Packaged entanglement states and particle teleportation

Rongchao Ma
Technical Institute of Physics and Chemistry, Chinese Academy of Sciences, Beijing 100190, China[*]

The entanglement states of particles are now widely used in quantum communication. However, these entanglement states usually relate to only one of the particles' physical quantities. Here we theoretically show that there exist packaged entanglement states which encapsulate all the necessary physical quantities for completely identifying the particles. We first show that a particle-antiparticle pair can form the packaged entanglement states in which the particles are indeterminate. Thereafter, we gave a possible experimental scheme for testing the packaged entanglement state. Finally, we proposed a protocol for teleporting a particle to an arbitrarily large distance using the packaged entanglement states. These packaged entanglement states could be important for particle physics and useful in matter teleportation, medicine, remote control, and energy transfer.

PACS numbers: 03.67.Ac, 03.67.Hk, 11.30.Er

I. INTRODUCTION

An entanglement state usually refers to the quantum state of a composite system which cannot be expressed as the direct product of the quantum states of its subsystems.[1–3] A common character of the entanglement states[4] is that a measurement on one of the particles will immediately change the state of other particles in the system via the "spooky action at a distance"[5] no matter how far these particles are spatially separated. The applications of the entanglement states in quantum communication[6] are now realized with photos[7,8], electrons[9–11], atoms[12,13], ions[14–16], and superconductors circuits[17,18]. These entanglement states usually relate to one[7–18] or two[19] of the particle's physical quantities, such as spin, or polarization (of photons).

However, one may ask whether the particles can form a special entanglement state that packages in multiple physical quantities capable of completely identifying the particles? We shall call such a state as a "packaged entanglement state" with which one can teleport the entire particles instead of just one of its physical properties[20–22]. The packaged entanglement state may enable matter teleportation which is well described in science fiction[23]. In the matter teleportation process, every intrinsic property of the particles needs to be transmitted for the purpose of rematerialization. Therefore, the packaged entanglement state could be a promising candidate for accomplishing the mission.

On the other hand, it is known that many physical quantities (or freedoms) of the particles usually are independent, such as charge, baryon number, lepton number etc.[24,25] But these physical quantities may be entangled together in a packaged entanglement state. Thus, it is highly desirable to carry out an in-depth investigation on the packed entanglement state and therefore answer at least the following questions: What interesting properties does this packaged entanglement state have? How to test it with experiments? Are there any other important applications for it?

In this paper, we first constructed the mathematical expression for the packaged entanglement states using the charge conjugation operator and discussed their physical properties in detail. Next, we proposed a possible experimental method for testing whether the particle-antiparticle pairs are in the packaged entanglement states. Thirdly, we discussed how to teleport a particle to a place at a distance using the packaged entanglement states. Finally, we discussed the possible applications of the particle teleportation process in medicine, remote control, and targeted energy transfer.

II. PACKAGED ENTANGLEMENT STATES OF A PARTICLE-ANTIPARTICLE PAIR

For simplicity, we will choose a particle-antiparticle pair to study the packaged entanglement states and their applications. First, let us discuss how to construct the mathematical expression of packaged entanglement states of the particle-antiparticle pair (with a zero total charge). This can be achieved by referring to charge conjugation.

Let $|P\rangle$ denote the particle's quantum state and $|\bar{P}\rangle$ denote the antiparticle's quantum state. It is known that $|P\rangle$ and $|\bar{P}\rangle$ are symmetrical in the sense of charge, i.e., the charge of $|P\rangle$ and $|\bar{P}\rangle$ are equal in quantity but with opposite signs. For this reason, we shall call the charge as the particle's gender. From particle physics[24–26] we know that $|P\rangle$ and $|\bar{P}\rangle$ can be interchanged by the charge conjugation operator C, i.e., $C|P\rangle = |\bar{P}\rangle$ and $C|\bar{P}\rangle = |P\rangle$. This mutual transformation indicates that $|P\rangle$ and $|\bar{P}\rangle$ could be mixed together in the sense of wave function under certain conditions.

