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The Special Problems of Euclidean Geometry , and Relativity .

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Abstract: **The Special Problems of E-geometry consist the , Mould Quantization , of Euclidean Geometry in it to become → Monad , through mould of Space –Anti-space in itself ,** ***which is the material dipole in inner monad Structure as the Electromagnetic cycloidal field*** **→ Linearly , through mould of Parallel Theorem ,** ***which are the equal distances between points of parallel and line*** **→ In Plane , through mould of Squaring the circle ,** ***where the two equal and perpendicular monads consist a Plane acquiring the common Plane-meter*** **→ and in Space (volume) , through mould of the Duplication of the Cube ,** ***where any two Unequal perpendicular monads acquire the common Space-meter to be twice each other*** **, as analytically all methods explained .**

Because of Geometers scarcity, I was instigated to republish this article. *Weakness created Non-Euclid geometries which deviated GR in Space-time confinement* , not conceiving the beyond Planck`s existence , *not explaining the WHY speed of light is constant*. In the manuscript is prooved that parallel postulate is only in Plane (*three points only and not a Spherical triangle*) , *and now is proved to be a Theorem* , where all properties of Euclidean geometry compactly exist as Extrema ***Quantization*** on , points , lines , planes , circles and spheres. Projective , Hyperbolic and Elliptic geometry is proved to be Extrema (deviations) in Euclidean geometry where on them Einstein's theory of general relativity is implicated to the properties of physical space . The universally outstanding denial perception that *Proof by geometric logic only is inaccessible , is now contradicted* . Furthermore consists the conceiving of Geometric logic and knowledge .. In Conclutions (9) is referred Geometrical mechanism of composing Spaces and the way of Quantization of Euclidean geometry to its constitutes i.e. *from point ,Segment ,Plane ,* to Volume , as the Physical world elements, through Extrema Principle. *It was attributed , → The Extrema in* { [7.1] → Zeno`s Paradox , [7-2] → Dichotomy Paradox , [7-3] → Arrow Paradox , [7-4] → Algebraic numbers , [7-5] → Natural numbers}, *and in* {[7-6] → *the Regular Polygons* was measured the side of Heptagon , [7-8] → Trisection of angle *by reducing the problem in monad Extrema type ,* even if the problem is not Plane , [7-6.1] → *the Doubling of the Cube ,* [7-8.2] → *the Special problem of Squaring the circle*, giving number π and are shown the Moulds and the Meters of ***Quantization of Euclidean geometry to the Physical world , and to Physics ,*** based on the Geometrical logic alone , which is according to Pythagoras , → **Unit** *is a Point without Position while* **a Point** *is a Unit having Position* . [43] . It is a provocation to all scarce today Geometers and Mathematicians to conceive the scientific depth of this article.

Keywords : Special Problems of Euclid Geometry and Relativity , Geometry and Physics , The Unsolved Special Problems

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CONTENTS :

1. Introduction

Euclid’s elements consist of assuming a small set of intuitively appealing axioms and from them, proving many other propositions (theorems). Although many of Euclid’s results have been stated by earlier Greek mathematicians, Euclid was the first to show how these propositions could be fit together into a comprehensive deductive and logical system self consistent. Because nobody until now succeeded to prove the parallel postulate by means of pure geometric logic and under the restrictions imposed to seek the solution, many self consistent non-Euclidean geometries have been discovered based on Definitions, Axioms or Postulates, in order that non of them contradicts any of the other postulates of what actually are or mean. In the manuscript is proved that parallel postulate is only in Plane (three points only) and is based on the four Postulates for Constructions, where all properties of Euclidean geometry compactly exist as Extrema on points, lines, planes, circles and spheres. Projective, Hyperbolic and Elliptic geometry is proved to be an Extrema (deviations) [16] in Euclidean geometry where on them Einstein's theory of general relativity is implicated and calls a segment as line and the disk as plane in physical space.

It have been shown that the only Space-Energy geometry is the Euclidean, on primary and on any vector unit AB, (AB = The Quantization of points and of Energy on AB vector) on the contrary to the general relativity of Space-time which is based on the rays of the non-Euclidean geometries and to the limited velocity of light. Euclidean geometry describes Space-Energy beyond Plank´s length level as monas in Space and also in its deviations which are described as Space-time in Plank´s length level. Quantization is holding only on points and Energy [Space-Energy] ,where Time is vanished [PNS], and not on points and Time [Space-time] which is the deviation of Euclidean geometry. [21]

2. Euclid Elements for a Proof of the Parallel Postulate (Axiom)

Axiom or Postulate is a statement checked if it is true and is ascertained with logic (the experiences of nature as objective reality).

Theorem or Proposition is a non-main statement requiring a proof based on earlier determined logical properties.

Definition is an initial notion without any sensible definition given to other notions.

Definitions, Propositions or Postulates created Euclid geometry using the geometrical logic which is that of nature, the logic of the objective reality.

Using the same elements it is possible to create many other geometries but the true uniting element is the before refereed.

2.1. The First Definitions (D) of Terms in Geometry

D1: A point is that which has no part (Position)

D2: A line is a breathless length (for straight line, the whole is equal to the parts)

D3: The extremities of lines are points (equation).

D4: A straight line lies equally with respect to the points on itself (identity).

D: A midpoint C divides a segment AB (of a straight line) in two. CA = CB any point C divides all straight lines through this in two.

D: A straight line AB divides all planes through this in two.

D: A plane ABC divides all spaces through this in two

2.2. Common Notions (Cn)

Cn1: Things which equal the same thing also equal one another.

Cn2: If equals are added to equals, then the wholes are equal.

Cn3: If equals are subtracted from equals, then the remainders are equal.

Cn4: Things which coincide with one another, equal one another.

Cn5: The whole is greater than the part.

2.3. The Five Postulates (P) for Construction

P1. To draw a straight line from any point A to any other point B .

P2. To produce a finite straight line AB continuously in a straight line.

P3. To describe a circle with any centre and distance. P1, P2 are unique.

P4. That, all right angles are equal to each other.

P5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side on which are the angles less than the two right angles, or (for three points on a plane)

5a. The same is plane’s postulate which states that, from any point M, not on a straight line AB, only one line MM´ can be drawn parallel to AB.

Since a straight line passes through two points only and because point M is the third then the parallel postulate it is valid on a plane (three points only).

3. The Method

AB is a straight line through points A, B , AB is also the measurable line segment of line AB , and M any other point . When MA+MB > AB then point M is not on line AB . (differently if MA+MB = AB , then this answers the question of why any line contains at least two points) ,

i.e. for any point M on line AB where is holding MA+MB = AB , *meaning that line segments* MA,MB *coincide on* AB , is thus proved from the other axioms and so D2 is not an axiom . → To prove that , one and only one line MM´ can be drawn parallel to AB.

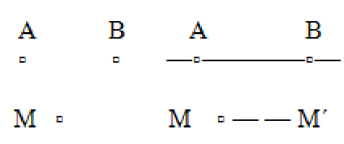


Figure 1. *The three points (a Plane)*

To prove the above Axiom is necessary to show:

a. The parallel to AB is the locus of all points at a constant distance **h** from the line AB, and for point M is MA1,
b. The locus of all these points is a straight line.

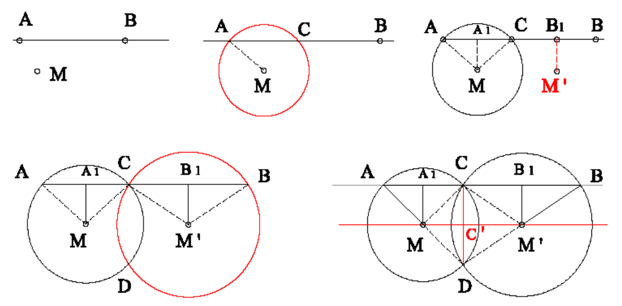


Figure 2. *The Method – (3)*

Step 1

Draw the circle (M, MA) be joined meeting line AB in C. Since MA = MC, point M is on mid-perpendicular of AC. Let A1 be the midpoint of AC, (it is A1A+A1C = AC because A1 is on the straight line AC. Triangles MAA1, MCA1 are equal because the three sides are equal, therefore angle < MA1A = MA1C (CN1) and since the sum of the two angles < MA1A+MA1C = 180° (CN2, 6D) then angle < MA1A = MA1C = 90 °.(P4) so, MA1 is the minimum fixed distance **h** of point M to AC.

Step 2

Let B1 be the midpoint of CB,(it is B1C+B1B = CB because B1 is on the straight line CB) and draw B1M′ = h equal to A1M on the mid-perpendicular from point B1 to CB. Draw the circle (M′, M′B = M′C) intersecting the circle (M, MA = MC) at point D.(P3) Since M′C = M′B, point M′ lies on mid-perpendicular of CB. (CN1)

Since M′C = M′D, point M′ lies on mid-perpendicular of CD. (CN1) Since MC = MD, point M lies on mid-perpendicular of CD. (CN1) Because points M and M′ lie on the same mid-perpendicular (This mid-perpendicular is drawn from point C′ to CD and it is the midpoint of CD) and because only one line MM′ passes through points M, M ′ then line MM′ coincides with this mid-perpendicular (CN4)

Step 3

Draw the perpendicular of CD at point C′. (P3, P1)

a. Because MA1 ⊥ AC and also MC′ ⊥ CD then angle < A1MC′ = A1 CC′. (Cn 2,Cn3,E.I.15) Because M′B1 ⊥ CB and also M′C′⊥ CD then angle < B1M′C′ = B1CC′. (Cn2, Cn3, E.I.15)
b. The sum of angles A1CC′ + B1CC′ = 180▫ = A1MC′ + B1M′C′. (6.D), and since Point C′ lies on straight line MM′, therefore the sum of angles in shape A1B1M′M are < MA1B1 + A1B1M ′ + [B1M′ M + M′MA1] = 90▫ + 90▫ + 180▫ = 360▫ (Cn2) , i.e. The sum of angles in a Quadrilateral is 360▫ and in Rectangle all equal to 90 ▫. (m)
c. The right-angled triangles MA1B1, M′B1A1 are equal because A1M = B1M′ and A1B1 common, therefore side A1M′ = B1M (Cn1). Triangles A1MM′,B1M′M are equal because have the three sides equal each other, therefore angle < A1MM′ = B1M′M, and since their sum is 180▫ as before (6D), so angle < A1MM′ = B1M′M = 90▫ (Cn2).
d. Since angle < A1MM′ = A1CC′ and also angle < B1M′M = B1CC′ (P4), therefore quadrilaterals A1CC′M, B1CC′M′, A1B1M′M are Rectangles (CN3). From the above three rectangles and because all points (M, M′ and C′) equidistant from AB, this means that C′C is also the minimum equal distance of point C′ to line AB or, h = MA1 = M′ B1 = CD / 2 = C′C (Cn1) Namely, line MM′ is perpendicular to segment CD at point C′ and this line coincides with the mid-perpendicular of CD at this point C′ and points M, M′ ,C′ are on line MM′. Point C′ equidistant ,h, from line AB, as it is for points M ,M′, so the locus of the three points is the straight line MM′, so the two demands are satisfied, (h = C′C = MA1 = M′B1 and also C′C ⊥ AB, MA1 ⊥ AB, M′B1 ⊥ AB). (o.ε.δ.)
e. The right-angle triangles A1CM, MCC′ are equal because side MA1= C′C and MC common so angle <A1CM = C′MC, and the Sum of angles C′MC + MCB1 = A1CM + MCB1 = 180▫

3.1. The Succession of Proofs

1. Draw the circle (M, MA) be joined meeting line AB in C and let A1, B 1 be the midpoint of CA, CB.
2. On mid-perpendicular B1M′ find point M′ such that M′B1 = MA1 and draw the circle (M′, M′B = M′C) intersecting the circle (M, MA = MC) at point D.
3. Draw mid-perpendicular of CD at point C′.
4. To show that line MM′ is a straight line passing through point C ′ and it is such that MA1 = M′B1 = C′C = h , i.e. a constant distance h from line AB or, also The Sum of angles C′MC + MCB1 = A1CM + MCB1 = 180 ▫

3.2. Proofed Succession

1. The mid-perpendicular of CD passes through points M, M ′.
2. Angle < A1MC′ = A1 MM′ = A1CC′, Angle < B1M′C′ = B1M′M = B1CC′ <A1MC′ =A1CC′ because their sides are perpendicular among them i.e.

   MA1⊥CA,MC′⊥CC′.

   a. In case < A1MM′ +A1CC′=180▫ and B1M′M +B1CC′ = 180▫ then < A1MM′ = 180▫ - A1CC′, B1M′M = 180▫ - B1CC′, and by summation < A1MM′ + B1M′M = 360▫ - A1CC′- B1CC′ or Sum of angles < A1MM′ + B1M′M = 360 - (A1CC′ + B1CC′) = 360 -180▫ = 180▫

3. The sum of angles A1MM′ + B1M′M = 180▫ because the equal sum of angles A1CC′ + B1CC′ = 180▫, so the sum of angles in quadrilateral MA1B1 M′ is equal to 360▫.

4. The right-angled triangles MA1B1, M′B1A1 are equal, so diagonal MB1 = M′A1 and since triangles A1MM′, B1M′M are equal, then angle A1MM′ = B1M′M and since their sum is 180 ▫, therefore angle < A1MM′ = MM′B1 = M′B1A1 = B1A1M = 90 ▫

5. Since angle A1CC′ = B1CC′ = 90▫, then quadrilaterals A1CC′M, B1CC′M′ are rectangles and for the three rectangles MA1CC′, CB1M′C′, MA1B1M′ exists MA1 = M′B1 = C′C
6. The right-angled triangles MCA1, MCC′ are equal, so angle < A1CM = C′MC and since the sum of angles < A1CM + MCB1 = 180 ▫ then also C′MC + MCB1 = 180 ▫ → which is the second to show, as this problem has been set at first by Euclid.

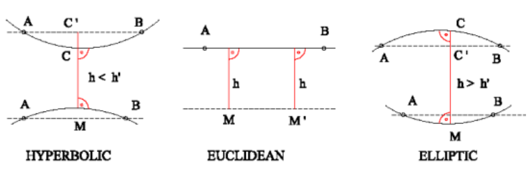
All above is a Proof of the Parallel postulate due to the fact that the parallel postulate is dependent of the other four axioms (***now is proved as a theorem from the other four***). Since line segment AB is common to ∞ Planes and only one Plane is passing through point M (Plane ABM from the three points A, B, M, then the Parallel Postulate is valid for all Spaces which have this common Plane , as Spherical, n-dimensional geometry Spaces. It was proved that it is a necessary logical consequence of the others axioms, agree also with the Properties of physical objects, d + 0 = d , d * 0 = 0 , now is possible to decide through mathematical reasoning , that the geometry of the physical universe is Euclidean . Since the essential difference between Euclidean geometry and the two non-Euclidean geometries , Spherical and hyperbolic geometry , is the nature of parallel line, i.e. the parallel postulate so , << ***The consistent System of the - non-Euclidean geometry - have to decide the direction of the existing mathematical logic*** >>.

The above consistency proof is applicable to any line Segment AB on line AB,(segment AB is the first dimensional unit, as AB = 0 → ∞), from any point M not on line AB, [MA + MB > AB for three points only which consist the Plane. For any point M between points A, B is holding MA+MB = AB i.e. from two points M, A or M, B passes the only one line AB. A line is also continuous (P1) with points and discontinuous with segment AB [14] ,which is the metric defined by non- Euclidean geometries ,and it is the answer to the cry about the < crisis in the foundations of Euclid geometry > (F.2)

3.3. A Line Contains at Least Two Points, is Not an Axiom Because is Proved as Theorem

Definition D2 states that for any point M on line AB is holding MA+MB = AB which is equal to < segment MA + segment MB is equal to segment AB > i.e. the two lines MA, MB coincide on line AB and thus this postulate is proved also from the other axioms, thus D2 is not an axiom, which form a system self consistent with its intrinsic real-world meaning. F.a → *F.2.*

4. The Types of Geometry



The structure of Euclidean geometry
Figure 3. *(Hyperbolic – Euclidean - Elliptic)- (4)*

Any single point A constitutes a Unit without any Position and dimension (non-dimensional = Empty Space) simultaneously zero , finite and infinite . The unit meter of Point is equal to 0.

Any single point B, not coinciding with A, constitutes another one Unit which has also dimension zero. Only one Straight line (i.e. the Whole is equal to the Parts) passes through points A and B, which consists another un-dimensional Unit since is consisted of infinite points with dimension zero. A line Segment AB between points A and B (either points A and B are near zero or are extended to the infinite), consists the first Unit with one dimensional, the length AB, beginning from Unit A and a regression ending in Unit B. Line segment AB = 0 → ∞, is the one-dimensional Space. The unit meter of AB is m = 2.(AB/2) = AB because only one middle point exists on AB and since also is composed of infinite points which are filling line, then nature of line is that of Point (the all is one for Lines and Points).

Adding a third point C , outside the straight line AB , (CA+CB >AB), then is constituted a new Unit (the Plane) without position, since is consisted of infinite points, without any position. Shape ABC enclosed between parts AB, AC, BC is of two dimensional, the enclosed area ABC, and since is composed of infinite Straight lines which are filling Plane, then, nature of Plane is that of Line and that of Points (the all is one for Planes, Lines and Points). Following harmony of unit meter AB=AC=BC, then Area ABC = 0 → ∞, is the two-dimensional Space with unit meter equal to $m = 2.(\pi.AB/\sqrt{2})^2 = \pi.\ AB^2$, i.e. one square equal to the area of the unit circle.

Four points A,B,C,D (....) not coinciding, consist a new Unit (the Space or Space Layer) without position also, which is extended between the four Planes and all included, forming Volume ABCD and since is composed of infinite Planes which are filling Space, then, nature of Space is that of Plane and that of Points (the all is one for Spaces, Planes, Lines and Points). Following the same harmony of the first Unit, shape ABCD is the Regular Tetrahedron with volume ABCD = 0 → ∞, and it is the three-dimensional Space. The dimension of Volume is 4 - 1 = 3. The unit measure of volume is the side X of cube X^3 twice the volume of another random cube of side a = AB such as $X^3 = 2.a^3$ and $X = \sqrt[3]{2}.a$. Geometry measures Volumes with side X related to the problem of doubling of the cube. In case that point D is on a lower Space Layer, then all Properties of Space, or Space Layer are transferred to the lower corresponding Unit , ***i.e. an inverse quantization from volume to Plane , from Plane the Straight line , and then to the Point, which is the quantization of units in E-Geometry.***

This Concentrated (Compact) Logic of geometry [CLG] exists for all Space – Layers and is very useful in many geometrical and physical problems. (exists, Quality = Quantity, since all the new Units are produced from the same, the first one, dimensional Unit AB).

N points represent the N-1 dimensional Space or the N-1 Space Layer, DL, and has analogous properties and measures. Following the same harmony for unit AB, (AB = 0 → ∞) then shape ABC...M (i.e. the ∞ spaces AB = 1, 2,..nth) is the Regular Solid in Sphere ABC...M = 0 → ∞. This N Space Layer is limiting to ∞ as N → ∞.

Proceeding inversely with roots of any unit AB = 0 → ∞ (i.e. the Sub-Spaces are the roots of AB, $\sqrt[2]{AB}$, $\sqrt[3]{AB}$,.. $\sqrt[n]{AB}$ then it is $\sqrt[n]{AB} = 1$ as n → ∞), and since all roots of unit AB are the vertices of the Regular Solids in Spheres then this n Space Layer is limiting to 0 as n→∞ The dimensionality of the physical universe is unbounded (∞) but simultaneously equal to (1) as the two types of Spaces and Sub-Spaces show.

Because the unit-meters of the N-1 dimensional Space Layers coincide with the vertices of the nth-roots of the first dimensional unit segment AB as AB = ∞ → 0 ,which is point, (the vertices of the n-sided Regular Solids), therefore the two Spaces are coinciding (the Space Layers and the Sub-Space Layers are in superposition on the same monads).[F.5]

That is to say, Any point on the Nth Space or Space-Layer, of any unit AB = 0 →∞, jointly exists partly or whole, with all Subspaces of higher than N Spaces, N = (N+1) -1 = (N+2) -2 = (N+N)-N...= (N+∞) - ∞, where (N+1),…(N+∞) are the higher than N Spaces, and with all Spaces of lower than N Subspaces, N = (N-1) +1 = (N-2)+2 = (N-N)+N = (N -∞) +∞, where (N-1), (N-2), (N-N), (N-∞) are the lower than N Spaces. The boundaries of N points, corresponding to the Space, have their unit meter of the Space and is a Tensor of N dimension (i.e. the unit meters of the N roots of unity AB), simultaneously, because belonging to the Sub-Space of the Unit Segments > N, have also the unit meter of all spaces. [F.5]

1. The Space Layers: (or the Regular Solids) with sides equal to line-segment AB = 0 → ∞ The Increasing Plane Spaces with the same Unit. (F.3)

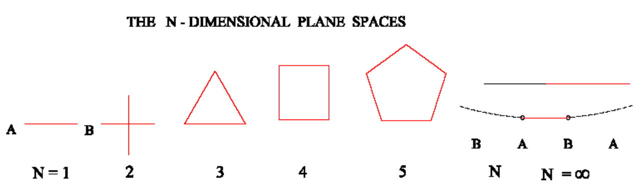


***Figure 4**. The Increasing Space Layers –(4)*

The Sub-Space Layers : (or the Regular Solids on AB) as Roots of AB = 0 → ∞ . The Decreasing Plane Spaces with the same Unit. (F.4)

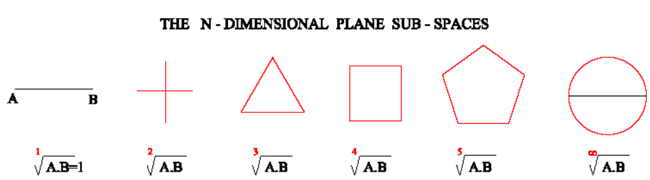


***Figure 5**. The decreasing (Sub-Spaces) Layers – (4)*

2. The superposition of Plane Space Layers and Sub-Space Layers: (F.6) The simultaneously co-existence of Spaces and Sub-Spaces of any Unit AB = 0 → ∞ , i.e.

   Euclidean, Elliptic, Spherical, Parabolic, Hyperbolic, Geodesics, metric and non-metric geometries have Unit AB as common. The Interconnection of Homogeneous and Heterogeneous Spaces, and Subspaces of the Universe. [F.6] . In the same monad AB , coexist the ± Spaces Layers and the ± Sub-Spaces and thus forming the united Unit , which is the monad or quaternion or any other complex magnitude .

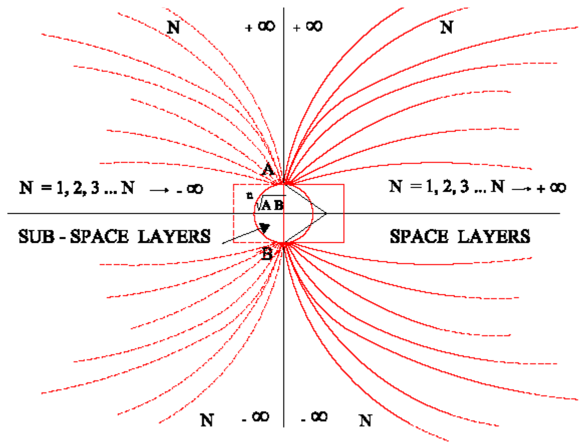


***Figure 6.** The Superposition of Spaces – (4)*

3. A linear shape is the shape with N points on a Plane bounded with straight lines. A circle is the shape on a Plane with all points equally distance from a fix point O. A curved line is the shape on a Plane with points not equally distance from a fix point O. Curved shapes are those on a Plane bounded with curved lines. Rotating the above axial-centrifugally (machine AB ⊥ AC) is obtained Flat Space, Conics, Sphere, Curved Space, multi Curvature Spaces, Curved Hyperspace etc. The fact that curvature changes from point to point, is not a property of one Space only but that of the common area of more than two Spaces, namely the result of the Position of Points. Euclidean manifold (Point, sectors, lines, Planes, all Spaces etc) and the one dimentional Unit AB is prooved to be the same thing (according to Euclid ἑν το πάν). [F.5]

Since Riemannian metric and curvature is on the great circles of a Sphere which consist a Plane, say AMA΄, while the Parallel Postulate is consistent with three points only, therefore the great circles are not lines (this is because it is MA + MA΄ > AA΄) and the curvature of Space is that of the circle in this Plane, i.e. that of the circle (O, OA), which are more than three points. Because Parallel Axiom is for three points only, which consist a Plane, then the curvature of < empty space > is equal to 0 , (Points have not any metric or intrinsic curvature). [F.6]

The physical laws are correlated with the geometry of Spaces and can be seen, using CLG, in Plane Space as it is shown in figures F3 - F5 and also in regular polygons which are Algebraic equations of any degree. A Presentation of the method is seen on Dr Geo-Machine Macro-constructions.

Perhaps, Inertia is the Property of a certain Space Layer, which is the conserved work as a field, and the Interaction of Spaces happening at the Commons (Horizon of Space, Anti-Space) or those have been called Concentrated Logic = Spin, and so create the motion. [42-43] .

Today has been shown that this common horizon is the common circle of Space , Anti-space equilibrium , which creates breakages and by collision all particles , dark matter and dark energy exist in these Inertial systems , STPL lines .

