Exploring Nonlinear Analysis in Cartography and Vexillology

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Abstract:

The nonlinear analysis techniques have accelerated and enhanced man's understanding of underlying system dynamics, evolution and patterns. The present work pertains to the unprecedented exploration of nonlinear analysis in two geography-oriented disciplines – cartography and vexillology. Specifically, the tools of fractal dimension and entropy are used to characterize the outline of around 200 countries, following which entropy for the red, green and blue color distributions of the corresponding flags are computed. Key inferences are listed, one of which is the remarkable correlation observed in all cases between the three entropies. Finally, it is hoped that the analysis techniques discussed and explored in the present work forms a preliminary step in applying the time tested successful techniques of nonlinear analysis to geography oriented disciplines.

Keywords: Nonlinear Analysis, Cartography, Vexillology, Entropy, Fractal Dimension

1. Introduction

The advancement of statistics and data analysis methodologies in recent times has accelerated growth in diverse disciplines ranging from genetics to nonlinear dynamics [1-2]. In the latter, progress has been made, thanks to establishment of standardized tools and techniques such as fractal dimensions, entropies and bifurcation analyses, which have enabled one to gain understanding of inherent patterns and underlying chaotic behavior in a system from time series and spatial distribution data [4-9].

It is in the spirit of this enhanced clarity using nonlinear analysis that the present work purports to exploration of nonlinear analysis methodologies in the related fields of cartography and vexillology. Specifically, the entropy and fractal dimension and computed for the geographical outlines of around two hundred countries, accompanied with the red, green and blue distribution entropies for the corresponding flags.

The formation of countries and country borders has been and still is a dynamic process, with the current national borders standing witness to years of conflict, war, agreements and treaties apart from natural borders such as mountains, rivers and coasts [10-12]. There are many instances in history where a rather insignificant change in the proceedings of events in the past could have entirely changed the course of the present, shaping the political world in a radically different way [13-14]. This property is a direct outcome of the 'butterfly effect', elucidated as sensitive dependence on initial conditions in chaos theory, underlying the formation of national borders, and explains why studying the fractal dimension and entropies have significance [9]. Another related aspect succinctly reflecting underlying chaotic behavior is in the national flags adopted by the countries. The story of the Napoleon and hence the tricolor's influence on most of Western Europe and its colonies are well-known, as is the use of the pan-African color sets of green/red/yellow and green/red/black by most African country flags [15-18]. While there are

few sights infusing as much patriotism as the unfurling of a nation's flag, it is an intriguing prospect that the study of entropies might give insight into that very history that shaped such patriotism. The results presented here are but a preliminary yet unprecedented step in introducing nonlinear analysis to the hitherto unexplored frontiers of cartography, vexillology and hence, geography.

2. Methodology and Results

In the present work, two main techniques, elaborated below are used for the nonlinear analysis in cartography and vexillology:

1. Fractal Dimension (D): A measure of complexity of a pattern, the fractal dimension is essentially a ratio of how detail changes with measuring scale. The presence of a non-integer fractal dimension indicates that the measured pattern/set fills space significantly different from ordinary, 'regular' geometrical sets, with higher mantissa values such as 0.8 and 0.9 corresponding to more folds and convolutions [3-5]. In the present work, Minkowski-Bouligand box counting method is used to compute the Fractal Dimension of the outlines of around 200 countries [19-20].
2. Entropy (H): This is a measure of uncertainty in the underlying system measured by the randomness of distribution in the pattern, with nearly equal distributions giving rise to higher entropy values [6]. In the present work, entropies are computed for both country outlines as well as the red, green and blue intensity distributions of the national flags.