Although the name "charge conjugation" is used, it not only reverses the sign of the particle's electric charge (Q), but also reverses the sign of all other internal quantum numbers[24], i.e., baryon number (B), lepton number (L), isospin (I_3), charm (C), strangeness (S), topness (T), and bottomness (B'). All these internal quantum numbers are packaged together under the charge conjugation. It should be also mentioned that[24] the charge conjugation does not change the sign of mass, energy, momentum, and spin.

It is known that not all particle (antiparticle) states, but only those neutral systems (with zero total charge) are the eigenstates of the charge conjugation operator C, such as γ, π^0, and a "particle-antiparticle" pair etc. It can be shown that[24], applying C to the bound state of a "particle-antiparticle" pair $|P\rangle\,|\bar{P}\rangle$, one has

$$C(|P\rangle\,|\bar{P}\rangle) = (-1)^J\,|P\rangle\,|\bar{P}\rangle\,. \tag{1}$$

where $J = L+S$ is the total angular momentum quantum number, L is the orbital angular momentum quantum number, and S is the total spin quantum number. For bosons $|b\rangle$, S is an even number and Eq.(1) reduces to $C(|b\rangle\,|\bar{b}\rangle) = (-1)^L\,|b\rangle\,|\bar{b}\rangle$.

Similar to Eq.(1), one can also show that the reversed configuration $|\bar{P}\rangle\,|P\rangle$ is an eigenstate of C. Mathematically, the linear combination (superposition) of $|P\rangle\,|\bar{P}\rangle$ and $|\bar{P}\rangle\,|P\rangle$ are also the eigenstates of C. For a "particle-antiparticle" pair A and B, therefore, one can construct the following four eigenstates of C, i.e.,

$$\begin{aligned}
|\Phi^+\rangle_{AB} &= |P\rangle_A\,|\bar{P}\rangle_B\,, && \text{(2a)}\\
|\Phi^-\rangle_{AB} &= |\bar{P}\rangle_A\,|P\rangle_B\,, && \text{(2b)}\\
|\Psi^+\rangle_{AB} &= \frac{1}{\sqrt{2}}\left[|P\rangle_A\,|\bar{P}\rangle_B + (-1)^J\,|\bar{P}\rangle_A\,|P\rangle_B\right], && \text{(2c)}\\
|\Psi^-\rangle_{AB} &= \frac{1}{\sqrt{2}}\left[|P\rangle_A\,|\bar{P}\rangle_B - (-1)^J\,|\bar{P}\rangle_A\,|P\rangle_B\right] && \text{(2d)}
\end{aligned}$$

The eigenstates $|\Phi^\pm\rangle_{AB}$ are separable states. A fundamental character of the separable states is that each particle in these states is determinate. But the eigenstates $|\Psi^\pm\rangle_{AB}$ are entanglement states because they cannot be expressed as the direct product of the particle state and antiparticle state.[1,2] A fundamental character of the entanglement states is that each particle in these states is indeterminate. In other words, each particle in the entanglement states is partially a particle and partially an antiparticle. Therefore, one cannot tell which one is a particle and which one is an antiparticle. When performing a measurement on A, it will collapse into either a particle, or an antiparticle. If A collapse into a particle, then B will collapse into an antiparticle. If A collapse into an antiparticle, then B will collapse into a particle.

On the other hand, one can easily show that[27] $C = C^\dagger$. This means that C is a Hermitian operator and is therefore an observable physical quantity. As the eigenstates of C, therefore, the entanglement states $|\Psi^\pm\rangle_{AB}$ must exist.