Gravity field is one of the finest existing Space and Anti-space quantization , which is restrained by gravity force.[41]

Hyperbolic geometry and straight lines:

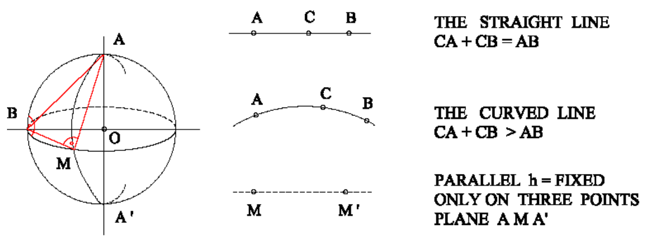


Figure 7. *The Euclidean straight line and the others – (4)*

The parallel axiom (the postulate) on any line Segment AB in empty Space is experimentally verifiable, and in this way it is dependent of the other Axioms and is logically consistent, and since this is true then is accepted and so the Parallel Postulate as has been shown is a Parallel Axiom, so all Nature (the Universe as objective reality) is working according to the Principles (the patterns), the Properties and the dialectic logic of the Euclidean geometry. [17]

Hyperbolic and Projective geometry transfers the Parallel Axiom to problem of a point M and a Plane AB-C instead of problem of three points only, a Plane, which such it is .(F.6)

Vast (the empty space) is simultaneously ∞ and 0 for every unit AB, as this is for numbers. Uniformity (P4 = Homogenous Plane) of Empty Space creates, all the one dimensional units, the Laws of conservation for Total Impulse, and moment of Inertia in Mechanics, independently of the Position of Space and regardless the state of motion of other sources. (Isotropic Spaces) Uniformity (Homogenous) of Empty Time creates, the Laws of conservation of the Total Energy regardless of the state of motion (Time is not existing here, since Timing is always the same as zero) and Time Intervals are not existing.[17]. It was shown in [38] that *Time is the conversion factor between the conventional units (second) and length units (meter).*

In Special Relativity events from the origin are determined by a velocity and a given unit of time, and the position of an observer is related with that velocity after the temporal unit.

Since all Spaces and Subspaces co-exist, then Past, Present and Future simultaneously exist on different Space Layer. Odd and Even Spaces have common and also opposite Properties,(the regular Odd and Even regular Polygons on any dimensional Unit) so for Gravity belonging to different Layers as that of particles, is also valid in atom Layers. Euclidean geometry with straight lines is extended beyond Standard Model (AB < 10^{-33} m) from that of general relativity where Spaces may be simultaneously Flat or Curved or multi-Curved , and according to the Concentrated, (Compact) Logic of the Space, are below Plank's length Level , so the changing curvature from point to point is possible in the different magnitudes of particles . In Planck length level and Standard Model, upper speed is that of light, while beyond Planck length a new type of light is needed to see what is happening.[43]

5. Respective Figures

5.1. Rational Figured Numbers or Figures

This document is related to the definition of “ Heron ” that gnomon is as that which, when added to anything, a number or figure, makes the whole similar to that to which it is added. In general the successive gnomonic numbers for any polygonal number, say, of n sides have n-2 for their common difference. The odd numbers successively added were called gnomons. See Archimedes (Heiberg 1881, page 142,ε΄.)

The Euclidean dialectic logic of an axiom is that which is true in itself.

This logic exists in nature (objective logic) and is reflected to our minds as dialectic logic of mind. Shortly for ancient Greeks was, (μηδέν εν τη νοήσει ειμή πρότερον εν τοί αισθήσοι) i.e. there is nothing in our mind unless it passes through.our senses . Since the first dimentional Unit is any line Segment AB, it is obvious that all Rational Segments are multiples of AB potentially the first polygonal number of any form, and the first is 2AB = AB +AB, which shows that multiplication and Summation is the same action with the same common base, the Segment AB. To Prove in F8 :

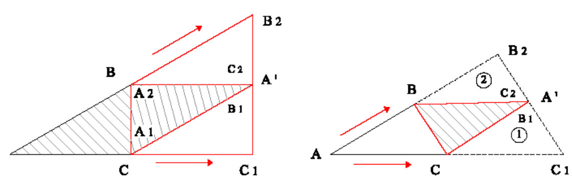


Figure 8. *The Rational figures – (5.1)*

The triangle with sides AC1,AB2,C1B2 twice the length of initial segments AC,AB,CB preserves the same angles < A = BAC, B = ABC, C = ACB of the triangle. Proof :

a. Remove triangle ABC on line AC such that point A coincides with point C (A1). Triangles CB1C1, ABC are equal, so CA΄ = AB, C1A΄ = CB
b. Remove triangle ABC on line AB such that point A coincides with point B (A2). Triangles BB2C2, ABC are equal, so BC2 = AC, B2C2 = BC
c. The two circles (C,CB1 = AB) and (B,BC2 = AC) determine by their intersection point A΄, so triangles CBA΄, CBA are equal, and also equal to the triangles CC1B1, BB2C2, and this proposition states that sides CB1 = CA΄, BC2 = BA΄. Point A΄ must simultaneously lie on circles (C1, C1 B1), (B2, B2C2), which is not possible unless point A΄ coincides with points B1 and C2.
d. This logic exists in Mechanics as follows : The linear motion of a Figure or a Solid is equivalent to the linear motion of the gravity centre because all points of them are linearly displaced, so 1st Removal ---- BB1 = AC, CB1 = AB, BC = BC 2nd Removal ---- CC2 = AB, BC2 = AC, BC = BC 1st +2nd Removal ---- CB1 = AB, BC2

= AC, BC = BC which is the same. Since all degrees of freedom of the System should not be satisfied therefore points B1, C2, A´ coincide.

e. Since circles (C1,C1B1 = C1A´= CB), (B2, B2C2 = B2A´= CB) pass through one point A´, then C1A´B2 is a straight line, this because C1A´+A´B2 = C1B2, and A´ is the midpoint of segment B2C1.

f. By reasoning similar to what has just been given, it follows that the area of a triangle with sides twice the initials, is four times the area of the triangle.

g. Since the sum of angles < C1A´C + CA´B + BA´B2 = 180▫ (6D) and equal to the sum of angles CBA + CAB + ACB then the Sum of angles of any triangle ABC is 180▫, which is not depended on the Parallel theorem or else-where.

This proof is a self consistent logical system .

Verification :

Let be the sides a=5, b=4, c =3 of a given triangle and from the known formulas of area S = (a + b + c) / 2 = 6, Area = √ 6.1.2.3 = 6 For a=10, b=8, c=6 then S = 24/2 = 12 and Area = √ 12.2.4.6.= 24 = 4 x 6 (four times as it is)

5.2. A given Point P and Any Circle (O, OA)

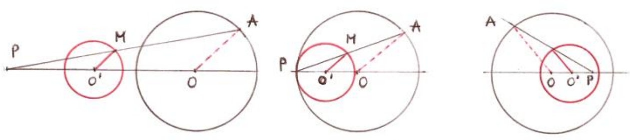


Figure 9. *A point and a circle – (5.2)*

1.. Point P is outside the circle.
2.. Point P on circle.
3.. Point P in circle.

To Prove :

The locus of midpoints M of segments PA, is a circle with center O´ at the middle of PO

and radius O´M = OA / 2 where, P is any point on a Plane A is any point on circle (O, OA) M is mid point of segment PA, Proof :

Let O´ and M be the midpoints of PO, PA. According to the previous given for Gnomon, the sides of triangle POA are twice the size of PO´M, or PO = 2.PO´ and PA = 2.PM therefore as before, OA = 2.O´M, or O´M = OA/2.

Assuming M found, and Since O´ is a fixed point, and O´M is constant, then (O´, O´M = OA/2) is a circle. For point P on the circle : The locus of the midpoint M of chord PA is the circle (O´,O´M = PO / 2) and it follows that triangles OMP, OMA are equal which means that angle < OMP = OMA = 90 ▫, i.e. the right angle < PMO = 90▫ and exists on diameter PO (on arc PO), and since the sum of the other two angles < MPO + MOP exist on the same arc PO = PM + MO, it follows that the sum of angles in a rectangle triangle is 90 + 90 = 180 ▫.(q.e.d)

5.3. The two Angles Problem

Any two angles α =AOB, β = A´O´B´ with perpendicular, sides are equal.

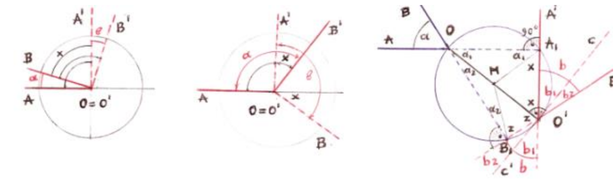


Figure 10. *The two perpendicular angles – (5.3)*

O = O ´ O = O ´O # O´

angle α ≤ 90 ▫ angle α > 90 ▫ any angle

Rotation of α=β Rotating of α=β Displacing of α = β

Let AOB = a be any given angle and angle A´O´B´ = b such that AO ⊥ O´A´, OB ⊥ O´B´.

To proof that angle b is equal to a.

Proof :

CENTRE O´ = O, α ≤ 90▫

Angle < AOA´ = 90▫ = AOB + BOA´ = α + x (1)

Angle < BOB ´ = 90▫ = BOA´ + A´OB´ = x + β (2), subtracting (1), (2) → angle β = α

CENTRE O´ = O, 90▫ < α < 180▫

Angle < AOA´ = 90▫ = AOB´ - B´OA´ = α - x (1)

Angle < BOB ´ = 90▫ = A´OB - A´OB´ = β - x (2), subtracting (1), (2) → angle β = α

CENTRE O´ # O.

Draw circle (M, MO = MO´) with OO´ as diameter intersecting OA,O´B´ produced to points A1,B1.

Since the only perpendicular from point O to O´A´ and from point O´ to OB is on circle (M, MO)

then, points A1, B1 are on the circle and angles O´A1O, O´B1O are equal to 90▫.

The vertically opposite angles a = a1 + a2, b = b1 + b2 where O´C ⊥ OO´.

Since MO = MA1 then angle < MOA1 = MA1O = a1.

Since MA1 = MO ´ then angle < MA1O´ = MO´A1 = x

Since MO´ = MB1 then angle < MO´B1 = MB1O´ = z

Angle MO´C = 90▫ = x + b1 = z + b2.

Angle O´A1O = 90▫ = x + a1 = x + b1 → a1 = b1

Angle O´B1O = 90▫ = z + a2 = z + b2. → a2 = b2

By summation a1 + a2 = b1 + b2 or b = a (o.ε.δ) i.e. any two angles a, b, having their sides perpendicular among them are equal.

From upper proof is easy to derive the Parallel axiom, and more easy from the Sum of angles on a right-angled triangle.

5.4. Any Two Angles Having their Sides Perpendicular among them are Equal or Supplementary [F.11]

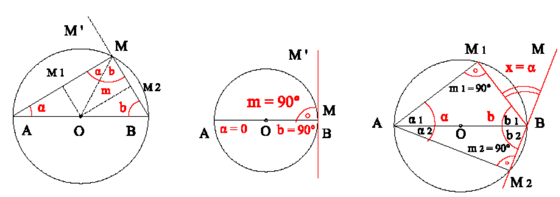


Figure 11. *The two perpendicular supplementary angles in Extrema presentation – (5.4)*

Let be AB = Diameter M → B , AB ⊥ BM´, AM1⊥BM1 ,

AM2 ⊥ BM2 = AM2 ⊥ BM

Let also angle < M1AM 2 = a and angle < M1BM 2 = b, which have side AM1 ⊥ BM1 and the side AM 2 ,

AM 2 ⊥ BM 2 ⊥ BM

Show :

1. Angle < M1AM2 = M1BM = a
2. Angle < M1AM2 + M1BM2 = a + b = 180▫.
3. The Sum of angles in Quadrilateral AM1BM2 is 360▫.
4. The Sum of angles in Any triangle AM1M2 is 180▫.

Proof :

1. In figure 10.3, since AM1 ⊥ BM1 and AM2 ⊥ BM2 or the same AM2 ⊥ BM, then according to prior proof, AB is the diameter of the circle passing through points M1, M2, and exists a1 +b1 = m1 = 90▫, a2 +b2 = m2 = 90▫ and by summation (a1 +b1) + (a2 + b2) = 180▫ or (a1 + a2) + (b1+ b2) = a + b = 180▫, and since also x + b = 180▫ therefore angle < x = a
2. Since the Sum of angles M1BM2 + M1BM = 180▫ then a + b = 180 ▫
3. The sum of angles in quadrilateral AM1BM2 is a + b + 90 +90 = 180 + 180 = 360 ▫
4. Since any diameter AB in Quadrilateral divides this in two triangles, it is very easy to show that diamesus M1M2 form triangles AM1M2, BM1M2 equal to 180▫ each.

so, Any angle between the diameter AB of a circle is right angle (90▫).

1. Two angles with vertices the points A, B of a diameter AB, have perpendicular sides
2. and are equal or supplementary.
3. Equal angles exist on equal arcs, and central angles are twice the inscribed angles.
4. The Sum of angles of any triangle is equal to two right angles.(o.ε.δ)

i.e two Opposite angles having their sides perpendicular between them, are Equal or Supplementary between them. This property has been used in proofs of Parallel Postulate and is also a key to many others .[20]

Many theorems in classical geometry are easily proved by this simple logic.

Conclutions, and how useful is this invention is left to the reader . Unfortunately not any reaction is noticed .

5.5. A Point M on any Circle

5.5.1. A Point M on a Circle of any Diameter AB = 0 → ∞

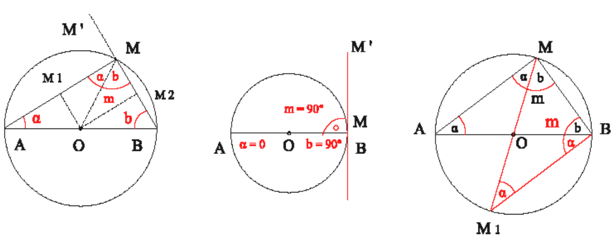


Figure 12. *An angle on a circle with Extrema cases - (5.5.1)*

AB = Diameter M → B, AB ⊥ BM′ Δ [AMB = MBM1]

Let M be any point on circle (O,OM = OA = OB) ,and M 1,M 2 the middle points of MA, MB and in second figure MM′ ⊥ BA at point B (angle AMM′ = 90 ▫).

In third figure MM1 is a diameter of the circle.

Show :

1. Angle < AMB = MAB + MBA = a + b = m
2. Triangles MBM1, MBA are always equal and angle < MBM1 = AMB = 90 ▫
3. The Sum of angles on triangle MAB are < AMB + MAB + MBA = 180▫.

Proof :

1. Since OA = OM and M1A = M1M and OM1 common, then triangles OM1A, OM1M are equal and angle < OAM = OMA = BAM = a → (a) Since OM = OB and M2B = M2M and OM2 common, then triangles OM2B, OM2M are equal and angle < OBM = OMB = ABM = b → (b) By summation (a), (b) BAM + ABM = (OMA + OMB) = AMB = a + b = m..(c) i.e. When a Point M lies on the circle of diameter AB, then the sum of the two angles at points A, B is constantly equal to the other angle at M. Concentrated logic of geometry exists at point B, because as on segment AB of a straight line AB, which is the one dimensional Space, springs the law of Equality, the equation AB = OA + OB i.e. The whole is equal to the parts, so the same is valid for angles of all points on the circumference of the circle (O,OM), [as Plane ABM and all angles there exist in the two dimensional Space], and it is m = a + b.
2. In figure (11), when point M approaches to B, the Side BM′ of angle < ABM tends to the perpendicular on BA and when point M coincides with point B, then angle < ABM = 90▫ and < OAM = BAM = 0, therefore angle < AMB = 90▫ and equation (c) becomes : BAM + ABM = AMB → 0 + 90▫ = AMB → AMB = 90▫, (i.e. AM ⊥ BM) and the sum of angles is (BAM + ABM) + AMB = 90▫ + 90 ▫ = 180▫, or BAM + ABM + AMB = 180▫
3. Triangles MBA, MBM1 are equal because they have diameter MM1 = AB, MB common and angle < OBM = OMB = b (from isosceles triangle OMB). Since Triangles MBA, MBM1 are equal therefore angle < MM1B = MAB = a, and from the isosceles triangle OM1B, angle < OBM 1 = OM1B = a The angle at point B is equal to MBM 1 = MBA + ABM 1 = b + a = m = AMB . Rotating diameter MM1 through centre O so that points M, M1 coincides with B, A then angle < MBM1 = MBA + ABM1 = BBA + ABA = 90▫ + 0 = 90▫ and equal to AMB i.e. The required connection for angle MBM1 = AMB = m = a + b = 90 ▫. (o.ε.δ)
4. Since the Sum of angles a + b = 90▫, and also m = 90 ▫ then a + b + m = 90+90 = 180 ▫. It is needed to show that angle m is always constant and equal to 90▫ for all points on the circle. Since angle at point B is always equal to MBM 1 = MBO + OBM 1 = b + a = m = AMB, by Rotating triangle MBM1 so that points M,B coincide then MBM1 = BBA +ABA = 90+0 = m Since angle < AMB = a + b = m and is always equal to angle < MBM1, of the rotating unaltered triangle MBM1, and since at point B angle < MBM1 of the rotating triangle MBM1 is 90 ▫, then is always valid, angle < AMB =

MBM1 = 90 ▫ (o.ε.δ),(q.e.d)

2a. To show, the Sum of angles a + b = constant = 90▫ = m. F.12-3, M is any point on the circle and MM1 is the diameter. Triangles MBA,MBM1 are equal and by rotating diameter MM1 through centre O, the triangles remain equal.

Proof :

a. Triangles MBA, MBM1 are equal because they have MM1 = AB, MB common and angle < OBM = OMB = b (from isosceles triangle OMB) so MA = BM1.
b. Since Triangles MBA, MBM1 are equal therefore angle < MM1B = MAB = a, and from isosceles triangle OM1B, angle < ABM1 = OBM 1 = OM1B = a
c. The angle at point B is always equal to MBM1 = MBO + OBM 1 = b + a = m = AMB Rotating triangle MBM1 so that points M,B coincide then MBM1 = ABB +ABA = 90+0 = m .Since angle AMB = a + b = m and is equal to angle <MBM1, of the rotating unaltered triangle MBM1 and which at point B has angle m = 90 ▫, then is valid angle <AMB = MBM1 = 90 ▫ i.e. the required connection for angle AMB = m = a + b = 90 ▫. (o.ε.δ), (q.e.d) - 22 / 4 /2010.

2b. When point M moves on the circle, Euclidean logic is as follows :

Accepting angle ABM΄ = b at point B, automatically point M is on the straight line BM΄ and the equation at point B is for (a = 0, b = 90▫, m= 90▫) → 0 + 90▫ = m and also equal to, 0 + b – b + 90▫ = m or the same → b + (90▫ - b) = m ….. (B)

In order that point M be on the circle of diameter AB, is necessary → m = b + a...(M) where, a, is an angle such that straight line AM (the direction AM) cuts BM΄, and is b + (90▫ - b) = m = b + a or → 90▫ - b = a and → a + b = 90▫ = constant, i.e. the demand that the two angles , a, b, satisfy equation (M) is that their sum must be constant and equal to 90 ▫. (o.ε.δ)-(q.e.d)

3. In figure F12-3, according to prior proof, triangles MBA,MBM1 are equal.Triangles AM1B, AMB are equal because AB is common, MA = BM1 and angle < MAB = ABM1, so AM1 = MB. Triangles ABM1, ABM are equal because AB is common MB = AM1 and AM = BM1 therefore angle < BAM1 = ABM = b and so, angle MAM1 = a + b = MBM1.

Since angle AMB = AM1B = 90▫ then AM ⊥ BM and AM1 ⊥ BM1.

Triangles OAM1, OBM are equal because side OA = OB, OM = OM1 and angle < MOB = AOM1, therefore segment M1A = MB.

Rotating diameter MM1 through O to a new position Mx, M1x any new segment is MxB = M1xA and the angle < MxBM1x = MxBA + ABM1x and segment BMx = AM1x.

Simultaneously rotating triangle MxBM1x through B such that BMx ⊥ AB then angle < MxBM1x = BBA + ABA = 90▫ + 0 = 90▫, i.e. in any position Mx of point M angle < AMxB = MxBM1x = 90▫

i.e. two Equal or Supplementary between them opposite angles, have their sides perpendicular between them. (the opposite to that proved) .

Followings the proofs, then any angle between the diameter of a circle is right angle (90 ▫), central angles are twice the inscribed angles, angles in the same segments are equal to one another and then applying this logic on the circumscribed circle of any triangle ABM, then is proofed that the Sum of angles of any triangle is equal to two right angles or < BAM + ABM + AMB = 180▫

5.5.2. The motion of a Point M on a Circle of any Diameter AB = 0 → ∞ F.13

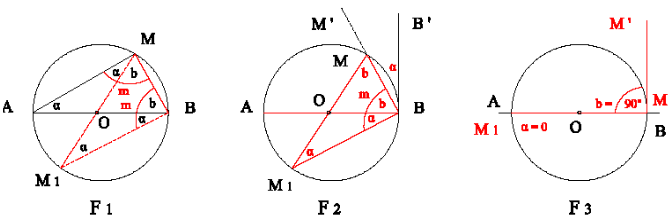


Figure 13. *All angles on a diameter of a circle are 90▫ - (5.5.2)*

To show that angle < AMB = m = 90 ▫ BB΄ ⊥ BA (angle ABB ΄ = 90 ▫) and MM “ ⊥ AB

F.13.1 : It has been proved that triangles AMB, MBM1 are equal and angle <AMB = MBM1 = m for all positions of M on the circle. Since triangles OMB, OAM1 are equal then chord BM = AM1 and arc BM = AM1.

F.13.2 : The rotation of diameter MM1 through centre O is equivalent to the new position Mx of point M and simultaneously is the rotation of angle < M΄MxBM1 = M΄BM1 through point B, and this because arc BM = AM1, BMx = AM1x, i.e. when point M moves with BMM΄ to a new position Mx on the circle, diameter MOM1 = MxOM1x is rotated through O, the points M,M1 are sliding on sides BMM΄, BM1 because point M1 to the new position M1x is such that AM1x = BMx and angle < M΄BM1 is then rotated through B. (analytically as below)

F.13.3 : When diameter MM1 is rotated through O, point M lies on arc MB = AM1 and angle < M΄BM1 is not altered (this again because MB = AM1) and when point M is at B, point M1 is at point A, because again arc BM = AM1 = 0, and angle a = BM1M = 0, or angle < M1BM = M1BM΄ = ABM΄ = 90▫ = m = a + b

Conclusion 1.

Since angle < AMB is always equal to MBM1 = M΄BM1 and angle M΄BM1 = 90▫ therefore angle < AMB = a + b = m = 90▫

Conclusion 2.

Since angle < ABB΄ = 90▫ = ABM + MBB΄ = b + a, therefore angle MBB΄ = a, i.e. the two angles < BAM, MBB΄ which have AM ⊥ BM and also AB ⊥ BB΄ are equal between them.

Conclusion 3.

Any angle < MBB΄ on chord BM and tangent BB΄ of the circle (O, OA = OB), where is holding (BB΄⊥ BA), is equal to the inscribed one, on chord BM.

Conclusion 4.

Drawing the perpendicular MM ” on AB, then angle BMM ” = MAB = MBB ´, because they have their sides perpendicular between them, i.e. since the two lines BB´, MM ” are parallel and are cut by the transversal MB then the alternate interior angles MBB´, BMM ” are equal.

Conclusion 5.

In Mechanics, the motion of point M is equivalent to, a curved one on the circle, two Rotations through points O, B, and one rectilinear in the orthogonal system M´BM1 = MBM1.

5.5.3. A Point M on a Circle of any Diameter AB = 0 → ∞

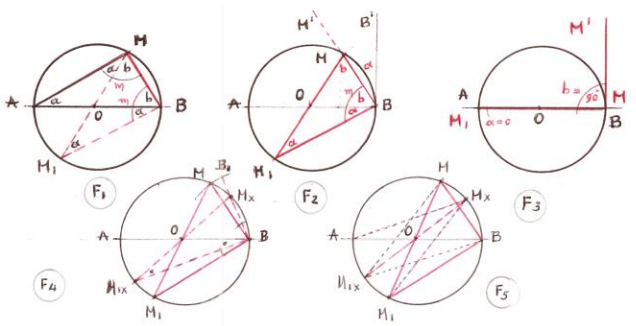


Figure 14. *An angle on any circle - (5.5.3)*

Show that angle MBM1 is unaltered when plane MBM1 is rotated through B to anew position MxBM1x

Let Plane (MBM1), (F14) be rotated through B, to a new position B1BM1x such that:

1. Line BM → BB1 intersects circle (O,OB) at point Mx and the circle (B,BM = BB1), at point B1.
2. Line BM1 → BM1x extended intersects circle (O,OB) at the new point M1x.
3. Angle < M1BM1x = MBB1 = MBMx, is angle of rotation.

Proof :

Since angle < M1BM1x = MBMx, therefore angle < M1BM is unaltered by rotation → i.e. Angle < M1BM = M1xBMx and diameter MM1 is sliding uniformly on their sides.

Data + Remarks.

