E whereas A is a pseudo parameter reduction. Hence pseudo parameter reduction does not maintain the optimal choice of objects. So pseudo parameter reduction is not the optimum one.

Since the pseudo parameter reduction is not optimum, we try to find a condition for which the pseudo parameter reduction becomes normal parameter reduction. For this, we introduce a new concept namely, equivalent decision partition.

Definition 2.4. Let $B \subseteq A$ and the decision partition of A and B be C_A and C_B respectively. If $C_A = C_B$ and $f_{A_i} - f_{A_j} = f_{B_i} - f_{B_j}$ for all i and j, then C_A and C_B are equivalent.

Let (F, E) be a soft set over U. Denote $V_E = \{u \in U | f_E(u) = \vee_{u_i \in U} f_E(u_i)\}$ [2]. Also, $N_E = \{f_E(u_i) | u_i \in U\}$. The following Theorem 2.5 gives a characterization for normal parameter reduction of soft sets.

Theorem 2.5. *For the soft set* (F, E), *the following hold.*

a. *A is a normal parameter reduction of (F, E).*
b. *A is a minimal subset of E such that C_A and C_E are equivalent.*
c. *$E - A$ is a maximal subset of E such that N_{E-A} is a singleton set and N_E and N_A are equivalent.*
d. *$E - A$ is a maximal subset of E such that $V_A = V_E$ and $V_{E-A} = U$.*

Proof. (a) $\Rightarrow$ (b). Suppose A is a normal parameter reduction of (F, E). Then $E - A$ is a maximal subset of E such that $f_A(u_i) \neq f_A(u_j)$ for some i and j and $f_{E-A}(u_i) = f_{E-A}(u_j)$ for all i and j. Since $f_{E-A}(u_i) = f_{E-A}(u_j) =$ constant (say, c), $f_E(u_i) = f_{E-A}(u_i) + f_A(u_i) = f_A(u_i) + c$. Therefore, the decision partition of C_A and C_E are equal and $f_{E_i} - f_{E_j} = f_{A_i} + c - f_{A_j} - c = f_{A_i} - f_{A_j}$. Since $E - A$ is a maximal subset of E such that $f_{E-A}(u_i) = f_{E-A}(u_j)$ for all i and j, A is a minimal subset of E such that C_A and C_E are equivalent.
(b) $\Rightarrow$ (c). Since $C_E = C_A$ and E is finite, f_{E_i} and f_{A_i} are equivalent. Therefore, N_E and N_A are equivalent. Since $f_E(u_i) = f_{E-A}(u_i) + f_A(u_i), f_{E_i} = f_{A_i} + f_{(E-A)_i}$. Also since $f_{E_i} - f_{E_j} = f_{A_i} - f_{A_j}, f_{(E-A)_i} = f_{(E-A)_j}$. Thus, $f_{E-A}(u_i) =$ constant (say, c) for all $u_i \in U$. Since A is a minimal subset of E such that C_A and C_E are equivalent, $E - A$ is a maximal subset of E such that $N_{E-A} = \{c\}$.
(c) $\Rightarrow$ (d). Since $N_{E-A} = \{c\}, f_{E-A}(u_i) = c$ for all $u_i \in U$. Hence $V_{E-A} = U$. Also, since $f_E(u_i) = f_{E-A}(u_i) + f_A(u_i) = f_A(u_i) + c, sup\{f_E(u_i) | u_i \in U\} = sup\{f_A(u_i) | u_i \in U\}$. Hence $V_A = V_E$.
(d) $\Rightarrow$ (a). Since $V_{E-A} = U, f_{E-A}(u_i) = f_{E-A}(u_j)$ for all i and j. Also, since $E - A$ is a maximal subset of E such that $V_{E-A} = U, V_E \neq U$. Therefore, $V_A \neq U$ and hence there exist $u_i \in U$ such that $f_A(u_i) < \vee_{u \in U} f_A(u)$. Let $\vee_{u \in U} f_A(u) = u_j$. Then $f_A(u_i) < f_A(u_j)$ and hence $f_A(u_i) \neq f_A(u_j)$. Thus, A is a normal parameter reduction of (F, E). □