The fractal dimensions and entropies for 196 nations are computed and tabulated in the Appendix. Certain key inferences obtained are elaborated below:

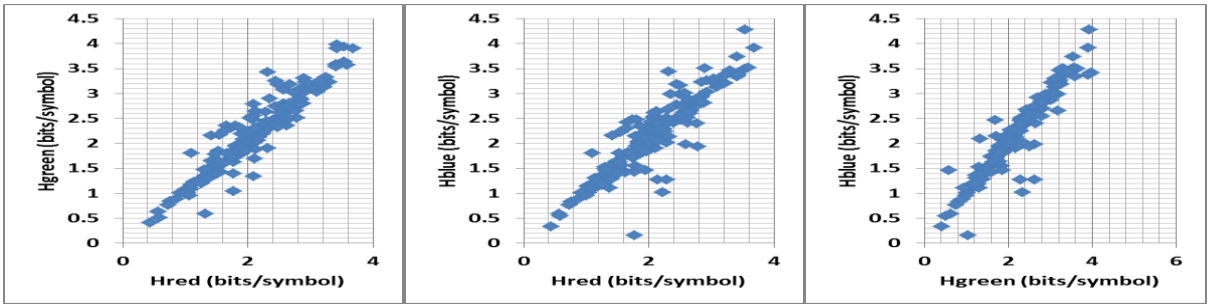

Figure 1 Correlations between the intensity entropies Hr, Hg and Hb

1. A remarkable correlation in trend is seen within the three entropies Hr, Hg and Hb. Consequently, it can be inferred that the three entropies increase or decrease in near-perfect harmony.
2. No discernible correlation is observed between the cartographic entropy and fractal dimension values.
3. Flags with simple designs, such as the single star in Vietnam, Somalia, Morocco and the single crescent-star designs of Mauritania and Turkey are seen to possess very low values of entropy fr all the three color distributions.
4. Flags with significant levels of complexity such as Belize, Sri Lanka and Eritrea, as well as the star spangled banner of the US are seen to have very high values of entropy.
5. The mean color averaged entropy for all the 196 country flags is obtained as 2.11, and the closest flags to this value are that of Greece and Mongolia. Similarly, the closest flags to the mean red,

green and blue entropies of 2.065, 2.145 and 2.146 are those of Trinidad & Tobago, Paraguay, Lebanon respectively.

6. From a cartographic perspective, it is seen that the lowest entropies belong to island nations such as Tonga, Tuvalu, Mauritius, Maldives and New Caledonia. On the other hand, nations with a large number of islands such as Canada, Indonesia, Philippines, and nations with a pronounced coastline such as Bangladesh, Greece, and Chile possess significantly high entropy values.
7. The mean value of the cartographic entropy is obtained as 0.609, which happens to be the entropy value of Uganda. Other nations in the vicinity of this value include Suriname, France, Sierra Leone and Laos.
8. It is seen that 27 out of 196 nations possess a fractal dimension of 1.000, signifying absence of a pronounced fractal or self-similar structures. On account of its extreme length to breadth ratio, Chile is the only nation with a fractal dimension less than 1.
9. Caribbean nations such as St. Kitts & Nevis, St. Vincent & The Grenadines and Bahamas are seen to possess high fractal dimensions, above 1.95, which indicates a reasonable amount of folding and convolutions in the corresponding borders.
10. The mean fractal dimension is obtained as 1.595, with 17 nations coming close to this at 1.585, most of them African nations including Liberia, Sierra Leone, Comoros, Libya, Chad, Djibouti, Gabon, Republic of the Congo and Egypt.

3. Conclusion

The present article explores the application of nonlinear analysis techniques to cartography and vexillology. Specifically, entropies and fractal dimensions are defined and computed for the country outlines of 196 nations, along with the entropy values for the three color distributions of red, green and blue in the corresponding national flags. Key inferences are listed, and it is hoped that the analysis techniques discussed and explored in the present work forms a preliminary step in applying the time tested successful techniques of nonlinear analysis to geography oriented disciplines.