1. Diameter MM1 is sliding in angle M1BM which means that points M1, M lie on the circle (O,OB) and on lines BM1, BM respectively, and also sliding to the other sides BM1x, BMx of the equal angle < M1xBMx. Any line segment M1xMw= MM1 is also diameter of the circle.
2. Only point Mx is simultaneously on circle (O,OB) and on line BB1.
3. The circle with point M1x as centre and radius M1xMw = MM1 intersect circle (O, OB) at only one unique point Mw.
4. Since angle < M1xBB1= M1BM and since segment M1xMw = MM1 then chord M1xMw must be also on sides of angles M1xBB1, M1BM, i.e. Point Mw must be on line BB1
5. Ascertain 2 and 4 contradict because this property belongs to point Mx, unless this unique point Mw coincides with Mx and chord MxM1x is diameter of circle (O,OB).

Point Mx is simultaneously on circle (O, OB), on angle < M1xBB1 = M1xBMx and is sliding on line BB1. We know also that the unique point Mw has the same properties as point Mx, i.e. point Mw must be also on circle (O, OB) and on line BB1, and the diameter M1xMw is sliding also on sides of the equal angles M1xBB1, M1xBMx, M1BM.

Since point Mw is always a unique point on circle (O,OB) and also sliding on sides of angle M1BM = M1xBMx and since point Mx is common to circle (O,OB) and to line BB1 = BMx , therefore, points Mw, Mx coincide and chord MxM1x is diameter on the circle (O,OB), i.e. The Rotation of diameter MM1 through O, to a new position MxM1x, is equivalent to the Rotation of Plane (MBM1) through B and exists angle < MBM1 = MxBM1x, so angle < MBM1 = MxBM1x = AMB = 90 ▫ = m = a + b ……… ο.ε.δ

Since angle < MBMx = M1BM1x is the angle of rotation, and since also arc MMx = M1M1x (this because triangles OMMx, OM1M1x are equal) then: Equal inscribed angles exist on equal arcs.

6. General Remarks

6.1. Axiom not Satisfied by Hyperbolic or other Geometry

It has been proved that quadrilateral MA1CC´ is Rectangle (F2) and from equality of triangles MA1C, MC´C then angle < C´MC = MCA1. Since the sum of angles < MCA1 + MCB = 180 ▫, also, the sum of angles < C´MC + MCB = 180 ▫ which answers to Postulate P5, as this has been set (F1.e). Hyperbolic geometry, Lobachevski, non-Euclidean geometry, in Wikipedia the free encyclopedia, states that there are TWO or more lines parallel to a given line AB through a given point M not on AB. If this is true, for second angle C2MC, exists also the sum of angles < C2MC + MCB = 180▫, which is Identity (C2MC = C´MC), i.e. all (the called parallels) lines coincide with the only one parallel line MM´, and so again the right is to Euclid geometry.

Definitions, Axioms or Postulates create a geometry, but in order this geometry to be right must follow the logic of Nature ,the objective reality, which is the meter of all logics, and has been found to be the first dimensional Unit AB = 0 → ∞ (F.3-4) i.e. the reflected Model of the Universe. Lobachevski’s and Riemann’s Postulate may seem to be good attempts to prove Euclid’s Fifth Postulate by contradiction, and recently by “compromising the opposites “in the Smarandache geometries. Non of them contradicts any of the other Postulates of what actually are or mean. From any point M on a straight line AB, springs the logic of the equation (the whole AB is equal to the parts MA, MB as well as from two points passes only one line –theorem-), which is rightly followed (intrinsically) in Euclidean geometry only, in contradiction to the others which are based on a confused and muddled false notion (the great circle or segment is line, disk as planes and others), so all non-Euclidean geometries basically contradict to the second definition (D2) and to the first Euclidean Postulate (P 1).

6.2. Hyperbolic Geometry Satisfies the Same 4 Axioms as Euclidean Geometry, and the Error if Any in Euclidean Derivation of the 5th Axiom

An analytical trial is done to answer this question.

Postulate 1: States that "Let it have been postulated to draw a straight-line from any point to any point". As this can be done by placing the Ruler on any point A to any point B, then this is not in doubt by any geometry. The world "line" in Euclid geometry is straight line (the whole is equal to the parts, where lines on parts coincide) and axioms require that line to be as this is (Black color is Black and White color is White). For ancient Greeks < Ευθεία γραμμή έστιν, ήτις εξ΄ ίσου τοίς εαυτής σημείοις κείται. >

Postulate 2 : states that, " And to produce a finite straight-line " Marking points A, B which are a line segment AB, and by using a Ruler then can produce AB in both sides continuously, not in doubt by any geometry.

Postulate 3: states that, "And to draw a circle with any center and radius" Placing the sting of a Compass at any point A (center) and the edge of pencil at position B and (as in definition 15 for the circle) Radiating all equal straight lines AB, is then obtained the figure of the circle (the circumference and the inside), not in doubt by any geometry.

Postulate 4 : states that, " And all right angles are equal to one another " In definition D8 is referred as Plane Angle, to be the inclination of two lines in a plane meeting one another, and are not laid down straight-on with respect to one another, i.e. the angle at one part of a straight line. In definition D9 is stated " And when the lines containing the angle are straight then the angle is called rectilinear " and this because straight lines divide the plane, and as plane by definition is 360▫ then the angles on a straight line are equal to 180▫ In definition D10 is stated that a perpendicular straight line stood upon (another) straight-line makes adjacent angles (which are) equal to one another, each of the equal angles is a right-angle and this because as the two adjacent angles are equal and since their sum is 180 ▫, then the two right-angles are 90 ▫ each and since this happen to any two perpendicular straight-lines, then all right angles are equal to one another, not in doubt by any geometry.

Postulate 5: This postulate is referred to the Sum of the two internal angles on the same side of a straight- line falling across two (other) straight lines, being produced to infinity, and be equal to 180▫. Because this postulate, beside all attempts to prove it, was standing for centuries, mathematicians created new geometries to step aside this obstruction. In my proposed article the followings have been geometrically proved:

From any point M to any line AB (the three points consist a Plane) is constructed by using the prior four Postulates, a system of three rectangles MA1CC΄, C΄CB1M΄, MA1B1M΄ which solve the problem. (3.d)

The Sum of angles C΄MC and MCB is < C΄MC + MCB = 180 ▫, which satisfies initial postulate P5 of Euclid geometry, and as this is now proved from the other four postulates, then it is an axiom.

The extended Structure of Euclidean-geometry to all Spaces (Spaces and Sub-spaces) resettles truth to this geometry, and by the proposed solution which is applicable to any point M, not on line AB, answers to the temporary settled age-old question for this problem.

Mathematical interpretation and all the relative Philosophical reflections based on the non-Euclid geometry theories must be properly revised and resettled in the truth one. For conceiving alterations from Point to sectors, *discrete*, lines , plane and volume is needed Extrema knowledge where there happen the inner transformations on geometry and the external transformations of Physical world .

6.3. The sum of Angles on Any Triangle is 180▫

Since the two dimensional Spaces exists on Space and Subspace (F.11) then this problem of angles must be on the boundaries of the two Spaces i.e. on the circumference of the circle and on any tangent of the circle and also to that point where Concentrated Logic of geometry exists for all units, as straight lines etc. It was proved at first that, the triangle with hypotenuse the diameter of the circle is a right angled triangle and then the triangle of the Plane of the three vertices and that of the closed area of the circle (the Subspace), measured on the circumference is 180 ▫.

6.4. Angles with Perpendicular Sides are Equal or Supplementary

In Proofed Succession (5.4-5) , is referred that two angles with perpendicular sides are equal (or supplementary). To avoid any pretext, a clear proof is given to this presupposition showing that, any angle between the diameter AB of a circle is right angle (90▫).

Any two angles with vertices the points A, B of a diameter AB, have perpendicular sides and are also equal or supplementary.

Equal angles exist on equal arcs, and central angles are twice the inscribed angles.

The Sum of angles of any triangle is equal to two right angles. So, there is not any error in argument of proofs.

The 5th Postulate is Depended on (derived from) the prior four axioms.

6.5. Questions and Answers on the fifth Postulate .

Question 1 : Which axiom is not satisfied by Hyperbolic or other geometry ? 15 / 4 / 2010

It has been proved that quadrilateral MA1CC΄ is Rectangle and from equality of triangles MA1C , MC΄C then angle < C΄MC = MCA1 .
Since the sum of angles < MCA1 + MCB = 180▫ , also , ***the sum of angles < C΄MC + MCB = 180 ▫ which answers to Postulate* P5** , as this has been set .

Hyperbolic geometry, *Lobachevsky , non-Euclidean geometry, Wikipedia the free encyclopedia* , states that there are TWO or more lines parallel to a given line AB through a given point M not on AB .

If this is true , *for second angle C2MC* , exists also the sum of angles < C2MC + MCB = 180▫ , which is Identity as (C2MC = C΄MC) , i.e. all (*the called parallels*) lines coincide with the only one parallel line MM΄ , and so again the right is to Euclid geometry .

Definitions , Axioms or Postulates create a geometry, but in order this geometry to be right must follow the logic of Nature , which is the meter of all logics , and has been found to be in the first dimensional Unit AB = 0 → ∞ (figure 7) i.e. the quantization of points as monads is the reflected Model of the Universe agreeing with E-Geometry

Lobachevsky's and Rieman's Postulate may seem to be good attempts to prove Euclid's Fifth Postulate by contradiction , and recently by " *compromising the opposites "* in the Smarandache geometries .

Non of them contradicts any of the other Postulates of what actually are or mean .

From any point M on a straight line AB , ***springs the logic of the equation*** (the whole line-segment AB is equal to the parts MA , MB) , which is rightly followed (*intrinsically*) ***in Euclidean geometry only*** , in contradiction to the others which are based on a confused and muddled false notion (the great circle or segment is line, disk as planes and others) ***so all non-Euclidean geometries contradict to the second (D2) definition and to the first (P1) Euclidean Postulate.***

Question 2 : Is it possible to show that the sum of angles on any triangle is 180▫ ? 1 / 5 / 2010

Yes by Using Euclidean Spaces and Subspaces .

Since the two dimensional Spaces exists on Space and Subspace then this problem of angles must be on the boundaries of the two Spaces i.e. on the circumference of the circle and on any tangent of the circle and also to that point where Concentrated Logic of geometry exists for all units , as straight lines etc. . It was proofed at first that, the triangle with hypotenuse the diameter of the circle is a right angled triangle and then the triangle of the Plane of the three vertices and that of the closed area of the circle (the Subspace) , measured on the circumference is 180 ▫ .

Question 3 : Why Hyperbolic geometry satisfies the same 4 axioms as Euclidean geometry , and the error in my Euclidean derivation of the 5^{th} axiom . 24 / 4 / 2010

An analytical trial is done to answer this question .

Postulate 1 : states that , " Let it have been postulated to draw a straight-line from any point to any point " .

As this can be done by placing the Ruler on any point A to any point B , then this is not in doubt by any geometry . The world " line" in Euclid geometry is straight line (and as was shown → the whole is equal to the parts) and axioms require that line to be as this is defined (as in colors , *Black color is Black and White color is White*) .
In ancient Greek < Ευθεία γραμμή έστιν , ήτις εξ΄ ίσου τοίς εαυτής σημείοις κείται > .

Postulate 2 : states that ,

" And to produce a finite straight-line "

Marking points A , B which are a line segment AB , and by using a Ruler then can produce AB line in both sides continuously , not in doubt by any geometry .

Postulate 3 : states that ,

" And to draw a circle with any center and radius "

Placing the sting of a Compass at any point A (center) and the edge of pencil at position B and (as in definition 15 for the circle) Radiating all equal straight lines AB , is then obtained the figure of the circle (the circumference and the inside) , not in doubt by any geometry .

Postulate 4 : states that ,

" And all right angles are equal to one another "

In definition D8 is referred as *Plane Angle* , to be the inclination of two lines in a plane meeting one another , and are not laid down straight-on with respect to one another , *i.e. the angle at one part of a straight line* .

In definition D9 is stated " And when the lines containing the angle are straight then the angle is called *rectilinear* " and this because straight lines divide the plane , and as plane by definition is 360▫ then the angles on a straight line are equal to 180 ▫

In definition D10 is stated that a perpendicular straight line stood upon (another) straight-line makes adjacent angles (which are) equal to one another ,each of the equal angles *is a right-angle* ,and this because as the two adjacent angles are equal and since their sum is 180▫ , then the two right-angles are 90▫ each and since this also happen to any two perpendicular straight-lines , then *all right angles* are equal to one another , not in doubt by any geometry .

Postulate 5 :

This postulate is referred to the Sum of the two internal angles on the same side ***of a straight- line falling across two*** (other) ***straight lines*** , being produced to infinity , and be equal to 180▫ .

Because this postulate , *beside all attempts to prove it* , was standing for centuries , mathematicians created new geometries *to step aside* this obstruction .
In my *proposed article* the followings have been geometrically proved :

1 . From any point M to any line AB (the three points consist a Plane) is constructed by using the prior four Postulates , a system of three rectangles MA1CC΄ , C΄CB1M΄ , MA1B1M΄ .

2 . The Sum of angles C΄MC and MCB is < C΄MC + MCB = 180▫ , which satisfies postulate P5 of Euclid geometry, and as this is now proved , *then it is a theorem* .

3 ***. The extended Structure of Euclidean-geometry to all Spaces (Spaces and Sub-spaces) resettles truth to this geometry , and by the proposed solution which is applicable to any point M , not on line AB , answers to the temporary settled age-old question for this problem .***

4 . Mathematical interpretation and all relative Philosophical reflections based on the non-Euclid geometry theories must properly revised and resettled in the truth one . Moreover ,

my dear professor (xxxxxxxxxx) , after giving you all these explanations you asked , I was waiting from you to promote my article contained in your Publication Org .That`s OK. Nevertheless I still believe that Relativity is a theory which has not elucidated the base of its Physical-content and has adopted the wrong base of the Non-Euclid geometries and is implicated and calls a circle segment as line segment ,and the spherical disk as plane in physical space, Spherical angle as Plane angle , for which I am quietly opposed .
I repeat again what is referred between us at the beginning of our understanding , that this today Work should be done in Euclidean-Geometry after Euler-Lagrange equations , *for Extrema of functional*s , and not after two hundred and more years since then .

Meanwhile , Mechanics and Physics have been developed and experimentally confirmed on the right basis of Euler-Lagrange and Hamilton works , in contradiction to Physics, *General Relativity of Einstein`s theory of gravity which ties , Time , with space* ***which is an enormous fault*** , which was based on the wrong Non-Euclid geometries and resulting to the confinement of Space and Energy (*velocity*) in Planck`s length and in light velocity without any scientific proof .[41]

Geometry in its Moulds , and present contribution of this Article to the Special problems of geometry.

7.1. Zeno`s Paradox and the nature of Points .

The world quantization has to do with the discrete continuity, which describes the Physical reality through the Euclidean conceptual ,for Points Straight lines, Planes , the Monads in Universe and the Dual Nature of Spaces as *discrete* and *continuous* **.**Euclidean Geometry is proved to be the Model of Spaces since it is Quantized as Complex numbers are so.

7.1.1. Achilles and the Tortoise :

The Problem :

(0m) → (100 m) (110 m)
A ------- C ------------------------- B → ------- D

D-7.1

< ***In a race , the Quickest runner can never overtake the Slowest , since the Pursuer must first reach the point whence the Pursued started , so that the Slower must always hold a lead*** >

This problem was devised by Zeno of Elea to support Parmenides´s doctrine that <*all is one in Euclidean Absolute Space* > , contrary to the evidence of our senses for plurality and change and to others arguing the opposite . Zeno´s arguments are as proof by contradiction or (*reduction ad absurdum*) which is a philosophical dialectic method . Achilles allows the Tortoise a head start 100 m and each racer starts running at some constant speed , *one very fast and one very slow* , the Tortoise say has further 10 m.

7.1.2 *The proposed Euclidean solution*

Straight line AB is continuous in Points between A and B [i.e. all points between line segment AB ***are the elements which fill AB*** , *which Points are also* , Nothing , or Everything else and are Anywhere as in Diagram (D-7.1)] , and Achilles in order to run the 100 m has to pass the infinite points between A and B **.** A point C is on line AB only when exists CA + CB = AB (*or the whole* AB *is equal to the parts* CA , CB , *and it is equation , i.e. an equality*) .

In case CA + CB > AB then point C is not on line AB , and this is the main difference between Euclidean and the Non-Euclidean geometries .On this is based the Philosophy of Parallel fifth Postulate which is proofed to be a Theorem.

Definition 2 (*a line AB is breathless length*) is altered as → *for any point C on line* AB *exists* CA + CB = AB i.e. it is the equation . [9-10]

Since points have not any dimension and since only AB has dimension (the length AB and for $\overline{AC}$ the length AC) and since on $\overline{AB}$ exist infinite line segments AC → AB , which have ***infinite Spaces , Anti-Spaces and Sub-Spaces*** [Fig.6] , then is impossible in--bringing Achilles to the Tortoise´s starting point B and also for Tortoise´s to 110 m , because as follows **,**

Straight line AB is not continuous unless* a Common Dimensional Unit AC > 0 *or AC = ds → AB is accepted and since in this way,

1.a. Straight line AB is continuous with points as filling (*Infinitively divisible* **) .**
1.b. Straight line AB is discontinuous (***discrete*** **) with dimensional Units , ds, as filling** (that is made up of finite indivisible parts the Monads ds → AB /n ,where n=1,2,→∞)
1.c. Straight line AB is discontinuous (*discrete*) with dimensional Units ds , and also continuous in ds with points as filling , Space ,Anti-space ,Sub-space , **where ,**

ds = quantum = AB / n (where n = 1,2,3 → ∞ , = [a+b.i] /n = complex number *and Infinitively divisible which is keeping the conservation of Properties at End Points A , B*) as filling , **and continuous with points as filling** *(for n = ∞ then ds = 0 i.e. the point in ds)* **. i.e**
Monads ds = 0 → ∞ are simultaneously (*actual infinity*) and (potential infinity) in Complex number form ,and this defines , infinity exists between all points which are not coinciding , and because **ds** comprises any two edge points with imaginary part then this property differs between the infinite points .**This is the Vector relation of Monads , ds , (or , as Complex Numbers in their general form w = a + b. i) , which is the Dual Nature of lines (discrete and continuous) .**

7.1.3. *The proposed Physical solution*

Following Euclidean geometry logic , short definitions and elucidations are made clear, and which are proceed .

***What is a Point and what is a segment* ?** [10-14]
Point is nothing and has not any Position and may be anywhere in Space , *therefore* , ***the Primary point*** ,A, ***being nothing also is in no Space, and it is the only Point and nowhere*** , i.e. Primary Point is the only *Space* and from this all the others which have Position , therefore and since this is the only Space, so to exist point ,A, at a second point ,B, somewhere else , point ,A, must move towards point ,B, where then A ≡ B . Point B is the Primary *Anti-Space* which Equilibrium point ,A , and both consist the Primary Neutral

Space → PNS = [A ≡ B] .The position of points in [PNS] creates the infinite dipole and all quantum quantities which acquire Potential difference and an Intrinsic momentum ± Λ in the three Spatial dimensions (x,y,z) , [26] and on the infinite points of the (n) Layers at these points , which exist from the other Layers of Primary Space , Anti-Space and Sub-Space , and this is because Spaces are monads in monads , i.e. quaternion .

Any magnitude having Position ,x,y,z *related to a coordinate system and Direction , Divergence* → ∇ ←, *is characterized as Vector and extensively as quaternion*). [9] . *From* [14] *a point C is on Straight line AB when AC+CB = AB , and continuous on dimensional unit AB when unit* AC > 0 *or AC = ds → AB* . Unit ds = AC is a discrete monad *so ,*

1a. Straight line AB is continuous with **points** as filling (*Infinitively divisible*) .

2a. Straight line AB is discontinuous (*discrete*) with dimensional Units ,**ds**, as filling , and (*that is made up of finite , indivisible parts the Monads* **ds** →AB / n , *where* n = 1,2,→∞)

3a. Straight line AB is discontinuous (*discrete*) also with its dimensional Units **ds** = AB , and from equality *ds = quantum = AB / n* (where n = 1,2,3 → ∞ , = quaternion [a + b.i] / n *Infinitively divisible keeping the conservation of properties at end points A , B*) as

filling and continuous with points as filling (*for n = ∞ → ds = 0 i.e. a point*) . i.e Monads where ds = 0 → ∞ are simultaneously (*Actual infinity*) and (*Potential infinity*) in Complex number form , and this defines , infinity exists between all points which are not coinciding and because ,ds, comprises any two edge points with imaginary part where then this property differs between the infinite points.

This is the Vector relation of Monads ,ds, (or as Complex Numbers (*quaternion*) in their general form w = a + b. i), which is the Dual Nature of lines (discrete and continuous) . It has been shown that Primary Neutral Space is not moving and Time is not existing , *so Points*, in Primary Space cannot move, *to where they are*, because are already there and motion is impossible. Since *Points C,D,,,* of Primary Neutral Space , PNS , are motionless (v = 0) at any Time (the composed instants are dt = 0) then motion is impossible . i.e. for monads issues [ds = a + b. i = v.dt] *and for ,*

a = 0 then ds = b.i = v.dt *and for* b ≠0 , dt = 0 then *ds = Constant = v . 0 → ∞* **x 0**

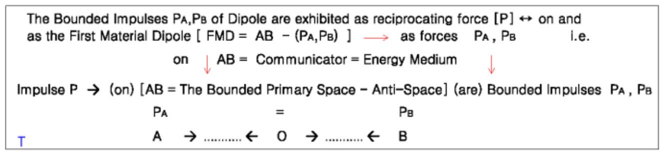
b = 0 then ds = a = v.dt *and for* dt = 0 then → *ds = a = Constant = v . 0 → ∞* **x 0** *therefore in PNS v = ∞ and T= 0 meaning infinite velocity ,v, and Time not existing*

, so since any Arrow (vector) moving from point A to point B , then exists a Numerical order A → B which is not valid for Temporal order (dt) . In case dt = 0 then motion from Point A to point B has not any concept , and distance CD and anywhere exist the Equal CD is unmovable ,

i.e. *The Motion of points C , D of PNS* is not existing *because time (d t = 0) and for ds = any constant exists with infinite velocity (v = ∞) while motion of the same points C , D* exists *in PNS out of a moving Sub -Space of AB* (Arrow CD is one of the ∞ roots of line segment AB) .

Monads ds = 0 → ∞ are Simultaneously, *actual infinity* (*because for n = ∞ then ds* = [*AB* / (*n* = ∞)] = 0 *i.e. a point*) *and , potential infinity* , (*because for n = 0 then ds* = [*AB / (n = 0)*] = ∞ *i.e. the straight line through AB* . *Infinity exists between all points which are not coinciding , and because Monads* ,ds, *comprises any two edge points with Imaginary part , then this property differs between the ,i, infinite points or* $d\bar{s}$ = λi + ∇i , i.e. quaternion .

On *Monad* AB which is = 0 ↔ AB ↔ ± ∞ exists < *a bounded State of energy for each of the Infinite Spaces and Anti-Spaces* called ***Energy monad*** in Space moulds > and this [Dipole AB = Matter] is the communicator of Impulse [P] *of Primary Space* . This Energy monad is modified as the Quanta of Energy , *a monad* , and is represented as the Dipole shown below , T , i.e.



This motion is Continuous and occurs on Dimensional Units , ds , *which is the Maxwell`s-Monads Displacement Electromagnetic current* [E+ $\bar{\nabla}$xP] , and not on Points which are dimensionless , upon these Bounded States of [PNS] *, Spaces and Anti-Spaces, and because of the different Impulses* P_A *, P* $_B$ *, of edge points A , B , and that of Impulses P i A , Pi B ,* of Sub-Spaces *, they are* either on straight lines AB or on tracks of the Spaces , Anti-Spaces and Sub-Spaces of AB . The range of *Relative velocities* is bounded according to the single slices of spaces (*ds*) . [14 -15] , [39-40] .

Since Primary point ,A, is the only Space then on this exists the Principle of Virtual Displacements W = $\int_A^B P.ds = 0$ or [ds.(P$_A$ + P $_B$) = 0] , i.e. for any monad ds > 0 Impulse P = (P$_A$ + P $_B$) = 0 and [ds. (PA + PB) = 0] , Therefore , Each Unit AB = ds > 0, exists by this Inner Impulse (P) where P$_A$ + P $_B$ = 0 , → i.e. The Position and Dimension of all Points which are connected across the Universe and that of Spaces exists , because of this equilibrium Static Inner Impulse , on the contrary should be one point only (Primary Point A = Black Hole → ds=0 and P = ∞).[17,22] ↔ Monad AB is dipole [{A(P$_A$) ←0→ (P$_B$)B}] and it is the symbolism of the two opposite forces (P$_A$) , (P$_B$) which are created at points A,B . This Symbolism of primary point (*zero 0 is nothing*) shows the creation of Opposites , A and B , points from this zero point which is Non-existence. [13] .

All points may exist with force P = 0 → { PNS the Primary Neutral Space} and also with P ≠ 0 , (P A + P B = 0) , { PS is the Primary Space} for all points in Spaces and Anti – Spaces , therefore [PNS] is self-created , and because at each point may exist also with P ≠ 0 , then [PNS] is a (perfectly Homogenous , Isotropic and Elastic Medium) Field with infinite points (i) which have a ± Charge with force Pi = 0 → P = Λ→ ∞ and containing everything .

