New sound in the superfluid helium

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Abstract

The friction sound is considered to be involved in the von Kármán vortex street. The non-relativistic and relativistic Strouhal numbers are derived from von Kármán's vortex street. The relativistic result follows from the relativistic addition formula for velocities. The friction tones generated by von Kármán's vortex street form the fifth sound in the liquid helium II. This sound was still not experimentally observed in superfluid helium II and it means that this sound is here predicted as the crucial step in the low temperature helium II physics of the low temperature laboratories.

The electron transport in graphene is supposed to be described by the hydrodynamic form of the viscous liquid allowing the existence of the vortex street. It is not excluded that the discovering of the vortex street in graphene can form one of the crucial discoveries in the graphene physics.

By analogy with helium II, we propose that photon is a quantum vortex, or, the Onsager vortexon.

1 Introduction

It is well known that there are four sounds in liquid superfluid helium II (He II). **The first sound** in helium II is caused by the pressure variations, where the supefluid and other normal components of helium II are in the concord motion. In other words, the sound is based on the longitudinal phonons in superfluid liquid. **The second sound** is formed by the entropic waves, which are oscillations of temperature and it was predicted by Landau (1941; 1944), explained rigorously by Lifshitz (1944) and experimentally confirmed by Peshkov (1944). **The third sound** is a sound, which exists only in thin films of He II. That is the surface sound in helium. **The forth sound** is the sound in the capillaries, where the normal component of superfluid helium II is equal to zero. **The fifth sound** is the so called the friction sound discovered by Bénard (1908) and Mallock (1907) in hydrodynamics and aerodynamics and theoretically explained and derived by Blokhintsev from the von Kármán vortex street. This sound was not still experimentally discovered in superfluid helium II and it means that this sound is here predicted as the new direction in the low temperature physics of the low temperature laboratories.

2 The friction sound in the classical aerodynamics

It is well known that the moving ballistic projectile generates not only the Mach cone but also the sound. Similarly, the moving projectile of a gun, moving asteroid in atmosphere, moving misails generate sound. The physical origin of this sound is not caused by the vibration of the surface of the projectile, or by the vibration of the Mach cone, or by the micro-structure of the Mach cone, but it is caused by the periodic generation of vortexes in the vicinity of the surface of the projectile during the air flowing around it. Such sound is generated also by the air flow around the cylinders, or strings. The system of strings generating the sound is named Aeolian's harp (Aeolus being God of winds in the Greek mythology) and the tones generated in a such a way are so called the friction tones. If the diameter of the string, or cylinder immersed in the flow is D and the velocity of the flow is v then the frequency f of the sound is given by the Strouhal formula:

$$f = \kappa(Re)\frac{v}{D}, \tag{1}$$

where κ is the Strouhal number named after Vincent Strouhal, a Czech physicist who experimented in 1878 with wires experiencing vortex shedding and singing in the wind (Strouhal, 1878; White, 1999). The symbol Re is the Reynolds number given by the formula $Re = vD/\nu$, where ν is the kinematic viscosity. The Strouhal number was late generalized to involve obertons, or

$$f = \kappa(Re)\frac{v}{D}n, \tag{2}$$

where n is the integer number of the oberton.

3 The von Kármán vortex street

The von Kármán vortex street is named after the engineer and fluid theorist Theodore von Kármán (1963; 1994). It is produced for instance by wind interacting with the suspended telephone, or, by the power lines, or, by a car antenna at certain speeds of a car. The von Kármán vortex street can be rigorously defined in the two-dimensional hydrodynamics with potential flow.

The plane potential flow of the ideal two-dimensional liquid can be described by the complex function $w(z) = \varphi(x, y) + i\psi(x, y)$ with $z = x + iy$ (Kočin et al., 1963). It is supposed that the function is analytical, which means that the Cauchy-Riemann conditions are fulfilled:

$$\frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial y}; \quad \frac{\partial\varphi}{\partial y} = -\frac{\partial\psi}{\partial x}. \tag{3}$$

The corresponding velocities of the two-dimensional liquid fluid are as follows:

$$v_x = \frac{\partial\varphi}{\partial x}; \quad v_y = \frac{\partial\varphi}{\partial y}. \tag{4}$$

Then, derivation of w gives:

$$\frac{dw}{dz} = \frac{\partial\varphi}{\partial x} + i\frac{\partial\psi}{\partial x} = \frac{\partial\varphi}{\partial x} - i\frac{\partial\varphi}{\partial y} = v_x - iv_y. \tag{5}$$

The vortex potential with center at point z_k was derived in the complex function theory of the fluid dynamics as

$$w(z) = \frac{\Gamma}{2\pi i}\ln\frac{(z - z_k)}{l}, \tag{6}$$

where Γ is so called circulation of liquid and l is the arbitrary constant with the dimensionality of length. Let us suppose that the centers of the vortexes are at points $z_0, \pm z_1, \pm z_2, \pm z_3, ...$ with $x_k = lk, k = 0, \pm 1, \pm 2, \pm 3, ...$ and $y_k = H/2$, where H is the arbitrary parameter.

