Canadian Journal of Physics, Vol. 92, No. 10, P. 1074 – 1081 (2014).

http://dx.doi.org/10.1139/cjp-2013-0683

Energy, Momentum, Mass and Velocity of a Moving Body in the Light of Gravitomagnetic Theory

Sergey G. Fedosin

Sviazeva Str., 22-79, Perm 614088, Perm Region, Russia

Tel. +7 912 987 04 08, Email: intelli@list.ru

Abstract : In the weak-field approximation of the covariant theory of gravitation the 4/3 problem is formulated for internal and external gravitational fields of a body in the form of a uniform ball. The dependence of the energy and the mass of the moving body on the energy of the field accompanying the body, as well as the dependence on the characteristic size of the body are described. Additions in the energy and the momentum of the system, defined by the energy and momentum of the gravitational and electromagnetic fields, associated with the body, are explicitly calculated. The conclusion is made that the energy and the mass of the system can be described through the energy of ordinary and strong gravitation and through the energies of electromagnetic fields of particles that compose the body.

Key words: Energy; Momentum; Theory of Relativity; Gravitation; Field Potentials; Gravitomagnetism.

PACS: 03.20.+i, 03.50.x, 12.10.-g

1. INTRODUCTION

In relativistic mechanics, there are standard formulas for the dependence of energy and momentum of a particle with the mass m on its velocity $\mathbf{v}$:

$$E = \gamma m c^2 , \qquad \mathbf{p} = \gamma m \mathbf{v} , \tag{1}$$

where $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$.

If the energy E and momentum $\mathbf{p}$ in Eq. (1) are known the mass and the velocity of the particle can be calculated:

$$m = \frac{1}{c^2}\sqrt{E^2 - p^2c^2} , \qquad \mathbf{v} = \frac{\mathbf{p}c^2}{E} . \tag{2}$$

In Eqs. (1) and (2) the speed of light c is included. For a particle in rest velocity and momentum are zero, and the energy of the particle equals the rest energy:

$$E_0 = m c^2 . \tag{3}$$

Equation (3) reflects the principle of proportionality of mass and energy. In elementary particle physics the energy and the momentum are usually measured parameters, and the mass and the velocity are found from Eq. (2) and are secondary parameters.

Now, we shall suppose that the measured parameters are the energy and the velocity of the particle. In this case, from Eqs. (1) and (3) we can calculate the mass and momentum:

$$m = \frac{E}{\gamma c^2} = \frac{E_0}{c^2}, \qquad \mathbf{p} = \frac{E\mathbf{v}}{c^2} = \frac{\gamma E_0 \mathbf{v}}{c^2}. \tag{4}$$

The case is also possible when the measured parameters are the momentum and the velocity of the particle and the calculated quantities are the mass and energy:

$$m = \frac{p}{\gamma v}, \qquad E = \frac{p c^2}{v}. \tag{5}$$

If the particle velocity v is given, then the mass can be found either through the energy according to Eq. (4), or through the momentum according to Eq. (5), in both cases, the mass should be the same.

From the above formulas it is not clear whether they contain the energy and the momentum of fields, which are inherent in the particles and the test bodies. In particular, the test bodies always have their proper gravitational field and can also carry an electrical charge and the corresponding electromagnetic field. In general theory of relativity (GTR) it is considered that relativistic energy and mass of a body decrease due to the contribution of gravitational energy. Although in GTR there is no unique definition of the gravitational energy and its contribution to the integral energy [1], in the weak-field approximation the following is assumed [2]:

$$E_{GTR} = E + U, \qquad M_{GTR} = \frac{E_{GTR}}{c^2}. \tag{6}$$

where E_{GTR} is the relativistic energy of system in the gravitational field, E is the energy in the absence of the field, U is the potential gravitational energy of the body.

Since the energy U is negative, then according to GTR the mass M_{GTR} as the mass of the system consisting of the body and its fields should decrease with increasing of the field.

The main purpose of this paper is to incorporate explicitly in the relativistic formulas for the energy and the momentum the additives, resulting from the energy and the momentum of fields associated with the test bodies. All subsequent calculations will be made in the framework of the covariant theory of gravitation (CTG) [3]. We will apply the weak-field approximation, when CTG is transformed into the Lorentz-invariant theory of gravitation (LITG), and it becomes possible to compare our results with the formulas of GTR in gravitomagnetic approximation.

2. 4/3 PROBLEM FOR ENERGY – MOMENTUM OF THE INTERNAL AND EXTERNAL ELECTROMAGNETIC FIELD OF THE CHARGED HEAVISIDE ELLIPSOID

When the spherical charge Q with the radius R is moving at the velocity v in empty space, its shape becomes according to the special theory of relativity an oblate ellipsoid. In this case, one axis of the ellipsoid, which is directed along the velocity of motion becomes shorter and equals kR, where $k=\sqrt{1-v^2/c^2}$. Such an ellipsoid is called the Heaviside ellipsoid.

2.1. External Electromagnetic Field

The scalar and vector potentials $(\varphi, \mathbf{A})$, the electric field strength and the magnetic induction $(\mathbf{E}, \mathbf{B})$ of a uniformly moving charge, the electromagnetic energy W_b and the momentum $\mathbf{P}_b$ of the field outside the charged ellipsoid, the electromagnetic energy W_i and the momentum $\mathbf{P}_i$ of the field inside a uniformly charged ellipsoid, other electromagnetic quantities in the case of the Heaviside ellipsoid are well studied. Relations of the special theory of relativity allow us to determine the relationship between the quantities for a resting spherical charge and the corresponding quantities for a moving charge.

