The relation between the ( hypothetical) intrinsic vibrational motion of particles and some of their fundamental properties.

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Abstract

The concept of an intrinsic vibrational-rotational motion of the electron( zitterbewegung) has been introduced by Schroedinger, and later developed by Kerson Huang and more recently by A. Barut, among others. These authors listed a series of features that should accompany such motion, although its very existence is usually regarded as merely speculative. In the present paper we investigate the consequences of the existence of this motion as far as measured properties of particles are concerned. A phenomenological model based upon the quantization of a classical vibrating system, in the lines of the old Correspondence Principle of Bohr, is applied to particles to mimic the effect of the zitterbewegung upon measurable dynamic properties like the magnetic energy, and the magnetic moment. Gauge invariance is inevitably imposed in the form of a quatization criterion needed for the passage from the classical to the quantum treatment, which results in the prediction of magnetic flux quantization within the area covered by the vibrations. The calculations are carried out for the electron, and also for the proton and the neutron by considering the electric charges of their constituent quarks. The conclusion is that if the zitterbewegung motion is real, the mass, the magnetic moment, the Compton wavelengths (or the measured sizes for the nucleons) for each of these particles, are gathered together in a single expression which is a function of the number of flux quanta trapped inside their "orbits". The treatment proposed is simple, self-contained, and quantitatively correct. We finish the paper making some remarks on the compatibility of these results with those obtained from first principles by QCD.

1. Introduction

The intrinsic vibrational-rotational motion of an electron was predicted by Schroedinger in 1930[1] as a consequence of solving the Dirac wave-equation including both positive and negative energy terms in the spectrum of states considered. Such subject was reassessed by Huang in 1952[2] in a more quantitative fashion for a wave packet, and again in 1981 by Barut and Bracken[3]. The properties that should accompany such motion according to these studies should be: 1) The permanent existence of a rotational motion of the electron with frequency $\omega = 2mc^2/\hbar$ and radius given by the Compton wavelength $\lambda = \hbar/(mc)$. 2) The association of the intrinsic magnetic moment of the electron $\mu_B = e\hbar/(2mc)$ with such a motion, as well as the existence of its spin angular momentum. 3) The rest energy of the motion should be $mc^2$ [3]—Such dramatic prediction in fact attributes the origin of free electrons to the mixture of positive and negative energy states from the vacuum reservoir, and implies that a situation of dynamic equilibrium should be established between what we call an electron and such vacuum background. In the introduction to his paper, Huang explains that observing the motion should be impossible since investigating deep inside the length-range of the Compton wavelength would create electron-positron pairs. The uncertainty principle also prohibits the direct observation of the isolated Bohr magneton of a single electron, so that we are left in a seemingly impossible position to actually check for the intrinsic motion in a direct experimental way. On the other hand, the magnitude of the effect has been deemed much smaller and short-lived for actual wave packets than predicted by Huang[4]. Recent work has argued that detailed field-theoretic reformulation of the treatment of the Dirac electron can be developed with no mention needed of hypothetical intrinsic vibrations[5].

What if the intrinsic motion of a charged particle is real, or otherwise ( as put in [3]) real because the corresponding formal description leads to measurable consequences? Are there other consequences of this apparently unaccessible and polemic effect that have not been included in previous work? Something that the present work is going to show is that indeed there is a very important feature of the periodic rotational-vibrational motion of a charged particle that should have been considered in so much previous discussion. In fact the inclusion of the magnetic flux associated with the hypothetical electron orbit provides a link between several of the quantities listed in the previous paragraph. The inclusion of such a quantity, indispensable from the point of view of sustaining the continuity of the particle wave function, in addition to giving results consistent with Huang´s and Barut´s conclusions, will permit the extension of this interpretation of the data to composite particles like the nucleons, as discussed in the following sections.