As mentioned before, the charge conjugation packages in a number of quantum numbers $(Q, B, L, I_3, C, S, T, B')$. Therefore, the entanglement states $|\Psi^\pm\rangle_{AB}$ should involve all these quantum numbers. Due to this reason, we call the quantum states $|\Psi^\pm\rangle_{AB}$ as packaged entanglement states. The physical quantities packaged in by the charge conjugation should entangle together in the packaged entanglement states. This feature can be embodied by rewriting Eq.(2c) and Eq.(2d), i.e.,

$$\begin{aligned}
|\Psi^\pm\rangle_{AB} =& \frac{1}{\sqrt{2}}\left[|Q,B,L,I_3,C,S,T,B'\rangle_A\,|-Q,-B,-L,-I_3,-C,-S,-T,-B'\rangle_B\right.\\
&\left.\pm(-1)^J\,|-Q,-B,-L,-I_3,-C,-S,-T,-B'\rangle_A\,|Q,B,L,I_3,C,S,T,B'\rangle_B\right].
\end{aligned} \tag{3}$$

Eq.(3) shows that the packaged entanglement states $|\Psi^\pm\rangle_{AB}$ cannot be written as the product of the sub quantum states related to the individual quantum numbers (freedoms). On the other hand, there are also other physical quantities, i.e., mass, energy, momentum, and spin, which are untouched by the charge conjugation. Thus, the sub quantum states related to these physical quantities should be factored out.

Finally, we would like to mention that the packaged entanglement states $|\Psi^\pm\rangle_{AB}$ are valid for both elementary particles and composite particles by referring to the charge conjugation. For composite particles, $|\Psi^\pm\rangle_{AB}$ can be further expressed by their component particles. For example, a proton is a composite particle made of three quarks[24,25], i.e., $|p\rangle = |uud\rangle$, where u is up quark and d is down quark. Thus, the packaged entanglement states of a $p-\bar{p}$ pair can be written as

$$\begin{aligned}
|\Psi^\pm\rangle_{AB} &= \frac{1}{\sqrt{2}}\left[|p\rangle_A\,|\bar{p}\rangle_B \pm (-1)^J\,|\bar{p}\rangle_A\,|p\rangle_B\right]\\
&= \frac{1}{\sqrt{2}}\left[|uud\rangle_A\,|\bar{u}\bar{u}\bar{d}\rangle_B \pm (-1)^J\,|\bar{u}\bar{u}\bar{d}\rangle_A\,|uud\rangle_B\right]
\end{aligned} \tag{4}$$

III. POSSIBLE VERIFICATION SCHEME

Although the experimental method for creating particle-antiparticle pairs in the packaged entanglement states $|\Psi^\pm\rangle_{AB}$ are unavailable at present, we can still use the following experimental scheme to test whether a particle-antiparticle pair is in one of the packaged entanglement states.

Eq.(2c) and Eq.(2d) show that each particle (A or B) in the packaged entanglement states is a mixture of a particle and an antiparticle. When A (or B) encounters an external particle X (from a particle source), the particle-antiparticle annihilation phenomenon[28,29] will force A (or B) to collapse into a particle conjugating to particle X (with every internal quantum number of A opposite to that of X) and then annihilate each other. More specifically, if X is a particle, then A will collapse into an antiparticle; if X is an antiparticle, then A will collapse into a particle. This indicates that each particle in a packaged entanglement state can annihilate with both a particle and an antiparticle. This property could be used to test the existence of the packaged entanglement states.[30]

Suppose that Alice has invented an experimental method[1] for creating particle-antiparticle pairs in the packaged entanglement states $|\Psi^{\pm}\rangle_{AB}$. Now Alice use this method to create a particle-antiparticle pair and then send one of the particles (particle A) to annihilate with an external similar particle X (see Fig. 1). Repeating this procedure for a number of times, Alice then starts to calculate the particle annihilating rate.

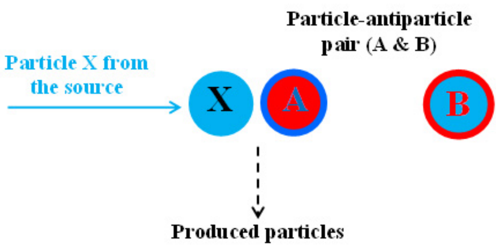

FIG. 1: (Color online) Schematic diagram for testing the existence of packaged entanglement states of particle-antiparticle pairs. If the annihilation rate is 1, then the pairs are in packaged entanglement states. If the annihilation rate is 0.5, then the pairs are in separable states.