It may be easy to see that the complex potential of the system of vortexes is (Kočin et al. 1963):

$$w(z) = \frac{\Gamma}{2\pi i}\left\{\ln\frac{(z - z_0)\pi}{l} + \sum_{k=1}^{\infty}\left[\ln\frac{(z - z_k)}{-lk} + \ln\frac{(z - z_{-k})}{lk}\right]\right\} + const, \tag{7}$$

where we have multiplied $z - z_0$ by π/l and $z - z_k$ by $1/(-kl)$, which leads to the change of additional constant in the complex potential and not to the change of physics following from the complex potential. After some mathematical manipulations, we get the last formula in the following form: with

$$w(z) = \frac{\Gamma}{2\pi i} \ln \left\{ \frac{(z - z_0)\pi}{l} \prod_{k=1}^{\infty} \frac{(z - z_k)}{-lk} \frac{(z - z_{-k})}{lk} \right\}. \tag{8}$$

Since we have used $z_k = z_0 - lk, z_{-k} = z_0 + lk$, then

$$w(z) = \frac{\Gamma}{2\pi i} \ln \left\{ \frac{(z - z_0)\pi}{l} \prod_{k=1}^{\infty} \left[1 - \left(\frac{(z - z_0)}{lk} \right)^2 \right] \right\}. \tag{9}$$

Now, using the formula

$$\sin \pi x = \pi x \prod_{k=1}^{\infty} \left(1 - \frac{x^2}{k^2} \right), \tag{10}$$

we get

$$w = \frac{\Gamma}{2\pi i} \ln \sin \frac{\pi}{l}(z - z_0). \tag{11}$$

The corresponding complex velocity is as follows:

$$v_x - iv_y = \frac{\Gamma}{2li} \cot \frac{\pi}{l}(z - z_0). \tag{12}$$

In case of two vortexes with circulation Γ_1, Γ_2, we get the complex velocities in the form:

$$v_x - iv_y = \frac{\Gamma_1}{2li} \cot \frac{\pi}{i}(z - z_1) + \frac{\Gamma_2}{2li} \cot \frac{\pi}{l}(z - z_2) = \frac{dw}{dz}. \tag{13}$$

It is possible to see that (Kočin et al., 1963)

$$v_{1x} - iv_{1y} = \frac{\Gamma_2}{2li} \cot \frac{\pi}{l}(z_1 - z_2) \tag{14}$$

and

$$v_{2x} - iv_{2y} = -\frac{\Gamma_1}{2li} \cot \frac{\pi}{l}(z_1 - z_2). \tag{15}$$

We have from equal complex velocities:

$$v_{1x} - iv_{1y} = v_{2x} - iv_{2y}, \tag{16}$$

the following evident relation

$$\Gamma_1 = -\Gamma_2. \tag{17}$$

In case that y-velocities of vortexes are zero, or, $v_{1y} = v_{2y} = 0$, then

$$z_1 - z_2 = b + Hi, \tag{18}$$

where b, H are some constants.

Now, let us use the formula:

$$\cot\frac{\pi}{l}(b + Hi) = \frac{\sin\frac{2\pi b}{l}}{\cosh\frac{2\pi H}{l} - \cos\frac{2\pi b}{l}} - i\frac{\sinh\frac{2\pi H}{l}}{\cosh\frac{2\pi H}{l} - \cos\frac{2\pi b}{l}}, \tag{19}$$

Then, it follows from the last equation that

$$v_{1,2x} = \frac{\Gamma}{2l}\frac{\sinh\frac{2\pi H}{l}}{\cosh\frac{2\pi H}{l} - \cos\frac{2\pi b}{l}}, \tag{20a}$$

$$v_{1,2y} = -\frac{\Gamma}{2l}\frac{\sin\frac{2\pi b}{l}}{\cosh\frac{2\pi H}{l} - \cos\frac{2\pi b}{l}}. \tag{20b}$$

We have from $v_{1y} = v_{2y} = 0$, that

$$\sin\frac{2\pi b}{l} = 0, \tag{21}$$

or, $b = 0$, $b = l/2$.

