ON COMPOSITES, AND PRIMES

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Abstract. In this paper, I introduce two concepts, first is number influence strength, and second is count of influenced multiples less than x. Going from there, we can calculate how many composites is less than x stepping to how many primes is less than x that denoted to π (x).

1. Introduction

In this paper, I talk about the least common multiple in the first section that can be just an introduction to coming sections. Then I introduce two concepts, first is number influence strength, and second is count of influenced multiples less than x. They both depend on least common multiple function heavily. The last section is for calculating composites and primes less than x that is possible using the two concepts.

2. Least Common Multiple

I will take you in a trip about least common multiple, because it's essential in our calculations. So what is the least common multiple?

What is a "Multiple"? We get a multiple of a number when we multiply it by another number. Such as multiplying by $\{1,2,3,4,5,\ldots\}$ but not $\{0\}$. What is a "Common Multiple"?

Common multiple is the shared multiple between two numbers or more. What is the "Least Common Multiple"?

Least common multiple is the smallest positive number that is a multiple of two or more numbers.

Least common multiple function is written as LCM. It can take many inputs as arguments and output one answer (result) that is the least common multiple. The next part is how to find it especially when arguments are more than two numbers.

There're many ways to find least common multiple, but I will discuss here one simple way I use that can be done by using simple calculator.

If you have many arguments $\{a_1, a_2, a_3, a_4, \ldots, a_n\}$, take two numbers and rise them to power -1 and subtract the largest fraction from the smallest. If the denominator is not divisible by the two numbers, then multiply it by their common devisor, otherwise LCM is the denominator. Repeat above steps with the answer and left numbers.

Example 2.1.

a. For $LCM(2,5)$, do the following, $\frac{1}{2} - \frac{1}{5} = \frac{3}{10}$. Denominator is divisible by 2 and 5, then $LCM(2,5) = 10$

b. For $LCM(2,4)$, do the following, $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$. Denominator is divisible by 2 and 4, then $LCM(2,4) = 4$

c. For $LCM(3,9)$, do the following, $\frac{1}{3}-\frac{1}{9}=\frac{2}{9}$. Denominator is divisible by 3 and 9, then $LCM(3,9)=9$
d. For $LCM(25,30)$, do the following, $\frac{1}{25}-\frac{1}{30}=\frac{1}{150}$. Denominator is divisible by 25 and 30, then $LCM(25,30)=150$
e. For $LCM(30,31)$, do the following, $\frac{1}{30}-\frac{1}{31}=\frac{1}{930}$. Denominator is divisible by 31 and 30, then $LCM(30,31)=930$
f. For $LCM(30,35)$, do the following, $\frac{1}{30}-\frac{1}{35}=\frac{1}{210}$. Denominator is divisible by 35 and 30, then $LCM(30,35)=210$
g. For $LCM(30,20)$, do the following, $\frac{1}{20}-\frac{1}{30}=\frac{1}{60}$. Denominator is divisible by 20 and 30, then $LCM(30,20)=60$
h. For $LCM(35,60)$, do the following, $\frac{1}{35}-\frac{1}{60}=\frac{5}{420}=\frac{1}{84}$. Denominator is not divisible by 35, and 60, then $LCM(30,20)=84\ \times GCD(35,60)=420$
i. For $LCM(20,30,35)$, $LCM(20,30)=60$, $LCM(60,35)=420$, $LCM(20,30,35)=420$
j. For $LCM(55,100,105)$, do the following, $\frac{1}{55}-\frac{1}{100}=\frac{9}{1100}$, $\frac{1}{105}-\frac{1}{1100}=\frac{199}{23100}$. Denominator is divisible by 55, 105 and 100, then $LCM(55,100,105)=23100$

You may wonder, why does it work? Well,

$$\frac{1}{a}-\frac{1}{b}=\frac{b-a}{a\times b}$$

If a and b share divisors say

$$\frac{1}{a}-\frac{1}{b}=\frac{c\left(\frac{b}{c}-\frac{a}{c}\right)}{c(d\times e)}$$

where $a=d\times c$, and $b=e\times c$, then c will be eliminated with $a\times b$. And what left in the dominator is LCM. If there's no shared divisors then LCM is $a\times b$. There are some situations when $\frac{b}{c}-\frac{a}{c}$ could result in a number is eliminated with $a\times b$, as in example 2.1.h, which results denominator less than a and b, or denominator is not divisible by a, and b. But these situations are rare and can be avoided by multiplying denominator by the great common devisor. So it can work most of the times. Make sure to check always your answer, by dividing denominator back by a, and b.