Since points A,B of [PNS] coincide with the infinite Points , of the infinite Spaces , Anti-Spaces and Sub-Spaces of [PNS] and exists rotational energy ± Λ and since Motion may occur at all Bounded Sub–Spaces (± Λ , λ), then this

Relative motion is happening between all points belonging to [PNS] and to those points belonging to the other Sub-Spaces (A≡B) . The Infinite points in [PNS] form infinite Units (monads = segment) AiBi = ds , which equilibrium by the Primary Anti-Space by an Inner Impulse (P) at edges A,B where $P\,i_A + P\,i_B \neq 0$, and ds= 0 →N→ ∞ .

Monad (Unit ds = Quaternion) ĀB is the ENTITY and [AB –P_A , P_B] is the LAW, therefore Entities are embodied with the Laws . Entity is quaternion ĀB , and law |AB| = Energy length (*the energy quanta*) of points |A,B| or the wavelength when AB=0 and imaginary part are the forces P_A, P_B, as fields in monads , (*This is distinctly seen for* ***Actions at a distance*** *, where there the continuity of all intermediate points being also nothing ,* ***is succeeded on*** *quantized , tiny energy volume which consists the* ***material point*** *i.e. a field ,* ***or*** *by the Exchange of energy in the Inner-monads field*) . [39-40] . Pythagoras definition for **Unit →** *is a Point without position while* **a Point →** *is a Unit having position .*

7.2. The dichotomy Paradox (*Dichotomy*) :

< *That which is in locomotion must arrive at the half-way stage before it arrives at the goal* >

(0m) → → (50 m) (100 m)

A ---------D-------- C-------------------- B D-7.2

As in **1-a,b,c.** Straight line AB is not continuous unless a Common Dimensional Unit AC > 0 or ds = 0 → AB / 2 → AB is accepted and this because point C is on line AB where then CA + CB = AB and since CA = CB then CD < CB therefore point D , (AD) will pass through C , (AC) before it arrives at the goal B , (AB).

7.3. The Arrow Paradox (*Arrow*) :

< *If everything when it occupies an equal Space is at rest , and if that which is in locomotion is always occupying such a Space at any moment , the flying Arrow is therefore motionless* >

(0m) (10 m) (10m) **ds = a+b.i = v.dt** (10 m)

A ---------------> B A -----C-D --------> B

D-7.3

The Arrow Paradox is not a simple mathematical problem because is referred to ***motion in Absolute Euclidean Space*** i.e. in a Space where ***issues Geometry*** *, Parallel Postulate the Squaring of circle etc,* ***and Physics*** where Space [PNS] is not moving and because of its Duality (*discrete and continuous as Complex numbers are*) ***,Time is not existing***. This Paradox ***is not*** in metaphysical problem since [15] is proved that , Complex numbers and Quantum Mechanics Spring out of ***the Quantized Euclidean Geometry.***
As in 1-**a,b,c)** Straight line AB is discontinuous (***discrete***) with dimensional Units , **ds=CD** as filling and ***continuous with points*** as filling (The Complex Numbers in the general form w = a+b. i) , ***which is the Dual Nature of lines*** (line= *discrete with ,Line-Segments, and continuous with points*).

It has been shown that Primary Neutral Space is not moving and Time is not existing , *so Points* , in Primary Space cannot move to where they are because are already there and motion is impossible. Since *Points C , D* of the Primary Neutral Space , PNS , are motionless (v = 0) at any Time (the composed instants are dt = 0) , and then motion is impossible , i.e. issues [ds = a + b. i = v.dt] *and for* ,
a = 0 then ds = b.i = v.dt *and for* b ≠0 , dt = 0 then *ds = Constant = v . 0* → i.e. v = ∞ **,**
b = 0 then ds = a = v.dt *and for* dt = 0 then → *ds = a = Constant = v . 0* → i.e. v = ∞ **,**

Therefore in PNS , v = ∞ , T = 0 , meaning infinite velocity and Time not existing , so

Since Arrow is moving from point A to point B , then exists the Numerical order A → B which is not valid for Temporal order (dt) . In case dt = 0 then motion from Point A to point B has not any concept , and distance CD and anywhere exist the Equal CD is unmovable **, i.e.**

Motion of points C, D of PNS **is not existing** ***because time (dt = 0) and infinite velocity (v = ∞) exists , while motion of the same points C, D exists in PNS out of a moving Sub-Space of AB*** **(** arrow CD is one of the ∞ roots of AB) **where (** *ds = CD = Monad in PNS* **)** [15] .

7.4. The Algebraic Numbers :

According to F.6 *Monad* **AB = 0 ↔ AB ↔ ± ∞ , is and also represents** the Spaces , Anti-Spaces , Sub-Spaces of AB which are **the Infinite Regular Polygons** , on circle with AB as Side , and on circle with AB as diameter and it is what is said , *monad in monad* . According to De Moivre's formula the n-th roots on the unit circle AB are represented by the vertices of these Regular n-sided Polygon inscribed in the circle which are Complex numbers in the general form as , **w = a+b.i = r.e** $^{(i\varphi)}$, **a** and **b** = Real Numbers **,** $r = \sqrt{a^2+b^2}$ **,** (±) **i** = Imaginary Unit.

We will show that since Complex Numbers are ***on Monad AB*** (*any two points are monads*) and it is the only manifold for the Physical reality , ***then Euclidean Geometry is also Quantized*** (Fig,15) . This geometrically is as follows,
a. Exists $^2\sqrt{1} = \pm 1$ **or [-1 ↔ +1]** , therefore **xx** (axis) coordinate system represents the one-dimensional Space and Anti-Space (the Straight line) , 1.1 = 1 , (-1).(-1) = 1
b. Exists $^2\sqrt{-1} = \pm i$ **or [+i ↕ -i]** , therefore **yy** (axis) coordinate system represents a perpendicular on
(-i).(- i) = + i ² = + (-1) = -1 , (+ i).(+ i) = + i ² = -1

c. Exists $^3\sqrt{1}$ **= [1 , - ½ + (√ 3.i) /2 , -½ - (√ 3.i) /2]** therefore **xx-yy** coordinate system represents two - dimensional ± Spaces and ± Complex numbers.(the Plane)
1.1.1 = 1 , [-½ +(√3.i) /2 **]** ³ = 1 , **[**-½ -(√3.i) /2] ³ = 1 + i

d. Exists ∜1 **=** $^2\sqrt{^2\sqrt{1}}$ **=** $^2\sqrt{\pm 1}$ **= [+1 , -1] , [** $^2\sqrt{-1}$ **= + i , - i]** or -1 ↔ +1 , ↕

therefore coordinate systems **xx - yy** represent all these Spaces , - i (± Real and ± Complex numbers) , where Monad = 1 = (*that which is one*) , represents the three-dimensional Space and Anti-Space. (the Sphere) which is , $[\pm 1]^4 = [\pm i]^4 = 1$

The fourth root of **1** ***are the vertices of Square in circle*** with **1** as diameter and since the Geometrical Visualization of Complex numbers , is formula $\sqrt[4]{1} = \pm 1 , \pm i$ …(d) and since ± 1 is the one-dimensional ***real Space*** (*the straight line*) , the vertical axis is the other one-dimensional ***Imaginary Space*** $\pm i$ **.** Since for dimension are needed N-1 points then (d) is representing the Space with three dimensions (dx ,dy ,dz) which are Natural , Real and Complex . Monads (Entities = AB) are the Harmonic repetition of their roots , and since roots are the combinations of purely real and purely Imaginary numbers , *which is a similarity with Complex numbers* (*Real and Image*) , then , ***Monads are composed of Real and Imaginary parts as Complex Numbers are .*** i.e. Objective reality contains both aspects (Real and Imaginary **,** discrete , AB , and Continuous **,** Impulses **PA , P B** , etc.) meaning that *Euclidean geometry is Quantized* . [15]

i.e. The Position and Dimension of all Points which are connected across Universe and that of Spaces exists , because of this Static Inner Impulse P, on the contrary should be one point only (Primary Point = Black Hole → ds = 0) . [43 - 45]

Impulse is ∞ and may be Vacuum , Momentum or Potential or Induced Potential.

Change (motion) and plurality are impossible in Absolute Space [PNS] and since is composed only of Points that consist an Unmovable Space , then neither Motion nor Time exists i.e. a constant distance AB = **ds** = monad anywhere existing is motionless . The discrete magnitude **ds =[AB /n] > 0** = *quantum* , and for infinite continuous **n** , **then ds** convergence to **0** . Even the smallest particle (say a photon) has mass [15] and any Bounded Space of ds > 0 is not a mass-less particle and occupies a small Momentum which is motion .

The Physical world is scale-variant and infinitely divisible , consisted of finite indivisible entities ds = AB → 0 called monads and of infinite points (ds = 0) , i.e.

All entities are Continuous with points and Discontinuous ,*discrete*, with ds > 0 . In PNS dt = 0 , *which is the meter of changes* , so motion cannot exist at all .

Since points A,B of PNS coincide with the infinite Points , of the infinite Spaces , Anti-Spaces and Sub-Spaces **of PNS** , and since Motion may occur at all Bounded Sub-Spaces then this ***Relative motion*** is happening on e - dimensional to xx Space and Anti-Space (the Straight line) between all points belonging to PNS and those belonging to other Spaces . *Time exists in Relative Motion and it is the numerical order of material changes in PNS - Space , and is not a fundamental entity as is said in Relativity*.

On *Monad* AB which is = 0 ↔ AB ↔ ± ∞ exists < *a bounded State of energy for each one of the Infinite Spaces and Anti-Spaces* > and the [Dipole AB = Matter = monad] is the communicator of Impulse [P] *of the Primary Space*.

This Energy monad is modified as the Quanta of Energy and is represented as the Dipole shown below , T , i.e.

The Bounded Impulses P_A,P_B of Dipole are exhibited as reciprocating force [P] ↔ on and as the First Material Dipole [FMD = AB – (P_A,P_B)] ⟶ as forces P_A , P_B i.e.

on ↓ AB = Communicator = Energy Medium ↓

Impulse P → (on) [AB = The Bounded Primary Space – Anti–Space] (are) Bounded Impulses P_A , P_B

P_A = P_B

T A → ……….. ← O → ……….. ← B

or **[P] ↔ [FMD = AB – PA , P B] → PA , P B .**

on **↓ Communicator = Medium ↓**

Impulse P → [Bounded *Primary Space- Anti-Space*]

→ Bounded Impulse PA

PÂ = PB

A → . . . . ← O → . . . . ← B

Motion is Continuous and occurs on and in Dimensional Units , *ds* **, and not on Points which are dimensionless , upon these Bounded States of [PNS] , *which are the Spaces and Anti-Spaces , and because of the different Impulses PA , P B of points A , B and that of Impulses Pi A , Pi B ,*** of Sub-Spaces ***, are*** **either on the straight lines AB or on tracks of Spaces , Anti-Spaces , and Sub-Spaces of monad AB . [14-15] .**

The range of *Relative velocities* is bounded according to the single slices of spaces (*ds*).

Remarks :

1. Spaces and Anti-Spaces are continuous and represent Real numbers , $\sqrt[2]{1} = \pm 1$

A Continuous Function is a Static Completed Entity while **ds** is a quality existing Entity conveyed through PNS .

2. The Model of nature is not built on Complex numbers because Complex numbers *spring out of Spaces* , Anti-Spaces and Sub-Spaces of the FDU **(ds = 0 → AB → ∞)** and represent reality . The roots of Monads are the *same Monads* in Space and Anti-Space as well as Imaginary Monads in Sub-Space i.e. The Harmonic repetition of the roots (Principle of Equality) composes units with *no need to be Image or real dimensions* .

Image or Real dimensions exist in Euclidean Geometry as the vertices of the Regular Polygons (and Anti-Polygons) on any First dimensional unit AB . The geometrical Visualization of Complex numbers , springs from formula $\sqrt[4]{1} = \pm 1 , \pm i$ (d)

and since ± 1 is the one dimensional ***real Space*** (*the straight line*) the vertical axis is on (*Harmonic repetition of Spaces*) the other one dimensional ***Imaginary Space*** which is conveyed . Since dimension needs (N-1) points then (d) is representing the Space with three dimensions (dx ,dy ,dz) which is Natural , Real and Complex numbers and it is not four dimensional Space as it is in " Space-Time " theory .

Position and Momentum are incompatible variables because any determination of either one of them , leaves the other completely undetermined i.e.

The Eingenvalues of Spatial Position are Incompatible with the Eingenvalues of Momentum (motion) , and so any **,ds,** in PNS has a definite Position and Momentum simultaneously . This is the Relative motion of Spaces.

7.5. The Natural Numbers :

Natural Numbers with their discrete nature Symbolize Discontinuity of Spaces , because Physical World is Continuous with Points (*motion*) and Discrete with Numbers = *Monads* = *ds . This is the Dual property of Physical World .* also ,

a.. From Nothing (i.e. the Point) to Existence (i.e. to be another Spherical Point) issues the zero Virtual work Principle , which zero Work is the equilibrium of two equal and opposite forces on points. Thus Space is the Point and Anti- space is the Other Point. Infinite points are between, the Point and Other Point ,and between the Infinite points also which consist the Primary Neutral Space , infinite and discrete.

b...In Physics ,Work as motion of opposite forces, exist on the infinite points between , the Point , and , the Other Point , which Opposite forces with different lever-arms exert the equal and opposite Momentums which equilibrium in a rest and of opposite motion system , the Work done in System is zero.

1a. ***Point*** is nothing , Everything , it is Anywhere , without Position and Magnitude.
2b. ***Straight line*** is 0 and ±∞ and since is composed of infinite points which are filling line , *then nature of line is that of Point (the all is one for Lines and Points).*
3c. ***Plane*** is Positive , Negative , ± Neutral and ± Complex and since is composed of infinite Straight lines which are filling Plane then *nature of Plane is that of Line and that of Points (the all is one for Planes , Lines and Points).*
4d. ***Space*** is Positive , Negative , ± Neutral and ± Complex and since is composed of infinite Planes which are filling Space then *nature of Space is that of Plane and that of Points (the all is one for Spaces , Planes , Lines and Points) .*

5e. ***The Bounded Spaces , Anti-Spaces , Sub-Spaces , of the First dimensional Unit AB = a + b.i , are composed of the two Elements that of* the [Dipole AB = Matter] *which is* [AB] *the communicator* and [P] the Impulse *of Primary Space ,with the Bounded Impulses (PA , P B) at edges of Dipole*** or as diagram ,

[P]↔[FMD = AB – PA , P B]→ PA ,PB .

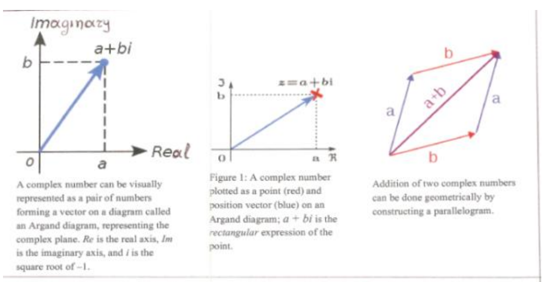


Figure 15 . *The Quaternion of Spaces – (7)*

6f. Achilles has to pass every point of line AB which is then as passing from the starting point A , ds = 0 , where Velocity of Achilles is v(A) = ds/dt = 0 .

The same happens for Tortoise at point B where Velocity v(T) = ds/dt = 0 .

On the contrary , Achilles passing AB on dimensional Units , ds , then Achilles velocity v(A) = ds/dt(A) is greater than that of Tortoise v(T) = ds / dt(T) . i.e.

A| ds = (AB/ n = ∞) = 0 |B   A| ds = → = AB / n = 11 |B
……..…………………….   →→→→→→→→→→→
***Continuous* (.)   *Discrete* (→)**

***Since in PNS , v = ∞ , T= 0 , meaning infinite velocity and Time not existing , then* Arrow AB in [PNS] is constant because AB = ds = Constant = v . 0 = ∞ . 0 . Straight line AB is discontinuous (*discrete*) with dimensional Units ds = AB / n where n = 1 → ∞ , and continuous with points [ds = 0] , (*This is the Dual Nature of lines* , *(in* geometry) , *discrete and continuous*) .**

The material Point is also continuous and equal to zero for N = - ∞ and discrete for any negative number N = 0↔∞ or $L_v = e^{i.\left(\frac{N\pi}{2}\right)b=10^{-N=-\infty}}$ = 0 , which is the Dual Nature of Euclidean geometry , i.e. nothing and number .

Spaces Anti-Spaces and Sub-Spaces are ***Homogenous*** because the Points of Monads are also so. Since all Directions ÃB , BÃ are equivalent , then PNS ***is also Isotropic in all directions*** ,

i.e all Relative Natural sizes and Laws remain Inalterable with Displacement and Rotation.
Since [*Spaces Anti-Spaces Sub-Spaces*] are the Roots of Monads and since Monads are composed of ***Real and Imaginary Parts*** , as Complex numbers are , then are Real and Imaginary , Discrete and Continuous and is not needed to be separate in Visualization .

Since velocity is ∞ and time is not existing ,T = 0 , then Curvature in PNS is zero and ∞ because happens at point..

***From all above , all changes in Geometry happen only in Extrema cases , where then qualities are becoming , i.e. Quantization of geometry from point (.) to discrete segment (-) , to line(….) , to Surface* (⊥) *, to Volume and to all Physical elements* (z=a+b.i) *exists through Extrema Principle .* [43]**

According to Pythagoras , → **Unit = ds** *is a Point without position while* **a Point = 0** *and it is a Unit having position. What is a Point and what is a Unit is clearly defined in* [43]

7.6. The Regular Polygons as Vertical Segments .

The Geometrical solution of this problem has been obtained, by extending Euclid logic of Units for Subspaces , *in the unit circle with AB = 2R as diameter , and radius R . Algebraic or Polynomial equations of any degree are represented with the Regular Polygons on the circle and thus is defined the Quantization mould of Polygons .*

In Diagram -1.(F-17) *is given the Geometrical solution of all Regular Polygons without presenting the G-Proof.*
It has been proved by De Moivre's , that the n-th roots on the unit circle AB are represented by the vertices of the Regular n-sided Polygon inscribed in the circle .
It has been proved that the Resemblance Ratio of Areas , of the circumscribed to the inscribed squares (Regular quadrilateral) which is equal to 2 , leads to the squaring of the circle.
It has been also proved that , Projecting the vertices of the Regular n-Polygon on any tangent of the circle , then the Sum of the heights y_n is equal to $n * R$. This property of the summation of heights correlates ***continuity*** as extrema to ***discrete*** and vice –versa .
This property on the circle yields to the Geometrical construction (As Resemblance Ratio of Areas which is now controlled) , and the Algebraic measuring of the Regular Polygons is as follows :

when : R = The radius of the circle , with a random diameter AB .
a = The side of the Regular **n** -Polygon inscribed in the circle
n = Number of sides , a , of the n -Polygon , then exists :

$$\mathbf{n \cdot R = 2 \cdot R + 2 \cdot y_1 + 2 \cdot y_2 + 2 \cdot y_3 + \ldots\ldots\ldots 2 \cdot y_n} \qquad \ldots\ldots\ldots\ldots \mathbf{(n)}$$

the heights y_n are as follows :

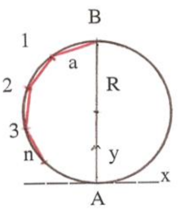


Figure 16 *.The Unit circle of the Polygons – (7)*

$y_B = [\ 2 \cdot R\ ]$

$y_1 = [\ 4.R^2 - a^2\ ] \ /\ (\ 2 \cdot R\ )$

$y_2 = [\ 4.R^4 - 4.R^2 \cdot a^2 + a^4\ ] \ /\ (\ 2.R^3\ )$

$$y_3 = \frac{[\ 8.R^6 - 10.R^4.a^2 + 6.R^2.a^4 - a^6\ ] - a.\sqrt{64.R^8 - 96.R^6.a^2 + 52.R^4.a^4 - 12.R^2.a^6 + a^8}}{2.R^5}$$

$y_n = [\ \ldots\ldots\ldots\ldots\ldots..\ ] \ /\ 2.R^n$, → The general equations are prepared .

THE ALGEBRAIC EQUATIONS OF THE REGULAR n - POLYGONS

(a) REGULAR TRIANGLE :

The Equation of the vertices of the Regular Triangle using (n = 3) is :

$$3.R = 2.R + \frac{[\ 4.R^2 - a^2\ ]}{R} \quad >>> \quad R^2 = 4 \cdot R^2 - a^2 \quad >>> \quad a^2 = 3 \cdot R^2$$

The side $\mathbf{a_3 = R \cdot \sqrt{3}}$ …….(1).

Using De Moivre's formula for complex numbers then complex cube roots of unit circle (R=1) then is $z_3 = \cos(360/3) + i.\sin(360/3) = [-1+ i.\sqrt{3}]/2$, therefore the side of the regular triangle a_3 becomes $(a_3)^2 = [\ R(1+1/2)]^2 + [\ R\sqrt{3}/2]^2 = 9.R^2/4 + 3.R^2/4 = 12.\ R^2/4 = 3.R^2$ and $a_3 = R\sqrt{3}$ as above .

(b) REGULAR QUADRILATERAL (SQUARE) :

The Equation of the vertices of the Regular Square using (n = 4) is :

$$4.R = 2.R + \frac{[\ 4.R^2 - a^2\ ]}{R} \quad >>> \quad a^2 = 2 \cdot R^2$$

The side $\mathbf{a_4 = R \cdot \sqrt{2}}$ …….(2)

It is z_4 = cos(360/4)+i.sin(360/4)= [0+ i.] and side $(a_4)^2 = R^2.(1^2+1^2) = 2.R^2$ and $a_4 = R\sqrt{2}$ as above (2).

(c) REGULAR PENTAGON ۞ :

The Equation of the vertices of the Regular Pentagon using (n = 5) is :

$$5.R = 2.R + [\frac{4.R^2 - a^2}{R}] + [\frac{4.R^4 - 4.R^2.a^2 + a^4}{R^3}] \quad >>> \quad a^4 - 5.R^2.a^2 + 5.R^4 = 0$$

Solving the equation gives :

$$a^2 = \frac{5.R^2 - \sqrt{25.R^4 - 20.R^4}}{2} = \frac{5.R^2 - R^2.\sqrt{5}}{2} = [\{5.R^2 - R^2.\sqrt{5}\}/2] = \frac{R^2}{2}.(5 - \sqrt{5})$$

$$a^2 = \{\frac{R^2}{4}\}.[10 - 2\sqrt{5}] \quad >>> \quad \text{The side} \quad \mathbf{a_5} = \left|\frac{R}{2}\right|.\sqrt{10 - 2.\sqrt{5}} \quad ......(3)$$

z_5 = cos(360/5) + i.sin(360/5) = [1 ±.√ 5 ± i.√ 10 ± √ 5] / 4 , therefore the side of the regular Pentagon a_5 becomes $(a_5)^2 = [R.\sqrt{(10+2\sqrt{5})}]^2 + [R(1+(1-\sqrt{5})/2]^2 = R^2\sqrt{(10 - 2\sqrt{5})/4}]$ and $a_5 = R.\sqrt{(10 - 2\sqrt{5})/2}$ as above (3) ..