## 2. Gauge invariance in a charged-particle closed orbit.

The subject of gauge invariance was introduced by Weyl in his attempts of unification of Gravitational and Electromagnetic fields. Schroedinger in 1922[6] and later, London [7], carried on with the implementation of Weyl´s ideas by admitting that the action in a Hamilton-Jacobi differential equation should include the electromagnetic potentials( as proposed by Weyl himself), and dropping off the metric tensor of the original formalism. With the advent of quantum mechanics London ( after Born) realized that the phase of the wave function for an electron should be complex and would replace the length of a measurement rod introduced in the original conception of Weyl. Such phase would include an action term, and to keep gauge invariance of the whole theory the canonic momentum in the action should be supplemented by the magnetic vector-potential in the form $e\mathrm{A}/c$. It follows that the nonintegrability of the phase of the wavefunction representing a particle in periodic confined motion requires that the line integral of the magnetic vector-potential A around a closed circuit (i.e., the magnetic flux across the area of the circuit) be an integer $n$ number of $\Phi_0 = hc/e$, otherwise the wavefunction will not be kept single-valued. Such point is discussed in detail for a particle in confined motion in the book by Frenkel[8], and in its simpler form this results in the Bohr-Sommerfeld-Wilson quantization rules.

Although no special attention was given to this detail at the time, those authors had in this way established that the gauge invariance of quantum theory in the presence of electromagnetic fields leads to flux quantization. The gauge invariance of quantum theory is fundamental in the interpretation of the Aharonov-Bohm effect, for instance. It should also be mentioned on passing that recent work by Yilmaz et al.[9] indicates that the Dirac electron should perform an intrinsic orbit enclosing an amount of flux $\Phi_0/2 = hc/(2e)$.

3. A classical model of a vibrating object and its application to particles.

As far as we know, there have been no previous attempts in the literature to include flux quantization as one of the features associated with zitterbewegung. The proposed program to be followed hereafter is therefore focused on the inclusion of such feature in the analysis of that motion, with the objective of seeking new predictions that might be comparable with experiments.

The author has been involved in the study of a fully classical oscillating system, called the Superconducting Electromechanical Oscillator (SEO) ( Schilling [10]). The SEO comprises a superconducting rectangular loop which performs (forced) harmonic oscillations under the effect of magnetic fields and gravity. The SEO deserves further attention in this case for the following reasons: 1) It develops a time-oscillating magnetic moment similar to that predicted by Huang for the zitterbewegung, due to a supercurrent that flows around the loop; 2) Similar to what has been proposed by Huang and others for a stable electron in a vacuum, the SEO rests in a stable energy state, in an equilibrium condition with the surrounding fields( cf. [10]). If the Correspondence Principle is applied to the SEO, with the quantization of terms in its energy expression, one might hopefully get a limiting case comparable to an actual oscillating particle performing zitterbewegung. From the analytsis of the SEO one obtains that the system rests in a state of total energy[10]

$$E = \tfrac{1}{2}c \;\Phi\, i \qquad (1)$$

In this equation $i$ is the maximum current that flows around the loop and $\Phi$ the corresponding magnetic flux( in [10] the time-averaged quantities were adopted). We note that such expression of the energy is of the expected form for a closed-circuit magnetic system. With this result in hand it is possible to go straight to the application of the Correspondence Principle. One inserts into (1) the magnetic flux trapped inside the zitterbewegung orbit, which from the previous paragraph should be an integer number of flux quanta, $nhc/e$. The zitterbewegung expression for the current( after solution of Dirac´s equation) is simply $\omega e/(2\pi)$. If such expression is inserted into (1) one obtains an expected result for a quantum oscillating system, which is $\frac{1}{2}\, n\, \hbar\, \omega$, including the ground state value for a single flux quantum, $\frac{1}{2}\, \hbar\, \omega$ ( $n = 0$ is meaningless) . Such result indicates the procedure makes sense, but the objective is to reproduce experimental data for particles, and the frequency of the oscillations is only *predicted* by Huang and others as $2\gamma mc^2/\, \hbar$. By inserting this value for the frequency with $n = 1$ flux quantum we immediately obtain that $E= \gamma mc^2$, that is, we recover the identity between the magnetic-vibrational energy associated with a single flux quantum and the total energy of the particle, which is a conclusion of [3]( which includes a relativistic correction for mass, $\gamma \geq 1$).

