General relativity and the other Solution

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ABSTRACT

In the general relativity theory, we find the other solution.For example, the other solution treats in Schwarzschild solution, Reissner-Nodstrom solution. Hence, the uniqueness of GR is denied by numberless solutions.

PACS Number:**04,04.90.+e**

Key words:General relativity theory,

The other solution,

Schwarzschild solution,

Reissner-Nodstrom solution

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1. Introduction

In the general relativity theory, our article's aim is that we find the other solution.

First, the gravity potential $g_{\mu\upsilon}$ is

$$ds^2 = g_{\mu\upsilon} dx^\mu dx^\upsilon \quad (1)$$

In gravity potential $g_{\mu\upsilon}$, we introduce tensor $f_{\mu\upsilon}$ and scalar K.

$$f_{\mu\upsilon} = K g_{\mu\upsilon} \ , \quad \frac{\partial K}{\partial x^\lambda} = 0$$

$$ds'^2 = f_{\mu\upsilon} dx^\mu dx^\upsilon = K g_{\mu\upsilon} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\upsilon}{\partial x'^\beta} dx'^\alpha \, dx'^\beta$$

$$= K g'_{\alpha\beta} \, dx'^\alpha \, dx'^\beta = f'_{\alpha\beta} \, dx'^\alpha \, dx'^\beta$$

$$g'_{\alpha\beta} = g_{\mu\upsilon} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\upsilon}{\partial x'^\beta} \ , \quad f'_{\alpha\beta} = f_{\mu\upsilon} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\upsilon}{\partial x'^\beta} \quad (2)$$

Inverse gravity potential $g^{\mu\upsilon}$,

$$f^{\mu\upsilon} f_{\mu\upsilon} = \delta_\mu{}^\upsilon = \left(\frac{1}{K} g^{\mu\upsilon}\right)(K g_{\mu\upsilon}), \ f^{\mu\upsilon} = \frac{1}{K} g^{\mu\upsilon} \quad (3)$$

In Christoffel symbol $\Gamma^\rho{}_{\mu\upsilon}$,

$$\Gamma'^\rho{}_{\mu\upsilon} = \frac{1}{2} f^{\rho\lambda} \left(\frac{\partial f_{\mu\lambda}}{\partial x^\upsilon} + \frac{\partial f_{\upsilon\lambda}}{\partial x^\mu} - \frac{\partial f_{\mu\upsilon}}{\partial x^\lambda}\right)$$

$$= \frac{1}{2}\left(\frac{1}{K} g^{\rho\lambda}\right)\left(K \frac{\partial g_{\mu\lambda}}{\partial x^\upsilon} + K \frac{\partial g_{\upsilon\lambda}}{\partial x^\mu} - K \frac{\partial g_{\mu\upsilon}}{\partial x^\lambda}\right) = \Gamma^\rho{}_{\mu\upsilon} \quad (4)$$

Therefore, in the curvature tensor $R^\rho{}_{\mu\upsilon\lambda}$,

$$R'^\rho{}_{\mu\upsilon\lambda} = \frac{\partial \Gamma'^\rho{}_{\mu\upsilon}}{\partial x^\lambda} - \frac{\partial \Gamma'^\rho{}_{\mu\lambda}}{\partial x^\upsilon} + \Gamma'^\sigma{}_{\mu\upsilon} \Gamma'^\rho{}_{\sigma\lambda} - \Gamma'^\sigma{}_{\mu\lambda} \Gamma'^\rho{}_{\sigma\upsilon}$$

$$= \frac{\partial \Gamma^\rho{}_{\mu\upsilon}}{\partial x^\lambda} - \frac{\partial \Gamma^\rho{}_{\mu\lambda}}{\partial x^\upsilon} + \Gamma^\sigma{}_{\mu\upsilon} \Gamma^\rho{}_{\sigma\lambda} - \Gamma^\sigma{}_{\mu\lambda} \Gamma^\rho{}_{\sigma\upsilon} = R^\rho{}_{\mu\upsilon\lambda} \quad (5)$$

In Ricci tensor $R_{\mu\upsilon}$,

$$R'_{\mu\upsilon} = R'^\rho{}_{\mu\rho\upsilon} = R^\rho{}_{\mu\rho\upsilon} = R_{\mu\upsilon} \quad (6)$$

In curvature scalar R

$$R' = f^{\mu\upsilon} R'_{\mu\upsilon} = \frac{1}{K} g^{\mu\upsilon} R_{\mu\upsilon} = \frac{1}{K} R \quad (7)$$