3. Numbers Strength and Influenced multiples Less Than X

Here, I'll discuss two important concepts that will let us understand more about primes and composites.

3.1. Number Influence Strength

Let me explain what I mean by the number influence strength. Let's say we have two numbers, a $<b$. Multiplying a times b results a multiple, but we say that the multiple is made by the strength of a, because a preceded b. In other words, multiples of a are $\{a\times 1, a\times 2, a\times 3, \ldots, a\times b, \ldots\}$. Let's take $a\times b=d$.

Multiples of b are $\{b \times 1, b \times 2, b \times 3, \ldots, b \times a, \ldots\}$. We know $b \times a = d$, but a had influenced d first, so we count it in Favor of a that a had influenced d before b. For example, we count 6 in Favor of 2 not 3, we count 15 in Favor of 3 not 5, and so on.

Notice, by saying influenced multiples by 2, we're not talking about composites resulted from 2. The first is $\{2,4,6,8,10,12, \ldots\}$, the second is $\{4,6,8,10, \ldots\}$. Also, influenced multiples by a aren't the same as multiples of a. Influenced multiples by 3 are $\{3,9,15,21, \ldots\}$, multiples of 3 are $\{3,6,9,12, \ldots\}$.

Definition 3.1. *Let's define $\iota(n)$ as a proportional denote to the number influence strength, then*

$$\iota(n) = \frac{1}{n} + \sum_{m=2}^{n-1} (-1)^{m+1} \left(\sum \{LCM(n,B)^{-1} : B \subset A \ \wedge \ |B| = m-1\} \right)$$

where $A = \{A \subset \mathbb{N} : 1 < A < n\}$, $n \in \mathbb{N}$, and $m \in \mathbb{N}$.

Explanation of the formula. let's start from inside summation, The second sum, sums for $\{B \subset A : |B| = m-1\}$, where it's all possible subsets of $A = \{A \subset \mathbb{N} : 1 < A < n\}$,since m starts from 2, then the second sum counts $LCM(n, [\{B \subset A : |B| = 1\}])^{-1}$, which is all subsets of A of cardinality 1. For example, if $n = 7$, then it sums $\frac{1}{LCM(2,7)} + \frac{1}{LCM(3,7)} + \frac{1}{LCM(4,7)} + \frac{1}{LCM(5,7)} + \frac{1}{LCM(6,7)}$. After that, the sum will be repeated but with different cardinality of subsets that is $\frac{1}{LCM(2,3,7)} + \frac{1}{LCM(2,4,7)} + \frac{1}{LCM(2,5,7)} + \frac{1}{LCM(2,6,7)} + \frac{1}{LCM(3,4,7)} + \frac{1}{LCM(3,5,7)} + \frac{1}{LCM(3,6,7)} + \frac{1}{LCM(4,5,7)} + \frac{1}{LCM(4,6,7)} + \frac{1}{LCM(5,6,7)}$. Then subsets with cardinality of 4, till we reach subsets with cardinality of 6 that is one subset $B = \{2,3,4,5,6,7\}$. Then the first summation sums all different cardinalities sums but with alternating sign. Since 7 is odd, the last sign will be negative. For $n = 2$, the sum will be empty sub that is 0.

Example 3.1.

a. $\iota(2) = \frac{1}{2} + 0 = \frac{1}{2}$

b. $\iota(3) = \frac{1}{3} - (LCM(2,3)^{-1}) = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

c. $\iota(4) = \frac{1}{4} - (LCM(2,4)^{-1} + LCM(3,4)^{-1}) + LCM(2,3,4)^{-1} = \frac{1}{4} - \left(\frac{1}{4} + \frac{1}{12}\right) + \left(\frac{1}{12}\right) = 0$

d. $\iota(5) = \frac{1}{5} - (LCM(2,5)^{-1} + LCM(3,5)^{-1} + LCM(4,5)^{-1}) + (LCM(2,3,5)^{-1} + LCM(2,4,5)^{-1} + LCM(3,4,5)^{-1}) - (LCM(2,3,4,5)^{-1}) = \frac{1}{5} - \left(\frac{1}{10} + \frac{1}{15} + \frac{1}{20}\right) + \left(\frac{1}{30} + \frac{1}{20} + \frac{1}{60}\right) - \left(\frac{1}{60}\right) = \frac{1}{5} - \left(\frac{1}{10} + \frac{1}{15}\right) + \frac{1}{30} = \frac{1}{15}$