(a) If the particle-antiparticle pairs are in one of the packaged entanglement states $|\Psi^{\pm}\rangle_{AB}$, then the annihilation rate would be 1. This is because each particle in the pairs are indeterminate and they can certainly annihilate with the external particles X (it does not matter whether X are particles or antiparticles).

(b) If the particle-antiparticle pairs are in one of the separable states $|\Phi^{\pm}\rangle_{AB}$, then the annihilation rate would be 0.5. This is because each particle in the pairs are determinate and the probability for A to be particles or antiparticles is 0.5, respectively. Thus, only half of the particles A can annihilate with the external particles X.

Finally, Alice can tell whether the particle-antiparticle pairs are in the packaged entanglement states $|\Psi^{\pm}\rangle_{AB}$ by checking the annihilation rate.

IV. PARTICLE TELEPORTATION PROTOCOL

We have constructed the mathematical expression for the packaged entanglement states. We shall now discuss the possible particle teleportation protocol using the packaged entanglement states and particle-antiparticle annihilation phenomenon (see Fig. 2). This is the foundation for matter teleportation. Here the particle teleportation does not mean to transmit a particle to the receiver (from Alice to Bob), but only transmit the packaged quantum information carried by the particle to the receiver.[23] For the convenience of discussion, the particle teleportation protocol is divided into 5 steps.

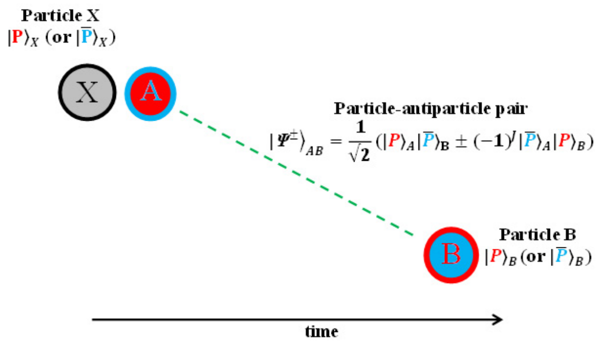

FIG. 2: (Color online) Schematic diagram for particle teleportation using the packaged entanglement states of a particle-antiparticle pair, $|\Psi^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}}\left[|P\rangle_A|\bar{P}\rangle_B \pm (-1)^J|\bar{P}\rangle_A|P\rangle_B\right]$, and particle-antiparticle annihilation phenomenon.

(1) Encoding. Consider that Alice has a particle X (or a sequence of particles) want to teleport to Bob. X can be either a particle or an antiparticle. Without losing generality, let us write out the quantum state of X as

$$|\phi\rangle_X = \alpha\,|P\rangle_X + \beta\,|\bar{P}\rangle_X\,. \tag{5}$$

If X is a particle, then we have $\alpha = 1$ and $\beta = 0$. If X is an antiparticle, then we have $\alpha = 0$ and $\beta = 1$.

(2) Quantum channel creation. To send the information stored on particle X to Bob, Alice needs a quantum channel, i.e., a particle-antiparticle pair in the packaged entanglement states $|\Psi^{\pm}\rangle_{AB}$. Let us first choose $|\Psi^{-}\rangle_{AB}$ (see Eq.(2d)) to carry out the calculation.

After the particle-antiparticle pair is created in the packaged entanglement state $|\Psi^{-}\rangle_{AB}$, one of them (particle A) is sent to Alice and another (particle B) is sent to Bob. Before Alice carry out any further operation, the three particles' complete state, $|\Psi^{-}\rangle_{XAB} = |\phi\rangle_X\,|\Psi^{-}\rangle_{AB}$, is

$$\left|\Psi^{-}\right\rangle_{XAB}=\frac{\alpha}{\sqrt{2}}\left[\left|P\right\rangle_{X}\left|P\right\rangle_{A}\left|\bar{P}\right\rangle_{B}-(-1)^{J}\left|P\right\rangle_{X}\left|\bar{P}\right\rangle_{A}\left|P\right\rangle_{B}\right]+\frac{\beta}{\sqrt{2}}\left[\left|\bar{P}\right\rangle_{X}\left|P\right\rangle_{A}\left|\bar{P}\right\rangle_{B}-(-1)^{J}\left|\bar{P}\right\rangle_{X}\left|\bar{P}\right\rangle_{A}\left|P\right\rangle_{B}\right]. \quad (6)$$