(d) REGULAR HEXAGON ۞ :

The Equation of the vertices of the Regular Hexagon using (n = 6) is :

$$6.R = 2.R + [\frac{4.R^2 - a^2}{R}] + [\frac{4.R^4 - 4.R.^2.a^2 + a^4}{R^3}] \quad >> \quad a^4 - 5.R^2.a^2 + 4.R^4 = 0$$

Solving the equation gives :

$$a^2 = \frac{5.R^2 - \sqrt{25.R^4 - 16.R^4}}{2} = [\frac{5-3}{2}].R^2 = R^2 \qquad \text{The side} \quad \mathbf{a_6 = R} \quad .....(4)$$

(e) REGULAR HEPTAGON ۞ :

The Equation of the vertices of the Regular Xeptagon using (n = 7) is :

$$7.R = 2.R + [\frac{4.R^2 - a^2}{R}] + [\frac{4.R^4 - 4.R^2.a^2 + a^4}{R^3}] + [\frac{8.R^6 - 10.R^4.a^2 + 6.R^2.a^4 - a^6}{2.R^5}]$$

$$- [\frac{a^2}{2.R^5}].\sqrt{64.R^8 - 96.R^6.a^2 + 52.R^4.a^4 - 12.R^2.a^6 + a^8}$$

Rearranging the terms and solving the equation in the quantity **a** , obtaining :

$$R^2.a^{10} - 13.R^4.a^8 + 63.R^6.a^6 - 140.R^8.a^4 + 140,R^{10}.a^2 - 49.R^{12} = 0 \qquad \text{for } a^2 = x$$

$$x^5 - 13.R^2.x^4 + 63.R^4.x^3 - 140.R^6.x^2 + 140.R^8.x^1 - 49.R^{10} = 0 \quad ....(7)$$

Solving the 5nth degree equation the Real roots are the following two :

$x_1 = R^2 . [3 - \sqrt{2}]$, $x_2 = R^2 . [3 + \sqrt{2}]$ which satisfy equation (7)

Having the two roots , the Sum of roots be equal to 13 , their combination taken 2 ,3, 4 at time be equal to 63 , - 140 , 140 , the product of roots be equal to – 49 , then equation (7) is reduced to the third degree equation as :

$$z^3 - 7. z^2 + 14. z - 7 = 0 \quad .....(7a)$$

by setting $\psi = z - (-7/3)$ into (7a) , then gives $\boldsymbol{\psi^3 + \rho. \psi + q = 0}$ **.... (7b)** where ,

$\rho = 14 - (-7)^2 / 3 = 14 - 49/3 = - 7 / 3$ > $\rho^2 = 49 / 9$ > $\rho^3 = - 343 / 27$
$q = 2. (-7)^3 / 27 + 14 .(-7) / 3 - 7 = 7 / 27$ > $q^2 = 49 / 729$

Substituting ρ , q then $$\boldsymbol{\psi^3 - (7 / 3) . \psi + (7 / 27) = 0 \ldots (7c)}$$

The solution of this third degree equation (7c) is as follows : $\rho = - 7 / 3$
$q = 7 / 27$

Discriminant $D = q^2 / 4 + \rho^3 / 27 = (49 / 729 .4) - (343 / 27.27) = - [49 / 108] < 0$

$$D = - 49 /108 = i^2 (3.21^2 / 4 .27^2) = i^2 (21 .\sqrt{3} / 2 .27)^2 = i^2 (21.\sqrt{3} / 54)^2$$

$$D = [7 . \sqrt{3} / 18]^2. i^2 \quad \text{also} \quad \sqrt[2]{D} = \left| \frac{7 .\sqrt{3}}{18} \right| . i$$

Therefore the equation has three real roots :

Substituting $\psi = w - \rho / 3.w = w + 7 / 9.w$ > $\psi^2 = w^2 + 49 / 81 .w^2 + 14 / 9$
> $\psi^3 = w^3 + 343/729w^3 + 49 /27w+7w / 3$
to (7b) then becomes $w^3 + 343 /729 w^3 + 7 / 27 = 0$
and for $z = w^3$ $z + 343 /729 z + 7 / 27 = 0$

$$\boldsymbol{z^2 + 7. z / 27 + 343 / 729 = 0} \quad \ldots(7c)$$

The Determinant $D < 0$ therefore the two quadratic complex roots are as follows :

$Z_1 = [-7/27 - \sqrt{49/27.27 - 4. 343/729}] / 2 = [-7/27 - \sqrt{49/27.27.4 - 49 .7. 4/27.27.4}] / 2$

$= [-7/27 - \sqrt{(49 -49.28)/ 27.27.4}] / 2 = [- 7 - 7 .\sqrt{- 27}] / 27 . 2$

$= [- 7 - 21 .\sqrt{- 3}] / 3^3 . 2$ $= \left[\frac{- 7}{2} \right] . (1 - 3 . i .\sqrt{3}) / 27 = (-7 / 54).[1 - 3.i.\sqrt{3}]$

$Z_2 = [-7/2 . (1 - 3 . i \sqrt{3}] / 27$ $= (- 7 / 54) . [1 + 3.i.\sqrt{3}]$

The Process is beginning from the last denoting quantities to the first ones :

Root $W_{1,2} = \sqrt[3]{Z} = \frac{1}{3}\sqrt[3]{\frac{-7 \pm 21 \cdot i \cdot \sqrt{3}}{2}} = \frac{1}{3}\sqrt[3]{-(7)\cdot\frac{[1 \pm 3 i \sqrt{3}]}{2}}$ .....(1)

Root $\psi = W + 7/9.W = \frac{1}{3}\sqrt[3]{\frac{-7 \pm 21 \cdot i \sqrt{3}}{2}} + \frac{7}{3\cdot\sqrt[3]{\frac{-7 \pm 21 i \sqrt{3}}{2}}}$ ....(2)

Root $X = \psi - \rho/3 = \psi + 7/3 = \frac{7}{3} + \frac{1}{3}\cdot\sqrt[3]{\frac{-7 \pm 21 \cdot i \cdot \sqrt{3}}{2}} + \frac{7}{3\cdot\sqrt[3]{\frac{-7 \pm 21 \cdot i \cdot \sqrt{3}}{2}}}$

$$X = \frac{1}{3}\left|\frac{7\cdot\sqrt{\frac{-7 \pm 21 \cdot i \cdot \sqrt{3}}{2}} + \sqrt{\frac{-7 \pm 21 \cdot i \cdot \sqrt{3}}{2}}^{\,3/2} + 7}{\sqrt[3]{\frac{-7 \pm 21 \cdot i \cdot \sqrt{3}}{2}}}\right| \cdot R^2 \quad \text{......... (3)}$$

The root a_7 of equation (7) equal to the side of the regular Heptagon is $a_7 = \sqrt{X}$

$$\mathbf{a_7 = \sqrt{\frac{1}{3}\left|\frac{7\cdot\sqrt[3]{\frac{-7 \pm 21 \cdot i \cdot \sqrt{3}}{2}} + \sqrt{\frac{-7 \pm 21 \cdot i \cdot \sqrt{3}}{2}}^{\,3/2} + 7}{\sqrt[3]{\frac{-7 \pm 21 \cdot i \cdot \sqrt{3}}{2}}}\right|} \cdot R} \quad \textbf{........ (4)}$$

Instead of substituting $\psi = w - \rho / 3.w$ into (7.b), is substituted $\boldsymbol{\psi = u + v}$ and then gives the equation of second degree as $z^2 + 7.z / 27 + 343 / 729 = 0$ which has the two complex roots as follows :

$Z_{1,2} = \frac{7}{54}\cdot[-1 \pm 3 \cdot i \cdot \sqrt{3}] = \frac{1}{27}\cdot[(-7 \pm 21 \cdot i \cdot \sqrt{3}) / 2]$ and the side a_7 is as :

$a_7 = \sqrt{\sqrt[3]{Z_1} + 7 / \sqrt[3]{Z_2} + \frac{7}{3}}$ and by substituting Z_1, Z_2 into (7b) becomes the same formula as in (4) .

It is easy to see that $\sqrt[3]{-(7/2)\cdot[1 - 3 \cdot i \cdot \sqrt{3}]} * \sqrt[3]{-(7/2)\cdot[1 + 3 \cdot i \cdot \sqrt{3}]} = 7$

Analytically is :

$x = R^2 \cdot [\ 0{,}753\ 020\ 375\ 967\ 025\ 701\ 777\ ]$ >> $x^2 = 0{,}56704$
$a_7 = \sqrt{x} = R \cdot [\ 0{,}867\ 767\ 453\ 193\ 664\ 601 \ldots\ ]$

By using the formula of the **real** root of equation (7a) then :

$a.x^3 + b.x^2 + c.x + d = 0$ >>> for $a = 1$, $b = -7$, $c = 14$, $d = -7$ then $x^3 - 7.x^2 + 14.x - 7 = 0$

and x equal to ,

$$x = -\frac{b}{3} - \frac{2^{1/3}.(-b^2 + 3.c)}{3\left[-2b^3+9bc-27d+\sqrt{4(-b^2+3c)^3+(-2b^3+9bc-27d)^2}\right]^{1/3}} + \frac{\left[-2b^3+9bc-27d+\sqrt{4(-b^2+3c)^3+(-2b^3+9bc-27d)^2}\right]^{1/3}}{32^{1/3}}$$

Substituting the coefficients to the upper equation becomes :

$-b^2 + 3.c = -(-7)^2 + 3.14 = -49 + 42 = -7$
$-2.b^3 + 9.b.c - 27.d = -2.(-7)^3 + 9.(-7).14 - 27.(-7) = 686 - 882 + 189 = -7$
$4.(-b^2+3.c)^3 = 4(-7)^3 = -1372$
$(-2.b^3 + 9.b.c - 27.d)^2 = (-7)^2 = 49$
$32^{1/3} = \sqrt[3]{8.4} = 2.\sqrt[3]{4}$

and for X is as follows ,

$$X = \frac{7}{3} - \frac{\sqrt[3]{2}.(-7)}{3.\sqrt[3]{-7 + 21.i.\sqrt{3}}} + \frac{\sqrt[3]{-7 + 21.i.\sqrt{3}}}{2.\sqrt[3]{4}} = A + B i \quad \text{and} \quad a7 = \sqrt{\mathbf{A^2 + B^2}}$$

$$\mathbf{a7} = \sqrt{\mathbf{X}} = \sqrt{\frac{7}{3} + \frac{7.\sqrt[3]{2}}{3.\sqrt[3]{-7 + 21.i.\sqrt{3}}} + \frac{\sqrt[3]{-7 + 21.i.\sqrt{3}}}{2.\sqrt[3]{4}}}$$

The Side of the **Regular Heptagon** **(4.a)**

Further Analysis to the Reader

(f) REGULAR OCTAGON :

The equation of vertices of the Regular Octagon is :

$$8.R = 2.R + \frac{(a^2)}{R} + \frac{(4.R^2.a^2 - a^4)}{R^3} + \frac{10.R^4.a^2 - 6.R^2.a^4 + a^6 + a^2.\sqrt{64.R^8 - 96.R^6 a^2 + 52.R^4.a^4 - 12.R^2.a^6 + a^8}}{2.R^5}$$

Rearranging the terms and solving the equation in the quantity , **a ,** is a 10th degree equation , and by reduction ($x = a^2$) is find the 5^{th} degree equation as follows :

$$A^{10} - 13.R^2.a^8 + 62.R^4.a^6 - 132.R^6.a^4 + 120.R^8.a^2 - 36.R^{10} = 0$$

$$x^5 - 13.R^2.x^4 + 62.R^4.x^3 - 132.R^6.x^2 + 120.R^8.x^1 - 36.R^{10} = 0 \qquad \text{…. (a)}$$

Solving the 5^{th} degree equation is find the known algebraic root of Octagon of side **a** as :

The roots are >>>>>>>>>> $x_1 = R^2 . [\, 2 - \sqrt{2}\,]$, $x_2 = R^2 . [\, 3 - \sqrt{3}\,]$

$$a_8 = \sqrt{x} = R . \sqrt{2 - \sqrt{2}} \qquad \text{........ (b)}$$

Verification :

$x = a^2 = R^2 (2 - \sqrt{2})$ $x^2 = R^4 . (6 - 4 \sqrt{2})$ $x^3 = R^6 . (20 - 14.\sqrt{2})$

$x^4 = R^8 . (68 - 48 \sqrt{2})$ $x^5 = R^{10} . (232 - 164 \sqrt{2})$ ....(c)

by substitution (c) in (a) becomes :

$$R^{10} . [\, 232 - 164 .\sqrt{2}\,] = R^{10} . [\, 232 - 164 .\sqrt{2}\,]$$
$$- R^{10} . [\, 884 - 624 .\sqrt{2}\,] = R^{10} . [\, -884 + 624 .\sqrt{2}\,]$$
$$R^{10} . [\, 1240 - 868 .\sqrt{2}\,] = R^{10} . [\, 1240 - 868 .\sqrt{2}\,]$$
$$- R^{10} . [\, 792 - 528 .\sqrt{2}\,] = R^{10} . [\, -792 + 528 .\sqrt{2}\,]$$
$$R^{10} . [\, 240 - 120 .\sqrt{2}\,] = R^{10} . [\, 240 - 120 .\sqrt{2}\,]$$
$$- R^{10} . [\, 36 \,] = R^{10} . [\, -36 \,]$$

----

and by summation becomes ,

$$R^{10} . [\, 1712 - 1712 + (1152 - 1152) . \sqrt{2}\,] = 0$$

$R^{10} . [\, 0+0\,] = 0$ therefore Side $\mathbf{a_8 = R . \sqrt{2 - \sqrt{2}}}$ ...... (b)

(g) CONCLUTION :

By summation the heights **,y,** on any tangent in a circle , which hold for every ***Regular n-sided Polygon*** inscribed in the circle as the next is gives :

$$\mathbf{n .R = 2.R + 2. y_1 + 2. y_2 + 2. y_3 + \ldots 2. y_n} \qquad \text{..................(n)}$$

the sides a_n of all these Regular n-sided Polygons are Algebraically expressed .

The Geometrical Construction of all Regular Polygons has been proved to be based on the Extrema solution of the moving Segment ZD of the Diagram 1 . Proof and solution presupposes something more than temporal mathematics , (the locus of a changeable segment in geometry of motion) so are not given in this article. Moving Segment ZD is one of the Master Keys for quantization in circle of E-Geometry , because so the nth degree Algebraic or Polynomial equations become the vertices of the n-polygons . The position of Segment ZD of the moving Point D on circle (K ,KA=KO) when formulates angle < ACB = 90° then segment defines the sides of the regular polygons which is the Quantization mould of Polygons . Heits**,** on any tangent in a circle , need a general approach because they are joining the length (Spaces) and the roots (Sub-spaces) of monads .

The Delian problem < Doubling the cube which is the construction of a line segment of length x such that $x^3 = 2$ or $x = \sqrt[3]{2}$ > is part of equation 4.a of the regular heptagon meaning that these problems are solved in Imaginary and Real part field .[47] . Now is given the solution of this age old unsolved problem in Pages 34 and 59.

In this way , all Regular p - gone are constructible and measureable .

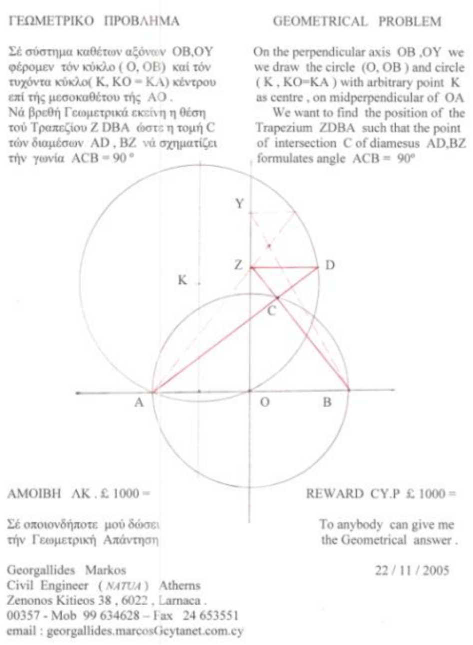
The mathematical reasoning is based on Geometrical logic exclusively alone . The methods used are ,

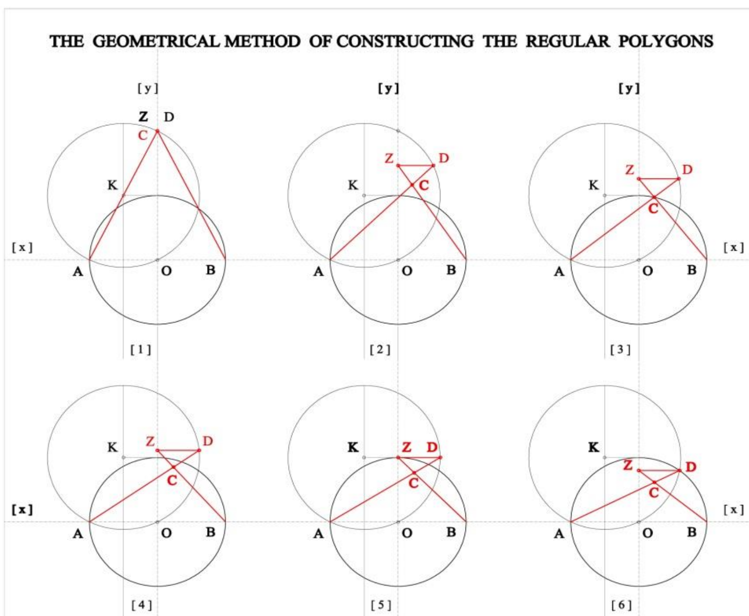
A.. ***Resemblance Ratio Method in Trial – 1*** : Says that when Areas of the circumscribed to the inscribed **Regular n –Polygons** is equal to 2 , then this ratio solves the problem of squaring the circle . It is also a problem of quantization of areas as that of regular polygons and has been approached and solved by extending Euclid logic of Units (*under the restrictions imposed to seek the solution which is , with a ruler and a compass*) on the unit circle AB , to unknown and now Geometrical elements . ***Resemblance Ratio*** *of Areas of the circumscribed to the inscribed* ***Regular n –Polygons maybe now controlled*** . [F-37]

B.. ***The Plane Procedure Method in Trial - 2*** : Mechanics is the study of motion , described by Kinematics , and which is caused by Dynamics . It is a moving geometrical machine producing squares such that the area of one of the changeable Squares , *or changeable Cubes* , to be equal to that of the circle [F42-45-46].

C.. ***The Extrema Method of the moving Segment*** : It is a moving geometrical segment alone either in a Rectangle or in a Square , producing segments such that are equal to the sides of regular polygons . Diagram 1 shows the Restrictions needed which show the way of constructing polygons without giving the Proof . [F17]

Diagram 1 . The Geometrical Way of Constructing Polygons , (GCRP Method)





→ The above Reward is still holding ,
→ markos

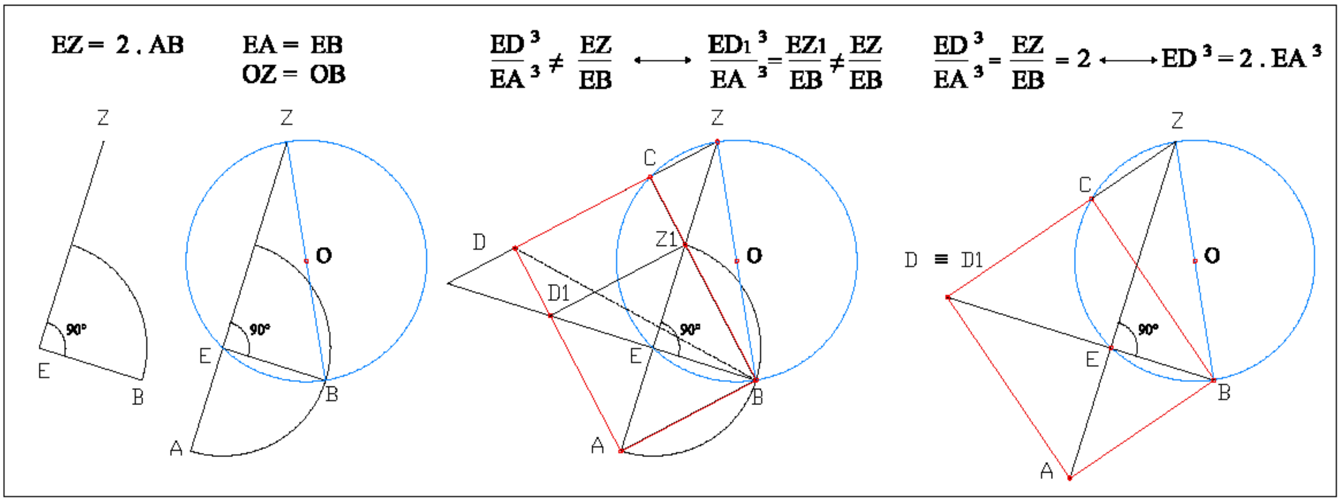
Figure 17 . The Extrema in Regular Polygons .

The Regular Heptagon :

According to Heron , the regular Heptagon is equal to six times the equilateral triangle with the same side has the approximate value of $\sqrt{3}$. R / 2 .
According to Archimedes , given a straight line AB we mark upon it two points C , D such that AD.CD = DB^2 and CB.DB = AC^2 , without giving the way of marking the two points .
According to the Contemporary Method , ***Point C being on circle*** defines the side of the Regular Heptagon and is the root of a third degree equation with three real roots , one of which is that of the regular Heptagon and as analytically presented .The method leads to to the equations of nth degree which are the (n-Spaces) on nth roots (Sub-spaces).
The relation of **a**7, to imaginary number ,i, defines the Imaginary quality of this space energy monad and further analysis of **a**7, as in (4.a) gives the relation of **i** and the Sub-Spaces of Units in geometry.

7.6.1 The Doubling of the Cube.

Let line segment AE is the edge and EA^3 volume of AE . It is required that we find the edge X = ED of a cube with twice this volume , or $X^3 = ED^3 = 2 . EA^3$. [F.18]



(1) (2) (3) (4)

Figure 18 . The doubling of the cube .

The Geometrical Solution :

1.. Let any line segment EZ be perpendicular to its half segment EB or as EZ = 2.EB ⊥ EB
2.. Draw circle (O,BZ/2) with diameter BZ
3.. Produce on ZE line-segment EA = EB or EA ≠ EB forming the Isosceles right-angled triangle AEB .
4.. Draw BC perpendicular to AB such that point C meet the circle (O,BZ/2) in point C .
5.. ZC and BE produced meet each other at common point D.
6.. Draw AD1 perpendicular to AB such that point D1 meet line BE produced .
7.. Make points D , D1 coinciding , by [Extrema method which is altering BA,BD1 by expanding squares and is completing ABCD Rectangle] , *the Method in* Maxima , or *by Archimedes method of Exhaustion* .

Show that $ED^3 = 2. EA^3$

Proof : F.18-4

1.. Since EZ = 2.EB then (EZ / EB) = 2 , and since angle < ZEB = 90° then BZ is the diameter of circle (O,OZ) and angle < ZEB = 90° on diameter .
2.. Since angle < ZEA = 180° and angle < ZEB = 90° therefore angle < BEA = 90° also .
3.. Since AD , BC are both perpendicular to AB , therefore are parallel , and since also each of the three angles < DAB , ABC , BCD = BCD1 are equal to 90° therefore angle < ADC = 90° and shape ABCD is a Rectangle .
4.. From right angle triangles ADZ , ADB we have ,

Δ ADZ → $ED^2 = EA . EZ$ …………… (a)
Δ ADB → $EA^2 = ED . EB$ …………… (b) and by division (a) / (b) then →

$$\frac{ED^2 = EA . EZ}{EA^2 = ED . EB} = \left|\frac{ED^2}{EA^2}\right| = \left|\frac{EA . EZ}{ED . EB}\right| \text{ or } \rightarrow \left|\frac{ED^3}{EA^3}\right| = \left|\frac{EZ}{EB}\right| = 2 \quad \text{……. (o.ε.δ),(q.e.d)}$$

i.e. **$ED^3 = 2 . EA^3$, *which is the Duplication of the Cube*** [47] .

All comments are left to the readers , markos 20/8/2015. Now Extrema Proof is in P-34. [L] Fig.30

7.7. The Extrema Principle in Euclidean geometry .

Extreme Principle or Extrema :

As in Calculus , *limit* , defines functions to a quantity so , *Extrema* , defines , *limit* , to the different qualities of Elements in Euclidean geometry (segments etc.) , *by the changes in limit cases for , Points –sectors –lines –Surfaces – Volumes , causing* ***the Euclidean Self Quantization*** **,** so

1.. All Principles are holding on any Point A

2.. For two points A , B not coinciding , exists Principle of Inequality which consists another quality . Any two Points exist in their Position under one Principle , *Equality of Stability ,(Virtual displacement which presupposes Work in a Restrain System , and represents the Quantization of points to Segments*) . [12]
This Equilibrium presupposes homogenous Space and Symmetrical Anti-Space .
For two points A , B which coincide , exists *Principle of Superposition* which is a Steady State containing Extrema for each point separately.

3.. For three points A , B , M (*Plane ABM*) not coinciding and not belonging in straight line ,exists Principle of Inequality which consists also another quality and in case of circle , AB *diameter and point* M *on the circumference , represents Quantization of Segments to Areas*) , the equal area square .

4.. For four points A , B , C, M (*Volume ABCM*) not coinciding and not belonging in straight line or plane, exists Principle of Inequality which consists also another quality and in case of cube with AB *side and point* M *on the Sphere , represents Quantization of Areas to Volumes ,* then is the duplication of the cube.

Extrema , for a *point A* is the Point , for a *straight line* the infinite points on line, either these coincide or not or these are in infinite, and for *a Plane* the infinite lines and points with all combinations and Symmetrical ones**,**

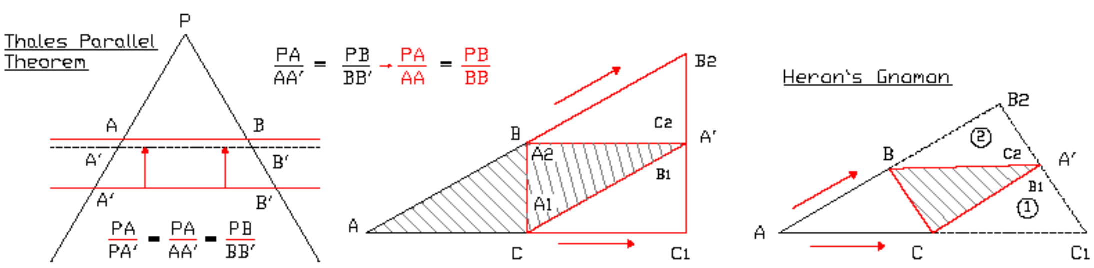
i.e. all Properties of Euclidean geometry, compactly exist in Extrema → Points ,Lines ,Planes ,Circles , Cubes Spheres , Polyhedrons , as the quality change Quantization . This Special property in E-geometry transforms continuity of points to the discrete of monads (The quantization of Energy as* ,Energy quanta, *and Space as* ,vectors and Quaternion) *and further to the Physical world elements through this Extrema Principle .
This is the deep meaning of the geometrical extrema .

Since Extrema is holding on Points , lines , Surfaces etc. therefore all their compact Properties (Principles of Equality , Arithmetic and Scalar , Geometric and Vectors , Proportionality, Qualitative , Quantities , Inequality , Perspectivity etc.) , exist in a common context .
Since a quantity is either a vector or a scalar and by their distinct definitions are ,

Scalars , are quantities that are fully described by a magnitude (or numerical value) alone ,
Vectors , are quantities that are fully described by both magnitude and a direction , and so
Quaternion , are quantities that are described by all , magnitude and a direction , therefore ,

In Superposition magnitude AB is equal to zero and direction , *any direction* , ≠ 0 i.e.

