Gedankenexperiment for looking at the role of acceleration at the beginning of expansion of the universe and its influence on the HUP

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Abstract. We will from first principles examine what adding acceleration does, and will not do as to the HUP previously derived. In doing so we will be examining a Friedmann equation for the evolution of the scale factor, using explicitly two cases, one case being when the acceleration of expansion of the scale factor is kept in, another when it is out, and the intermediate cases of when the acceleration factor, and the scale factor is important but not dominant. In doing so we will be tying it in our discussion with the earlier work done on the HUP. $\left(\frac{\ddot{a}}{a}\right)$ set equal to zero would mean that the ratio $\left(\frac{\dot{a}}{a}\right)$ could have complex solution roots, whereas $\left(\frac{\ddot{a}}{a}\right)$ not equal to zero, constant, but large would frequently imply $\left(\frac{\dot{a}}{a}\right)$ would have three dissimilar real valued roots. From the sake of physical analysis, the situation with $\left(\frac{\ddot{a}}{a}\right)$ not equal to zero yields more tractable result for $\left(\frac{\dot{a}}{a}\right)$ which will have implications for the HUP inequality

Introduction

We will be examining a Friedmann equation for the evolution of the scale factor, using explicitly two cases, one case being when the acceleration of expansion of the scale factor is kept in, another when it is out, and the intermediate cases of when the acceleration factor, and the scale factor is important but not dominant. In doing so we will be tying it in our discussion with the earlier work done on the HUP but from the context of how the acceleration term will affect the HUP, and making sense of [1]

$$
\begin{aligned}
&\left\langle \left(\delta g_{uv}\right)^2 \left(\hat{T}_{uv}\right)^2 \right\rangle \geq \frac{\hbar^2}{V_{Volume}^2} \\
&\xrightarrow[uv\to tt]{} \left\langle \left(\delta g_{tt}\right)^2 \left(\hat{T}_{tt}\right)^2 \right\rangle \geq \frac{\hbar^2}{V_{Volume}^2} \\
&\& \quad \delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+
\end{aligned}
\tag{1}
$$

Namely we will be working with [2]

$$
\begin{aligned}
&\delta t \Delta E = \frac{\hbar}{\delta g_{tt}} \equiv \frac{\hbar}{a^2(t)\cdot\phi} << \hbar \\
&\Leftrightarrow S_{initial}\left(with\left[\delta g_{tt}\right]\right) = \left(\delta g_{tt}\right)^{-3} S_{initial}\left(without\left[\delta g_{tt}\right]\right) >> S_{initial}\left(without\left[\delta g_{tt}\right]\right)
\end{aligned}
\tag{2}
$$

I.e. the fluctuation $\delta g_{tt} << 1$ dramatically boost initial entropy. Not what it would be if $\delta g_{tt} \approx 1$. The next question to ask would be how could one actually have [1, 2, 3]

$$\delta g_{tt} \sim a^2(t) \cdot \phi \xrightarrow[\phi \sim Very\,Large]{} 1 \tag{3}$$

In short, we would require an enormous 'inflaton' style ϕ valued scalar function, and $a^2(t) \sim 10^{-110}$ How could ϕ be initially quite large? Within Planck time the following for mass holds, as a lower bound[1]

$$m_{graviton} \geq \frac{2\hbar^2}{(\delta g_{tt})^2 l_P^2} \cdot \frac{(E-V)}{\Delta T_{tt}^2} \tag{4}$$

Here, [1]

$$K.E. \sim (E-V) \sim \dot{\phi}^2 \propto a^{-6} \tag{5}$$

Then [1]

$$\dot{\phi} \sim a^{-3} \Leftrightarrow \phi \approx t \cdot a^{-3} + H.O.T \tag{6}$$

The question to ask, now, is about the acceleration of the scale factor, due to time. Which will be the subject of our inquiry as to the next section of this document.

2. How could anyone get the acceleration of the Universe factored into our scale factor?

Begin looking at material from page 483-485 of [4]

$$\left(\frac{\dot{a}}{a}\right)^3 - \frac{3}{2} \cdot \left(\frac{\dot{a}}{a}\right)^2 - 2 \cdot \left(\frac{\ddot{a}}{a}\right) \cdot \left(\frac{\dot{a}}{a}\right) + \left[\frac{-1}{64\pi G \cdot t^2} + \frac{\Lambda}{2}\right] = 0 \tag{7}$$

Then, consider two cases of what to do with the ration of $\left(\frac{\dot{a}}{a}\right)$ and solve the above as a cubic equation.