Properties 3.1.

a. $0 \le \iota(n) < 1$ for all $n \in \mathbb{N}$.

b. $\iota(n) = \frac{1}{n} \times \left(1 - \sum_{m=1}^{n-1} \iota(p_m)\right)$, where p_m is the nth prime, $n \in \mathbb{N}_p$, and $\mathbb{N}_p$ are prime numbers.

c. $\iota(n) = 0$ for all $n \in \mathbb{N}_c$, where $\mathbb{N}_c$ are composite numbers.

d. $0 < \iota(n) < 1$ for all $n \in \mathbb{N}_p$, where $\mathbb{N}_p$ are prime numbers.

Example 3.2.

a. $\iota(4) = 0$, from property 3.1.c.
b. $\iota(6) = 0$, from property 3.1.c.
c. $\iota(9) = 0$, from property 3.1.c.
d. $\iota(5) = \frac{1}{5} - (LCM(2,5)^{-1} + LCM(3,5)^{-1}) + (\ LCM(2,3,5)^{-1}) = \frac{1}{5} - \left(\frac{1}{10} + \frac{1}{15}\right) + \frac{1}{30} = \frac{1}{15}$
e. $\iota(5) = \frac{1}{5} - ((2 \times 5)^{-1} + (3 \times 5)^{-1}) + (\ (2 \times 3 \times 5)^{-1}) = \frac{1}{5} - ((10)^{-1} + (15)^{-1}) + (\ (30)^{-1}) = \frac{1}{5} - \left(\frac{1}{10} + \frac{1}{15}\right) + \frac{1}{30} = \frac{1}{15}$
f. $\iota(5) = \frac{1}{5} \times (1 - \iota(2) - \iota(3)) = \frac{1}{5} \times \left(1 - \frac{1}{2} - \frac{1}{6}\right) = \frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$, from property 3.1.b.
g. $\iota(7) = \frac{1}{7} \times (1 - \iota(2) - \iota(3) - \iota(5)) = \frac{1}{7} \times \left(1 - \frac{1}{2} - \frac{1}{6} - \frac{1}{15}\right) = \frac{1}{7} \times \frac{4}{15} = \frac{4}{105}$, from property 3.1.b.
h. $\iota(11) = \frac{1}{11} \times (1 - \iota(2) - \iota(3) - \iota(5) - \iota(7)) = \frac{1}{11} \times \left(1 - \frac{1}{2} - \frac{1}{6} - \frac{1}{15} - \frac{4}{105}\right) = \frac{1}{11} \times \frac{8}{35} = \frac{8}{385}$, from property 3.1.b.

Notice. For primes, composites can be dropped as in example 3.2.d. LCM can be dropped as in example 3.2.e.

Table 3.1.

$\iota(n) =$	
$\iota(2) =$	$\frac{1}{2}$
$\iota(3) =$	$\frac{1}{6}$
$\iota(4) =$	0
$\iota(5) =$	$\frac{1}{15}$
$\iota(6) =$	0
$\iota(7) =$	$\frac{4}{105}$
$\iota(8) =$	0
$\iota(9) =$	0
$\iota(10) =$	0

Table 3.2.

$\iota(p) =$	
$\iota(2) =$	$\frac{1}{2}$
$\iota(3) =$	$\frac{1}{6}$
$\iota(5) =$	$\frac{1}{15}$

$\iota(7) =$	$\frac{4}{105}$
$\iota(11) =$	$\frac{8}{385}$
$\iota(13) =$	$\frac{16}{1001}$
$\iota(17) =$	$\frac{192}{17017}$
$\iota(19) =$	$\frac{3072}{323323}$

Glimpse on $\boldsymbol{\iota(1)}$. If we include 1 in our group A, where $A = \{A \in N: 1 \leq A < n\}$ for all n, then 1 will gain the whole power and all other numbers will have no power that is 0. Since all numbers are multiples influenced by 1 first, and influenced multiples by less numbers can't influence others. Then $(n) = 0$. For example $\iota(2) = \frac{1}{2} - \left(\frac{1}{LCM(1,2)}\right) = \frac{1}{2} - \frac{1}{2} = 0$. $\iota(3) = \frac{1}{3} - \left(\frac{1}{LCM(1,3)} + \frac{1}{LCM(2,3)}\right) + \left(\frac{1}{LCM(1,2,3)}\right) = \frac{1}{3} - \left(\frac{1}{3} + \frac{1}{6}\right) + \frac{1}{6} = 0$. And to avoid this, we discarded 1 from the group, and counted the strength of n excluding 1 influence.