From Heaviside's works [4, 5] we know that if the center of a charged ellipsoid passes the origin of the Cartesian coordinate system at the time $t=0$, moving at a constant velocity along the axis OX, the scalar and vector potentials of the field at the point with the radius vector $\mathbf{r}=(x, y, z)$ outside of the ellipsoid will equal:

$$\varphi = \frac{Q}{4\pi\varepsilon_0\sqrt{(x-vt)^2 + k^2(y^2+z^2)}}, \qquad \mathbf{A} = \frac{\varphi\mathbf{v}}{c^2}, \tag{7}$$

where ε_0 is the vacuum permittivity.

The electric field strength $\mathbf{E}$ and the magnetic induction $\mathbf{B}$ of the Heaviside ellipsoid at the point with the radius vector $\mathbf{r} = (x, y, z)$ are calculated as follows:

$$\mathbf{E} = -\nabla\varphi - \frac{\partial\mathbf{A}}{\partial t} = \frac{Q(\mathbf{r}-\mathbf{v}t)k^2}{4\pi\varepsilon_0\left[(x-vt)^2 + k^2(y^2+z^2)\right]^{3/2}}, \qquad \mathbf{B} = \nabla\times\mathbf{A} = \frac{\mathbf{v}\times\mathbf{E}}{c^2}. \tag{8}$$

In Eq. (8) it is assumed that the velocity of the ellipsoid's motion $\mathbf{v}$ is directed along the axis OX and has the components $v_x = v$, $v_y = v_z = 0$.

Based on the results obtained by Heaviside [4] and Searle [6], we will write the expression for the electromagnetic energy outside the charged Heaviside ellipsoid [6, page 340, eq. (24)]:

$$W_b = \frac{\gamma Q^2(1+v^2/3c^2)}{8\pi\varepsilon_0 R} = \gamma W_{0b}(1+v^2/3c^2), \tag{9}$$

where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, $W_{0b} = \frac{Q^2}{8\pi\varepsilon_0 R}$ is the field energy around a stationary charged sphere; at $v=0$ the Heaviside ellipsoid turns into this sphere.

We will assume that Eq. (4), connecting the mass and the energy of the particle, is also valid for the electromagnetic field. In this case, the effective mass of the electromagnetic field associated with the external field energy will be:

$$\mu_{eb} = \frac{W_b}{\gamma c^2} = \frac{W_{0b}\,(1+v^2/3c^2)}{c^2}\,. \tag{10}$$

The momentum of the electromagnetic field outside the charged Heaviside ellipsoid was calculated in [7]:

$$\mathbf{P}_b = \frac{\gamma Q^2\,\mathbf{v}}{6\pi\varepsilon_0 R c^2}\,. \tag{11}$$

From Eq. (11) similarly to Eq. (5) we obtain the effective electromagnetic mass associated with the momentum of the external electromagnetic field:

$$\mu_{pb} = \frac{P_b}{\gamma v} = \frac{Q^2}{6\pi\varepsilon_0 R c^2} = \frac{4W_{0b}}{3c^2}\,. \tag{12}$$

Comparing Eqs. (10) and (12) we obtain:

$$\mu_{eb} = \frac{3(1+v^2/3c^2)\,\mu_{pb}}{4}\,. \tag{13}$$

2.2. Internal Electromagnetic Field

It is well known that the electromagnetic energy within the charged Heaviside ellipsoid is equal to one-fifth of the external energy [8]. Using Eq. (9), for the electromagnetic energy and the effective mass of the field inside the Heaviside ellipsoid we have the following:

$$W_i = \frac{W_b}{5} = \frac{\gamma Q^2(1+v^2/3c^2)}{40\pi\varepsilon_0 R} = \gamma W_{0i}\,(1+v^2/3c^2)\,, \tag{14}$$

$$\mu_{ei} = \frac{W_i}{\gamma c^2} = \frac{W_{0i}\,(1+v^2/3c^2)}{c^2}\,, \tag{15}$$

where $W_{0i} = \dfrac{Q^2}{40\pi\varepsilon_0 R}$ is the field energy inside the fixed uniformly charged ball.

Similarly to Eq. (14) for energy, the momentum of the electromagnetic field inside the charged Heaviside ellipsoid is five times less than in Eq. (11):

$$\mathbf{P}_i = \frac{\gamma Q^2\,\mathbf{v}}{30\pi\varepsilon_0 R c^2}\,. \tag{16}$$

From Eq. (16) we obtain the effective mass of the field associated with the momentum of the electromagnetic field inside the charged ellipsoid:

$$\mu_{pi} = \frac{P_i}{\gamma v} = \frac{Q^2}{30\pi\varepsilon_0 R c^2} = \frac{4W_{0i}}{3c^2}\,. \tag{17}$$

From Eqs. (15) and (17) we obtain the relation for the masses of the field inside the ellipsoid, similar to the relation for the masses of the field outside the ellipsoid in Eq. (13)

$$\mu_{ei} = \frac{3(1+v^2/3c^2)\,\mu_{pi}}{4}\,. \tag{18}$$

The difference between the masses μ_{eb} and μ_{pb} in Eq. (13), and the masses μ_{ei} and μ_{pi} in Eq. (18) is the essence of the so-called 4/3 problem, according to which the field masses μ_{pb} and μ_{pi}, calculated through the field momentum at low velocities are approximately 4/3 more than the corresponding field masses μ_{eb} and μ_{ei}, found through the field energy.

3. ENERGY – MOMENTUM OF THE EXTERNAL AND INTERNAL GRAVITATIONAL FIELD OF THE HEAVISIDE ELLIPSOID

The characteristic feature of the fundamental fields, which include the gravitational and electromagnetic fields, is the similarity of their equations for the potentials and the field strengths. This follows from the equations of gravitomagnetism, which are the consequence of the general theory of relativity in consideration of phenomena in a weak field. In the Lorentz-invariant theory of gravitation [3, 9, 10] the similarity of equations for both fields is even more apparent. Accordingly, 4/3 problem also takes place for the gravitational field. We considered this issue previously with respect to the gravitational field of a moving ball [11-13]. We will present here the obtained results in order to compare them with the formulas for the effective masses of the electromagnetic field and then to include the masses of the gravitational and electromagnetic fields in the total mass of the system which consists of the body and its fields.