3. Drawbacks in [12]

In 2014, Wang et. al [12] gave the reduct definition for fuzzy information system. We observe that the reduct definition in [12] do not retain the optimal choice of objects. In this section, we give an example to show that the reduct definition given

by Wang et. al [12] does not give the optimal choice.
In [12], Wang et. al defined lower reduct and the upper reduct as "$\cap P = \cap C$ and $\cap P \subset \cap(P - \{A_i\})$ for all $A_i \in P$" and "$\cup P = \cup C$ and $\cup P \subset \cup(P - \{A_i\})$ for all $A_i \in P$", respectively. But for all $A_i \in P, \cup P \supseteq \cup(P - \{A_i\})$. So the upper reduct definition given by Wang et. al need not be true in general. Also, Wang et. al use the definitions of lower reduct and upper reduct as "$\cap P = \cap C$ and $\cap P \neq \cap(P - \{A_i\})$ for all $A_i \in P$" and "$\cup P = \cup C$ and $\cup P \neq \cup(P - \{A_i\})$ for all $A_i \in P$", respectively, for proving Theorem 4.4 and Example 4.1 in [12]. Therefore, we redefine the lower reduct and upper reduct as follows.

Definition 3.1. Let (U, C) be a fuzzy information system and $P \subseteq C$. Then

i. P is referred to as a lower reduct of C if P satisfies $\cap P = \cap C$ and $\cap P \neq \cap(P - \{A_i\})$ for all $A_i \in P$
ii. P is referred to as an upper reduct of C if P satisfies $\cup P = \cup C$ and $\cup P \neq \cup(P - \{A_i\})$ for all $A_i \in P$
iii. P is referred to as a reduct of C if P is both an upper and a lower reduct of C.

Using Definition 3.1, Wang et. al proved Lemma 1.4. But the following Example 3.2 shows that the reduct definition does not maintain the optimal choice of objects. Therefore, we conclude that the 1.4 does not give the optimal choice of objects and hence there is no use for using the reduct definition of [12] given by Wang et. al.

Example 3.2. Consider the fuzzy information system (U, C) where $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and $C = \{A_1, A_2, A_3\}$ where $A_1 = \{(x_1, 0.4), (x_2, 0.6), (x_3, 0.5), (x_4, 0.1), (x_5, 0), (x_6, 0.3)\}$, $A_2 = \{(x_1, 0.2), (x_2, 0.1), (x_3, 0.3), (x_4, 0.4), (x_5, 0.7), (x_6, 0.4)\}$, $A_3 = \{(x_1, 0.1), (x_2, 0), (x_3, 0.2), (x_4, 0.7), (x_5, 0.8), (x_6, 0.6)\}$. Let $P = \{A_1, A_3\}$. Then $\cup P = \{(x_1, 0.4), (x_2, 0.6), (x_3, 0.5), (x_4, 0.7), (x_5, 0.8), (x_6, 0.6)\} = \cup C$ and $\cap P = \{(x_1, 0.1), (x_2, 0), (x_3, 0.2), (x_4, 0.1), (x_5, 0), (x_6, 0.3)\} = \cap C$. Also, $\cup P \neq \cup(P - \{A_i\})$ and $\cap P \neq \cap(P - \{A_i\})$. Therefore, P is a reduct of C. But for P, the optimal choice is x_6 whereas the optimal choice of C is x_5. Thus, this reduct definition do not maintain the optimality of objects.

Remark 3.3. Take $\{A_i | A_i \in C\}$ as parameter set E and U as universe. Then every fuzzy information system can be considered as a fuzzy soft set. Thus, we use normal parameter reduction of fuzzy soft sets for fuzzy information system. Also, all the results in Section 2 hold for fuzzy soft sets. Therefore, using these characterizations also, one can find the reduction of the fuzzy information system.

Example 3.4. Consider the fuzzy information system as in Example 3.2. Let $E = \{A_1, A_2, A_3\}$ be the set of parameters and $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the universe. Then the fuzzy information system is considered as the fuzzy soft set with tabular representation as given below.

U	A_1	A_2	A_3	$f_E(.)$
x_1	0.4	0.2	0.1	0.7
x_2	0.6	0.1	0	0.7
x_3	0.5	0.3	0.2	1
x_4	0.1	0.4	0.7	1.2
x_5	0	0.7	0.8	1.5
x_6	0.3	0.4	0.6	1.3

Since there is no $A \subseteq E$ such that $f_{E-A}(x_1) = f_{E-A}(x_2) = f_{E-A}(x_3) = f_{E-A}(x_4) = f_{E-A}(x_5) = f_{E-A}(x_6)$, the whole set E is the reduction for the information system given in Example 3.2.