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Appendix – Table of H and D values of Countries

Name	D	H	Hr	Hg	Hb	Name	D	H	Hr	Hg	Hb
Afghanistan	1.322	0.361	2.140	2.304	1.271	Liberia	1.585	0.621	3.117	3.198	3.233
Albania	1.000	0.461	1.765	1.400	1.433	Libya	1.585	0.687	2.099	1.344	2.088
Algeria	1.904	0.398	1.779	1.839	1.547	Liechtenstein	1.000	0.580	1.751	1.806	1.794
Andorra	1.904	0.387	2.754	3.011	2.865	Lithuania	1.696	0.632	1.756	1.717	1.846
Angola	1.857	0.561	1.958	1.878	1.844	Luzembourg	1.696	0.510	1.735	1.714	1.720
Antigua & Barbuda	1.000	0.340	2.438	2.751	2.782	Macedonia	1.453	0.630	2.592	2.625	1.979
Argentina	1.000	0.536	1.698	1.660	1.747	Madagascar	1.793	0.451	1.868	1.869	1.913
Armenia	1.000	0.629	1.988	2.037	2.177	Malawi	1.793	0.459	3.535	3.936	4.281
Australia	1.000	0.579	2.099	1.691	2.472	Malaysia	1.793	0.487	2.613	2.356	2.411
Austria	1.000	0.615	1.151	1.231	1.311	Maldives	1.696	0.130	1.630	1.608	1.414
Azerbaijan	1.953	0.680	1.733	2.319	2.481	Mali	1.322	0.394	1.915	1.898	1.832
Bahamas	1.953	0.557	1.411	2.155	2.155	Malta	1.453	1.143	1.473	1.519	1.535
Bahrain	1.953	0.479	1.385	1.449	1.402	Mauritania	1.700	0.443	0.757	0.841	0.834
Bangladesh	1.857	0.931	1.204	1.251	1.187	Mauritius	1.000	0.247	2.561	2.552	2.620
Barbados	1.857	0.364	1.618	1.602	1.739	Mexico	1.754	0.572	2.490	2.521	2.549
Belarus	1.857	0.615	2.140	2.293	2.247	Moldova	1.565	0.742	2.670	2.816	2.700
Belgium	1.857	0.709	1.887	1.783	1.950	Monaco	1.453	0.740	1.270	1.280	1.191
Belize	1.661	0.690	3.527	3.641	3.496	Mongolia	1.661	0.615	2.112	2.103	2.155
Benin	1.793	0.706	1.974	2.013	2.063	Montenegro	1.904	1.537	2.823	2.781	2.796