Let´s return to equation (1). Rather than inserting in it the zitterbewegung theoretical frequency and current, one should use an independent estimate for this current. Let´s utilize the usual definition of the magnetic moment, which is valid independently of the mathematical formalism:

$$\mu = i\,(\pi R^2)/c \tag{2}$$

In equation (2) a classical picture ( consistent with Huang´s own vision) may be considered in which the magnetic moment $\mu$ of the particle is given by an effective loop current $i$ times the area of the loop of radius $R$. Independently of the calculations of the zitterbewegung we know $\mu$ is the Bohr magneton $e\hbar/(2\gamma mc)$, with $\gamma \geq 1$ the relativistic correction for mass. We know also that QED calculations estimate the size of the extension of vacuum polarization( which might represent $R$) due to the electron charge as given by a corrected Compton wavelength $\lambda = \hbar\,/(\gamma mc)$. In a purely formal interpretation one might discard the classical picture proposed and simply consider $\omega$ as the rate of transition between the positive and negative energy states involved in the formation of a free electron state. The freed electron should therefore be in a mixture of the two states for virtual particles within its range $\lambda$. Inserting these expressions in equation (2) and then back into equation (1) one obtains $E = ½\, \gamma mc^2$, which again identifies the total energy with the trapped magnetic energy, within a factor of two. It seems that the relativistic correction in this particular case should already be an intrinsic part of what is considered the observed rest energy, and will be dropped hereafter.

The conclusion is that the model, which attributes the rest energy of an electron to magnetic energy confined in a vibrating-rotating charged loop containing one magnetic flux quantum is quantitatively consistent, at least within a factor of two. This result is in agreement with the calculations of Barut and Bracken, and Huang for zitterbewegung.

4.Extension of the model to nucleons.

The calculations carried out in the previous section should be valid for any charged fundamental particles. In order to analyze the specific cases of the proton and the neutron we will take the simple assumption that they are formed by the combination of three fundamental particles, named up- and down- quarks, with fractionary charges of $2e/3$ and $-e/3$, respectively.

First of all, let´s combine equations (1) and (2), taking account of the quantization of flux, and then make the obtained expression for $E$ equal to $mc^2$, as indicated by the foregoing results. Measurable properties for particles are therefore gathered in the formula

$$mR^2/\mu = nh/(2\pi ec) \tag{3}$$

4a. Application of the model to the proton

In equation (3) all terms are known for the proton and neutron, with the exception of $n$. The rest masses, and the magnetic moments are known, whereas the radius of the proton has recently been determined as 0.84 fm( Antognini et al. [11]).

Figure 1 is the reproduction of the theoretical transverse planar charge density profiles in the proton and neutron( Miller [12]), which indicate a wide charge distribution with its “tail” at about 1.5 fm. Such *planar* distributions are useful in the present case since a loop of current is being considered. In the analysis below we will consider the parameter $R$ as given by the *averaged size* for these 2DIM distributions taken from the plots, which is 0.6 fm (what Miller calls $R^*$, see [12]).

In the case of the proton the left side of (3) requires $m = 1.67\mathrm{x}10^{-24}$ g, $\mu=1.41\mathrm{x}10^{-23}$ erg/Oe, and we take $R= 0.6\ \mathrm{x}10^{-13}$ cm. Replacing the constants on the right side of (3) one obtains $n$=5.8 which is very close to 6.

Considering that the conditions of gauge invariance *must be valid also for wave functions representing the quarks*, we conclude that due to its fractionary charge the up-quark orbit would contain 3/2 $\Phi_0$. For the down-quark would correspond 3 $\Phi_0$. It is not clear how these numbers obtained for the individual quarks would combine. One possibility is that they might follow the rules for angular momentum vectors addition which includes the use of Clebsch-Gordan coefficients in a weighted average that would produce spin-1/2 nucleons from their combinations. Another possibility[13] is that they are just summed as scalar numbers, taking only account of the sign of their charges. In a nucleon the two similar quarks combine in a triplet S=1 state( in a pictorial view, they would turn in the same direction), so that it is the oppositely charged quark the one whch will turn in the opposite direction. The result is that the absolute number of flux quanta should simply be summed for the three quarks( two up and one down), giving 6 flux quanta. This exactly agrees with the $n$ derived from the application of (3) to the data for the proton.

4b. Application of the model to the neutron.