Inferences from $\boldsymbol{\iota(n)}$.

a. Depending on our $\iota(n)$ function, we can say with 100% of confidence that "for every sequential six numbers, there're only 2 numbers that can't be divisible by 2 or 3". Why? We know that for every 2×3, 2 and 3 repeat the cycle in meeting as they started from 0. To calculate non-influenced multiples by 2 or 3, we calculate $1 - \iota(2) - \iota(3) = \frac{2}{6} = \frac{1}{3}$. So for every cycle that is 2×3, $\frac{1}{3}$ of it is not divisible by 2 or 3 which are 2 numbers from 6.

b. For every 30 numbers 8 numbers are not divisible by 2, 3, or 5. Since $\iota(4) = 0$, including it doesn't matter because it has no power. $1 - \iota(2) - \iota(3) - \iota(5) = \frac{4}{15}$, $2 \times 3 \times 5 = 30$, $\frac{4}{15} \times 30 = 8$

c. but what about numbers that cannot be divisible by 3 , 5, or 4, but not 2. Here we cannot use the same way, because $\iota(n)$ calculate the influence strength from 2. But we can change that by using $\iota_{x>m}(n)$, which means start counting the influence from $x > m$. $\iota_{x>3}(3) = \frac{1}{3}$, here you can see, we've ignored 2 strength**.** $\iota_{x>3}(4) = \frac{1}{6}$, since 2 is ignored, then 4 hasn't been influenced yet. $\iota_{x>3}(5) = \frac{1}{5} - \left(\frac{1}{LCM(3,5)} + \frac{1}{LCM(4,5)}\right) + \frac{1}{LCM(3,4,5)} = \frac{\mathbf{1}}{\mathbf{5}} - \frac{\mathbf{1}}{\mathbf{15}} - \frac{\mathbf{1}}{\mathbf{20}} + \frac{\mathbf{1}}{\mathbf{60}} = \frac{\mathbf{1}}{\mathbf{10}}$. $1 - \iota_{x>3}(3) - \iota_{x>3}(4) - \iota_{x>3}(5) = \frac{\mathbf{2}}{\mathbf{5}}$. That means with respect only to $x > 3$, for every $3 \times 4 \times 5$, there is $60 \times \frac{2}{5} = 24$, that are not divisible by 3,4, or 5. As you've noticed 4 had some value, but not other composites. $\iota_{x>3}(4) = \frac{1}{6}$, $\iota_{x>3}(6) = 0$, $\iota_{x>3}(8) = 0$, $\iota_{x>3}(9) = 0$.

d. For 5, and 3 only, we calculate $\iota(5)$ with respect to 3 only. $\iota_{x=3}(3) = \frac{1}{3}$. $\iota_{x=3}(5) = \frac{1}{5} - \left(\frac{1}{LCM(3,5)}\right) = \frac{1}{5} - \frac{1}{15} = \frac{2}{15}$. So for every 15 numbers 8 cannot be divisible by 3 or 5. Those functions can be obtained by changing the A group. The last example A was $A = \{3\}$.

Primes and $\iota(n)$.