Bhutan	1.804	0.518	2.757	2.823	2.814	Morocco	1.754	0.365	0.443	0.416	0.331
Bolivia	1.696	0.666	1.938	2.082	2.116	Mozambique	1.754	0.525	2.664	3.190	2.652
Bosnia-Herzegovina	1.696	0.895	1.896	2.198	2.201	Myanmar	1.000	0.555	2.647	2.641	2.605
Botswana	1.696	0.629	1.434	1.364	1.382	Namibia	1.754	0.439	2.449	3.250	3.184
Brazil	1.000	0.418	1.660	2.357	2.430	Nauru	1.850	1.392	1.308	1.308	1.532
Brunei	1.754	0.567	2.846	2.876	2.957	Nepal	1.793	0.474	2.834	2.790	2.842
Bulgaria	1.754	0.488	1.766	1.748	1.768	Netherlands	1.585	0.835	1.842	1.866	1.865
Burkina Faso	1.696	0.669	1.460	1.518	1.542	New Caledonia	1.453	0.229	2.491	3.192	2.993
Burundi	1.696	0.488	2.878	2.801	2.816	New Zealand	1.804	0.584	1.982	2.218	2.335
Cambodia	1.696	0.792	2.643	2.478	2.638	Nicaragua	1.696	0.674	1.375	1.492	1.541
Cameroon	1.453	0.600	2.052	2.074	2.124	Niger	1.754	0.457	2.003	1.960	2.074
Canada	1.322	1.365	1.416	1.648	1.576	Nigeria	1.754	0.493	1.099	1.091	1.093
Cape Verde	1.754	0.285	2.282	2.326	2.480	Norway	1.857	0.793	2.095	2.064	2.269
Central African	1.793	0.661	3.078	3.111	3.131	Oman	1.754	0.477	2.290	2.406	2.387
Chad	1.585	0.704	1.888	1.861	1.868	Pakistan	1.754	0.517	1.502	1.426	1.445
Chile	0.792	0.845	1.870	1.814	1.915	Palau	1.000	1.710	1.067	1.091	1.128
China	1.453	0.489	0.572	0.638	0.588	Palestine	1.696	0.298	2.094	2.561	2.617
Colombia	1.904	0.507	1.884	1.815	1.921	Panama	1.696	0.723	2.030	1.993	2.043
Comoros	1.585	0.347	2.892	3.313	3.511	Papua N Guinea	1.804	0.677	2.246	2.245	2.235
Congo DR	1.696	0.557	2.227	2.231	2.282	Paraguay	1.696	0.552	2.102	2.143	2.052
Congo R	1.585	0.786	2.114	2.171	1.907	Peru	1.793	0.548	1.129	1.187	1.179
Costa Rica	1.696	0.640	2.389	2.473	2.414	Philippines	1.793	0.980	2.306	2.450	2.678
Croatia	1.696	0.746	2.573	2.798	2.803	Poland	1.754	0.475	1.185	1.237	1.136
Cuba	1.565	0.656	2.279	2.354	2.515	Portugal	1.661	0.615	2.224	2.333	1.017
Cyprus	1.661	0.505	1.257	1.245	1.362	Puerto Rico	1.000	1.344	2.506	2.468	2.471
Czech	1.696	0.620	2.087	2.009	2.079	Qatar	1.661	0.580	1.397	1.418	1.286
Denmark	1.804	0.829	1.320	1.357	1.357	Romania	1.585	0.583	1.856	1.866	1.910
Djibouti	1.585	0.729	2.473	2.329	2.514	Russia	1.000	0.829	1.953	1.942	1.969
Dominica	1.585	0.382	2.925	3.039	3.035	Rwanda	1.696	0.619	2.543	2.414	2.503
Dominican Republic	1.585	0.596	2.260	2.345	2.379	Samoa	1.453	0.431	1.316	1.346	1.384
Ecuador	1.585	0.672	2.831	2.916	2.919	San Marino	1.696	0.395	2.671	2.706	2.686
Egypt	1.585	0.797	2.019	2.206	2.308	Sao Tome &P	1.000	0.280	2.763	2.659	2.406
El salvador	1.453	0.671	1.798	1.770	1.847	Saudi Arabia	1.793	0.454	1.313	1.474	1.491
Equatorial Guinea	1.696	0.456	2.778	2.774	2.916	Senegal	1.793	0.625	2.105	2.083	2.118
Eritrea	1.696	0.523	3.588	3.578	3.523	Serbia	1.754	0.629	3.205	3.136	3.211
Estonia	1.696	0.631	1.916	1.881	1.965	Sierra Leone	1.585	0.613	1.965	1.851	1.834
Ethiopia	1.696	0.482	3.299	3.227	3.466	Singapore	1.696	0.557	1.537	1.646	1.624
Fiji	1.696	0.453	3.054	3.109	3.208	Slovakia	1.453	0.555	2.516	2.486	2.558
Finland	1.793	0.507	1.392	1.412	1.486	Slovenia	1.696	0.514	2.309	2.297	2.456
France	1.754	0.610	1.811	1.820	1.827	Solomon Islands	1.904	0.329	2.586	2.708	2.924
French Guiana	1.585	0.627	1.811	1.820	1.827	Somalia	1.904	0.414	0.581	0.511	0.547