According to the Lorentz-invariant theory of gravitation (LITG), when a ball with the radius R is moving at the velocity v in empty space, the surface of the ball must be replaced with the Heaviside ellipsoid. The ball becomes somewhat compressed along the velocity of motion, one axis becomes shorter and is assumed to be $R\sqrt{1-v^2/c_g^2}$. We will remind that in LITG in all the formulas the gravitation propagation speed c_g is used instead of the speed of light c. In LITG not only the theory of gravitation, but also the theory of relativity as part of LITG is constructed so that the speed of light everywhere is replaced by c_g. Thus it is assumed that space-time measurements can be carried out by means of gravitational waves in the same way as it is done by means of electromagnetic waves.

In gravitomagnetism, which follows from the general theory of relativity, in the weak field limit it is assumed that the speed of gravitation is equal to the speed of light. This leads to the fact that for the

Heaviside ellipsoid one axis along the velocity of motion is $R\sqrt{1-v^2/c^2}$, as in the case of the electromagnetic field, discussed in Section 2.

3.1. External Gravitational Field

We will assume that the ball with the gravitational mass M is moving along the axis OX of some reference frame. As in the case of the electromagnetic field, we can introduce for the gravitational field in LITG the scalar and vector potentials $(\psi, \mathbf{D})$ at an arbitrary point in space (x, y, z), which for the ball are as follows:

$$\psi = -\frac{GM}{\sqrt{(x-vt)^2 + (1-v^2/c_g^2)(y^2+z^2)}}, \qquad \mathbf{D} = \frac{\psi\,\mathbf{v}}{c_g^2}, \tag{19}$$

where G – the gravitational constant.

In Eq. (19) it is assumed that at $t=0$ the center of the ball (the center of the Heaviside ellipsoid) is located in the origin of the coordinate system. We can notice that the gravitational potentials in Eq. (19) are similar by their form to the potentials in Eq. (7) of the electromagnetic field.

Further we will consider that $c_g = c$, then the subsequent results will have the same form both in LITG and in gravitomagnetism. With the help of field potentials in Eq. (19) it is easy to determine the gravitational field strength and the torsion field (gravitomagnetic field), which are the analogues of the electric field strength and the magnetic induction, respectively. The energy of the gravitational field outside the moving ball is written similarly to Eq. (9):

$$U_b = -\frac{\gamma G M^2 (1+v^2/3c^2)}{2R} = \gamma U_{0b}\,(1+v^2/3c^2), \tag{20}$$

where $U_{0b} = -\dfrac{GM^2}{2R}$ is the field energy around the stationary ball.

The effective mass of the field, associated with energy, is found similarly to Eq. (10):

$$m_{gb} = \frac{U_b}{\gamma c^2} = \frac{U_{0b}\,(1+v^2/3c^2)}{c^2}\,. \qquad (21)$$

The momentum of the gravitational field outside the Heaviside ellipsoid equals:

$$\Pi_b = -\frac{2\gamma GM^2\,\mathbf{v}}{3Rc^2}\,, \qquad (22)$$

from this the effective mass of the field associated with the momentum is as follows:

$$m_{pb} = \frac{\Pi_b}{\gamma v} = -\frac{2GM^2}{3Rc^2} = \frac{4U_{0b}}{3c^2}\,. \qquad (23)$$

Comparing Eqs. (21) and (23) gives:

$$m_{gb} = \frac{3(1+v^2/3c^2)\,m_{pb}}{4}\,. \qquad (24)$$

The difference between the masses of the gravitational field in Eq. (24) is the same as for the masses of the electromagnetic field in Eq. (13). This means that the 4/3 problem takes place in case of the gravitational field.

3.2. Internal Gravitational Field

The potentials of the gravitational field inside the uniform ball, which takes the form of the Heaviside ellipsoid due to the motion, were calculated in [12, 13] by adding the retarded potentials of all the point masses that made up the ball. As a result the gravitational energy inside this Heaviside ellipsoid equals:

$$U_i = -\frac{\gamma G M^2 (1+v^2/3c^2)}{10R} = \gamma U_{0i} (1+v^2/3c^2)\,, \tag{25}$$

where $U_{0i} = -\frac{GM^2}{10R}$ is the field energy inside a stationary ball with radius R.

The effective mass of the field associated with energy is obtained similarly Eq. (4):

$$m_{gi} = \frac{U_i}{\gamma c^2} = \frac{U_{0i}\,(1+v^2/3c^2)}{c^2}\,. \tag{26}$$

For the momentum and the effective mass of the gravitational field inside the Heaviside ellipsoid we find:

$$\Pi_i = -\frac{2\gamma G M^2\,\mathbf{v}}{15Rc^2}\,, \tag{27}$$

$$m_{pi} = \frac{\Pi_i}{\gamma v} = -\frac{2GM^2}{15Rc^2} = \frac{4U_{0i}}{3c^2}\,. \tag{28}$$

From Eqs. (26) and (28) the relation follows for the effective masses of the field, which is similar to Eq. (24) and leads to the 4/3 problem:

$$m_{gi} = \frac{3(1+v^2/3c^2)\,m_{pi}}{4}\,. \tag{29}$$

4. THE CONTRIBUTION OF GRAVITATIONAL FIELD IN ENERGY AND MOMENTUM OF A MOVING BODY

We shall try to include in equation (1) the relations found above for the energy and the momentum of the gravitational field of a moving body in the form of a ball. We shall suppose as a first approximation that in static case instead of Eq. (3) there is the following relation for the relativistic energy of system:

$$E_0 = E'_0 - U_0 = M'c^2 + \frac{3GM^2}{5R}, \tag{30}$$

where $U_0 = U_{0b} + U_{0i} = -\frac{3GM^2}{5R}$ – the integral energy of static gravitational field inside and outside the ball with uniform density,

M – the gravitational mass of the ball,

$E'_0 = M'c^2$ – the rest energy, found in such a way that it does not depend on the energy of the macroscopic gravitational field. To determine the energy E'_0 the ball's substance should be divided into pieces and spread to infinity while the total mass of all pieces of the body is M'.