4. Image of a normal parameter reduction

In soft set, Maji et. al [5] introduced the reduct concept using rough sets. But, Chen et. al [1] pointed out the problems involved in the reduct definition using rough sets given by Maji et. al [5] and they initiated the concept of parameterization reduction. In the parameterization reduction, the optimal choice objects of the soft set are maintained. But the suboptimal choice of objects are not maintained. To overcome these drawbacks in parameterization reduction, Kong et. al [4] introduced the concept of normal parameter reduction using choice value of objects. Till now, normal parameter reduction is the only method which maintains the optimal choice as well as the suboptimal choice of objects

Importance: Normal parameter reduction is the best tool for finding reduction of parameters. But if the size of objects and the parameters are very large, then the problem of finding normal parameter reduction becomes a complicated one. So we try to find out whether the image of normal parameter reduction of one soft set is normal parameter reduction of another soft set. In general, the image of normal parameter reduction of one soft set is not a normal parameter reduction of another soft set. But if we are able to find the consistency map between these two soft sets, then it holds good. Thus, if our soft set has larger parameter collection, it is enough to find the normal parameter reduction of another soft set having lesser parameter collection which was induced from the given soft set.

Definition 4.1. Let (F, A) be a soft set over U. Any two elements e_i and e_j in A are said to be *consistent* if (i) $F(e_i) \cup F(e_j) \neq U$ (ii) $F(e_i) \cap F(e_j) \neq \varnothing$ (iii) For each $u_k \in F(e_i) \cap F(e_j), F(e_i) \cap (F(e_s) - u_k) \neq \varnothing$ or $F(e_j) \cap (F(e_s) - u_k) \neq \varnothing$ where $e_s \neq e_i, e_j$.

The following Example 4.2 shows the existence of consistent and non-consistent parameters.

Example 4.2. Consider the universe $U = \{u_1, u_2, u_3, u_4, u_5\}$ and the parameter set $E = \{e_1, e_2, e_3, e_4\}$.
(a) Let (F, E) be the soft set defined by $F(e_1) = \{u_1, u_3, u_5\}, F(e_2) = \{u_1, u_3\}, F(e_3) = \{u_3, u_4, u_5\}, F(e_4) = \{u_2, u_5\}$. Consider the parameters e_2 and e_3. Then $F(e_2) \cup F(e_3) = \{u_1, u_3, u_4, u_5\} \neq U$ and $F(e_2) \cap F(e_3) = \{u_3\} \neq \varnothing$. Also, $F(e_1) - \{u_3\} = \{u_1, u_5\}$ and $F(e_4) - \{u_3\} = \{u_2, u_5\}$. Therefore, $F(e_2) \cap (F(e_1) - \{u_3\}) = \{u_1\} \neq \varnothing, F(e_3) \cap (F(e_1) - \{u_3\}) = \{u_5\} \neq \varnothing$ and $F(e_3) \cap (F(e_4) - \{u_3\}) = \{u_5\} \neq \varnothing$.

Thus, for each $e_i \neq e_2, e_3$ and for $u_3 \in F(e_2) \cap F(e_3), F(e_2) \cap (F(e_i) - \{u_3\}) \neq \varnothing$ or $F(e_3) \cap (F(e_i) - \{u_3\}) \neq \varnothing$. Therefore, e_2 and e_3 are consistent.
(b) Let (G, E) be the soft set defined by $G(e_1) = \{u_1, u_2, u_3, u_5\}, G(e_2) = \{u_1, u_3\}$, $G(e_3) = \{u_3, u_4\}, G(e_4) = \{u_2, u_5\}$. Consider the parameters e_2 and e_3. Then $G(e_2) \cup G(e_3) = \{u_1, u_3, u_5\} \neq U$ and $G(e_2) \cap G(e_3) = \{u_3\} \neq \varnothing$. Now, $G(e_4) - \{u_3\} = \{u_2, u_5\}$. Then $G(e_2) \cap (G(e_4) - \{u_3\}) = G(e_3) \cap (G(e_4) - \{u_3\}) = \varnothing$. Therefore, e_2 and e_3 are not consistent parameters.