The situation with $b = 0$ is called the symmetrical configuration which is non-stable (Kočin et al., 1963) and the situation with $b = l/2$ which is the chess stable configuration. We have two velocities:

$$v_{1x} = \frac{\Gamma}{2l}\coth\left(\frac{\pi H}{l}\right); \quad (b = 0), \tag{22}$$

$$v_{2x} = \frac{\Gamma}{2l}\tanh\left(\frac{\pi H}{l}\right); \quad (b = l/2). \tag{23}$$

4 The derivation of the Strouhal number from the vortex street

The period forming by the vortex street, where the relative velocities is $v - u$, is (Blokhintsev, 1981):

$$T = \frac{l}{v - u} \tag{24}$$

and the frequency f is

$$f = \frac{v-u}{l} = \left(1 - \frac{u}{v}\right)\frac{D}{l}.\frac{v}{D} \tag{25}$$

It means in the last formula that the non-relativistic Strouhal number κ is

$$\kappa = \left(1 - \frac{u}{v}\right)\frac{D}{l}. \tag{26}$$

5 The relativistic Strouhal number

The rigorous derivation of the relativistic Strouhal number follows from the relativistic hydrodynamics (Landau et al. 1987), together with the derivation of the relativistic von Kármán's vortex theory. However, we here suppose that the relativistic Strouhal number follows immediately from the non-relativistic formula by the operation of the relativistic generalization.

The Strouhal formula contains quantity D with the dimensionality of length, and velocities v and u. According to special theory of relativity, length is not contracted when the cylinder or string is placed perpendicularly to the direction of motion, and it means that it is not contracted if it is placed perpendicularly to the air flow in the considered experiment. On the other hand, the special relativity addition theorem is necessary to apply for velocities v and u. In other words, the relativistic formula is as follows (with $v \oplus u$ being the relativistic addition) :

$$v \oplus u = \frac{v+u}{1+\frac{uv}{c^2}}. \tag{27}$$

Using the formula (25) for non-relativistic frequency generated by the vortexes, we get after some algebraic operations, the relativistic Strouhal number in the form:

$$\kappa = \frac{\left(1-\frac{u}{v}\right)\frac{D}{l}}{\left(1-\frac{uv}{c^2}\right)}. \tag{28}$$

Let us remark, that if we consider the Strouhal effect in the inertial system moving with velocity V with regard to the laboratory system, then it is necessary still transform the last formula according the relativistic Doppler formula.

6 Discussion

We have considered the aerodynamic and hydrodynamic situations where the friction sound is generated. The non-relativistic and relativistic Strouhal numbers were derived from so called von Kármán's vortex street. The relativistic derivation of this formula followed from the relativistic addition formula for velocities.

The physical phenomenon called the friction sound can be extended to the cosmic rays moving in the relic photon sea, or, motion of asteroids in atmosphere. In case of cosmic rays we must consider the moving bunch of cosmical particles with its effective diameter D and not the individual particles. The cosmic space is the black-body of the photon sea (Pardy, 2013a; 2013b), which enables the formation of the von Kármán photon vortex street. The detection of the generated sound is possible by special microphones.

While the von Kármán vortexes and friction tones are generated by the motion of bodies in liquid helium, forming the fifth sound and it can be verified by the experiments in the low temperature laboratories, there are no von Kármán's vortexes in vacuum. It means, no satellite produces von Kármán's vortexes in vacuum, which is the experimental proof, that there is no classical hydrodynamics of ether-vacuum medium. The satellite experiment is equivalent to the Michelson-Morley experiment proving the nonexistence of the relative inertial motion with regard to vacuum.

The wave function ψ in quantum mechanics forms no vortex in the two-slit experiment. On the other hand the vortexes can exist in the hydrodynamic form of quantum mechanics (Madelung, 1926; Bohm et al., 1954; Wilhelm, 1970; Rosen, 1974;), (Pardy, 2001; 1994), if and only if, we introduce viscosity into the hydrodynamic equations of quantum mechanics. To our knowledge this logical possibility was not considered in quantum theory till this time.

The Bohr mechanism generates photon according to equation $E_2 - E_1 = \hbar\omega$. It is not excluded that the photon generated by this mechanism is quantized vortex named vortexon, with regard to Onsager idea (Onsager, 1949) on the existence of quantum vortexes in superfluid helium II.

The electron theory with fluid mechanics have connection with the viscosity of Fermi liquids (Lifshitz et al., 1981) and the electronic Poiseuille flow (Gurzhi, 1968). Andreev, Kivelson, and Spivak (2011) argued that hydrodynamic contributions can be dominating in systems with a large disorder correlation length. In contrast, the nonlocal response considered by Levitov and Falkovich in graphene (Levitov et al., 2015) is directly sensitive to the collective momentum transport mode which underpins viscous flow. The vorticity of the shear flows generated by viscosity can result in a backflow of electrical current that can run against the applied field. The resulting negative nonlocal voltage can serve as a clear signature of the collective viscous behavior. Spatial patterns of electric potential can be used directly to image vorticity and shear flows in electron systems with modern capacitance scanning microscopy techniques (Yoo, et al., 1997; Yacoby et al., 1999)

So, the electron transport in graphene can be described by the hydrodynamic form of the viscous liquid allowing the existence of the vortex street. It is not excluded that the discovering of the vortex street in graphene will form one of the crucial discoveries in the graphene physics.

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