A SpatioTemporal Rogue Wave Generator

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Abstract:

Rogue Waves are highly unstable waves extremely localized in space and time, appearing from nowhere and disappearing without leaving a trace. The observation of such waves in oceanography has led to increased interest in the physics community where optical rogue waves generated using microstructured fibers are studied for their interactions and the underlying physical phenomena. In the present article, an extremely simple rogue wave generator is proposed, where a hyperbolic secant based temporal profile evolves according to a hyperbolic secant square spatial profile. The derivative of this spatial profile gives the spatial evolution equation, and this is implemented using FPGA. It is seen that a single rogue wave exponentially rising to its peak amplitude at mid time instant and then quickly decaying is formed. The extremely simple signal-oriented design of rogue wave generator, effectively capturing the spatial and temporal propagation properties of such rogue waves, coupled with the potential applications it enables using the generated electrical rogue waves forms the novelty of the present work.

Keywords: Rogue Waves, Spatiotemporal Generator, Hyperbolic Secant, FPGA

1. Introduction

Rogue Waves, also called Freak Waves, are large, spontaneous, surface waves threatening ships and ocean liners in the far out in open water, and are thus the stuff that make up maritime folklore [1-2]. Distinct from tsunamis, which are massive displacements of water, rogue waves were often considered mythical and lacking in evidence until the "Draupner Wave" incident in 1995, where a rogue wave of a maximum height of 25.6 meters was observed in the North Sea [3-5]. The distinguishing property of a rogue wave is that it lasts very briefly in time in a very limited region in space, essentially appearing from nowhere and disappearing without leaving a trace [2].

A plethora of mechanisms have been suggested explaining rogue wave phenomena, including diffractive focusing, shoaling due to focusing of currents, thermal expansion, wind waves and nonlinear effects such as modulational instability causing an unusual, unstable wave to suck the energy from nearby waves, growing quickly to a monster and collapsing soon after [5-10]. In the last case, rogue waves are seen as the spatiotemporal solutions of the Nonlinear Schrodinger Equation, which has also known to give tsunami-like solutions, called solitons [11-12].

The observations of rogue waves have stirred up interest in the topic within the physics community, where microstructured optical fibers studied near thresholds of modulational instability based soliton-supercontinuum generation have given rise to rogue wave phenomena [13]. These waves have been associated with another solution of the Nonlinear Schrodinger Equation, termed the Peregrine Breather, which differs from a conventional soliton in that it possesses a double spatio-temporal localization, with the peak intensity at maximum compression as much as nine times stronger than the surrounding background [14-15].

In the present work, a spatiotemporal rogue wave generator is proposed and implemented using Field Programmable Gate Array (FPGA). Specifically, an initial time-varying hyperbolic secant signal (sech(t)) is considered, and a spatial evolution profile representing a hyperbolic sceant square ($sech^2(x)$) is defined. By differentiating the spatial profile, a spatial evolution equation is obtained. This is implemented using FPGA, and the surface plot is obtained, illustrating the occurrence of a single rogue wave, with minimal trace. The extreme simplicity of the proposed rogue wave generator coupled with the immense potential of applications using electrical rogue waves thus enabled forms the novelty of the present work.

2. Results and Discussion

As a starting step of the proposed rogue wave generator, a temporal hyperbolic secant function is defined as follows:

$$A(t) = A_0 sech\left(\frac{t-S}{W}\right) \tag{1}$$

where A_0 denotes the peak amplitude and S and W denote the pulse shift (time offset) and width (measured at half-peak value) respectively. This signal represents a bell-shaped curve and is plotted in Fig. 1.