One thing I like about $\iota(n)$ is that it can predict some of the first primes distribution, well, exactly is the appearance of first primes. Since 1 is not calculated for $\iota(n)$. We will behave as 1 does not exist. First let's define what primes are in respect to $\iota(n)$. Primes are all non-influenced multiples by less numbers. Using that definition, we can say if $\iota(2) = \frac{1}{2}$, then non-influenced multiples are located in the second half $1 - \iota(2) - \frac{1}{2}$, and primes are non-influenced multiples. From here, we can srart chasing primes. Starting from 2, non-influenced multiples are $1 - 0 = 1$, that is. For every coming number one is a prime, that holds true for 2 and 3. But then $\iota(2)$ will be added. So non-influenced multiples are $1 - \iota(2) = \frac{1}{2}$. For every 2 numbers one is prime. Holds true for $\{(3,4), (5,6), (7,8)\}$. $\iota(3)$ is added, non-influenced multiples are $1 - \iota(2) - \iota(3) = \frac{1}{3}$. For every 3 numbers 1 is prime, holds true for $\{(9,10,11), (12,13,14), (15,16,17), (18,19,20), (21,22,23)\}$. Then $\iota(5)$ is added, so for every 15 numbers there will be 4 primes. Then for every 35 numbers 8 will be primes, nearly till 121. Then for every 77 numbers 16 will be primes, till 169. But as you see, here we lost trace. Because $(169 - 121) < 77$. That primes square will be closer than the product of primes, the steps that we need to count for $\iota(n)$.

Table 3.3.

	Holds true in
Every one number is prime	$\{(2), (3)\}$
Every 2 numbers, one is prime	$\{(3,4), (5,6), (7,8)\}$
Every 3 numbers, one is prime	$\{(9,10,11),$ $(12,13,14),$ $(15,16,17),$ $(18,19,20),$ $(21,22,23)\}$
Every 15 numbers, 4 are primes	$\left\{\begin{pmatrix}23,24,25,26,27,28,\\29,30,31,32,33,34,\\35,36,37,38,39\end{pmatrix}\right\}$
Every 35 numbers, 8 are primes	$\left\{\begin{pmatrix}39,40,41,42,43,\\44,45,46,47,48,\\49,50,51,52,53,\\54,55,56,57,58,\\59,60,61,62,63,\\64,65,66,67,68,\\69,70,71,72,73,74\end{pmatrix}\right\}$

3.2. Influenced Multiples Less Than X

Now we can discuss the second concept, which is very similar to the first. Influenced multiples by $n \in \mathbb{N}$ less than x has the same idea. It counts influenced multiples in a specified region. For example, influenced multiples less than 15 by 2 are $\{2,4,6,8,10,12,14\}$. But influenced multiples by 3 in the same region are $\{3,9,15\}$. That's because $\{6,12\}$ are already taken by 2. So influenced multiples by 2 less than x is 7, and by 3 is 3. That is the idea. You can obtain count of multiples of let's say 7 by $\left\lfloor \frac{x}{7} \right\rfloor$, where the $\lfloor \quad \rfloor$ denote floor function. But what about influenced multiples by 7.

Definition 3.2. *Let's define $\iota(x,n)$ as a count of influenced multiples by n less than x, then*

$$\iota(x,n) = \frac{x}{n} + \sum_{m=2}^{n-1} (-1)^{m+1} \left(\sum \left\{ \left\lfloor \frac{x}{LCM(n,B)} \right\rfloor : B \subset A \ \wedge \ |B| = m-1 \right\} \right)$$

where $x \in \mathbb{N}$, $n \in \mathbb{N}$, $m \in \mathbb{N}$, $n > 1$, and $A = \{A \subset \mathbb{N}: 1 < A < n\}$.

Well, $\iota(x,n)$ is the same concept of $\iota(n)$, yet its function is similar to it. Except that we use $floor(x)$ for x divided by LCM of every subset.

Notice. $floor(x)$ returns the integer part only for $x \geq 0$.

Properties 3.2.

a. $\iota(x,n) = 0$, if $n > x$.
b. $\iota(x,n) = 0$, if $n \in \mathbb{N}_c$, where $\mathbb{N}_c$ are composite numbers.
c. $\iota(x,n) = 1$, if $x = n$, and $n \in \mathbb{N}_p$, where $\mathbb{N}_p$ denoted for prime numbers.
d. $\iota(x,n) = \iota(n) \times x$, if $x = a \times n!$ for $a \in \mathbb{N}$.
e. $\iota(x,n) = \iota(n) \times x$, if $= a \times \prod_{k=1}^{m} p_k$, where $a \in \mathbb{N}$, and $p_k \in \mathbb{N}_p$.

Notice. $p_n\# \equiv \prod_{k=1}^{n} p_k$, is the product of primes. For example, $\prod_{k=1}^{13} p_k = 2 \times 3 \times 5 \times 7 \times 11 \times 13$.