The choice of the minus sign in front of U_0 in Eq. (30) will be substantiated in Section 5, where the relativistic energy of the system is reduced to the binding energy of the system, and all the energy components are included in the energy expression in Eq. (45) with negative signs. In Eq. (30) all terms must be associated with the relativistic energy either of the system, or of the body, or of the body field. We believe that the field energy U_0 is a component of the total, not the relativistic energy. Due to the relation between the total energy and the binding energy, which are equal in the absolute value but differ in signs, we take U_0 with the minus sign.

Similarly to Eqs. (1) and (4) we define the relativistic energy of the moving system:

$$E = \gamma E_0 = \gamma(M'c^2 - U_0)\,. \qquad (31)$$

On the other hand, the gravitomagnetic energy as the integral energy of the gravitational field inside and outside the ball, taking into account Eqs. (20) and (25) is negative and equals:

$$U = U_b + U_i = -\frac{3\gamma GM^2(1+v^2/3c^2)}{5R} = \gamma U_0\,(1+v^2/3c^2)\,.$$

For the relativistic energy of the system in the form of the moving ball and its field, we have as in Eq. (30):

$$E = E' - U = E' - \gamma U_0\,(1+v^2/3c^2)\,. \qquad (32)$$

From Eqs. (31) and (32) it follows:

$$E' = \gamma M'c^2 + \frac{\gamma U_0 v^2}{3c^2}\,. \qquad (33)$$

Since the energy of the static field is negative: $U_0 = -\frac{3GM^2}{5R}$, then in Eq. (33) in the energy E' of the moving ball the negative additive from field energy will appear, and the energy $E'_0 = M'c^2$ does not depend on U_0.

We shall consider now the law of conservation of momentum. The momentum of the system consists of the momentum of the ball $\mathbf{P}_b$ and the momentum of the gravitational field, and taking into account Eq. (22) for the field momentum outside the ball, and Eq. (27) for the momentum of the field inside the ball, the total momentum of the field is:

$$\mathbf{P}_f = -\frac{4\gamma GM^2\,\mathbf{v}}{5Rc^2}\,.$$

Then for the momentum of the system we can write down:

$$\mathbf{P} = \mathbf{P}_b + \mathbf{P}_f = M_V \, \mathbf{v} - \frac{4\gamma G M^2 \, \mathbf{v}}{5Rc^2} = M_V \, \mathbf{v} + \frac{4\gamma U_0 \, \mathbf{v}}{3c^2}, \qquad (34)$$

where M_V is the mass of moving ball as a function of the velocity $\mathbf{v}$.

Momentum of the system can also be expressed as in Eq. (4), taking into account Eq. (30) we find:

$$\mathbf{P} = \frac{\gamma E_0 \, \mathbf{v}}{c^2} = \frac{\gamma (M' c^2 - U_0) \, \mathbf{v}}{c^2}. \qquad (35)$$

From comparing Eqs. (34) and (35) it follows:

$$M_V = \gamma M' - \frac{7\gamma U_0}{3c^2}. \qquad (36)$$

From Eq. (33) it follows that at $v = 0$ the rest energy E'_0 of the pieces of ball at infinity does not include the field energy, but with the addition of pieces in the ball and subsequent movement of the ball in the energy E' an additive appears, related with the energy U_0 of the field. The field energy U_0 also makes contribution to the mass M_V of the moving ball in Eq. (36).

Comparing Eqs. (31) and (35) with Eq. (1) shows that taking into account the gravitational field the role of the total mass of the body and its field is played by the quantity $M_\Sigma = M' - U_0 / c^2$. If we know the energy E in Eq. (31) and the momentum $\mathbf{P}$ in Eq. (35), it follows from these relations that we can express the mass M_Σ of the system and the velocity v of the body. In case of a uniform ball with radius R we can write down:

$$M_\Sigma = \frac{1}{c^2}\sqrt{E^2 - P^2c^2} = M' + \frac{3GM^2}{5Rc^2}, \qquad \mathbf{v} = \frac{\mathbf{P}c^2}{E}. \tag{37}$$

According to Eq. (37), the invariant system mass depends not only on the energy and momentum of the body, but also depends on the average body size due to the contribution of the gravitational field mass to the constant value of the mass M'.

We shall note also that the problem of 4/3 for the gravitational field (inequality of the mass of the field, found from the energy, and the mass of the field, calculated by the momentum of the field) was compensated by the dependence of the energy E' in Eq. (33) and the mass M_V in Eq. (36) of the moving ball on the field energy U_0. As a result, the field energy U_0 in Eqs. (31) and (35) is included symmetrically in both the relativistic energy and momentum of the system. In this case, our task was not to solve the 4/3 problem as such, but to take into account the energy and momentum components of the gravitational field associated with the system.