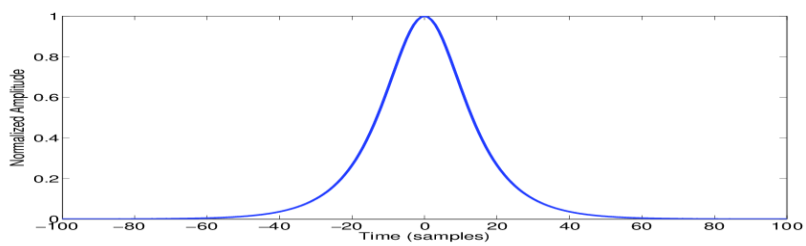

Figure 1 The Hyperbolic Secant Signal

The next step is to define a spatial profile that determines how the temporal signal $A(t)$ evolves during propagation. A one-dimensional space consisting of the x direction alone is considered, and the spatial profile $C(x)$ is given as a hyperbolic secant square function.

$$C(x) = sech^2\left(\frac{x-\Delta}{\Omega}\right) \tag{2}$$

with Δ and Ω denoting the spatial offset and localization respectively. The choice of hyperbolic secant square function is due to the fact that this spatial profile enables the temporal signal to start off extremely small, grow up exponentially in a localized space corresponding to the central region of $C(x)$, and decay rapidly to a very negligible value. For this purpose, the value of A_0 is set to an extremely small amplitude of 0.01.

The space derivative of $C(x)$ yields $C'(x)$, which can be discretized to obtain $C'(i)=C(i+1)-C(i)$, which when rewritten as $C(i+1)=C(i)+C'(i)$, gives the spatial evolution equation as follows describing the relation of the output at each spatial point *i+1* in terms of the corresponding value at the previous spatial point *i*.

$$C(i+1) = C(i) - \frac{2}{\Omega} tanh\left(\frac{x-\Delta}{\Omega}\right) sech^2\left(\frac{x-\Delta}{\Omega}\right) \quad (3)$$

This spatial evolution equation is implemented using the Altera Cyclone 2 FPGA, where 'space' and 'time' is discretized to 200 cells each, with the peak of *A(t)* being launched at *x=100* during *t=1*.

The evolution of the signal through space and time is obtained as a spatiotemporal surface plot, shown in Fig. 2.

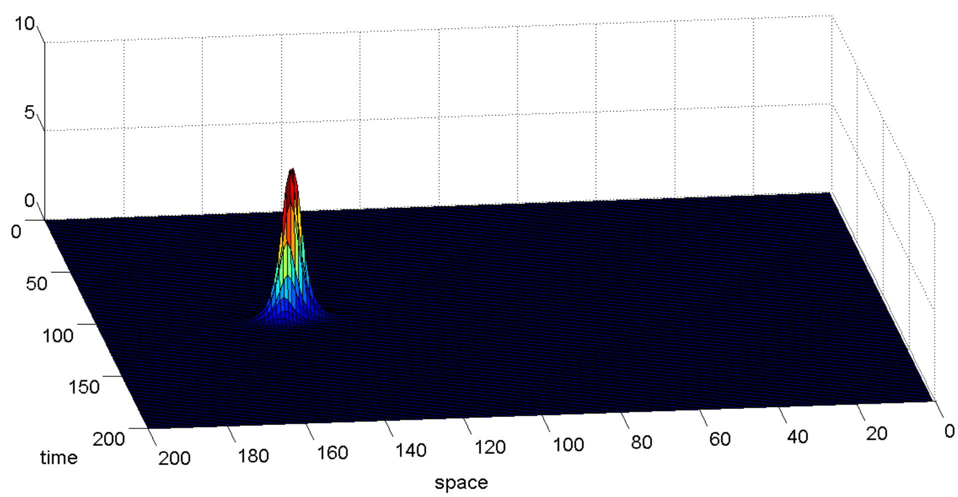

Figure 2 Spatiotemporal Evolution of the Rogue Wave

It is seen from the plot that the signal starts off as an extremely small, almost negligible blip at *x=100* during *t=0*. With the passing of time, the signal then propagates leftwards towards *x=200*, and reaches its peak at around *x=150*, during *t=100*. Following this, the signal soon decays and reaches *x=200*, *t=200* as a small negligible blip.

Thus, it is seen that the spatial and temporal propagation properties of a rogue wave are successfully captured in this extremely simple spatial evolution equation given by Eq. (3), which fulfills theobjective of the present work.