Definition 4.3. Let (F, A) be a soft set over U and $i\colon A \to B$ be a mapping where $B \subseteq E$. The equivalence class $[e]_i$ of A with respect to i is *consistent equivalence class* if (i) $\cup_{e_k \in [e]_i} F(e_k) \neq U$ and (ii) any one of the pair of $e_i, e_j \in [e]_i, e_i$ and e_j are consistent.

Example 4.4. Consider the soft set (F, A) in Example 4.2(a) and let $B = \{t_1, t_2\}$ be another parameter set on U. Define a mapping $i\colon A \to B$ by $i(e_1) = i(e_2) = i(e_3) = t_1$ and $i(e_4) = t_2$. Then the equivalence classes of A with respect to the mapping i is given by $\{e_1, e_2, e_3\}$ and $\{e_4\}$. Consider the equivalence class $[e_1]_i = [e_2]_i = [e_3]_i = \{e_1, e_2, e_3\}$. Then $\cup_{e_k \in [e_1]_i} F(e_k) = F(e_1) \cup F(e_2) \cup F(e_3) = \{u_1, u_3, u_4, u_5\} \neq U$. Also, e_2 and e_3 are consistent. Hence $\{e_1, e_2, e_3\}$ is the consistent equivalence class of A with respect to i.

Definition 4.5. Let (F, A) be a soft set over U and $i\colon A \to B$ be a mapping where $B \subseteq E$. Then the equivalence class E_2 of A with respect to i is said to be *core equivalence class* if E_2 is the minimal subset of A such that $\cup F(E_2) = U$ and $F(E_2)$ are mutually disjoint.

Example 4.6. Consider the universe set $U = \{u_1, u_2, u_3, u_4, u_5\}$ and the parameter set $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, t_1, t_2, t_3, t_4\}$. Let $A = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ and $B = \{t_1, t_2, t_3, t_4\}$. Consider the soft set (F, A) which is defined by $F(e_1) = \{u_1, u_3, u_5\}$, $F(e_2) = \{u_2, u_5\}$, $F(e_3) = \{u_1, u_3\}$, $F(e_4) = \{u_1, u_3\}$, $F(e_5) = \{u_3, u_4, u_5\}$, $F(e_6) = \{u_2, u_5\}$, $F(e_7) = \{u_4\}$. Let the mapping $i\colon A \to B$ be defined by $i(e_1) = t_1$, $i(e_2) = \{t_2\}$, $i(e_3) = i(e_6) = i(e_7) = t_3$ and $i(e_4) = i(e_5) = t_4$. Then the equivalence classes of A with respect to the mapping i is given by $\{e_1\}, \{e_2\}, \{e_3, e_6, e_7\}, \{e_4, e_5\}$. Consider the equivalence class $X = \{e_3, e_6, e_7\}$. Then $\cup_{x \in X} F(x) = F(e_3) \cup F(e_6) \cup F(e_7) = U$ and $F(e_3) \cap F(e_6) = F(e_6) \cap F(e_7) = F(e_3) \cap F(e_7) = \varnothing$. Therefore, $\cup_{x \in X} F(x) = U$ and $F(X)$ is mutually disjoint. Hence $\{e_3, e_6, e_7\}$ is a core equivalence class.

Definition 4.7. Let (F, A) be a soft set over U and $i\colon A \to B$ be a onto function defined by $i(e_i) = t_j$ for all $e_i \in A$. Then i induces a soft set (G, B) over U, defined by $G(t_i) = \cup_{e_i \in i^{-1}(t_i)} F(e_i)$. The soft set (G, B) is called the $i-$ *induced soft set* of (F, A).

Example 4.8. Consider the soft set (F, A) and the mapping i which are defined in Example 4.6. Then the $i-$ induced soft set (G, B) is given by $G(t_1) = F(e_1) = \{u_1, u_3, u_5\}, G(t_2) = F(e_2) = \{u_2, u_5\}, G(t_3) = \cup_{e_i \in i^{-1}(t_3)} F(e_i) = F(e_3) \cup F(e_6) \cup F(e_7) = U, G(t_4) = \cup_{e_i \in i^{-1}(t_4)} F(e_i) = F(e_4) \cup F(e_5) = \{u_1, u_3, u_4, u_5\}$. Therefore, $(G, B) = \{(t_1, \{u_1, u_3, u_5\}), (t_2, \{u_2, u_5\}), (t_3, U), (t_4, \{u_1, u_3, u_4, u_5\})\}$.