Gabon	1.585	0.744	2.207	2.186	2.127	South Africa	1.754	0.669	2.307	3.432	3.445
Gambia	1.000	1.010	2.601	2.513	2.516	South Sudan	1.696	0.077	3.162	3.268	3.331
Georgia	1.793	0.657	1.533	1.852	1.811	Spain	1.754	0.557	2.280	2.361	2.019
Germany	1.696	0.634	1.771	1.047	0.156	Sri Lanka	1.754	0.471	3.417	3.980	3.415
Ghana	1.696	0.706	2.188	2.262	2.269	St Kitts & Nevis	1.953	1.665	2.489	3.181	3.166
Greece	1.696	0.932	2.313	1.907	2.141	St Lucia	1.453	0.404	1.370	1.303	1.105
Greenland	1.661	0.868	1.563	1.620	1.565	St Vincent Grenadines	1.966	1.311	1.799	2.360	2.480
Grenada	1.661	0.299	3.247	3.336	3.401	Sudan	1.754	0.414	2.048	2.509	2.437
Guatemala	1.696	0.493	1.764	1.625	1.725	Suriname	1.754	0.606	2.604	2.490	2.525
Guinea	1.793	0.572	1.761	1.750	1.786	Swaziland	1.696	0.494	3.408	3.585	3.344
Guinea Bissau	1.793	0.672	2.195	2.218	2.284	Sweden	1.000	0.621	1.501	1.384	1.387
Guyana	1.565	0.763	2.844	3.094	3.234	Switzerland	1.793	0.598	0.924	1.010	1.011
Haiti	1.696	0.750	1.805	1.751	1.783	Syria	1.857	0.429	1.957	2.037	2.100
Honduras	1.696	0.727	1.564	1.429	1.479	Taiwan	1.565	0.494	1.323	0.596	1.468
Hungary	1.793	0.478	1.914	1.941	2.025	Tajikistan	1.696	0.692	2.231	2.375	2.387
Iceland	1.793	0.727	1.842	1.975	2.034	Tanzania	1.696	0.541	2.350	2.896	2.990
India	1.322	0.537	1.646	2.254	2.304	Thailand	1.000	0.536	2.169	2.129	2.288
Indonesia	1.000	0.938	1.098	1.173	1.204	Timor	1.585	0.299	2.018	2.039	1.983
Iran	1.907	0.451	3.099	3.034	3.223	Togo	1.000	0.591	2.591	2.614	2.722
Iraq	1.907	0.563	2.232	2.190	2.289	Tonga	1.696	0.122	1.000	1.004	0.961
Ireland	1.585	0.803	1.105	1.804	1.806	Trinidad	1.000	0.402	2.072	2.076	2.069
Israel	1.793	0.649	2.124	1.885	2.075	Tunisia	1.565	0.651	1.071	0.956	1.115
Italy	1.000	0.623	1.818	1.843	1.800	Turkey	1.000	0.577	0.854	0.894	0.879
Ivory Coast	1.696	0.744	1.966	1.924	1.901	Turkmenistan	1.000	0.670	2.150	2.064	2.044
Jamaica	1.793	0.321	2.098	2.791	2.547	Tuvalu	1.565	0.227	3.029	3.110	3.298
Japan	1.322	0.416	1.039	1.011	1.013	Uganda	1.696	0.609	2.774	2.517	1.938
Jordan	1.857	0.460	2.117	2.644	2.641	Ukraine	1.793	0.692	1.253	1.203	1.170
Kazakhstan	1.000	0.541	1.785	1.713	2.152	UAEs	1.696	0.484	1.865	2.329	2.344
Kenya	1.696	0.545	2.284	2.630	1.281	UK	1.793	0.805	3.412	3.545	3.735
Kiribati	1.661	1.038	2.929	3.233	3.266	USA	1.453	0.697	3.680	3.903	3.917
Korea North	1.696	0.723	2.315	2.339	2.456	Uruguay	1.904	0.489	2.622	2.531	2.635
Korea South	1.000	0.748	1.798	1.946	1.950	Uzbekistan	1.000	0.530	2.516	2.610	2.729
Kuwait	1.696	0.638	2.080	2.477	2.515	Vanuatu	1.565	0.423	2.575	3.078	3.015
Kyrgyzstan	1.565	0.627	1.486	1.779	1.639	Venezuela	1.793	0.642	2.128	2.315	2.391
Laos	1.565	0.614	1.833	1.899	2.001	Vietnam	1.793	0.589	0.731	0.767	0.768
Latvia	1.696	0.794	1.022	1.090	1.148	Yemen	1.565	0.521	1.681	1.801	1.831
Lebanon	1.000	0.660	2.105	2.024	2.147	Zambia	1.696	0.532	1.948	1.853	1.460
Lesotho	1.322	0.520	1.540	2.169	2.230	Zimbabwe	1.696	0.551	3.419	3.906	3.370