3. Conclusion

The observation of rogue wave phenomena in oceanographic waves has caused considerable interest in optical community regarding the simulation of such waves using optical fibers to study their properties. In this context, the present work proposes, designs and implements an extremely simple spatiotemporal rogue wave generator whose starting step is a hyperbolic secant based temporal signal. Following this, a hyperbolic secant square spatial profile is formulated, whose derivative is seen to yield the spatial evolution equation. The implementation of this design is carried out using FPGA, and it is seen that the surface plot depicts a single rogue wave starting as a small blip during the initial time instant and exponentially crescendoing to a peak at mid time instant, followed by a rapid exponential decay of amplitude back to the small blip. Thus, the spatial and temporal propagation aspects of rogue waves have been reproduced using a spatiotemporal generator whose extreme simplicity of design coupled with the plethora of potential applications thus opened up using electrical rogue waves forms the novelty of the present work.

References

[1] Müller, P. E. T. E. R., Chris Garrett, and A. Osborne. "Rogue waves." Oceanography 18, no. 3 (2005): 66.

[2] Kharif, Christian, and Efim Pelinovsky. Freak waves phenomenon: physical mechanisms and modelling. Springer Vienna, 2006.

[3] Adcock, T. A. A., P. H. Taylor, S. Yan, Q. W. Ma, and P. A. E. M. Janssen. "Did the Draupner wave occur in a crossing sea?." In Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, vol. 467, no. 2134, pp. 3004-3021. The Royal Society, 2011.

[4] Haver, Sverre. Freak wave event at Draupner jacket Januar 1 1995. Statoil, Tech. Rep. PTT-KU-MA, 2003.

[5] Constantin, Adrian, and David Henry. "Solitons and tsunamis." Zeitschrift für Naturforschung A 64, no. 1-2 (2009): 65-68.

[6] Dysthe, Kristian, Harald E. Krogstad, and Peter Müller. "Oceanic rogue waves." Annu. Rev. Fluid Mech. 40 (2008): 287-310.

[7] Zakharov, Vladimir E., A. I. Dyachenko, and A. O. Prokofiev. "Freak waves as nonlinear stage of Stokes wave modulation instability." European Journal of Mechanics-B/Fluids 25, no. 5 (2006): 677-692.

[8] Kharif, Christian, and Efim Pelinovsky. "Physical mechanisms of the rogue wave phenomenon." European Journal of Mechanics-B/Fluids 22, no. 6 (2003): 603-634.

[9] Kharif, Christian, Efim Pelinovsky, and Alexey Slunyaev. Quasi-Linear Wave Focusing. Springer Berlin Heidelberg, 2009.

[10] Onorato, Miguel, S. Residori, U. Bortolozzo, A. Montina, and F. T. Arecchi. "Rogue waves and their generating mechanisms in different physical contexts." Physics Reports 528, no. 2 (2013): 47-89.

[11] Akhmediev, Nail, Adrian Ankiewicz, and J. M. Soto-Crespo. "Rogue waves and rational solutions of the nonlinear Schrödinger equation." Physical Review E 80, no. 2 (2009): 026601.

[12] Dai, Chao-Qing, Yue-Yue Wang, Qing Tian, and Jie-Fang Zhang. "The management and containment of self-similar rogue waves in the inhomogeneous nonlinear Schrödinger equation." Annals of Physics 327, no. 2 (2012): 512-521.

[13] Yeom, Dong-Il, and Benjamin J. Eggleton. "Photonics: rogue waves surface in light." Nature 450, no. 7172 (2007): 953-954.

[14] Kibler, Bertrand, Julien Fatome, Christophe Finot, Guy Millot, Frédéric Dias, Goëry Genty, Nail Akhmediev, and John M. Dudley. "The Peregrine soliton in nonlinear fibre optics." Nature Physics 6, no. 10 (2010): 790-795.

[15] Shrira, Victor I., and Vladimir V. Geogjaev. "What makes the Peregrine soliton so special as a prototype of freak waves?." Journal of Engineering Mathematics 67, no. 1-2 (2010): 11-22.