5. ANALYSIS OF THE COMPONENTS OF MASS AND ENERGY OF THE SYSTEM

5.1. Gravitational Field

Until now we have not specified of which components the mass M_Σ of the system consists, and whether other energies except the energy of gravitational field contribute to it. For example, what shall happen if the body is heated? From the standpoint of kinetic theory, an increase of temperature leads first to an increase of the average velocity of the particles that makeup the body. In this case, according to Eq. (1) the average energy of each particle of the body would increase, and due to the additivity of energy the energy E_0 of the system should change. For the case of the body at rest and its gravitational field $E_0 = M'c^2 - U_0 = M_\Sigma c^2$, and for Eqs. (31) and (35) for moving body we can write down the following:

$$E = \gamma E_0 , \qquad \mathbf{P} = \frac{\gamma E_0 \mathbf{v}}{c^2} . \tag{38}$$

Heating of the body from an external source leads to the change of E_0 in Eq. (38), and the heat as a form of energy is distributed between the kinetic energy of substance and the energy U_0 of the gravitational field. When heated, the mass density could decrease and the body radius could increase.

Any interaction between particles of the body with each other or with the environment, which changes the energy of the particles, also changes the energy $E_0 = M_\Sigma c^2$ of the system at rest. In accordance with Eq. (37) the mass M_Σ of the system with the ball depends not only on M', but also on the radius of the ball R.

5.2. Electromagnetic Field and Internal Kinetic Energy

Suppose that some charge Q is uniformly distributed within a stationary ball. In this case, taking into account Eqs. (9) and (14) the total energy of the electric field is:

$$W_{0e} = \frac{3Q^2}{20\pi \varepsilon_0 R} . \tag{39}$$

The electromagnetic energy may include the energy of the magnetic field W_{0m}, if the ball is magnetized or if there are electric currents. The energies W_{0e} and W_{0m} together constitute the total energy W_0 of the electromagnetic field of the body, which should contribute to the energy of the system.

We assume that other forms of energy (e.g. heat) can change the body mass, but can not change the charge of the body, because it is necessary to transfer the charged particles to the body (or from the body). This is one of the differences between the electromagnetic and gravitational fields, in addition to the unipolarity of gravitational charges (which are the masses) and the bipolarity of electromagnetic charges.

The mass M' in Eq. (30) is the total mass of all body parts, separated to infinity. As in [14] we can assume that in this case the substance is at zero degrees according to Kelvin temperature scale. When integrating all parts into a single body the substance temperature increases up to the value T and a certain mass M'_T appears, which presents the additional mass of the internal kinetic energy E_k of the body. This energy includes the kinetic energy of motion of atoms and molecules, the energy of turbulent motion of the substance fluxes, as well as the energy of oscillations and rotations of atoms and molecules and the energy of their additional interaction as a result of substance heating.

If V_T is the average velocity of particles in the body at temperature T, then the following approximate relations would hold: $E_k \approx \frac{M' V_T^2}{2}$, $M'_T = \frac{E_k}{c^2} \approx \frac{M' V_T^2}{2c^2}$.

Since we intend to include the electromagnetic energy W_0 of the ball and the kinetic energy E_k of the set of atoms and molecules of the ball's substance to the total energy of the system, we introduce new notation: E_0 will be replaced by $E_{0\Sigma}$, E will be replaced by E_Σ, $\mathbf{P}$ will be replaced by $\mathbf{P}_\Sigma$. Similarly to Eq. (30) we can then write:

$$E_{0\Sigma} = M' c^2 - E_k - U_0 - W_0 . \tag{40}$$

As the energy of field, we include the energy E_k in Eq. (40) with the negative sign.

For the body that is only under influence of its proper gravitational and electromagnetic field, the virial theorem is satisfied, according to which the absolute value of potential energy of the field on the average is twice as much than the kinetic energy of body particles:

$$2E_k + U_0 + W_0 \approx 0 , \qquad E_{tot} = E_k + U_0 + W_0 \approx -E_k \approx \frac{U_0 + W_0}{2} , \tag{41}$$

here E_{tot} is the total energy excluding the rest energy of the particles of the body.

Substituting Eq. (41) in Eq. (40) gives the approximate equality:

$$E_{0\Sigma} = M' c^2 - E_{tot} \approx M' c^2 - \frac{U_0 + W_0}{2} . \tag{42}$$

6. MASS OF THE BODY AT 0^0 KELVIN

We shall now consider the essence of the mass M' related to the total mass of body particles excluding the contribution from the mass of the internal kinetic (thermal) energy and the energy of macroscopic fields. The contributions in mass M' are made by the masses of various types of energy associated with atoms and molecules at the temperature near absolute zero: strong interaction, binding the substance of the elementary particles and retaining the nucleons in atomic nuclei; electromagnetic interaction of particles; the energy of motion of electrons in atoms; rotational energy of atoms and molecules; vibrational energy of atoms in molecules, energy of atoms in molecules, etc.

6.1. Strong Interaction

In Standard Model it is assumed that the strong interaction arises due to the action of the gluon field between the quarks located in the hadrons (mesons and baryons), and the strong interaction between leptons is absent.

There is also a hypothesis that the strong interaction is a manifestation of strong gravitation at the level of elementary particles and atoms [15]. According to the Lorentz-invariant theory of gravitation, there are two components, in the form of gravitational field strength and the torsion field, and the stability of nucleons in nuclei can be described as the balance of forces from the attraction of the nucleons to each other due to strong gravitation, and the repulsion of nucleons due to the torsion field [3]. The same idea is applied to describe the structure and the stability of a number of hadrons, considered as the composition of nucleons and mesons [12]. Strong gravitation differs from the ordinary gravitation by replacing of the gravitational constant G by the constant of strong gravitation Γ, and acts between all particles, including leptons. The estimation of the quantity Γ can be obtained from the balance of four forces acting on the electron in the hydrogen atom: 1. The force of electric attraction between the electron and the atomic nucleus. 2. The force of electric repulsion of the charged matter of the electron from itself (the electron is represented as a cloud around the nucleus). 3. The centripetal force from the rotation of the electron around the nucleus. 4. The attraction of the electron to the nucleus under the influence of strong gravitation. These forces are approximately equal to each other, so the relations for the forces of attraction from strong gravitation and the electric force are satisfied [9]:

$$-\frac{\Gamma M_p M_e}{R_e^2}=-\frac{e^2}{4\pi\varepsilon_0 R_e^2}\,, \qquad \Gamma=\frac{e^2}{4\pi\varepsilon_0 M_p M_e}=1.514\cdot 10^{29}\ \text{m}^3\cdot\text{kg}^{-1}\cdot\text{c}^{-2}\,, \tag{43}$$

where M_p and M_e – the mass of proton and electron, respectively,

R_e – the radius of rotation of the electron cloud,

e – the elementary electric charge as the proton charge equal to the absolute value of the negative charge of electron,

ε_0 – the vacuum permittivity.