(3) Sending. With the quantum channel, Alice can send out her information stored on particle X by letting particle X to annihilate with particle A (dematerialization in matter teleportation). Due to the particle-antiparticle annihilation phenomenon, A will collapse into a particle conjugating to X and then they will annihilate each other. Thus, the $\left|\Psi^{-}\right\rangle_{XAB}$ in Eq.(6) will collapse into a state $\left|\Psi^{-}\right\rangle'_{XAB}$ which only has terms $\left|P\right\rangle_{X}\left|\bar{P}\right\rangle_{A}$ and $\left|\bar{P}\right\rangle_{X}\left|P\right\rangle_{A}$, i.e.,

$$\begin{aligned}&\left|\Psi^{-}\right\rangle'_{XAB}\\&=-(-1)^{J}\alpha\left(\left|P\right\rangle_{X}\left|\bar{P}\right\rangle_{A}\right)\left|P\right\rangle_{B}+\beta\left(\left|\bar{P}\right\rangle_{X}\left|P\right\rangle_{A}\right)\left|\bar{P}\right\rangle_{B}\\&=(-1)^{J+1}\alpha\left|P\bar{P}\right\rangle_{XA}\left|P\right\rangle_{B}+\beta\left|\bar{P}P\right\rangle_{XA}\left|\bar{P}\right\rangle_{B}.\end{aligned} \quad (7)$$

where $\left|P\bar{P}\right\rangle_{XA}$ and $\left|\bar{P}P\right\rangle_{XA}$ are the particles produced by the $\left|P\right\rangle_{X}\left|\bar{P}\right\rangle_{A}$ and $\left|\bar{P}\right\rangle_{X}\left|P\right\rangle_{A}$ annihilation[28], respectively.

(4) Receiving. Bob can receive the packaged information of particle X because Bob's particle B becomes related to X after Alice sent out her information by annihilating X with A. More specifically, if X is a particle, i.e., $\left|\phi\right\rangle_{X}=\left|P\right\rangle_{X}$ (see Eq.(5)), then Eq.(7) becomes

$$\left|\Psi^{-}\right\rangle'_{XAB}=(-1)^{J+1}\left|P\bar{P}\right\rangle_{XA}\left|P\right\rangle_{B}, \quad (8)$$

and B becomes a particle identical to X (with a sign $(-1)^{J+1}$); if X is an antiparticle, i.e., $\left|\phi\right\rangle_{X}=\left|\bar{P}\right\rangle_{X}$, then Eq.(7) becomes

$$\left|\Psi^{-}\right\rangle'_{XAB}=\left|\bar{P}P\right\rangle_{XA}\left|\bar{P}\right\rangle_{B}, \quad (9)$$

and B becomes an antiparticle identical to X.

(5) Decoding. After received Alice's packaged information, Bob needs to decode it (rematerialization in matter teleportation). Eq.(8) and Eq.(9) show that Bob's particle B is identical to particle X. This means that Bob can successfully decode the packaged information sent to him by Alice (carried by X).

Similarly, one can repeat the above particle teleportation process using the packaged entanglement state $\left|\Psi^{+}\right\rangle_{AB}$ (see Eq.(2c)). Therefore, Eq.(7) becomes

$$\left|\Psi^{+}\right\rangle'_{XAB}=(-1)^{J}\alpha\left|P\bar{P}\right\rangle_{XA}\left|P\right\rangle_{B}+\beta\left|\bar{P}P\right\rangle_{XA}\left|\bar{P}\right\rangle_{B}. \quad (10)$$

If $\left|\phi\right\rangle_{X}=\left|P\right\rangle_{X}$, then $\left|\Psi^{+}\right\rangle'_{XAB}=(-1)^{J}\left|P\bar{P}\right\rangle_{XA}\left|P\right\rangle_{B}$. If $\left|\phi\right\rangle_{X}=\left|\bar{P}\right\rangle_{X}$, then $\left|\Psi^{+}\right\rangle'_{XAB}=\left|\bar{P}P\right\rangle_{XA}\left|\bar{P}\right\rangle_{B}$.