Definition 4.9. Let (F, A) be a soft set over U and $i\colon A \to B$ be a mapping where $B \subseteq E$. Then the map i is a *consistent map* if all the equivalence classes of A with respect to i is either in E_1 or in E_2 or in E_3 where E_2 is the core equivalence class, E_1 is the consistent equivalence class of $A - E_2$ and E_3 is the equivalence class of single element.

Example 4.10. Consider the soft set (F, A) and i as in Example 4.6. Then the equivalence classes of A with respect to the mapping i are $\{e_1\}, \{e_2\}, \{e_3, e_6, e_7\}, \{e_4, e_5\}$. By Example 4.6, $\{e_3, e_6, e_7\}$ is a core equivalence class. Also, $\{e_4, e_5\}$ is a consistent equivalence class of $A - \{e_3, e_6, e_7\}$. Thus, all the equivalence classes of A with respect to i are in either E_1 or E_2 or E_3. Therefore, i is a consistent map.

Definition 4.11. Let (F, A) be a soft set over U and $i\colon A \to B$ be an onto function from A to B. Then i induces a map $h\colon D \to B$ where D is the set of all equivalence classes of A with respect to i. The map h is called an *$i-$ induced map*. Clearly, h is a bijection.

Example 4.12. Consider the Example 4.10. Here the $i-$ induced map h is given by $h(\{e_1\}) = t_1, h(\{e_2\}) = t_2, h(\{e_3, e_6, e_7\}) = t_3, h(\{e_4, e_5\}) = t_4$.

Remark 4.13. If i is a consistent map, then the $i-$ induced map h is also consistent.

Theorem 4.14. *Suppose (F, A) is a soft set over U and i is an onto consistent map from A to $B \subseteq E$. Let (G, B) be an $i-$ induced soft set of (F, A) over U and h be an $i-$ induced map. Then P is a normal parameter reduction of (F, A) if and only if $h(P)$ is a normal parameter reduction of (G, B).*

Proof. Suppose (F, A) is a soft set over U and h is an $i-$ induced map. Let P be a normal parameter reduction of (F, A). Then $A - P$ is a maximal subset of A such that $f_P(u_i) \neq f_P(u_j)$ for some $u_i, u_j \in U$ and $f_{A-P}(u_i) = constant$ (say, c) for all $u_i \in U$. Since $f_{A-P}(u_i) = c$ for all $u_i \in U, F(A - P) = U$ or $\varnothing$. Since $\varnothing$ is neither in E_1 nor in E_2 and nor in E_3 and h is consistent, $F(A - P) = U$ and hence $A - P$ is a core equivalence class. Therefore, all the elements in $A - P$ is mapped into the single image t_i and hence $G(t_i) = \cup_{e_i \in h^{-1}(t_i)} F(e_i) = \cup_{e_i \in A-P} F(e_i) = U$. That is, $G(h(A - P)) = U$. Since $h(A - P) = \{t_i\}, g_{h(A-P)}(u_i) =$ constant, c_1 (in particular, $c_1 = 1$) for all $u_i \in U$. This implies $g_{h(A)-h(P)}(u_i) = c_1$ and hence $g_{B-h(P)}(u_i) = c_1$ for all $u_i \in U$. Therefore, $g_{B-h(P)}(u_i) = g_{B-h(P)}(u_j)$ for all $u_i, u_j \in U$. Since h is consistent, any $e_i \notin P, h(e_i) \notin h(P)$ (otherwise, if e_i is in the reduction $h(P)$, then $F(P \cup \{e_i\}) = U$. Thus, $P \cup \{e_i\}$ is a core equivalence class and hence $P \cup \{e_i\}$ is the subset of the normal parameter reduction). Thus, $B - h(P)$ is a maximal subset of B such that $g_{B-h(P)}(u_i) = g_{B-h(P)}(u_j)$ for all $u_i, u_j \in U$. Therefore, $g_{h(P)}(u_i) \neq g_{h(P)}(u_j)$ for some $u_i, u_j \in U$. Hence $h(P)$ is a normal parameter reduction of B.