Another way to estimate Γ is based on the theory of similarity of matter levels and the use of coefficients of similarity. These coefficients are defined as follows: $\Phi=1.62\cdot 10^{57}$ – coefficient of similarity by mass (the ratio of the mass of neutron star to the proton mass); $P=1.4\cdot 10^{19}$ – the coefficient of similarity by size (the ratio of the radius of neutron star to the proton radius); $S=0.23$ – the coefficient of similarity by speed (the ratio of the characteristic speed of the particles of neutron star to the speed of light as the typical speed of the proton matter). For strong gravitational constant a formula is obtained: $\Gamma=G\frac{\Phi}{PS^2}$, where exponents of similarity coefficients correspond to the dimension of gravitational constant G according to the dimensional analysis.

If we understand the strong interaction as the result of strong gravitation, the main contribution to the proton rest energy should be made by the positive kinetic energy of its matter and the negative energy of the strong gravitation (the electrical energy of the proton can be neglected due to its smallness). The sum of these energies gives the total energy of the proton, and due to the virial theorem in Eq. (41) this sum of energies is approximately equal to half of the energy of strong gravitation. Since the energy of the strong gravitation is negative, then the total energy of the proton is negative too. The total energy of the proton up to the sign can be regarded as the binding energy of its matter; the binding energy equals to the work that should be done to spread the matter to infinity so that there total energy of the matter (potential and kinetic) should be equal to zero. According to its meaning, the positive proton rest energy must be equal to the binding energy or the absolute value of the total energy of the proton. This gives the equality between the rest energy and the absolute value of half of the energy of strong gravitation:

$$M_p c^2 = \frac{\eta \Gamma M_p^2}{2R_p}, \tag{44}$$

where $\eta = 0.6$ for the case if the proton was uniform density ball with the radius R_p.

If we substitute Eq. (43) in Eq. (44), we obtain another equation, which allows estimating the radius of the proton:

$$M_e c^2 = \frac{\eta e^2}{8\pi\varepsilon_0 R_p}, \qquad R_p = \frac{\eta e^2}{8\pi\varepsilon_0 M_e c^2} = \frac{\eta r_0}{2},$$

where r_0 is the classical electron radius.

In self-consistent model of the proton [16] we find that in Eq. (44) the radius of the proton is $R_p = 8.73 \cdot 10^{-16}$ m, and the coefficient $\eta = 0.62$ due to a small increase in the density in the center of the proton. At the same time, in the assumption that positive charge is distributed over the volume of proton similar to the mass distribution and the maximum angular frequency of the proton rotation is limited by the condition of its integrity in the field of strong gravitation, we can find the magnetic moment of the proton as a result of rotation of its charged matter:

$$P_m = \delta e \sqrt{\Gamma M_p R_p},$$

where $P_m = 1.41 \cdot 10^{-26}$ J/T is the magnetic moment of the proton,

$\delta = 0.1875$ (in the case of the uniform density and the charge of proton it should be $\delta = 0.2$).

The fact that the rest energy of the proton is associated with strong gravitation, also follows from the modernized Fatio-Le Sage theory of gravitation [17]. In this theory, based on the absorption of the fluxes of gravitons in the matter of bodies with transfer of the momentum of gravitons to the matter, the exact formula for Newton's gravitational force (the law of inverse squares) is derived; the energy density of the flux of gravitons ($4 \cdot 10^{34}$ J/m^3), the cross section of their interaction with the substance ($7 \cdot 10^{-50}$ m^2) and other parameters are deduced.

In the theory of infinite hierarchical nesting of matter [3], [9] it is shown that at each main level of matter the corresponding type of gravitation appears: there is a strong gravitation at the level of elementary particles, but at the level of stars it is the ordinary gravitation. The gravitation reaches a maximum in the densest objects – in nucleons and in neutron stars. In the substance of the earth's density the range of strong gravitation is less than a meter, and at such sizes of bodies strong gravitation is replaced by the ordinary gravitation. This corresponds to the fact that the masses and the sizes of objects at different levels of matter increase exponentially, and the point of replacing of the strong gravitation by the ordinary gravitation lies near the middle of the range of masses from nucleons to the stars on the axis of the masses on the logarithmic scale.