V. DISCUSSION

In early quantum teleportation protocols[6,31], Alice sends out the information of particle X by performing a Bell measurement on particles X and A. This measurement has four possible results. Thereafter, Alice needs a classical channel to inform Bob about her Bell measurement result. In the present particle teleportation protocol, however, Alice sent out the information of X by annihilating X with A. Alice's experimental result is fixed by the particle-antiparticle annihilation phenomenon. Thus, Bob's result has a fixed relationship with that of X and he can decode the information directly. The classical channel between Alice and Bob is then removed.

In the packaged entanglement states, every additive quantum number is symmetrical (with opposite sign). Thus, the particle teleportation process satisfies a number of conservation principles, i.e., charge (Q) conservation, baryon number (B) conservation, lepton number (L) conservation, isospin (I_3) conservation, charm (C) conservation, strangeness (S) conservation, topness (T) conservation, and bottomness (B') conservation. In addition, it also satisfies the principles of linear momentum conservation, total energy conservation, and angular momentum conservation.

In the particle teleportation process, Alice can control particle B to be a particle or an antiparticle at a distance. In other words, Alice can control the gender of Bob's particle B at a distance. This means that Alice can control particle B whether to annihilate or not to annihilate with its environment particles at will. This property may be found important applications in medicine, such as positron emission tomography (PET)[32], positron annihilation spectroscopy (PAS), and the band structure measurements in solid state physics as well. It could also be applied in remote control. Furthermore, the particle-antiparticle annihilation process can release a large amount of energy. Thus, the particle teleportation process could be used to transport energy.

VI. CONCLUSION

The properties of packaged entanglement states and their applications in matter teleportation are studied. It is shown that the packaged entanglement states of a particle-antiparticle pair are also the eigenstates of charge conjugation operator. The species of particles in the packaged entanglement states are indeterminate. They are mixtures of particles and antiparticles. An experimental scheme for confirming the existence of packaged entanglement states based on the particle-antiparticle annihilation was proposed. A protocol for particle (matter) teleportation using the packaged entanglement states was also proposed. Different to early studies, the particle teleportation protocol introduced here does not need a classical channel due to the particle-antiparticle annihilation phenomenon.