Any Segment AB between two points A , B consist a Vector described by the magnitude AB and directions ÃB , BÃ and in case of Superposition ÃA , AÃ . i.e. Properties of Vectors, Proportionality, Symmetry , etc , exists either on edges A , B or on segment AB as follows to Thales in F-19 :



(1) (2) (3)

Figure 19 . The Extrema in Thales theorem and Rational Figures – (7)

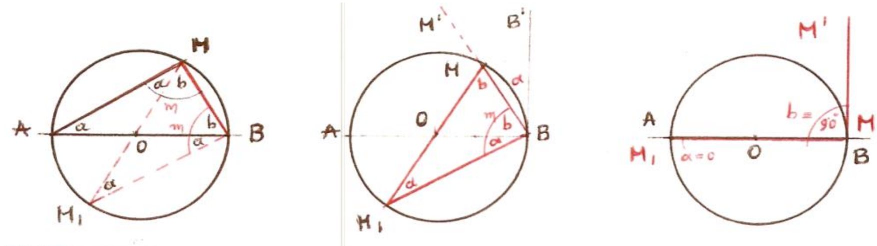
A . Thales Theorem as Extrema . F19 - (1)

According to Thales F19-(1) , if two intersecting lines PA , PB are intercepted by a pair of Parallels AB // A′B′ , then ratios PA/AA′ , PB/ BB′ , PA/PA′ , PB/ PB′ of lines , or ratios in similar triangles PAB , PA′B′ are equal or λ = [PA / AA′] = [PB / BB′] .
In case line A′ B′ coincides with AB , then AA′ = AA , BB′ = BB , i.e. λ = [PA / 0] = [PB / 0] and exist **Extrema** where then ratio λ is ,
λ = [PA / AA] = [PB / BB] = [PA / 0] = [PB / 0] , (Principle of Superposition).

B. Extrema Rational Figured Numbers of Figures F19 - (2-3) .

The definition of " Heron " that gnomon is as that which , when added to anything , a number or figure , makes the whole similar to that to which it is added . (Principle of Proportionality) .
It has been proven that the triangle with sides twice the length of the initial , preserves the same angles of the triangle. [9] i.e. exists **Extrema** for Segment BC at B1C1 .

C. Extrema for a given Point **M** on any circle with AB as diameter . [8] .

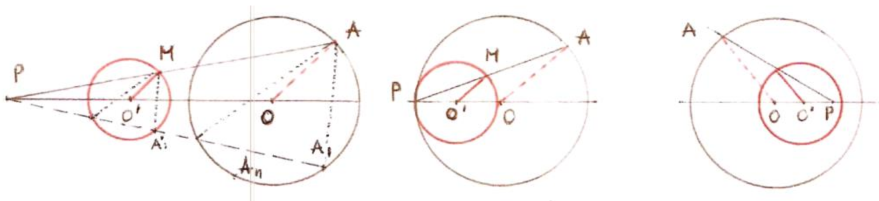


(1) (2) (3)

Figure 20 . A point M on any circle of diameter the monad AB :

It has been proved [9] that angle AMB = 90° for all points of the circle .This when point M is on B where then exists **Extrema** at point B for angle AMB . (Segments MA , MB) i.e.
Angle AMB = 90° = AMM1 + M1MB = ABA + ABB = 0 + 90° = 90° …. F20. (3)

D. Extrema for a given Point **P** and any circle (O , OA) . [16]



(1) (2) (3)

Figure 21 . The position of a point M to any circle of diameter AB :

It has been proved that the locus of midpoints M of any Segments PA , is a circle with center O′ at the middle of PO and radius O′ M = OA / 2 . (F.21)
Extending the above for point M to be any point on PA such that PM / PA = a constant ratio equal to λ , then the locus of point M is a circle with center O′ on line PO , PO′ = λ .PO , and radius O′ M = λ . OA .
Extrema exists for points on the circle .The above extrema is very useful in Kinematics where any motion can be analyzed as , Translation on Lines and Rotation on Points (*the poles*) .

Remarks :

1. In case PO′ = λ . PO and O′M = λ . PO and this is holding for every point on circle , so also for 2 , 3 ,.. **n** points i.e. any Segment AA1 , triangle A,A1,A2 , Polygon A,A1 …An ,… the circle, is represented as a Similar Figure (*Segment , triangle , Polygon or circle*) .

2. In case PO′ = λ . PO and PM1 = PA . λ1 , then for every point A on the circle (O , OA) , in or out the circle , exist a point M1 such that line AM1 passes through point P . i.e. every segment AA1 , Triangle A,A,A2 , Polygon A,A1……An , or circle , is represented as segment M,M1,M2 , the Polygon M,M1 ….. Mn or a closed circle [Perspective or Homological shape] .

E. Extrema of Parallel Postulate . [9] .

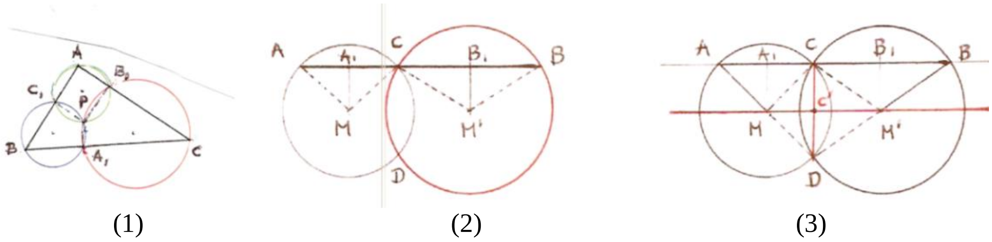


Figure 22 . A point M and any line directional to AB :

Any point M , not coinciding with points A , B consists a Plane (the Plane MAB) and from point M passes only one Parallel to AB . *This parallel and the symmetrical to AB is the* **Extrema** , because all altitudes are equal and of minimum distance , Segment MA1 = CC` . When point M lies on line AB then parallel is on AB and all properties of point P are carried on AB and all of M on line AB . F22.(3) .

Segment MA1 = CC`= M`B1 is the Linear Quantization of Euclidean Geometry through Mould of Parallel Theorem . [The Linear Quantization of E-Geometry i.e. Segment MA1 = Segment M`B1 = Constant].

F. Extrema on Symmetry (Central , Axial) . F.23

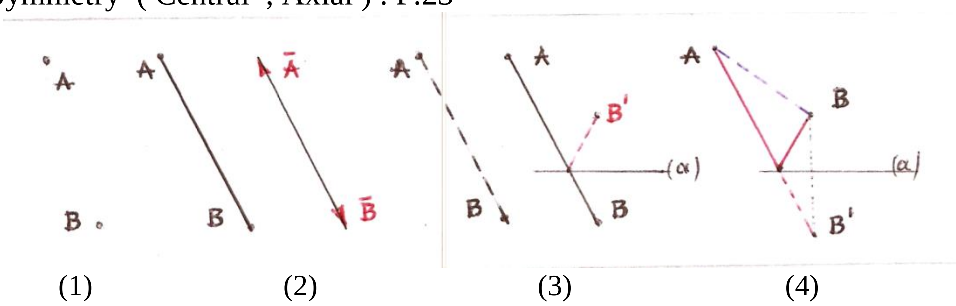


Figure 23 . A point and a line of any direction :

Since any two points A,B consist the first dimensional Unit (magnitude AB and direction ÃB ,B Ã) F23.(1-2) equilibrium at the middle point of A , B , *Central Symmetry* . Since the middle point belongs to a Plane with infinite lines passing through and in case of altering central to axial symmetry , then equilibrium also at *axial Symmetry* F23.(3), so Symmetrical Points are in **Extrema .** *Nature* follows this property of points of Euclidean geometry (*common context*) as Fermat`s Principle for Reflection and Refraction . F23.(4) .

G. Extrema for a point P and a triangle ABC . (F24)

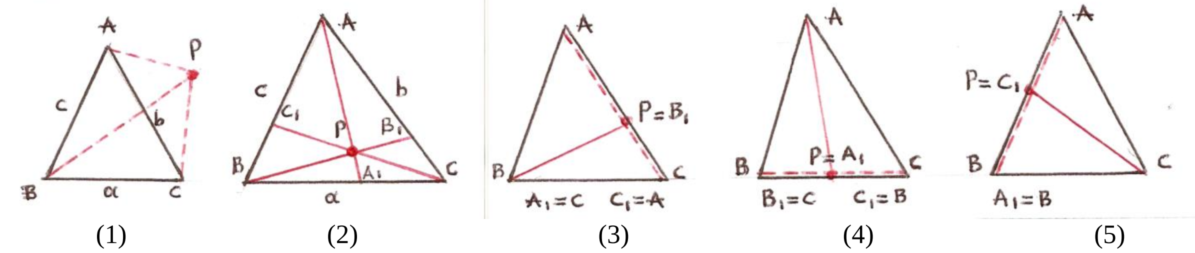


Figure 24 . The position of a point P and any triangle ABC :

If a point P is in Plane ABC and lines AP , BP , CP intersect sides BC , AC , AB , of the triangle ABC at points A1 ,B1 ,C1 respectively F24. (1–2) , then the product of ratios λ is , λ = (AB1 / B1C) . (CA1 / A1B) . (BC1 / C1A) = 1 , and the opposite . Proof :

Since **Extrema of Vectors** exist on edge Points A ,B ,C , and lines AC , BC , AB then for ,

a. Point P on line AC , F23.(3) → A1 = C , B1 = P , C1 = A and for
λ = (AB1 / B1C).(CA1 / A1B).(BC1 / C1A) → λ1 = (PA / PC).(CC / BC).(AB / AA)

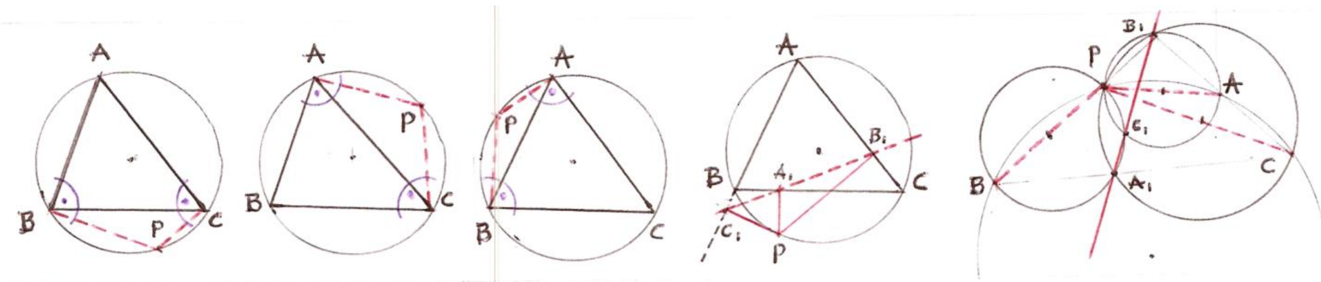
b. Point P on line BC , F23.(4) → A1 = P , B1 = C , C1 = B and for
λ = (AB1 / B1C).(CA1 / A1B).(BC1 / C1A) → λ2 = (AC / CC).(PC / PB).(BB / AB)

c. Point P on line BA , F23.(5) → A1 = B , B1 = A , C1 = P and for
λ = (AB1 / B1C).(CA1 / A1B).(BC1 / C1A) → λ3 = (AA / AC).(BC / BB).(PB / PA) and for

λ1 . λ2 . λ3 = [PA.CC.AB.AC.PC.BB.AA.BC.PB] : [PC.BC.AA .CC.PB.AB .AC.BB.PA] = 1

i.e. Menelaus *Theorem* , and for Obtuse triangle λ1 . λ2 . λ3 = - 1 → Ceva`s Theorem

H. Extrema for a point P , a triangle ABC and the circumcircle .



(1) (2) (3) (4) (5)

Figure 25 . The position of a point P and any circumcircle on any triangle ABC :

a. Let be A1 ,B1 ,C1 the feet of the perpendiculars (altitudes are the Extrema) from any point P to the side lines BC , AC , AB of the triangle ABC , F-25
Since Properties of Vectors exist on lines AA1 , BB1 ,CC1 then for **Extrema Points** ,

F25.(1) → Point A1 on points B , C of line BC respectively , formulates perpendicular lines BP , CP which are intersected at point P . Since Sum of opposite angles PBA+PCA = 180° therefore the quadrilateral PBAC is cyclic on circumcircle of triangle ABC and also the feet of the perpendiculars of point P on sides of triangle ABC are points B , C .

F25.(2) → Point B1 on points A , C of line AC respectively , formulates perpendicular lines AP , CP which are intersected at point P . Since Sum of opposite angles PAB + PCB = 180° therefore the quadrilateral PABC is cyclic on circumcircle of triangle ABC and also the feet of the perpendiculars of point P on sides of triangle ABC are points A , C .

F25.(3) → Point C1 on points A , B of line AB respectively, formulates perpendicular lines AP , BP which are intersected at point P . Since Sum of opposite angles PAC + PBC = 180° therefore the quadrilateral PACB is cyclic on circumcircle of triangle ABC and also the feet of the perpendiculars of point P on sides of triangle ABC are points A , B .

So **Extrema** for the three sides of the triangle is point P which is on the circumcircle of the triangle and the feet A1 ,B1 ,C1 of the perpendiculars of point P on sides of triangle ABC are respectively on sides BC , CA , AB i.e. on a line **.** **[** F25.4] → *Simson`s line.*
Since altitudes PA1 , PB1 , PC1 are also perpendiculars on the sides BC,AC,AB and segments PA,PB,PC are diameters of the circles, then points A1,B1,C1 are collinear .[F25.5] . → *Saimon Theorem*

b. Let be A1 ,B1 ,C1 any three points on sides of a triangle ABC (these points are considered **Extrema** because they maybe on vertices A , B , C or to ∞) . F- 25 .

If point P is the common point of the two circumcircle of triangles AB1C1 , BC1A1 , (a vertex and two adjacent sides) then circumcircle of the third triangle CA1B1 passes through the same point P . *Proof* :

Since points A , B1 , P , C1 are cyclic then the sum of opposite angles BAC + B1PC1 = 180°.
Since points B , C1 , P , A1 are cyclic then the sum of opposite angles ABC + C1PA1 = 180°.
and by Summation BAC + ABC + (B1PC1 + C1PA1) = 360 ° ……… (1)
Since BAC+ABC+ACB = 180 ° then BAC+ABC = 180° - ACB and by substitution to (1)
(180° - ACB) + 360 - A1PB1 = 360° or , the sum of angles ACB + A1PB1 = 180° , therefore points C , B1 , P , C1 are cyclic i.e.

Any three points A1 , B1 , C1 on sides of triangle ABC , forming three circles determined by a Vertex and the two Adjacent sides , meet at a point P . (Miquel`s Theorem)

c. In case angle PA1B = 90° then also PA1C = 90° . Since angle PA1C + PB1C = 180° then also PB1C = 90° (*angles PA1B , PC1B , PB1C are extreme of point P*) .

d. ABC is any triangle and A1 ,B1 ,C1 any three points on sides opposite to vertices A ,B, C Show that Perimeter C1B1 +B1A1+A1C1 is minimized at Orthic triangle A1B1C1 of ABC.
Since point P gets Extrema on circumcircle of triangle ABC , so sides A1B1 ,B1C1 ,C1A1 are

Extrema at circumcircle determined by a vertex and the two adjacent sides . Since adjacent sides are determined by sides AA1 , BB1 , CC1 then maximum exists on these side . **[** F25-5] *Proof* :

In triangle AA1B , AA1C the sum of sides p1

p1 = (AA1+A1B+AB) + (AA1+AC+A1C) = 2.(AA1) + AB+AC+BC = 2. (AA1) + a+b+c
In triangle BB1A , BB1C the sum of sides p2
p2 = (BB1+B1A+AB) + (BB1+BC+B1C) = 2.(BB1) + AB+AC+BC = 2. (BB1) + a+b+c
In triangle CC1A , CC1B the sum of sides p3
p3 = (CC1+C1A+AB) + (CC1+AC+C1C) = 2.(CC1) + AB+AC+BC = 2. (CC1) + a+b+c
The sum of sides **p** = p1 + p2 + p3 = 2. [AA1+BB1+CC1] + 3.[a+b+c] and since a+b+c is constant then **p** becomes minimum when AA1+BB1+CC1 or when these are the altitudes of the triangle ABC , where then are the vertices of orthic triangle . ***i.e.***

The Perimeter C1B1 + B1A1 + A1C1 of orthic triangle A1B1C1 is the minimum of all triangles in triangle ABC and it is an extrema .

I. Perspectivity :

In Projective geometry , (*Desargues` theorem*) , two triangles are in perspective *axially* , if and only if they are in perspective *centrally* . Show that , Projective geometry is an **Extrema** in Euclidean geometry.

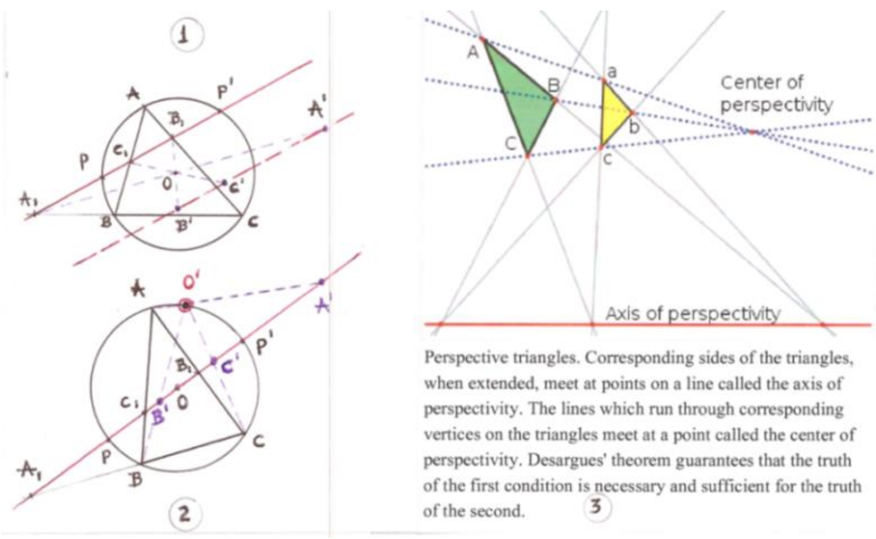


Figure 26 . The two Perspective triangles ABC – abc :

Two points P , P′ on circumcircle of triangle ABC , form **Extrema** on line PP′ . Symmetrical axis for the two points is the mid-perpendicular of PP′ which passes through the center O of the circle, *therefore* Properties of axis PP′ are transferred on the Symmetrical axis in rapport with the center O (central symmetry) , *i.e.* the *three points of intersection A1 ,B1 ,C1 are Symmetrically placed as A′ , B′ , C′ on this Parallel axis .* F26.(1)

a. In case points P , P′ are on **any diameter** of the circumcircle F26.(2) , then line PP′ coincides with the parallel axis , the points A′ , B′ , C′ are Symmetric in rapport with center O , and *the Perspective lines AA′ , BB′ , CC′ are concurrent in a point O′ situated on the circle .*

*When a pair of lines of the two triangles (ABC , abc) are parallel F26.(3), where the point of intersection recedes to infinity , axis PP′ passes through the circumcenters of the two triangles ,(Maxima) and is not needed " **to complete** " the Euclidean plane to a projective plane .i.e.*

Perspective lines of two Symmetric triangles in a circle are concurrent in a point , on the diameters and through the vertices of corresponding triangles .

b. *When all pairs of lines of two triangles are parallel ,* equal triangles *, then points of intersection recede to infinity , and axis PP` passes through the circumcenters of the two triangles* (*Extrema*).

c. *When second triangle is a point* ***P*** *then axis PP′ passes through the circumcenter of triangle .*

Now is shown that *Perspectivity* exists between a triangle ABC , a line PP′ and any point P where then exists Extrema , i.e. ***Perspectivity in a Plane is transferred on line and from line to Point .This is a compact logic in Euclidean geometry which holds in Extrema Points .***

J. Extrema of point A1 and triangle ABC in a circle of diameter AA′ .

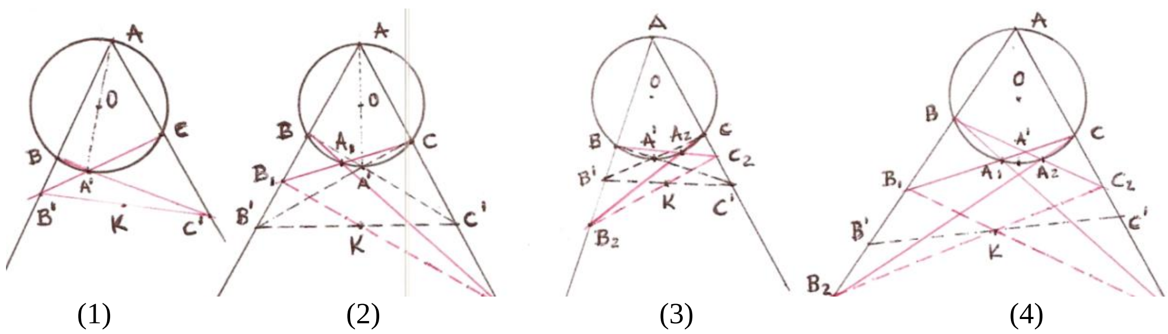


Figure 27. Extrema points on any circumcircle of triangle ABC :

In F27 . Lines CA′, BA′ produced intersect lines AB , AC at points B′, C′ respectively .
A1 is any point on the circle between points B , A′.
CA1 , BA1 produced intersect lines AB , AC at points B1 , C1 respectively .

Show that lines B_1C_1 are concurrent at the circumcenter K of triangles CC′B′ , BB′C′ .

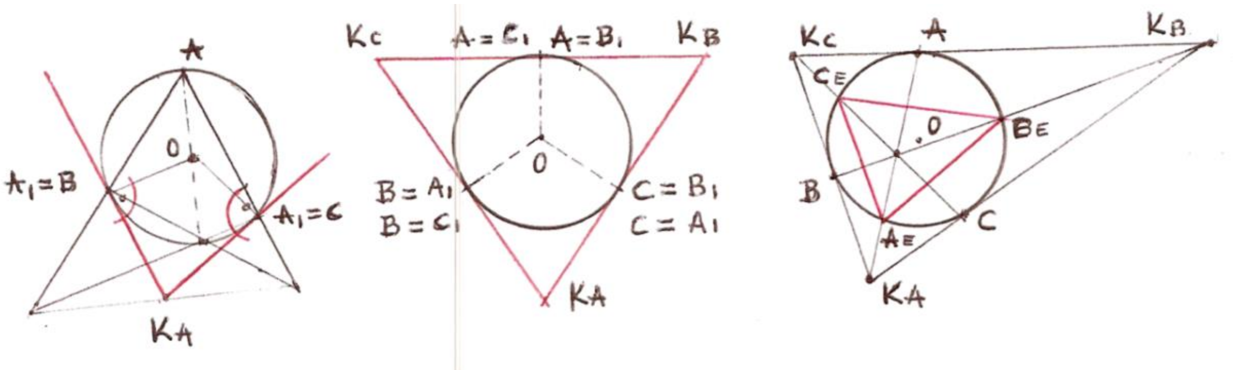
Proof :

Angle < C′CA′= C′ CB′ = 90° therefore circumcenter of triangle CC′B′ is point K , the middle point of B′C ′.
Angle < B′BA′= B′BC′ = 90° therefore circumcenter of triangle BB′C′ is point K , the middle point of B′C′ .
Considering angle < C′CA′= 90° as constant then all circles passing through points C , A′, C′ have their center on KC .
Considering angle B′BA′= 90° as constant then all circles passing through points B , A′, B′ have their center on KB .
Considering both angles < C′CA′= B′BA′= 90° then lines BA′, CA′ produced meet lines AB′, AC′ at points B′, C′ such that line B′C′ passes through point K (*common to KC , KB*)

i.e. On the contrary , In any angle < BAC of triangle ABC exists a constant point K_A such that all lines passing through this point intersect sides AB , AC at points C_1 , B_1 so that internal lines CB_1 , BC_1 concurrence on the circumcircle of triangle ABC and in case this point is point A , then lies on line AK_A . F27 . (1 - 3) .

The case of an angle equal to 180° is examined at next K as the general extrema .

K. Extrema of circumcircle triangle ABC on its vertices .



F28.(1) F28.(2) F28.(3)

Figure 28 . Concurrency points in and out of any circumcircle of triangle ABC :

a) When point A_1 is on point B (Superposition of points A_1 , B) then line BA_1 is the tangent at point B , where then angle < OBK_A = 90° . When point A_1 is on point C (Superposition of points A_1 , C) then line CA_1 is the tangent at point C , where then angle < OCK_A = 90° . F28.(1) Following the above for the three angles BAC , ABC , ACB F28.(2) then ,

K_A B , K_AC are tangents at points B and C and angle < OBK_A = OCK_A = 90 °.
K_B C , K_BA are tangents at points C and A and angle < OCK_B = OAK_B = 90° .
K_C A , K_CB are tangents at points A and B and angle < OAK_C = OBK_C = 90° .