In the above picture the rest energy of proton in Eq. (44) is approximately equal to the absolute value of the total energy of the proton in its proper field of strong gravitation (for increased accuracy we should also take into account the electromagnetic energy of the proton), and the energy $M'c^2$ in Eq. (42) consists of the rest energy of nucleons and electrons of the matter of the body, with the addition of the energy of their gravitational and electromagnetic interactions and the mechanical motion in atoms and molecules. Consequently, the energy $M'c^2$ of the body, taking into account the virial theorem in Eq. (41) can be reduced to the half of the absolute value of the sum of the energy of strong gravitation U_{0g} and electromagnetic energy W_{0g} of the nucleons, electrons, atoms and molecules involved in formation of the binding energy. As a result, the relativistic energy of the stationary body and its fields instead of Eq. (42) can be written down as follows:

$$E_{0\Sigma} \approx -\frac{U_{0g}+W_{0g}}{2}-\frac{U_0+W_0}{2}. \qquad (45)$$

To understand the meaning of energy $E_{0\Sigma}$ better, we shall consider the energy balance in the process of merging of matter, with formation of elementary particles at the beginning, passing then to confluence of the elementary particles into atoms and finally in the formation of a body of many atoms. Initially, the matter is motionless at infinity and its parts do not interact with each other, so that total energy of the system is zero (we do not consider here the rest energy of matter in its condition when it is fragmented and was not yet included into the composition of the elementary particles). If the matter particles will draw together under the influence of strong then ordinary gravitation, the negative energy U of gravitational field and the positive kinetic energy E_k of motion of particles will appear, and due to the energy conservation law the integral energy should not change, remaining equal to zero. In the energy balance it is necessary to take into account the electromagnetic energy W and the energy E_r leaving the system due to the emission of field quanta such as photons and neutrinos:

$$E_r+E_k+U+W=0, \qquad E_r=-(E_k+U+W)=-E_{Tot}=-\frac{U+W}{2}. \qquad (46)$$

In Eq. (46) the virial theorem in the form of Eq. (41) is used for the components of the total energy E_{Tot} of the system. According to Eq. (46), the energy E_r of the emission that left the system equal up to a sign to the total energy E_{Tot}, i.e. the energy of emission E_r equals the binding energy of the system. By comparing Eqs. (46) and (45) we now see that the relativistic energy $E_{0\Sigma}$ of body and its field is the same as the energy extracted from the body by different emission during the formation of the body.

As a rule in the energy $E_{0\Sigma}$ only those components are taken into account that are associated with formation of elementary particles, atoms and macroscopic molecular substance; and the binding energies of the particles of which the matter of elementary particles is built are not taken into account and are assumed to be constant. Heating the body due to gravitation according to Eqs. (46) and (45) leads to an increase of body energy $E_{0\Sigma}$. This conclusion is based on the fact that although the internal kinetic energy of the body E_k is part of Eq. (40) with the negative sign, but the change of the potential energy $U+W$ by the virial theorem compensates the contribution of the energy E_k. An example is the star, which is heated and accelerates its rotation during compression by gravitation, and the absolute value of the gravitational energy of the star increases.

According to Eq. (45), the relativistic energy $E_{0\Sigma}$ of the system consists mainly of the energies of two fundamental fields – gravitational and electromagnetic, responsible for the integrity of the particles of the body and for the composition of the body of the individual particles. In this case, the strong interaction between the particles is taken into account by the energy of strong gravitation U_{0g} and the electromagnetic energy W_{0g}.

6.2. Weak Interaction

As for the weak interaction it is assumed to be the result of transformation of matter, which was for a long time under the influence of the fundamental fields. An example is the long-term evolution of a star massive enough to form a neutron star in a supernova outburst, when the neutrino burst is emitted with the energy of about the binding energy of the star (the gravitational energy of the matter compression into a small-sized neutron star is converted into the energy of neutrinos, the energy of photon emission, the kinetic energy and the heating of the expelled shell). At the level of elementary particles, this corresponds to the process of formation of a neutron with the emission of neutrino.

If in the weak interaction the body at rest emits (the body absorbs) neutrinos, photons and other particles, it leads to a change of the relativistic energy $E_{0\Sigma}$ of the system. In general, the energy $E_{0\Sigma}$ is the function of time and speed with which the separate particles or units of matter are emitted from the body or absorbed by it. Due to the laws of conservation of energy and momentum, if some particles bring into the system the energy and momentum, then after some time they are distributed in the system and according to virial theorem they can be taken into account through the energy and the momentum of the fundamental fields.

Therefore, we can state that according to Eq. (45), the source of the energy of the system, and of its mass M_Σ as the measure of inertia are the gravitational and electromagnetic fields associated with the masses and charges, as well as electric currents and mass flows. In Fatio-Le Sage theory of gravitation it is supposed that the fields are the consequence of the interaction of the masses and the charges with the fluxes of gravitons and tiny charged particles that penetrate the space.

If we define the total mass of the system in the form $M_\Sigma = \frac{E_{0\Sigma}}{c^2}$, then Eq. (38) for energy and momentum of a moving body is as follows:

$$E_\Sigma = \gamma M_\Sigma c^2, \qquad \mathbf{P}_\Sigma = \gamma M_\Sigma \mathbf{v}. \tag{47}$$

7. CONCLUSIONS

Equations (47) look exactly the same as Eq. (1) for a small test particle. However, the mass M_Σ of system in Eq. (47) takes fully into account the field energies, whereas for the mass m of a small particle in Eq. (1) it is only expected. The appearance in the mass M_Σ of the contribution from the energy of fields has occurred because we have used the energy of mutual interaction of many small particles in a massive body. Hence, by induction, we should suppose that not only the mass of body, but the mass of any isolated small particle should be determined taking into account the contribution from the energy of proper fundamental fields of the particle. The described concept of mass in the covariant theory of gravitation (CTG) is confirmed by the analysis of the Hamiltonian [18] and of the Lagrangian in the principle of least action [19].

We should note the difference between the results of CTG and general theory of relativity (GTR) with respect to mass and energy. In CTG the mass of system with the uniform spherical body at rest with the radius R including effective mass of its fields is expressed with the help of Eqs. (39) and (45):

$$M_\Sigma = \frac{E_{0\Sigma}}{c^2} = -\frac{U_{0g} + W_{0g}}{2c^2} - \frac{U_0 + W_0}{2c^2} = M' + \frac{3GM^2}{10Rc^2} - \frac{3Q^2}{40\pi\varepsilon_0 Rc^2}, \tag{48}$$

where the mass M' sets the total mass of body parts at zero absolute temperature, excluding the potential energy of the fields, the mass M is obtained through the density and the volume and represents the gravitational mass, the expression for the energy U_0 is given after Eq. (30).