Hence, in the gravity field equation of Einstein,

$$R'_{\mu\upsilon}-\frac{1}{2}f_{\mu\upsilon}R'=R_{\mu\upsilon}-\frac{1}{2}Kg_{\mu\upsilon}(\frac{1}{K}R)$$

$$=R_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}R=-\frac{8\pi G}{c^4}T_{\mu\upsilon},$$

$$\text{Of course } T'_{\mu\upsilon}=T_{\mu\upsilon} \quad (8)$$

Therefore, tensor $f_{\mu\upsilon}$ satisfy the gravity field equation of Einstein.

If $T_{\mu\upsilon}=0$,

$$R'_{\mu\upsilon}-\frac{1}{2}f_{\mu\upsilon}R'=R_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}R=0$$

$$f^{\mu\upsilon}[R'_{\mu\upsilon}-\frac{1}{2}f_{\mu\upsilon}R']=Kg^{\mu\upsilon}[R_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}R]=0$$

$$\rightarrow\ -R'=-KR=0\ \rightarrow\quad R'_{\mu\upsilon}=R_{\mu\upsilon}=0 \quad (9)$$

2. The other solution in Schwarzschild solution, Reissner-Nodstrom solution

$$f_{\mu\upsilon}=Kg_{\mu\upsilon}\ ,\quad \frac{\partial K}{\partial x^{\lambda}}=0$$

$$ds^2=g_{\mu\upsilon}dx^{\mu}dx^{\upsilon},\ ds'^2=f_{\mu\upsilon}dx^{\mu}dx^{\upsilon} \quad (10)$$

For example, K is

$$K=1+\exp(-n\frac{hc}{GM^2})\ \text{ or }\ K=1+\exp(-m\frac{hc}{kQ^2})$$

$$n,\ m>0,\ n,\ m\ \text{is number} \quad (11)$$

Schwarzschild solution(vaccum solution) is

$$ds^2=-c^2(1-\frac{2GM}{rc^2})dt^2+\frac{dr^2}{1-\frac{2GM}{rc^2}}+r^2d\theta^2+r^2\sin^2\theta d\phi^2 \quad (12)$$

The new solution is

$$ds'^2=f_{\mu\upsilon}dx^{\mu}dx^{\upsilon}=Kg_{\mu\upsilon}dx^{\mu}dx^{\upsilon}=K\cdot ds^2$$

$$=[1+\exp(-n\frac{hc}{GM^2})]\,[-c^2(1-\frac{2GM}{rc^2})dt^2+\frac{dr^2}{1-\frac{2GM}{rc^2}}+r^2d\theta^2+r^2\sin^2\theta d\phi^2]$$

$$= -c^2(1-\frac{2GM}{rc^2})dt'^2 + \frac{dr'^2}{1-\frac{2GM}{rc^2}} + r'^2\, d\theta^2 + r'^2 \sin^2 \theta d\phi^2] \text{ , } r' = r[1+\exp(-n\frac{hc}{GM^2})]^{\frac{1}{2}}$$

(13)

Reissner-Nodstrom solution is

$$ds^2 = g_{\mu\upsilon}dx^\mu dx^\upsilon$$

$$= -c^2(1-\frac{2GM}{rc^2}+\frac{kGQ^2}{r^2c^4})dt^2 + \frac{dr^2}{1-\frac{2GM}{rc^2}+\frac{kGQ^2}{r^2c^4}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (14)$$

The new solution is

$$ds'^2 = f_{\mu\upsilon}dx^\mu dx^\upsilon = Kg_{\mu\upsilon}dx^\mu dx^\upsilon = K \cdot ds^2$$

$$= [1+\exp(-n\frac{hc}{GM^2})]\cdot[1+\exp(-m\frac{hc}{kQ^2})]\cdot[-c^2(1-\frac{2GM}{rc^2}+\frac{kGQ^2}{r^2c^4})dt^2 + \frac{dr^2}{1-\frac{2GM}{rc^2}+\frac{kGQ^2}{r^2c^4}}$$

$$+ r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \quad , \quad K = [1+\exp(-n\frac{hc}{GM^2})]\cdot[1+m\exp(-\frac{hc}{kQ^2})]$$

(15)

3. Conclusion

We find the other solution in the General relativity theory. Hence, the uniqueness of GR is denied by numberless solutions.

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