Conversely, suppose $h(P)$ is a normal parameter reduction of (G, B). Then $h(P)$ is a maximal subset of B such that $g_{h(P)}(u_i) \neq g_{h(P)}(u_j)$ for some $u_i, u_j \in U$ and $g_{B-h(P)}(u_i) = g_{B-h(P)}(u_j)$ for all $u_i, u_j \in U$. That is, $g_{B-h(P)}(u_i) = constant$ (say, c). Since $g_{B-h(P)}(u_i) = c, \sum_{e_i \in h^{-1}(B-h(P))} f_{\{e_i\}}(u_i) = c$. This implies that $\sum_{e_i \in h^{-1}(B)-h^{-1}h(P)} f_{\{e_i\}}(u_i) = c$ and hence $f_{A-P}(u_i) = c$ for all $u_i \in U$. Since h is consistent, $A - P$ is a maximal subset of A such that $f_{A-P}(u_i) = f_{A-P}(u_j)$ for all

$u_i, u_j \in U$. Since $g_{h(P)}(u_i) \neq g_{h(P)}(u_j)$ for some $u_i, u_j \in U$, $\sum_{e_i \in h^{-1}h(P)} f_{\{e_i\}}(u_i) \neq \sum_{e_i \in h^{-1}h(P)} f_{\{e_i\}}(u_j)$ for some $u_i, u_j \in U$. That is, $f_P(u_i) \neq f_P(u_j)$ for some $u_i, u_j \in U$. Hence P is a normal parameter reduction of (F, A). □

Example 4.15. Consider the soft set (F, A) and i as in Example 4.6. By Example 4.10, i is a consistent map. That is, i satisfies all the conditions of the Theorem 4.14. The normal parameter reduction of (F, A) is $P = \{e_3, e_6, e_7\}$. Then $h(P) = \{t_3\}$ is the normal parameter reduction of (G, B).

The following Example 4.16 shows that consistency property in Theorem 4.14 cannot be dropped.

Example 4.16. Consider the universe $U = \{u_1, u_2, u_3, u_4, u_5\}$ and parameter sets $A = \{e_1, e_2, e_3, e_4\}, B = \{t_1, t_2, t_3\}$. Let (F, A) be a soft set defined by $(F, A) = \{(e_1, \{u_1, u_3, u_5\}), (e_2, \{u_1, u_3\}), (e_3, \{u_3, u_4\}), (e_4, \{u_2, u_5\})\}$ and i be a mapping from A to B which is defined by $i(e_1) = t_1, i(e_2) = t_2, i(e_3) = t_2, i(e_4) = t_3$. Then the equivalence classes of A with respect to i is $\{e_1\}, \{e_2, e_3\}, \{e_4\}$. Therefore, the i induced map h is given by $h(\{e_1\}) = t_1, h(\{e_2, e_3\}) = t_2, h(\{e_4\}) = t_3$. Also, the induced soft set (G, B) of (F, A) is given by $(G, B) = \{(t_1, \{u_1, u_3, u_5\}), (t_2, \{u_1, u_3, u_4\}), (t_3, \{u_2, u_5\})\}$. Consider the equivalence class $\{e_2, e_3\}$. Clearly, $F(e_2) \cap F(e_3) = \{u_3\} \neq \varnothing$ and $F(e_2) \cup F(e_3) \neq U$. Therefore, it is not a core equivalence class. Now $F(e_4) - \{u_3\} = \{u_2, u_5\}$. Also, $(F(e_4) - \{u_3\}) \cap F(e_2) = \varnothing$ and $(F(e_4) - \{u_3\}) \cap F(e_3) = \varnothing$. Therefore, $\{e_2, e_3\}$ is not a consistent equivalence class. Thus, i is not a consistent map. Also, the normal parameter reduction of (F, A) is $\{e_1, e_2, e_3, e_4\}$ whereas the normal parameter reduction of (G, B) is $\{t_1\} = h(\{e_1\})$. Thus, consistency property cannot be dropped in the Theorem 4.14.

5. Conclusion

In this paper, we give the drawback in the reduct definition given by Wang et. al and give some characterizations of normal parameter reduction of soft sets. Also, we prove that the image and inverse image of normal parameter reduction is normal parameter reduction using the concept of consistency.

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