Example 3.4.

a. $\iota(a \times 2!,2) = a \times 2 \times \frac{1}{2} = a$, from property 3.2.d.
b. $\iota(a \times 3!,3) = a \times 6 \times \frac{1}{6} = a$, from property 3.2.d.
c. $\iota(a \times 5!,5) = a \times 120 \times \frac{1}{15} = a \times 8$, from property 3.2.d.
d. $\iota(a \times 7!,7) = a \times 5040 \times \frac{4}{105} = a \times 192$, from property 3.2.d.
e. $\iota(a \times \prod_{n=2}^{7} n, 7) = a \times 210 \times \frac{4}{105} = a \times 8$, from property 3.2.d.
f. $\iota(18,3) = 3, 18 = 3 * 3!$, from property 3.2.d.
g. $\iota(10080,7) = 2 \times 192 = 384, 10080 = 2 * 7!$, from property 3.2.d.
h. $\iota(120,5) = 30 \times 5 = 120 \ \times \frac{1}{15} = 8, 120 = 4 \times \prod_{k=1}^{5} p_k$, from property 3.2.e.
i. $\iota(10080,7) = 48 \times 8 = 384, 10080 = 48 \times \prod_{k=1}^{7} p_k$, from property 3.2.e.
j. $\iota(10,100) = 0$, from property 3.2.a.
k. $\iota(100,10) = 0$, from property 3.2.b.

l. $\iota(101,101) = 1$, from property 3.2.c.

m. $\iota(100,5) = \left\lfloor \frac{100}{5} \right\rfloor - \left(\lfloor LCM(2,5)^{-1} \times 100 \rfloor + \lfloor LCM(3,5)^{-1} \times 100 \rfloor + \lfloor LCM(4,5) \times 100 \rfloor\right) + \left(\lfloor LCM(2,3,5)^{-1} \times 100 \rfloor + \lfloor LCM(2,4,5)^{-1} \times 100 \rfloor + \lfloor LCM(3,4,5)^{-1} \times 100 \rfloor\right) - \left(\lfloor LCM(2,3,4,5)^{-1} \times 100 \rfloor\right) = \left\lfloor \frac{100}{5} \right\rfloor - \left(\left\lfloor \frac{100}{10} \right\rfloor + \left\lfloor \frac{100}{15} \right\rfloor + \left\lfloor \frac{100}{20} \right\rfloor\right) + \left(\left\lfloor \frac{100}{30} \right\rfloor + \left\lfloor \frac{100}{20} \right\rfloor + \left\lfloor \frac{100}{60} \right\rfloor\right) - \left\lfloor \frac{100}{60} \right\rfloor = 20 - (10 + 6 + 5) + (3 + 5 + 1) - 1 = 20 - 21 + 9 - 1 = 7$

n. $\iota(100,7) = \left\lfloor \frac{100}{7} \right\rfloor - \left(\lfloor LCM(2,7)^{-1} \times 100 \rfloor + \lfloor LCM(3,7)^{-1} \times 100 \rfloor + \lfloor LCM(5,7) \times 100 \rfloor\right) + \left(\lfloor LCM(2,3,7)^{-1} \times 100 \rfloor + \lfloor LCM(2,5,7)^{-1} \times 100 \rfloor + \lfloor LCM(3,5,7)^{-1} \times 100 \rfloor\right) - \left(\lfloor LCM(2,3,5,7)^{-1} \times 100 \rfloor\right) = \left\lfloor \frac{100}{7} \right\rfloor - \left(\left\lfloor \frac{100}{14} \right\rfloor + \left\lfloor \frac{100}{21} \right\rfloor + \left\lfloor \frac{100}{35} \right\rfloor\right) + \left(\left\lfloor \frac{100}{42} \right\rfloor + \left\lfloor \frac{100}{70} \right\rfloor + \left\lfloor \frac{100}{105} \right\rfloor\right) - \left\lfloor \frac{100}{210} \right\rfloor = 14 - (7 + 4 + 2) + (2 + 1 + 0) - 0 = 14 - 13 + 3 = 4$

Notice. Composites can be dropped for primes as in example 3.4.n, and so LCM can be.