Since at points A , B , C of the circumcircle exists only one tangent then ,
The sum of angles OCK_A + OCK_B = 180° therefore points K_A , C , K_B are on line K_AK_B .
The sum of angles OAK_B + OAK_C = 180° therefore points K_B , A , K_C are on line K_BK_C .
The sum of angles OBK_C + OBK_A = 180° therefore points K_C , B , K_A are on line K_AK_C . i.e.

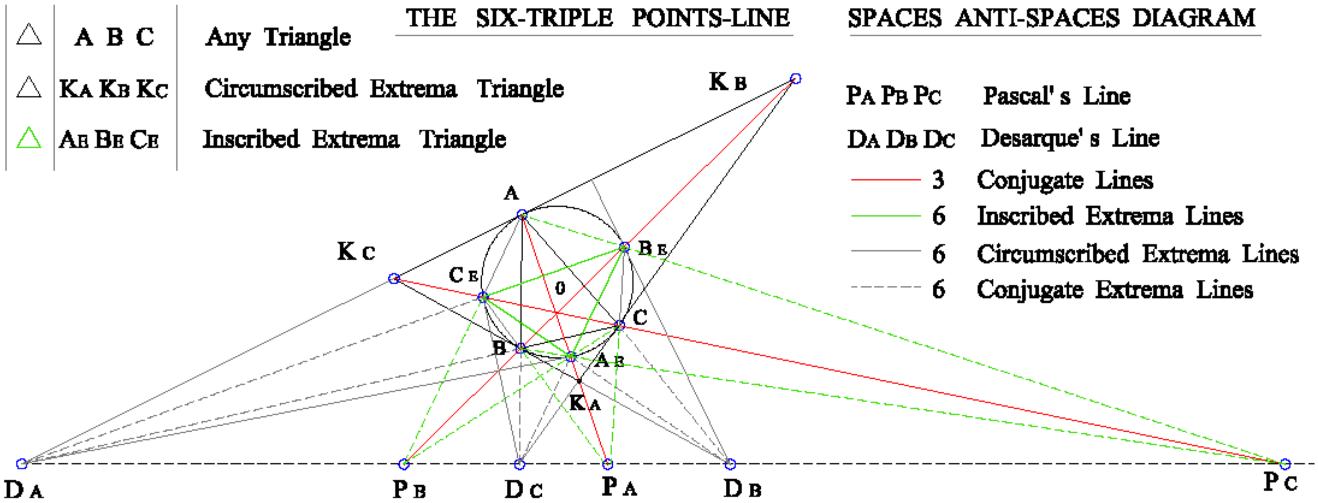
Circle (O , OA = OB = OC) is inscribed in triangle $K_AK_BK_C$ and circumscribed on triangle ABC .

b) Theorem : *On any triangle* **ABC** *and the circumcircle exists one inscribed triangle* **$A_EB_EC_E$** *and another one circumscribed* ***Extrema*** **triangle** **$K_AK_BK_C$** *such that the* ***Six*** *points of intersection of the six pairs of* ***triple lines*** *are collinear* → (3+3) . 3 = 18 Fig – 29

The six-triple points-line [STPL] → D_A , D_B , D_C - P_A , P_B , P_C

where :

Triangle **ABC** → is the Space
Triangle **$A_EB_EC_E$** → is the Anti-Space
Triangle **$K_AK_BK_C$** → is the Sub-Space



F.29. ***The Six , Triple Concurrency Points , Line .*** [STPL] → **D_A , D_B , D_C - P_A , P_B , P_C**

Proof : F.28. (1 – 2 – 3) , F29

Let ABC be any triangle (The Space) , the $A_EB_EC_E$ be the Anti-triangle (The Anti-space) A_E ,B_E ,C_E are the points of intersection of circumcircle and the lines AK_A ,BK_B ,CK_C respectively.

1. When points A_1 , A coincide , then internal lines CB_1 , BC_1 coincide with sides CA , BA , so line K_AA is constant . Since point A_E is on Extrema line AK_A then lines C_EB ,B_EC concurrent on line AK_A . The same for tangent lines K_AK_B , K_AK_C of angle < $K_BK_AK_C$.
2. When points A_1 , B coincide , then internal lines CA_1 , AC_1 coincide with sides CB , AB , so line K_BB is constant . Since point B_E is on Extrema line BK_A then lines A_EC ,C_EA concurrent on line BK_B . The same for tangent lines K_BK_C , K_BK_A of angle < $K_CK_BK_A$.
3. When points A_1 , C coincide , then internal lines AB_1 , BA_1 coincide with sides AC , BC , so line K_CC is constant . Since point C_E is on Extrema line CK_C then lines B_EA ,A_EB concurrent on line CK_C .The same for tangent lines K_CK_A , K_CK_B of angle < $K_AK_CK_B$, i.e.

Triangles ABC , $A_EB_EC_E$, $K_AK_BK_C$ are Perspective between them .

Since Triangles ABC ,$A_EB_EC_E$ are Perspective between them , therefore the pairs of Perspective lines [AA_E , BC_E , CB_E] , [BB_E , CA_E , AC_E] , [CC_E , AB_E , BA_E] are concurrent in points P_A , P_B , P_C respectively .

Since Triangles ABC ,$K_AK_BK_C$ are Perspective between them , therefore the pairs of Perspective lines [K_BA , CB , C_EB_E] , [K_AB , AC , A_EC_E] , [K_BC, BA , B_EA_E] , are concurrent in points D_A , D_B , D_C respectively .

Since lines (K_AK_B , K_BK_C , K_CK_A) are Extrema (tangents to circumcircle) for both triangles ABC and $A_EB_EC_E$, of sides (BC, B_EC_E) , (AB , A_EB_E) , (AC , A_EC_E) , then , the points of intersection of these lines lie on the same line . i.e.

This compact logic of the points* [A , B , C] , [A_E , B_E , C_E] , [K_A , K_B , K_C] *when is applied on the three lines* K_AK_B , K_AK_C , K_BK_C , *then the* SIX *pairs of the corresponding lines which extended are concurrent at points* P_A , P_B , P_C *for the triple pairs of lines* (*Pascal`s Perspectivity of points in Euclidean geometry*) [AA_E , BC_E , CB_E] , [BB_E ,CA_E ,AC_E] , [CC_E ,AB_E ,BA_E] *and at Points* D_A , D_B , D_C *for the triple pairs of lines* [K_AK_B , AB , A_EB_E] , [K_AK_C , AC , A_EC_E] , and [K_BK_C , BC , B_EC_E] , (*Desargues`s Perspectivity of points in Euclidean geometry*) *and all the 18 common points lie on a straight line the STPL .*

The Physical meaning of this geometrical property is further analyzed .

Remarks on → The [STPL] Mechanism as the Geometrical mould on Physical world :

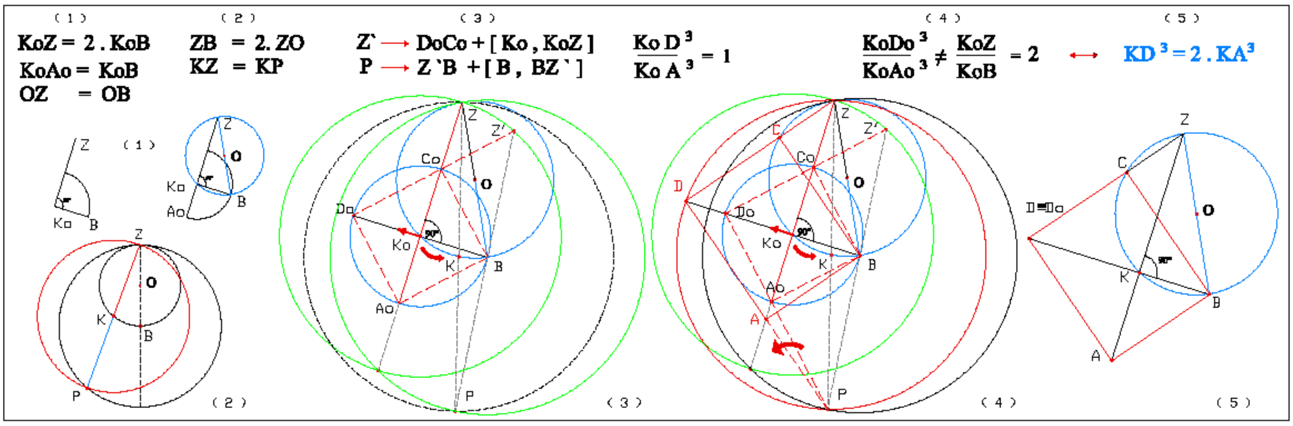
1.. [STPL] is a ***Geometrical Mechanism*** that produces and composite all opposite space Points from Spaces (A-B-C) , Anti-Spaces (A_E ,B_E,C_E) and Sub-Spaces ($K_AK_BK_C$) in a Common Circle , *Sub-Space* , line or cylinder .

2.. Points A,B,C and lines AB,AC,BC of ***Space , communicate with*** the corresponding A_E ,B_E,C_E and A_EB_E, A_EC_E ,B_EC_E , of ***Anti-Space*** , separately or together with bands of three lines at points P_A ,P_B,P_C , and with bands of four lines at points D_A,D_B,D_C ***on common circumscribed circle*** (O,OA) ***the Sub-Space***. [17]

3.. If any monad AB (*quaternion*) , [s, $\bar{v}$.∇i] , *all or parts of it* , somewhere exists at points A ,B ,C or at segments AB ,AC ,BC then [STPL] line or lines ,is the Geometrical expression of the Action of External triangle $K_AK_BK_C$, *the tangents as extrema is the Subspace* , on the two Extreme triangles ABC and $A_EB_EC_E$ (*of Space Anti- space*). of 1,3,5, spin, *the minimum Energy - Quanta* .(this is the How Opposites combine to produce the Neutral) . [29]

When a monad (quaternion with real part = s = 2r and Imaginary part $\bar{v} = \nabla i = \bar{\Lambda} = \Omega = m.v.r$) is in the recovery equilibrium (*a surface of a cylinder with 2r diameter*) , and because velocity vector is on the circumference , then the two quaternion elements identify with points A,B,C (of the extreme triangles ABC of Space ABC) and Imaginary part with points A_E , B_E, C_E (of the extreme triangles $A_E.B_E.C_E$ of Anti-Space) , on the same circumference of the prior formulation and are rotated with the same angular velocity vector $\bar{w}$. The inversely directionally is rotated Energy $\pm\bar{\Lambda}$ equilibrium into the common circle, so Spaces and Anti-Spaces meet in this circle which is the common Sub-space .Extreme Spaces (the Extreme triangles ABC) meet Anti-Spaces (the Extreme triangles $A_E.B_E.C_E$), through the only Gateway which is the Plane Geometrical Formulation Mechanism (mould) of the [STPL] line . [43]

L . Extrema on Duplication of the Cube .



(1) (2) (3) (4) (5)

Figure 30. Extrema Poles on any circumcircle of triangle ZKoB :

In F30. Draw Line segment KoZ tobe perpendicular to its half segment KoB or as KoZ = 2.KoB ⊥ KoB and the circle (O,BZ/2) of diameter BZ. Line-segment ZKo produced to KoAo = KoB (*or and KoXo≠KoB*) is forming the Isosceles right-angled triangle AoKoB . Draw segments BCo , AoDo equal to BAo and be perpendicular to AoB such that points Co , Do meet the circle (Ko,KoB) in points Co , Do respectively , and thus forming the inscribed square BCoDoAo . Draw circle (Ko ,KoZ) intersecting line DoCo produced at point Z` and draw the circle ( B ,BZ) intersecting diameter Z`B produced at point P (*the Pole*) . Draw line ZP intersecting (O,OZ) circle at point K , and draw the circle (K , KZ) intersecting line BDo produced at point D . Draw line DZ intersecting (O,OZ) circle at point C and Complete Rectangle CBAD on diamesus BD .

Show that this is an Extrema Mechanism on where ***The Three dimensional Space KoA*** →

is Quantized to $KoD^3 = 2. KoA^3$

Analysis :

In (1) KoZ = 2.KoB and KoAo = KoB , KoB ⊥ KoZ and KoZ / KoB = 2.

In (2) Circle (B,BZ) with radius twice of circle (O,OZ) is ***the extrema*** case where circles with radius KZ = KP are formulated and are the locus of all moving circles on arc BK , as in page 27-D. (Fig.21-2)

In (3) Inscribed square BCoDoAo passes through middle point of KoZ so CoKo = CoZ and since angle < ZCoO = 90▫ , then segment OCo // BKo and BKo = 2.OCo .
Since radius OB of circle (O,OB = OZ) is ½ of radius OZ of circle (B,BZ=2.BO) then ,**D**, is ***Extrema*** case where circle (O,OZ) is the the ***locus of the centers*** of all circles (Ko,KoZ) , (B,BZ) moving on arc ,KoB, as this was proved before . All circles ***centered on this locus*** are common to circle (Ko,KoZ) and (B,BZ) separately. The only case of being together is the common point of these circles which is their common point P , where then ***centered circle exists on* ZP *diameter .***

In (4) Initial square AoBCoDo , ***Expands and Rotates*** through point B , while segment DoCo limits to DC , where ***extrema poin***t Z `moves to Z . Simultaneously , the circle of radius KoZ moves to circle of radius BZ on the locus of ½ chord KoB . Since angle < Z`DoAoP is always 90^{o} so , exists on the diameter Z`P of circle (B,BZ`) and it is the limit point of chord DoAo of the rotated square BCoDoAo , not surpassing the common point Z . Rectangle BAoDoCo in angle < PDoZ` is expanded to Rectangle BADC in angle < PDZ by existing on the two limit circles (B,BZ`= BP) and (Ko,KoZ) and point Do by sliding to D . On arc KoB of these limits is ***centered circle on* ZP *diameter ,***
i.e. ***Extrema*** happens to → ***the common Pole of rotation through a constant circle*** centered ***on KoB arc*** and since point Do is the intersection of circle (Ko, KoB = KoDo) which limit to D therefore the intersection of the common circle (K, KZ = KP) and line KoDo denotes that extrema point where the expanding line DoCoZ ` with leverarm DoAoP ***is rotating through Pole P*** , and limits to line DCZ ,
i.e. point P is the common Pole of the Expanding and simultaneously rotating Rectangles .

In (5) rectangle BCDA formulates the two right-angled triangles ADZ , ADB which solve the problem .

Segments KoD , KoA are the two Quantized magnitudes in Space (volume) such that Euclidean Geometry Quantization becomes through the *Mould of Doubling of the Cube* **. [*This is the Space Quantization of E-Geometry i.e. → The cube of Segment KoD is the double magnitude of KoA cube , or monad KoD^3 = 2 times monad KoA^3*].**

Proof : F.30. 3-4

1.. Since KoZ = 2.KoB then (KoZ / KoB) = 2 , and since angle < ZKoB = 90^{o} then BZ is the diameter of circle (O,OZ) and angle < ZKoB = 90^{o} on diameter ZB .
2.. Since angle < ZKoAo = 180^{o} and angle < ZKoB = 90^{o} therefore angle < BKoAo = 90^{o} also .
3.. Since BKo⊥ZKo then Ko is the midpoint of chord on circle (Ko,KoB) which passes through Rectangle (*square*) BAoDoCo . Since angle < ZDP = 90^{o} (*because exists on diameter ZP*) and since also angle <BCZ =90^{o} (*because exists on diameter ZB*) therefore triangle BCD is right-angled and BD the diameter .Since Expanding Rectangles BAoDoCo , BADC rotate through Pole ,**P**, then points Ao , A lie on circles with BDo , BD diameter , therefore point D is common to BDo line and (K,KZ = KP) circle , and BCDA is Rectangle .
i.e. Rectangle BCDA possess AKo ⊥ BD and DCZ line passing through point Z .

4.. From right angle triangles ADZ , ADB we have ,

Δ ADZ → $KoD^2 = KoA . KoZ$ …………… (a)
Δ ADB → $KoA^2 = KoD . KoB$ …………… (b) and by division (a) / (b) then →

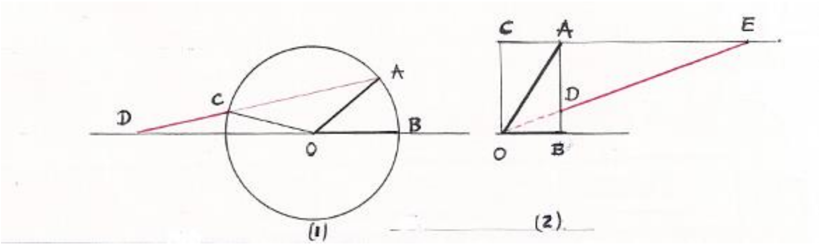
$$\frac{KoD^2 = KoA . KoZ}{KoA^2 = KoD . KoB} = \left|\frac{KoD^2}{KoA^2}\right| = \left|\frac{KoA . KoZ}{KoD . KoB}\right| \text{ or } \rightarrow \left|\frac{KoD^3}{KoA^3}\right| = \left|\frac{KoZ}{KoB}\right| = 2 \quad \text{... (o.ε.δ),(q.e.d)}$$

i.e. **$KoD^3 = 2 . KoA^3$,** ***which is the Duplication of the Cube*** .

In terms of Mechanics , ***Spaces Mould happen through ,*** *Mould of Doubling the Cube* ***, where for any monad ds =KoA analogus to KoAo , the Volume or The cube of segment KoD is the double the volume of KoA cube ,or monad KoD^3 =2.KoA^3. This is one of the basic Geometrical Euclidean Geometry Moulds , which create the METERS of monads → Linear is the Segment*** **MA1** ***, Plane is the square*** **CMNH** ***equal to the circle , and in Space is volume*** **KoD^3** ***, in all Spaces , Anti-spaces and Sub -spaces of monads ← i.e The Expanding square*** **BAoDoCo** ***is Quantized to*** **BADC** ***Rectangle by*** **Translation** ***to point Z `, and by*** **Rotation** ***through point*** **P (*the Pole of rotation) .***The Constructing relation between segments KoX , KoA is → $(KoX)^2 = (KoA)^2.(XX1 /AD)$ as in Fig.53 of P-63 . ***All comments are left to the readers***, markos 30/8/2015. In P-61 all meters .

7.8. ***Trisection of any Angle .***

1. **Archimedes method** 2. **Pappus method**



F.31. ***Archimedes* (1) *and the Pappus* (2) *Trisection method .***

Consider the angle < AOB .

Draw circle (O , OA) with its center at the vertex O and produce side BO to D .

Insert a straight line AD such that point C is on the circle and point D on line BO and length DC **such that** it is equal to the radius of the circle .

Proof :

Since CD = CO then triangle CDO is isosceles and angle < CDO = COD

The external angle OCA of triangle CDO is < OCA = CDO + COD = 2. CDO

and equal to angle ADO and since angle < OAC = OCA then < OAC = 2.ODA

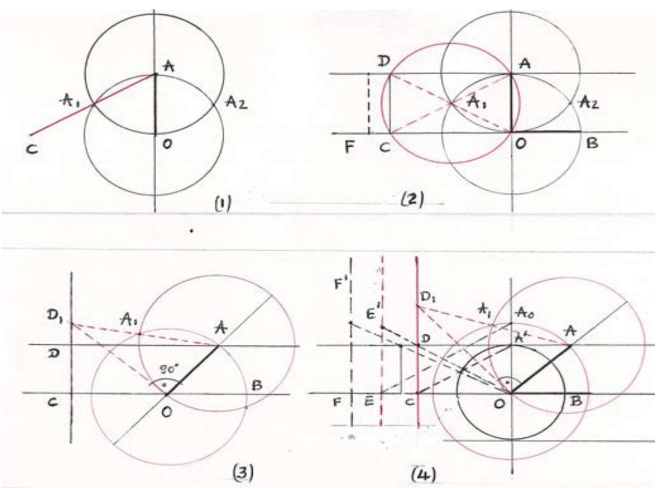
The external angle AOB of triangle OAD is < AOB = OAD + ODA = 2. ODA + ODA = 3 . ODA

2. **Pappus method** :

It is a slightly different of Archimedes method can be reduced to a neusis as follows :

Consider the angle < AOB .
Draw AB perpendicular to OB .
Complete rectangle ABOC .
Produce the side CA to E .
Insert a straight line ED of given length 2 .OA between AE and AB
in such a way that ED verges towards O . Then angle < AOB = 3 . DOB

3. **The Present method :**



F.32. ***The proposed Contemporary Trisection method .***

We extend Archimedes method as follows :

a . (F.33. 1 - 2) Given an angle < AOB = AOC = 90▫

1. Draw circle (A , AO = OA) with its center at the vertex A intersecting circle (O , OA = AO) at the points A_1 , A_2 respectively .
2. Produce line AA_1 at C so that A_1C = A_1 A = AO and draw AD // OB .
3. Draw CD perpendicular to AD and complete rectangle AOCD .
4. Point F is such that OF = 2 . OA

b . (F.32. 3 -4) Given an angle < AOB < 90▫

1. Draw AD parallel to OB .
2. Draw circle (A , AO = OA) with its center at the vertex A intersecting circle (O , OA = AO) at the points A1 , A2 .
3. Produce line A A1 at D1 so that A1D1 = A1 A = OA .
4. Point F is such that OF = 2 . OA = 2 . OAo .
5. Draw CD perpendicular to AD and complete rectangle A´OCD .
6. Draw Ao E Parallel to A´ C at point E (or sliding E on OC) .
7. Draw AoE´ parallel to OB and complete rectangle AoOEE´ .
8. Draw AF intersecting circle (O , OA) at point F1 and insert on AF segment F1 F2 equal to OA → F1 F2 = OA .
9. Draw AE intersecting circle (O , OA) at point E1 and insert on AE segment E1 E2 equal to OA → E1 E2 = OA = F1 F2 .

Show that :

a) For all angles equal to 90▫ Points C and E are at a constant distance OC = OA . √ 3 and OE = OAo . √ 3 , from vertices O , and also A´C //AoE .
b) The geometrical locus of points C , E is the perpendicular CD , EE´ on AB.
c) All equal circles with their center at the vertices O , A and radius OA = AO have the same geometrical locus EE´ ⊥ OE for all points A on AD , or All radius of equal circles drawn at the points of intersection with its Centers at the vertices O , A and radius OA = AO lie on CD , EE´ .
d) Angle < D_1OA is always equal to 90▫ and angle AOB is created by rotation of the right-angled triangle AOD_1 through vertex O .
e) Angle < AOB is created in two ways , By constructing circle (O , OA = OAo) and by sliding of point A´ on line A´ D Parallel to OB from point A´ to A .
f) The rotation of lines AE , AF on circle (O , OA = OAo) from point E to point F which lines intersect circle (O , OA) at the points E_1 , F_1 respectively , **fixes a point** G on line EF and a point G_1 common to line AG and to the circle (O , OA) **such that** G G_1 = OA .
Proof :

a) .. (F.32 .1 - 2)

Let OA be one-dimensional Unit perpendicular to OB such that angle < AOB = AOC = 90▫
Draw the equal circles (O,OA) , (A , AO) and let points A_1 , A_2 be the points of intersection .
Produce AA_1 to C .
Since triangle AOA_1 has all sides equal to OA (AA_1 = AO = OA_1) then it is an equilateral triangle and angle < A_1AO = 60 ▫
Since Angle < CAO = 60 ▫ and AC = 2. OA then triangle ACO is right-angled and angle < AOC = 90▫ , and so the angle ACO = 30 ▫ .
Complete rectangle AOCD
Angle < ADO = 180 – 90 – 60 = 30 ▫ = ACO = 90 ▫ / 3 = 30 ▫
From Pythagoras theorem $AC^2 = AO^2 + OC^2$ or $OC^2 = 4.OA^2 - OA^2 = 3. OA^2$
and ***OC = OA . √ 3 .***
For OA = OAo then AoE = 2. OAo and ***OE = OAo . √ 3 .***

Since OC / OE = OA / OAo → then line CA′ is parallel to EAo

b) .. (F.32.3 - 4)

Triangle OAA_1 is isosceles , therefore angle < A_1AO = 60 ▫ . Since $A_1D_1 = A_1O$, triangle D_1A_1O is isosceles and since angle < OA_1A = 60 ▫ , therefore angle < OD_1A = 30 ▫ or , Since $A_1A = A_1D_1$ and angle < A_1AO = 60 ▫ then triangle AOD1 is
also right-angle triangle and angles < D_1OA = 90 ▫ , angle < OD_1A = 30 ▫ .
Since the circle of diameter D_1A passes through point O and also through the foot of the perpendicular from point D_1 to AD , and since also ODA = ODA′ = 30 ▫ ,
then this foot point coincides with point D , therefore the locus of point C is the perpendicular CD_1 on OC . For $AA_1 > A_1D_1$, D'_1 is on the perpendicular D'_1E on OC.

c) .. (F.32 . 3 - 4)

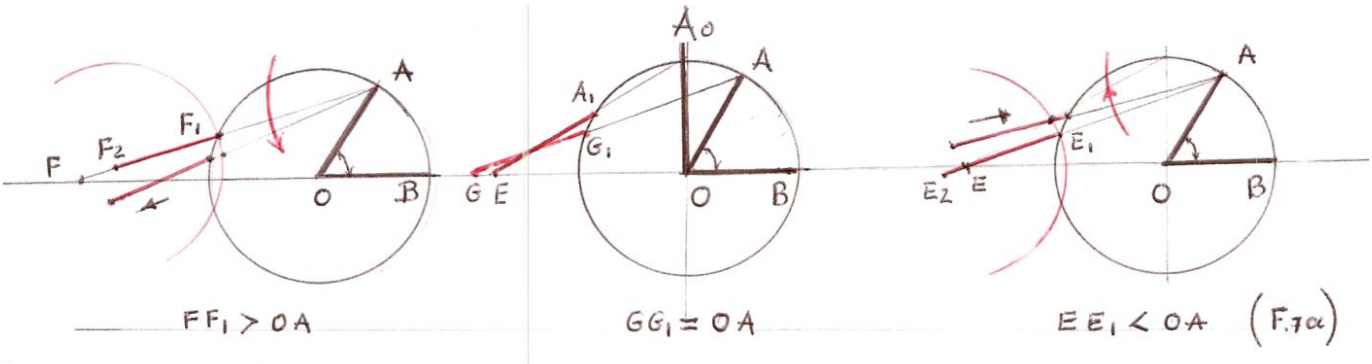
Since the Parallel from point A_1 to OA passes through the middle of OD_1 , *and in case where AOB = AOC = 90 ▫ through the middle of AD* , then the circle with diameter D_1A passes through point D which is the base of the perpendicular , **i.e.**

The geometrical locus of points C , or E , is the perpendicular CD , EE′ on OB .

d) .. (F.32 .3 - 4)

Since $A_1A = A_1D_1$ and angle < A_1AO = 60 ▫ then triangle AOD_1 is a right-angle triangle and ***angle < D_1OA = 90*** ▫ .