In the case of the neutron the left side of (3) requires $m = 1.67\mathrm{x}10^{-24}$ g, $\mu=0.966\mathrm{x}10^{-23}$ erg/Oe, and we take $R= 0.6\ \mathrm{x}10^{-13}$ cm. Replacing the constants on the right side of (3) one obtains $n$=8.5. In view of the half integer number of flux quanta for the up quark, fractionary values for $n$ might be expected. Repeating the procedure and arguments of the previous paragraph [13] for the( two down and one up) quarks combination in a neutron one obtains $n$= 7.5.

5.Analysis and Conclusions.

The set of results indicates the following. If the intrinsic vibration-rotation motion of fundamental particles exists in nature, such motion must trap quantized amounts of magnetic flux, which is required by gauge invariance of the theory. The values of the flux depend on the actual charge of the particle. We have shown that if these particles are simply considered as loops of current( a useful image, as noted earlier), the corresponding magnetodynamic trapped energy matches their rest energies to a precision of a factor of at most 2. This seems to be valid for fundamental as well as to composite particles. In particular, the accurate results for the nucleons are rather unexpected in view of the literature on the origin of mass for the baryons which relies on QCD calculations [14]. One should notice however that the present phenomenological treatment *by no means* replaces QCD calculations for nucleons, which will actually determine the range $R$ from first principles, and the properties of quarks inside the nucleons(see [12], for instance). However, the possibility of quantitatively analysing the electron and the nucleons within the same theoretical model, albeit simple, is indeed an interesting result. The justification is certainly related to the general application of the principle of gauge invariance both in the present treatment and in gauge theories like QCD, which results in the present case in magnetic flux quantization for the electrons and also for the quarks inside the nucleons. One must remember that ( the equivalent of) flux quantization within the strong-interaction formalism of SU(3) is a quite old concept, which has become a center piece of QCD. For instance, just to mention one of several seminal papers, Mandelstam used the concept in the 1970s to justify quarks confinement[15] in SU(3), which has been followed by a stream of publications on the subject up to this day[16]. It is interesting that in that early paper Mandelstam comments that his conclusions should be independent of quarks colors, which essentially reduces the problem of confinement to its topological aspects. Although the theory has evolved considerably ever since[16], this seems to justify the good results of  the present phenomenological model, but the demonstration ( if possible!) lies beyond the scope of this treatment.

In summary, if the zitterbewegung motion is real, the rest mass, the sizes, the magnetic moments, are all tied together through gauge invariance of the theory, which imposes flux quantization inside the orbit.  This is valid for all particles tested. This is this works conclusion.

References:


1. E. Schroedinger, Sitzber. Preuss. Akad. Wiss., physik-math. Kl. **24** (1930) 418

2. K. Huang, Am.J.Phys. **20**(1952) 479.

3. A.O.Barut and J.A.Bracken , Phys Rev D**23**(1981) 2454.

4. J.A. Lock, Am.J.Phys. **47**(1979)797

5. D. Sepunaru, HAIT J. Sci. Eng**, 1**(2004)411

6. E. Schroedinger, Z. Phys. **12** (1922), 13 .

7. F. London, Z. Phys. **42**(1927), 375

8. J. Frenkel, Wave Mechanics: Advanced General Theory, Dover, New York(1950).

9. O. Yilmaz et al., Concepts in Physics, **IV**(2007),141.

10. O.F.Schilling, J.Electromag. Anal.Appl, **5**(2013)91.

11. A. Antognini et al., Science, **339**(2013), 417.

12. G.A.Miller, Phys. Rev. Lett**. 99**(2007),112001 ( see corrections in arxiv 0705.2409, and the definition of *R* in 0908.1535 v1).

13. K. Moriyasu, Am.J.Phys. 49(1981), 819

14. F.Wilczek, Origins of mass, arxiv: 1206.7114.

15. S. Mandelstam, Phys.Lett.**53B**(1975), 476.

16. For a recent review on the subject see K.Konishi in M.Gasperini and J. Maharana , editors, String Theory and Fundamental Interactions, Lecture Notes on Physics vol. 737( pp. 471 and ff.), Springer, Berlin(2008).

Figure and Figure Caption

Figure 1: Theoretical transverse charge densities for the nucleons( from [12]).