Influenced multiples, $\boldsymbol{\iota(x,n)}$, in a specified range. To obtain $\iota(x,n)$ in a specified range $a < x < b$. Then $\iota(a < x < b, n) = \iota(b,n) - \iota(a,n)$

Example 3.5.

a. $\iota(2310 < x < 6930,11) = \iota(6930,11) - \iota(2310,11) = \left(6930 \times \iota(11)\right) - \left(2310 \times \iota(11)\right) = 144 - 48 = 96.$

3.3. Proofs.

Proof 3.1. Property 3.1.c.

$\iota(n) = 0$, for all $n \in \mathbb{N}_c$. Say $a < n$ and is divisor of n, For every $\frac{1}{n}$ is subtracted by $\frac{1}{LCM(a,n)}$. Say $b < n$ but not divisor for a. Then for every $\frac{1}{LCM(b_1,b_2,b_3,\ldots,n)}$ is eliminated by $\frac{1}{LCM(a,b_1,b_2,b_3,\ldots,n)}$, because $\frac{1}{LCM(a,n)} = \frac{1}{n}$ and because of the alternating sign for every cardinality.

Proof 3.2. Property 3.1.b.

$\iota(n) = \frac{1}{n} \times \left(1 - \sum_{m=1}^{n-1} \iota(p_m)\right)$. Since Proof 3.1 holds true, then all composite numbers can be eliminated from calculations, and we can calculate only for primes. If a, and n are primes, then $LCM(a,n) = a \times n$. So $\iota(n) = \frac{1}{n} - \left(\frac{1}{(n-1)n} + \frac{1}{(n-2)n} + \cdots + \frac{1}{(2)n}\right) + \left(\frac{1}{(n-1)(n-2)n} + \frac{1}{(n-1)(n-3)n} + \cdots + \frac{1}{(2)(3)n}\right) - \cdots$ taking $\frac{1}{n}$ as common devisor we have

$$\iota(n) = \frac{1}{n} \times \left(\left(1 - \frac{1}{(n-1)} + \frac{1}{(n-2)} + \cdots + \frac{1}{(2)}\right) + \left(\frac{1}{(n-1)(n-2)} + \frac{1}{(n-1)(n-3)} + \cdots + \frac{1}{(2)(3)}\right) - \cdots\right)$$

$$\iota(n) = \frac{1}{n} \times \left(1 - \left(\frac{1}{(n-1)} + \frac{1}{(n-2)} + \cdots + \frac{1}{(2)}\right) + \left(\frac{1}{(n-1)(n-2)} + \frac{1}{(n-1)(n-3)} + \cdots + \frac{1}{(2)(3)}\right) - \cdots\right)$$

$$\iota(n) = \frac{1}{n} \times \left(1 - \iota(2) - \left(\frac{1}{(n-1)} + \frac{1}{(n-2)} + \cdots + \frac{1}{3}\right) + \left(\frac{1}{(n-1)(n-2)} + \frac{1}{(n-1)(n-3)} + \cdots + \frac{1}{(2)(3)}\right) - \cdots\right)$$

$$\iota(n) = \frac{1}{n} \times \left(1 - \iota(2) - \frac{1}{3} + \frac{1}{(2)(3)} - \left(\frac{1}{(n-1)} + \frac{1}{(n-2)} + \cdots + \frac{1}{5}\right) + \left(\frac{1}{(n-1)(n-2)} + \frac{1}{(n-1)(n-3)} + \cdots + \frac{1}{(2)(5)}\right) - \cdots\right)$$

$$\iota(n) = \frac{1}{n} \times \left(1 - \iota(2) - \iota(3) - \left(\frac{1}{(n-1)} + \frac{1}{(n-2)} + \cdots + \frac{1}{5}\right) + \left(\frac{1}{(n-1)(n-2)} + \frac{1}{(n-1)(n-3)} + \cdots + \frac{1}{(2)(5)}\right) - \cdots\right)$$

$$\iota(n) = \frac{1}{n} \times \left(1 - \iota(2) - \iota(3) - \cdots - \frac{1}{(n-1)} + \left(\frac{1}{(n-1)(n-2)} + \frac{1}{(n-1)(n-3)} + \cdots + \frac{1}{(2)(n-1)}\right) - \left(\frac{1}{(n-1)(n-2)(n-3)} + \frac{1}{(n-1)(n-2)(n-4)} + \cdots + \frac{1}{(n-1)(n-2)2}\right) + \cdots\right)$$

$$\iota(n) = \frac{1}{n} \times \left(1 - \iota(2) - \iota(3) - \cdots - \iota(n-1)\right)$$

Proof 3.3. Definition 3.2.