---

* Electronic address: marongchao@yahoo.com

[1] R. Horodecki, P. Horodecki, M. Horodecki and K. Horodecki, Rev. Mod. Phys. 81, 865-942 (2009).
[2] Hoi-Kwong Lo, S. Popescu and T. Spiller (Editors), Introduction to Quantum Computation and Information (World Scientific, River-Edge, 1998).
[3] Michael A. Nielsen & Isaac L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, New York, 2010).
[4] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935).
[5] Letter from Einstein to Max Born, 3 March 1947, p158. The Born-Einstein Letters; Correspondence between Albert Einstein and Max and Hedwig Born from 1916 to 1955 (Macmillan Press Ltd., New York, 1971).
[6] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, W. K. Wootters, Phys. Rev. Lett. 70, 1895-1899 (1993).
[7] Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter & Anton Zeilinger, Nature 390, 575-579 (1997).
[8] D. Boschi, S. Branca, F. DeMartini, L. Hardy, & S. Popescu, Phys. Rev. Lett. 80, 1121-1125 (1998).
[9] Michael N. Leuenberger, Michael E. Flatte, and D. D. Awschalom, Phys. Rev. Lett. 94, 107401 (2005).
[10] W. Pfaff, B. J. Hensen, H. Bernien, S. B. van Dam, M. S. Blok, T. H. Taminiau, M. J. Tiggelman, R. N. Schouten, M. Markham, D. J. Twitchen, R. Hanson, Science 345, 532 (2014).
[11] R. S. Deacon, A. Oiwa, J. Sailer, S. Baba, Y. Kanai, K. Shibata, K. Hirakawa, S. Tarucha, Nature Communications 6, 7446 (2015).
[12] H. Krauter, D. Salart, C. A. Muschik, J. M. Petersen, Heng Shen, T. Fernholz & E. S. Polzik, Nature Physics 9, 400-404 (2013).
[13] Julian Hofmann, Michael Krug, Norbert Ortegel, Lea Gerard, Markus Weber, Wenjamin Rosenfeld, Harald Weinfurter, Science 337, 72-75 (2012).
[14] M. D. Barrett, J. Chiaverini, T. Schaetz, J. Britton, W. M. Itano, J. D. Jost, E. Knill, C. Langer, D. Leibfried, R. Ozeri, D. J. Wineland, Nature 429, 737-739 (2004).
[15] M. Riebe, H. Haffner, C. F. Roos, W. Hnsel, J. Benhelm, G. P. T. Lancaster, T. W. Karber, C. Becher, F. Schmidt-Kaler, D. F. V. James & R. Blatt, Nature 429, 734-737 (2004).
[16] S. Olmschenk, D. N. Matsukevich, P. Maunz, D. Hayes, L.-M. Duan, C. Monroe, Science 323, 486-489 (2009).
[17] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature 398, 786-788 (1999).
[18] M. Baur, A. Fedorov, L. Steffen, S. Filipp, M. P. da Silva, and A. Wallraff, Phys. Rev. Lett. 108, 040502 (2012).
[19] Xi-Lin Wang, Xin-Dong Cai, Zu-En Su, Ming-Cheng Chen, Dian Wu, Li Li, Nai-Le Liu, Chao-Yang Lu & Jian-Wei Pan, Nature 518, 516 (2015).
[20] T. Opatrny and G. Kurizki, Phys. Rev. Lett. 86, 3180 (2001).
[21] J. M. Torres, J. Z. Bernad, and G. Alber, Phys. Rev. A 90, 012304 (2014).
[22] Jacob F. Sherson, Hanna Krauter, Rasmus K. Olsson, Brian Julsgaard, Klemens Hammerer, Ignacio Cirac and Eugene S. Polzik, Nature 443, 557-560 (2006).
[23] Lawrence M. Krauss, The Physics of Star Trek (Flamingo, Reissue edition, 1995).
[24] D. J. Griffiths, Introduction to Elementary Particles (Wiley-VCH, 2nd ed., 2008).
[25] Donald H. Perkins, Introduction to High Energy Physics (Cambridge University Press, 4th Edition, 2000).
[26] M. E. Peskin, D. V. Schroeder, An introduction to quantum field theory (Addison-Wesley, 1995).
[27] Apply the charge conjugation operator C to the particle state $|P\rangle$ twice, we have $C^2 |P\rangle = C |\bar{P}\rangle = |P\rangle$. This gives $C^2 = 1$. On the other hand, the normalization condition $\langle P | C^\dagger C | P\rangle = \langle \bar{P} | \bar{P}\rangle = 1$ gives $C^\dagger C = 1$. Thus, we have $C = C^\dagger$.
[28] When a low-energy elementary particle-antiparticle pair annihilates, say an electron annihilates with a positron, they can only produce two or more photons. But when a composite particle-antiparticle pair annihilates, say a proton annihilates with an antiproton, they will produce multiple particles, such as photons, electrons, positrons, and neutrinos.
[29] Eberhard Klempt, Chris Batty, Jean-Marc Richard, Physics Reports 413 (45), 197317 (2005).
[30] Jon Magne Leinaas, Jan Myrheim, and Eirik Ovrum, Phys. Rev. A 74, 012313 (2006).
[31] W. Dur and J. I. Cirac, J. Mod. Opt. 47, 247 (2000).
[32] Gopal B. Saha, Basics of PET Imaging: Physics, Chemistry, and Regulations (Springer, 2nd ed., 2010)