As a result the relativistic mass M_Σ of the system by combining the body parts into a whole increases due to the energy of the gravitation field U_0, and decreases due to the electric energy W_0.

In general theory of relativity in order to determine the mass of a stationary system we can integrate the timelike component of the stress-energy tensor of the system T^{00} over the volume and divide the result by the squared speed of light. According to [20] for the gravitational field contribution and [21] with respect to the electromagnetic field, the mass of the system M_Σ in the first approximation is:

$$M_\Sigma = \frac{1}{c^2}\int(\rho c^2 + \rho\Pi + \rho\psi + \rho_q\varphi - \frac{1}{2}\varepsilon_0 E^2)dV = M - \frac{6GM^2}{5Rc^2} + \frac{3Q^2}{20\pi\varepsilon_0 Rc^2} + \frac{1}{c^2}\int \rho\Pi dV, \tag{49}$$

where $M = \int \rho dV$ is the body mass, ρ and ρ_q are the density of mass and charge, respectively, ψ and φ define the scalar potentials of the gravitational and electric field, respectively, ε_0 is the vacuum permittivity, E is the electric field intensity, Π is the elastic energy per unit mass. In this case the mass density ρ is associated with the scalar potential by the Poisson equation $\Delta\psi = 4\pi G\rho$ and satisfies the continuity relation of the special theory of relativity.

Since the potential ψ is negative and the potential φ is positive, in Eq. (49) the substance energy in the gravitational field reduces the mass M_Σ, while the energy of the charges in the electric field and the elastic energy increase the system mass M_Σ.

In GTR the gravitational field potentials are described by the metric tensor components, and the field and metric always exist in the presence of masses, therefore instead of ρ the invariant mass density ρ^* is used. The invariant density is part of the continuity relation in the curved spacetime: $\partial_\alpha(\sqrt{-g}\,\rho^* u^\alpha) = 0$, here g is the determinant of the metric tensor, u^α is the 4-velocity. In a weak field for a fixed body we can approximately write the following:

$$\rho^* = \rho\left(1 + \frac{3\psi}{c^2}\right).$$

If based on this we express ρ and substitute it in (49), we will obtain for the mass-energy of the system an expression similar to those presented in [22] and [23] (in contrast to [20], in [23] ρ is the invariant density and ρ^* denotes the mass density in special theory of relativity).

In Eq. (49) not the mass M' but the mass M is used, which is expressed through the mass density and volume and included in the formula for the gravitational energy. As it was shown in [18], for three masses associated with the system, the following relation holds: $M' < M_\Sigma < M$, which also follows from Eqs. (48) and (49). In our opinion, the reason of difference between Eqs. (48) and (49) is associated with different positions of the two theories: in CTG there is explicit stress-energy tensor of gravitational field, included in the Lagrangian and contributing to the spacetime metric and the energy-momentum of the considered system. This allows us to define all the three masses M', M_Σ, M and to find their meaning, and the mass M' is associated with the cosmological constant in the equation for the metric of the system. In GTR the principle of equivalence is used instead of this, the gravitational field is reduced to the metric field, and correspondingly, the energy and the momentum do not form tensor and can be found only indirectly, through the spacetime metric.

Eqs. (48) and (49) imply consistency of positions CTG and GTR, as these theories determine the mass and energy from different standpoints.

REFERENCES

1. C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, CA, 1973).
2. L. B. Okun, Nucl. Phys. B (Proc. Suppl.). **110**, 151 (2002).
3. S. G. Fedosin, *Fizicheskie teorii i beskonechnaia vlozhennost' materii* (Perm, 2009).
4. O. Heaviside, The Electrician. **22**, 147 (1888).
5. O. Heaviside, Philos. Mag. Series 5. **27**, 324 (1889).
6. G. F. C. Searle, Philos. Mag. Series 5. **44**, 329 (1897).
7. S. Hajra, J. Mod. Phys. **3**, 187 (2012).
8. S. Hajra, Ind. J. Theor. Phys. **42**, 164 (1991).

9. S. G. Fedosin, *Fizika i filosofiia podobiia: ot preonov do metagalaktik* (Perm, 1999).

10. S. G. Fedosin, Apeiron. **14**, 385 (2007).

11. S. G. Fedosin, Journal of Vectorial Relativity. **3**, 30 (2008).

12. Comments to the book: S. G. Fedosin, *Fizicheskie teorii i beskonechnaia vlozhennost' materii* (Perm, 2009).

13. S. G. Fedosin, Advances in Physics Theories and Applications. **23**, 19 (2013).

14. S. G. Fedosin, Caspian Journal of Applied Sciences Research. **1**, 1 (2012).

15. C. Sivaram and K. P. Sinha, Phys. Rev. D. **16**, 1975 (1977).

16. S. G. Fedosin, Hadronic Journal. **35**, 349 (2012).

17. S. G. Fedosin, Journal of Vectorial Relativity. **4**, 1 (2009).

18. S. G. Fedosin, Advances in Natural Science. **5**, 55 (2012).

19. S. G. Fedosin, Hadronic Journal. **35**, 35 (2012).

20. V. A. Fock, *The Theory of Space, Time and Gravitation* (Pergamon Press, London, 1959).

21. L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields* (Vol. 2, 4th ed. Butterworth-Heinemann, 1975).

22. A. Papapetrou, Proc. Phys. Soc. A. **64**, 57 (1951).

23. S. Chandrasekhar, Ap. J. **142**, 1488 (1965); Ap. J. **158**, 45 (1969).