Since set of influenced multiples D of n less than x are not divisible by $a < n$, then $D = multiples\ of\ n - divisibles\ by\ a$. So for each point $a_1 \times a_2 \times a_3 \ldots \times n \times b_1 \times b_2 \times b_3$ is subtracted by all possible subsets of $x \times LCM(a_1, a_2, \ldots, n)^{-1}$ times b_n that can be obtained by using combinations. $D = multiples\ of\ n - \binom{n-2}{1} + \binom{n-2}{2} - \binom{n-2}{3} + \binom{n-2}{7} - \cdots \pm \binom{n-2}{n-2} = multiples\ of\ n + (-1)(b_n)$, where $\binom{n}{r}$ denote for all possible subsets of LCM, and b_n for divisors bigger than n. This is also a proof for definition 3.1.

Proof 3.4. Properties 3.2.

a. **Property a.** $\iota(x, n) = 0$, if $n > x$. For any $a < n$, $LCM(a, n) > x$, then $\left\lfloor \frac{x}{LCM(a,n)} \right\rfloor = 0$

b. **Property b.** $\iota(x, n) = 1$, if $x = n$. For any $a < n$, $LCM(a, n) > x$, then $\left\lfloor \frac{x}{LCM(a_1, a_2, \ldots, n)} \right\rfloor = 0$. $\iota(x, n) = \frac{x}{n} = 1$

c. **Property d.** $\iota(x, n) = \iota(n) \times x$, if $x = a \times n!$. $\iota(n)$ holds true for $n!$, since $n!$ Is the end of one cycle and then the cycle will start again. So for every complete cycle this statement holds true where a denote to the number of complete cycles,

4. Composites and Primes less than x

Theorem 4.1. *Let's define $c(x)$ as a count of composites less than x, then*

$$c(x) = \sum_{n=2}^{\lfloor\sqrt{x}\rfloor} \big(\iota(x,n) - \iota(n,n)\big)$$

Example 4.1.

a. $c(100) = \sum_{n=2}^{10}\big(\iota(100,n) - \iota(n,n)\big) = \iota(100,2) + \iota(100,3) + \iota(100,4) + \iota(100,5) + \iota(100,6) + \iota(100,7) + \iota(100,8) + \iota(100,9) + \iota(100,10) - \big(\iota(2,2) + \iota(3,3) + \iota(4,4) + \iota(5,5) + \iota(6,6) + \iota(7,7) + \iota(8,8) + \iota(9,9) + \iota(10,10)\big) = 50 + 17 + 0 + 7 + 0 + 4 + 0 + 0 + 0 - (1 + 1 + 0 + 1 + 0 + 1 + 0 + 0 + 0) = 78 - 4 = 74$

b. $c(1000) = \sum_{n=2}^{31}\big(\iota(1000,n) - \iota(n,n)\big) = \iota(1000,2) + \iota(1000,3) + \iota(1000,5) + \iota(1000,7) + \iota(1000,11) + \iota(1000,13) + \iota(1000,17) + \iota(1000,19) + \iota(1000,23) + \iota(1000,29) + \iota(1000,31) - \big(\iota(2,2) + \iota(3,3) + \iota(5,5) + \iota(7,7) + \iota(11,11) + \iota(13,13) + \iota(17,17) + \iota(19,19) + \iota(23,23) + \iota(29,29) + \iota(31,31)\big) = 500 + 167 + 67 + 38 + 21 + 17 + 11 + 9 + 7 + 3 + 2 - 11(1) = 842 - 11 = 831$

Theorem 4.2. *Let's denote $\pi(x)$ to the count of primes less than x, then*

$$\pi(x) = x - (\,c(x) + 1) = x - \left(\sum_{n=2}^{\lfloor\sqrt{x}\rfloor} [\iota(x,n) - \iota(n,n)] + 1\right)$$

Example 4.2.

a. $\pi(100) = 100 - (\,c(100) + 1) = 100 - (74 + 1) = 25$
b. $\pi(1000) = 1000 - (\,c(1000) + 1) = 1000 - (831 + 1) = 168$

At the end, I thank everyone that has read my paper. I welcome all of your notices on my first paper sent to my Email below.

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