2a. *What if $\left(\frac{\ddot{a}}{a}\right)$ ~ vanishingly small contribution.* (*low acceleration*)

$$\left(\frac{\dot{a}}{a}\right)^3 - \frac{3}{2} \cdot \left(\frac{\dot{a}}{a}\right)^2 + \left[\frac{-1}{64\pi G \cdot t^2} + \frac{\Lambda}{2}\right] \cong 0 \tag{8}$$

Then, using the idea of a 'repressed cubic' we will have the following solution for $\left(\frac{\dot{a}}{a}\right)$, namely [5]

$$\left(\frac{\dot{a}}{a}\right) = Solution = \xi \tag{9}$$

2a.1**: Solutions for Eq. (8), in reduced Cubic form for Eq.(8)**

$$\xi = A+B, \frac{\sqrt{-3}}{2}\cdot(A-B)-\left(\frac{A+B}{2}\right), \frac{-\sqrt{-3}}{2}\cdot(A-B)-\left(\frac{A+B}{2}\right) \tag{10}$$

$$A = \left(\left(\frac{-1}{128\pi G\cdot t^2}+\frac{\Lambda}{4}\right)+\sqrt{\frac{1}{4}\cdot\left(\frac{-1}{64\pi G\cdot t^2}+\frac{\Lambda}{2}\right)^2+\frac{1}{8}}\right)^{1/3}$$
$$B = -\left(\left(\frac{1}{128\pi G\cdot t^2}-\frac{\Lambda}{4}\right)+\sqrt{\frac{1}{4}\cdot\left(\frac{-1}{64\pi G\cdot t^2}+\frac{\Lambda}{2}\right)^2+\frac{1}{8}}\right)^{1/3} \tag{11}$$

Then by [ ] page 9

$$\Theta = \frac{1}{8}\cdot\left[2\cdot\left(\frac{-1}{64\pi G\cdot t^2}+\frac{\Lambda}{2}\right)^2-1\right] \tag{12}$$

$$\begin{aligned} &\Theta > 0 \Rightarrow \xi \quad has, 1st-real, 2nd-imaginary, 3rd-imaginary \\ &\Theta = 0 \Rightarrow \xi \quad has, 3-real-roots, 2-of-3-roots-equal \\ &\Theta < 0 \Rightarrow \xi \quad has, 3-real-roots, all-roots-unequal \end{aligned} \tag{13}$$

The situation to watch is when the time, t, is extremely small. Then one is having to work with the situation where $\Theta > 0 \Rightarrow \xi \quad has, 1st-real, 2nd-imaginary, 3rd-imaginary$. I.e. the situation is then dominated with one real root and two imaginary roots. The value of what happens to $\left(\frac{\dot{a}}{a}\right) = Solution = \xi$ is one which will be commented upon if there is one real root, and two imaginary. What would be a possible constraint upon would be if we had, for non-dimensionalized units

$$\begin{aligned} &\left[2\cdot\left(\frac{-1}{64\pi G\cdot t^2}+\frac{\Lambda}{2}\right)^2-1\right] \approx 0 \Leftrightarrow \left(\frac{-1}{64\pi G\cdot t^2}+\frac{\Lambda}{2}\right) \approx \frac{1}{\sqrt{2}} \\ &\Leftrightarrow \Lambda \approx \sqrt{2}+\frac{1}{32\pi G\cdot t^2} \end{aligned} \tag{14}$$

I.e. for the case that one uses non-dimensionalized units we would have, then

$$\Theta \le 0 \Leftrightarrow \Lambda \ge \sqrt{2}+\frac{1}{32\pi G\cdot t^2} \tag{15}$$

How likely is this to happen, in the Pre Planckian regime ? Not likely. Secondly what we get is

$$\Theta \le 0 \Rightarrow \xi \quad has, 3-real-roots \tag{16}$$

So if we neglect having the acceleration of the scale factor, by abandoning $\left(\frac{\ddot{a}}{a}\right)$ acceleration, we get a weird family of solutions for Eq.(8)

2a.2: **Solutions for Eq. (7), in Cubic form for Eq.(7) gained by NOT abandoning** $\left(\frac{\ddot{a}}{a}\right)$

Following [4] look first at

$$
\begin{aligned}
\tilde{a}_1 &= -2\cdot\left[\left(\frac{\ddot{a}}{a}\right)+3/2\right] \\
\tilde{b}_1 &= -\frac{1}{4}\cdot\left[1+4\cdot\left(\frac{\ddot{a}}{a}\right)+\frac{1}{256\pi G\cdot t^2}-\frac{\Lambda}{8}\right]
\end{aligned}
\tag{17}
$$

Our approximation is, to set $\left(\frac{\ddot{a}}{a}\right)$ as a constant, but not zero. If so then set $\left(\frac{\ddot{a}}{a}\right)$ as a non dimensional but very large quantity. Then a solution exists as given as for a reduced cubic version of Eq.(7) which can be given by

$$
\xi_1 = A_1 + B_1, \frac{\sqrt{-3}}{2}\cdot\left(A_1 - B_1\right) - \left(\frac{A_1+B_1}{2}\right), \frac{-\sqrt{-3}}{2}\cdot\left(A_1 - B_1\right) - \left(\frac{A_1+B_1}{2}\right)
\tag{18}
$$

And

$$
\begin{aligned}
A_1 &= \sqrt[3]{-\frac{\tilde{b}_1}{2}+\sqrt{\left(\frac{\tilde{b}_1}{2}\right)^2+\frac{\tilde{a}_1^{\,3}}{27}}} \\
B_1 &= -\sqrt[3]{\frac{\tilde{b}_1}{2}+\sqrt{\left(\frac{\tilde{b}_1}{2}\right)^2+\frac{\tilde{a}_1^{\,3}}{27}}}
\end{aligned}
\tag{19}
$$

And when $\left(\frac{\ddot{a}}{a}\right)$ is set as a non dimensional constant quantity and possibly quite large, then

$$
\left(\frac{\dot{a}}{a}\right)_1 = Solution = \xi_1 + \frac{1}{2}
\tag{20}
$$

If so then

$$
\Theta_1 = \frac{\left(\tilde{b}_1\right)^2}{4} + \frac{\left(\tilde{a}_1\right)^3}{27}
\tag{21}
$$

If $\left(\frac{\ddot{a}}{a}\right)$ is constant and very large, the results of the sign of Eq.(21) are as follows

$$
\begin{aligned}
&\Theta_1 > 0 \Rightarrow \xi_1 \quad has, 1st - real, 2nd - imaginary, 3rd - imaginary \\
&\Theta_1 = 0 \Rightarrow \xi_1 \quad has, 3 - real - roots, 2 - of - 3 - roots - equal \\
&\Theta_1 < 0 \Rightarrow \xi_1 \quad has, 3 - real - roots, all - roots - unequal
\end{aligned}
\tag{22}
$$

Here, with very large constant initial $\left(\frac{\ddot{a}}{a}\right)$ we have that the third outcome is by far most likely to happen, in contrast to what would happen in the situation with $\left(\frac{\ddot{a}}{a}\right)$ = 0 . This in doing so is a bridge to fully implementing Giovannini's [3]

3. Conclusion: Making sense of if $\delta g_{tt} \sim a^2(t) \cdot \phi \xrightarrow[\phi \sim Very\,Large]{} 1$ holds, or does not hold.

The following will be asserted , i.e. in a regime of space-time delineated as Pre Planckian, that there is an interval of time, Δt which is less than Planck time ~ 10^-44 seconds for which the following will hold, if we use the case given by Eq.(17) to Eq.(22) then due to $\left(\frac{\ddot{a}}{a}\right)$ being large and not negative

$$
\delta g_{tt} \sim \left.\frac{t}{a(t)}\right|_t \xrightarrow[t \to t + \Delta t]{} \theta \propto \xi^+ << 1 \tag{23}
$$

Thereby we will have [1]

$$
\begin{aligned}
&\delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \\
&Unless \quad \delta g_{tt} \sim O(1)
\end{aligned}
\tag{24}
$$

Similarly, if Eq. (9) to Eq.(16) hold, then we will have, due to $\left(\frac{\ddot{a}}{a}\right)$ equal zero, an increased likely hood of $\delta g_{tt} \sim a^2(t) \cdot \phi \xrightarrow[\phi \sim Very\,Large]{} 1$ holding. Thereby falsifying the conditions for which Eq.(24) hold. This above will give additional meaning to the following, namely from [1] the below Eq.(25) goes to Eq.(24).This adopting not just from [1] but from [6,7]

$$
\begin{aligned}
&V^{(4)} = \delta t \cdot \Delta A \cdot r \\
&\delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} \geq \frac{\hbar}{2} \\
&\Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}
\end{aligned}
\tag{25}
$$

As well as give a starting point for the speed of a graviton, in early space-time conditions as written as , with $v_{graviton}$ the speed of a graviton, and $m_{graviton}$ the rest mass of a graviton, and $E_{graviton}$ in the inertial rest frame given as: [8,9]

$$\left(\frac{v_{graviton}}{c}\right)^2 = 1 - \frac{m_{graviton}^2 c^4}{E_{graviton}^2} \tag{26}$$

Verification of this formula, Eq.(26) from relic conditions will be part of our future research endeavor. In addition we also hope to answer some of the issues Barbour raised in [10, 11] from our modified HUP stand point as written above. As well as formulate officially more the derivations given in [12]

Acknowledgements

This work is supported in part by National Nature Science Foundation of China grant No. 11375279

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