Since angle < AD_1O is always equal to 30 ▫ and angle D_1OA is always equal to 90▫ , therefore angle < AOB is created by the rotation of the right - angled triangle AOD_1 through vertex O .
Since tangent through Ao to circle (O , OA′) lies on the circle of half radius OA then this is perpendicular to OA and equal to A′A . (F.32)



F_2F_1 = OA A1E = OAo ,,, GA_1 = OA E_2E_1 = OA

F.33. ***The three cases of the Sliding segment OA between a line OB and a circle O,OA .***

e) .. (F.32 .3 - 4) - (F.33)

Let point **G** be sliding on OB between points **E** and **F** where lines AE , AG , AF intersect circle (O , OA) at the points E_1 , G_1 , F1 respectively where then exists FF_1 > OA , GG_1 = OA , EE_1 < OA .

Points E , F are the limiting points of rotation of lines AE , AF (because then for angle < AOB = 90 ▫ → $A_1C = A_1A$ = OA , $A_1Ao = A_1E$ = OAo and for angle < AOB = 0▫ → OF = 2 . OA) . Exists also E_1E_2 = OA , F_1F_2 = OA and point G_1 common to circle (O , OA) and on line AG such that GG_1 = OA .
AE_2 Oscillating to AF_2 passes through AG so that GG_1 = OA and point G on EF. When point G_1 of line AG is moving (rotated) ***on circle (E_2 , E_2E_1 = OA)*** and Point G_1 of ***G_1G is stretched on circle (O , OA)*** then $G_1G \neq$ OA .

A position of point G_1 is such that , when $GG_1 = OA$ point G lies on line EF.
When point G_1 of line AG is moving (rotated) ***on circle (F_2 , $F_2F_1 = OA$***) and point G_1 of ***G_1G is stretched on circle (O , OA)*** then $G_1G \neq OA$.

A position of point G_1 is such that , when $GG_1 = OA$ point G lies on line EF .
For both opposite motions there is only one position where point G lies on line OB and is not needed point G_1 of GA ***to be stretched*** on circle (O , OA) .

This position happens at the common point P of the two circles which is their point of intersection . At this point P exists only rotation and is not needed G_1 of GA to be stretched on circle (O , OA) so that point G to lie on line EF. This means that point P lies on the circle (G , $GG_1 = OA$) , or GP = OA .

Point A of angle < BOA is verged through two different and opposite motions , i.e.

1 . From point A′ to point Ao where *is done a parallel translation* of CA′ to the new position EAo , *this is for all angles equal to 90▫* , and from this position to the new position EA by rotating EAo to the new position EA having always the distance $E_1 E_2 = OA$.
This motion is taking place on a circle of center E_1 and radius $E_1 E_2$.

2. From point F , *where OF = 2. OA , is done a parallel translation of A′F” to FAo,* and from this position to the new position FA by rotating FAo to FA having always the distance $F_1 F_2 = OA$.
The two motions coexist again on a point **P** which is the point of intersection of the circles (E_2 , $E_2E_1 = OA$) and (F_2 , $F_2F_1 = OA$) .

f) .. (F.32 .3 - 4) **-** (F.33 -7a) Remarks – Conclusions .

1 . Point E_1 is common of line AE and circle (O, OA) and point E_2 is on line AE such that $E_1 E_2 = OA$ and exists $E E_1 < E_2 E_1$ **.** $E_1 E_2 = OA$ is stretched *,moves* on EA so that point E_2 is on EF . Circle (E , $E E_1 < E_2 E_1 = OA$) cuts circle (E_2 , $E_2 E_1 = OA$) at point E_1 . There is a point G_1 on circle (O ,OA) such that $G_1G = OA$, *where point G is on EF* , ***and is not needed G_1G to be stretched*** on GA where then , circle (G , $GG_1= OA$) cuts circle (E_2 , $E_2 E_1 = OA$) at a point P .

2. Point F_1 is common of line AF and circle (O,OA) and point F_2 is on line AF such that $F_1 F_2 = OA$ and exists $F F_1 > F_2 F_1$ **.** $F_1 F_2 = OA$ is stretched *,moves* on FA so that point F_2 is on F E . Circle (F , $F F_1 > F_2 F_1 = OA$) cuts circle (F_2 , $F_2 F_1 = OA$) at point F_1 . There is a point G_1 on circle (O ,OA) such that $G_1G = OA$, *where point G is on FE* , ***and is not needed G_1G to be stretched*** on OB where then , circle (G , $GG_1= OA$) cuts circle (F_2 , $F_2 F_1 = OA$) at a point P .

3. ***When point G is at such position on EF that $GG_1= OA$, then point G must be at A COMMON , to the three lines EE_1 , GG_1 , FF_1 , and also to the three circles (E_2 , $E_2 E_1 = OA$) , (G , $GG_1= OA$) , (F_2 , $F_2 F_1 = OA$) . This is possible at the common point P of Intersection of circle (E_2 , $E_2E_1 = OA$) and (F_2 , $F_2F_1 = OA$) and since GG_1 is equal to OA without G_1G be stretched on GA , then also GP = OA***

4. In additional , for point G_1 :

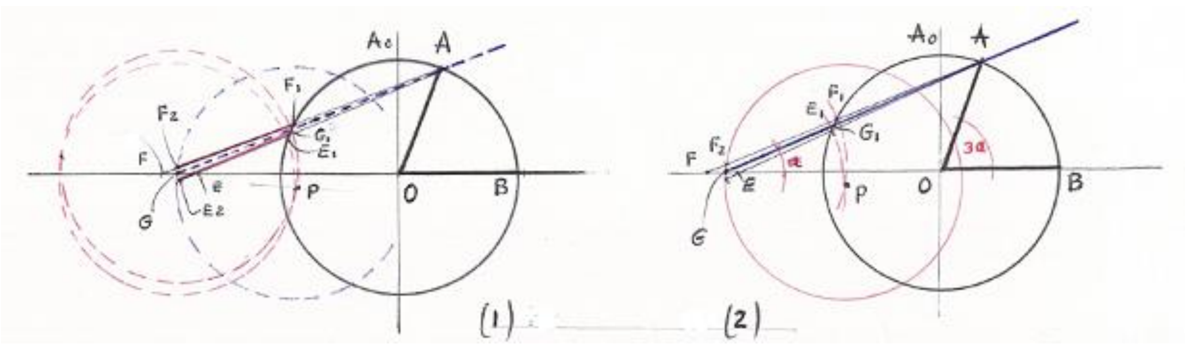
a. Point G_1 , *from point E_1,* moving on circle (E_2 , $E_2 E_1 = OA$) formulates AE_1E such that $E_1E = G_1G < OA$, for G moving on line GA . There is a point on circle (E_2 , $E_2 E_1 = OA$) such that $GG_1 = OA$.
b. Point G_1 , *from point F_1,* moving on circle (F_2 , $F_2 F_1 = OA$) formulates AF_1F such that $F_1F = GG_1 > OA$, for G moving on line GA . There is a point on circle (F_2 , $F_2 F_1 = OA$) such that $GG_1 = OA$.

c. Since for both Opposite motions there is a point on the two circles that makes $GG_1 = OA$ then this point say P , is common to the two circles .

d. Since for both motions at point P exists $GG_1 = OA$ then circle (G , $GG_1 = OA$) passes through point P , and since point P is common to the three circles , then fixing point P as common to the two circles (E_2 , $E_2 E_1 = OA$) ,(F_2 , $F_2 F_1 = OA$) , point G is found as the point of intersection of circle (P , PG = OA) and line EF. This means that the common point P of the three circles is constant to this motion

e. Since also happens , motion of a constant Segment on a line and a circle , then it is Extrema Method of the moving Segment as stated . The method may be used for part or Blocked figures either sliding or rotating .

From all above the geometrical trisection of any angle is as follows , Fig.34



F.34. ***The Trisection method of any angle < AOB***

5. ***The steps of Trisection of any angle < AOB =*** 90 ▫ → 0 ▫ (F.32 -4) - (F34.1-2)

1. Draw circle (O , OA) and line AD parallel to OB .
2. Draw OAo ⊥ OB where point Ao is on the circle (O , OA) and the circle (Ao , AoE = 2.OA) which intersects line OB at the point E .
3. Fix point F on line OB such that OF = 2 . OA
4. Draw lines AF , AE intersecting circle (O , OA) at points F_1 , E_1 respectively .
5. On lines F_1A , E_1A fix points F_2 , E_2 such that $F_2F_1 = OA$ and $E_2 E_1 = OA$
6. Draw circles (F_2 , $F_2 F_1 = OA$) , (E_2 , $E_2 E_1 = OA$) and fix point P as their common point of intersection .
7. Draw circle (P , PG = OA) intersecting line OB at point G and draw line GA intersecting circle (O , OA) at point G_1 .

Then Segment $GG_1 = OA$, and angle < AOB = 3. AGB .

Proof :

1 . Since point P is common to circles (F_2 , $F_2 F_1 = OA$) , (E_2 , $E_2 E_1 = OA$) , then $PG = PF_2 = PE_2 = OA$ and line AG between AE , AF intersects circle (O ,OA) at the point G_1 such that $GG_1 = OA$. (F34.1 -2)

2 . Since point G_1 is on the circle (O , OA) and since $GG_1 = OA$ then triangle GG_1O is isosceles and angle < AGO = G_1OG .

3 . The external angle of triangle GG_1O is < AG_1O = AGO + G_1OG = 2. AGO .

4 . The external angle of triangle GOA is < AOB = AGO + OAG = 3.AGO .

Therefore angle < AGB = (1 / 3) . (AOB) ………….. (ο.ε.δ.)

Conclusions : ….

1. Following the dialectic logic of ancient Greeks (Αναξίμανδρος) « τό μή Ον , Ον Γίγνεσθαι » ‘ The Non-existent , Exists when is done ’ ,

‘ The Non - existent becomes and never is ’ and the Structure of Euclidean geometry in a Compact Logic Space Layer , as this exists in a known Unit (***case of 90 ▫ angle***) , then we may find a new machine that produces the 1/3 of angles .

Since Non-Existent (*Points*) is found everywhere and as angle , then Existence (*Quantization to 1/3 angle*) is found and is done everywhere .

In Euclidean geometry points do not exist , but their position and correlation is doing geometry .

The universe cannot be created , because becomes and never is . [43]

According to Euclidean geometry and since the position of points (*empty Space*) creates geometry and Spaces , Zenon Paradox is the first concept of Quantization .

2. It has been proved [8] that two equal and perpendicular one-dimensional Units OA , OB formulate a machine which produces squares and One of them is equal to the area of the circle (O , OA = OB)
3. It has been proved [9] that three points formulate a Plane and from the one point passes only one Parallel to the other straight line (three points only) .
4. It has been proved [10] that all Subspaces in a unit circle of radius the one - Dimensional unit OA are the Vertices of the Regular Polygons in the unit circle.
5. Now is proved [11] that one-dimensional Unit OA rotating and lying on two parallel lines OB , AD formulate all angles < AOB = 90 ▫ → 0 and a new geometrical machine exists , mould , which divides angle < AOB to three equal angles .

7.8-1 ***A Simplified Approach of Squaring the circle using Resemblance Ratio*** **→ (Trial -1-) [8]**

THE KNOWN EUCLID GEOMETRICAL ELEMENTS

1 . Any single **point A** , constitutes a Unit which has Dimension zero without any position (non-dimensional = The Empty space) , (F.35-1).

Any single **point B not coinciding with A** ,constitutes another Unit which has also dimension zero

Only **one straight line** (ie. The Whole is equal to the Parts , the equation CA + CB = AB) passes through points A and B which consists another non-dimensional Unit , since it is consisted of infinite points with dimension zero . (F.35-1).

A line segment AB between points A and B , (either points A and B are near zero or are extended to the infinite) , ***consists the first Unit with one dimensional*** , the length AB , beginning from Unit A and a regression ending in Unit B , (F.35-1).

Adding a third **point C not on the straight line AB** , then is constituted a new Unit (the Plane) without any dimension and position , since is consisted of infinite points without any position .

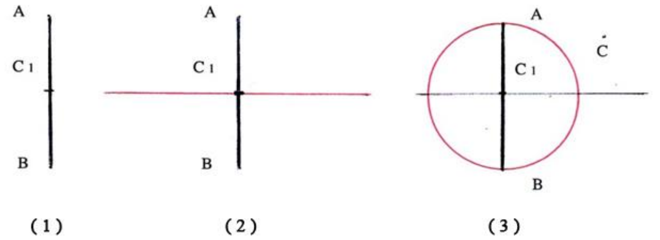
For point C is valid the Inequality CA + CB > AB and line AB on both sides , divides Plane ABC in two equal parts . Shape ABC enclosed between parts AB , AC , BC is of two dimensional , the enclosed area (F.35-2).

F.35. *A Point C , a Line AB , a Segment |AB| , a Plane ABC .*

2 . The first **Property of length AB** (which is the first Unit) is the middle point C1 , that is a point equally be distant from points A and B . Point C1 is on line AB because C1A + C1B = AB and **inversely**, since on Segment AB exists C1A = C1B then point C1 is on segment AB , (F.36-1)

Second **Property of length AB** is the locus of points equally be distant from points A and B which is the mid-perpendicular to AB from point C1 , (F.36- 2). **Inversely** , since C1A = C1B point C1 is on mid-perpendicular to AB with the minimum distance .

Third **Property of length AB** is the construction (drawing) of a circle in Plane ABC with AB as diameter and the point C1 as center. On this circle , the n-th roots of the length AB (the inscribed **n** Regular Polygons ۞) , are existing with all their properties (and for n = 4) . (F.36 - 3). (έν το πάν) .



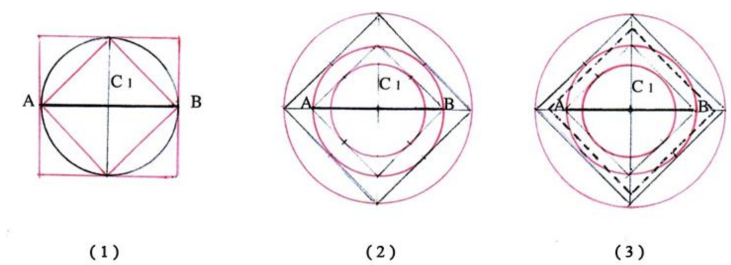
F.36. *A Segment , the middle , and the Circle on Segment.*

Fourth **Property** of length **AB** is the construction of the inscribed and the circumscribed Square on the circle with **AB** as diameter .The circumscribed square is inscribed to the circumscribed circle and the inscribed square is circumscribed to the inscribed circle (F.36-3) .
According to the upper Properties of the length **AB** , the respective ratio of areas so for squares as for circles is always equal to 2 , that is to say , **the area of the circumscribed shapes is twice the area of the inscribed ones** . (De .Moivre´s Formula for n= 4) , (F.37, 2-3) ,

This property of Segment **AB** , *extended to the circle on AB is diameter* , was called :
" Resemblance Ratio of Areas to the circle equal to 2 " and is obtained from the following shapes:

1 . **The circumscribed square** which area is twice the area of the inscribed one .
2 . **The circumscribed circle** which area is twice the area of the **circle** .
3 . **The circle** which area is twice the area of **the inscribed one** .
4 . **That square of area equal to the circle** , is of area twice the area of the inscribed circle .

(this property on that square is transferred simultaneously by the equality of the two areas , when square □ = o circle , then that square is twice the area of the inscribed one .) , (F.37-3).
When the upper shapes meet to one point , then this point of intersection has the property of the shapes, that is to say : " *The Resemblance Ratio of Areas to the circle be equal to 2", R.R=2.*



F.37. *The circumscribed Circle and Square on a circle .*

It has been proved that on a segment AB , the upper Property (R.R =2) is represented by a constant point G with its symmetrical one , because of the twin System – Image , (F.38-4).
We can Geometrically construct the three shapes having the fourth property .

In order to construct the shape (***the square*** **) with the fourth Property (** ***having the Resemblance Ratio of Areas equal to 2*** **), is necessary to find a Geometrical Formation of Constructing Squares as well as the point or the points on this Formation which have this Property of ,** ***the Resemblance Ratio be equal to 2*** **, and also this Property can be transferred to the shapes formed , (F.38-3).**

THE PROVED , UNKNOWN GEOMETRICAL ELEMENTS .

1 . **Each segment AB** , *extended also to the circle on AB as diameter* , **has one Point G** (and its symmetrical one) **with this Property of Resemblance Ratio be equal to 2** . (R.R = 2) , (F.38-1)
2 . **On each triangle ABC** with sides AB ,AC ,BC, *extended to the circles with sides as diameters* ,**are existing three points G** and also a common one **F**, ***having Resemblance Ratio equal to 2 ,*** (F.38-2)

3 . **On each triangle ABC** with sides AB , AC , BC **are existing three straight lines GF** passing through these three points G and their common one F having R.R = 2 , **which** (straight lines) **have the Property of the Resemblance Ratio be equal to 2** , (F.38-3),

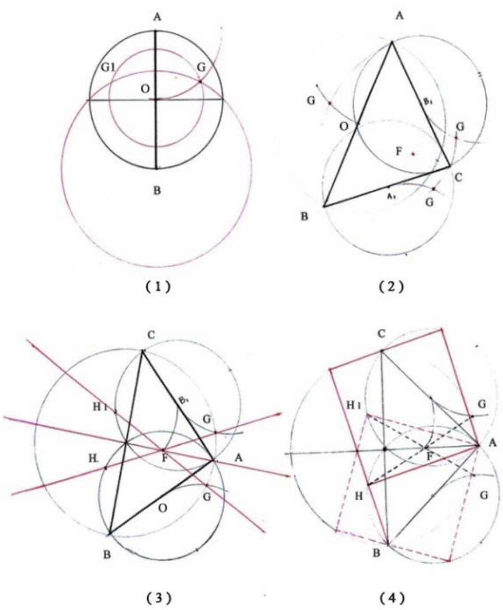
4 . **A segment AB** with the equal and perpendicular one AC=AB , **constructs an isosceles right-angle triangle ABC and** on this triangle are drawn **the three circles with the sides as diameters** both (triangle and circles) ***consist* the " *Plane Formation of Constructing Squares* ", *from the zero one , to the inscribed and the circumscribed square ,(Machine AC⊥ AB).* (F.38-4)** The triangle with the three circles is , ***the Steady Formation ,*** and the designed squares on this formation is , ***the Changeable Formation ,*** of the two and perpendicular units AB , AC . (F.38-4)

5 .**The three straight lines GF** , on the isosceles (side AB = AC) right-angle triangle ABC **having** the Property of **Resemblance Ratio be equal to 2 , cut the Plane Formation Constructing Squares on two points H , H 1 ,** *symmetrically placed to the third straight line of RR = 2* , (F.38-3).

6 .**The Changeable Geometrical Plane Formation** of this , System–Image **(ie. the changeable squares of side AH and the anti-squares (** ***Idle*** **) of side AH 1**) , passing from these two points H , H1, **get the Property of the Resemblance Ratio be equal to 2** , which means : , (F.38-4)

Area of Square–Anti Square = 2 . [Area of inscribed circle] , that is on the circle with radius R ,

$$\square = 2 \cdot [\, \pi \cdot (R / \sqrt{2})^2 \,] = 2 \cdot \pi \cdot R^2 / 2 = \pi \cdot R^2 = ©$$



F.38. *The circumscribed Circle and Square on a circle* . [□ = AH²] = [O = π.(AB/2) ²]

The above unknown but now proved Geometrical Elements are true and since the simple rules of Ordinary Logic are accepted as a basic Principle of mathematical reasoning , then is true.

3. On each triangle **ABC** exist the following Properties :

1 . The triangle is consisted of three vertices ,the Points A ,B ,C and the three sides AB,AC,BC ,(F.39-1)

2 . On each side there is one middle point C 1 , B 1 , A 1 and there are three diamesus AA1 , BB1 , CC1 which are passing through one point called (common point of diamesus) , (F.39-2).

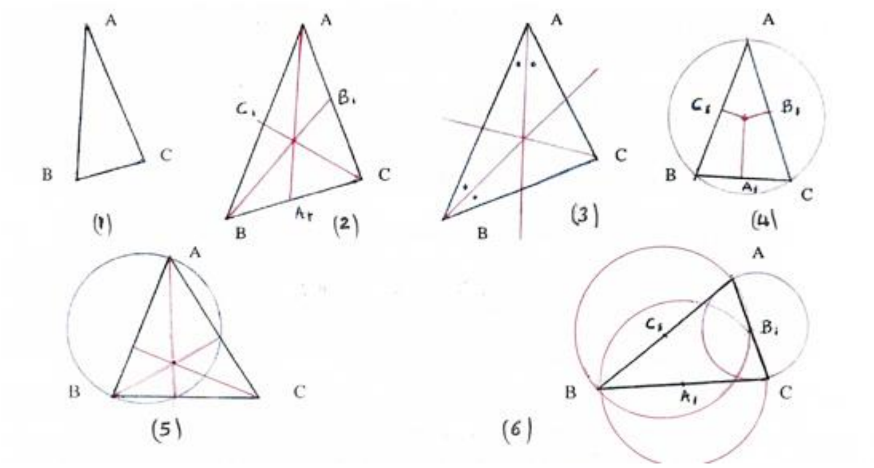
3 . Every two sides form an angle having one bisector , and the three bisectors are passing through one point called (common point of bisectors) , (F.39-3).

4 . Only one mid-perpendicular is drawn from midpoints A1 , B1 , C1 which are intersected to one point called (common point of mid-perpendiculars) , and the circle with this point as center passes through the vertices of the triangle (the circumscribed circle of the triangle) , (F.39-4).

5 . Only one perpendicular is drawn from the three vertices A,B,C to the opposite sides BC, CA, AB The three perpendiculars meet at one point called , centroid , (F.39-5).

6 . Only one circle is constructed on sides AB , AC , BC as diameters , which passes through the bases of the perpendiculars drowned from the two vertices of each side , (F.39-6).

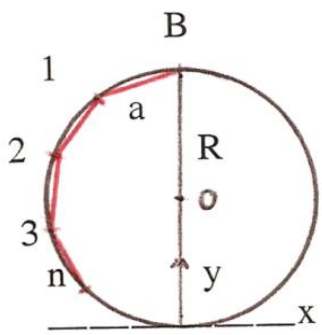
7 . ***In Euclid geometry , logical consequence of geometrical Proofs valid also and Inversely . The following Unknown (now proved) Geometrical elements on the first dimensional Unit AB " under Euclid restrictions imposed to seek for the solution " , (*** *Using a Ruler and a Compass* ***) , Solve approximately the problem . (F.39- 1-2-3-4) . All proofs are in next pages*** .



F.39. *The different properties on any triangle ABC* .

GENERAL GEOMETRICAL ANALYSIS AND PROOF OF REGULAR POLYGONS

F.40. *The circumscribed Circle , and the ,* **n** Roots on AB , [*The Polygons on circle*].



F.40. The **n** roots of R = OB

By an nth root of Line Segment AB we mean a segment **On** such that **[On]** ^{n} **= AB .**
In particular , on line segment AB equal to the number 1 , *exist two square roots , 1 and -1* .
The number 1 has three cube roots , *the number 1 and two imaginary* , four four.*th* roots , *the numbers 1 and - 1 and the imaginary number i and – i* , five *the fifth ones* , **n** roots for the *nth roots* , and the ∞ points on the circumference of the unit circle for *the ∞ th roots* .
The **n** roots of line segment AB are represented by the vertices of the regular n-sided polygon inscribed in the circle of diameter AB and in the case of ∞ roots , the points on the circumference of the circle , $^{n}\sqrt{AB} = 1$ as $n \to \infty$. All above have been proved by De . Moivre's .

The referred properties* of the roots *exist on any* Line Segment AB *and are represented on the circle with diameter AB .(έν το πάν = everything is one) . This Property on Segment AB yields to the* Geometrical construction *and the Algebraic measuring of all Regular n - Polygons .

For … > R.R = 2 > square → ■ = 2 . ● → Inscribed circle

When the Resemblance Ratio of Areas so for squares as for circles *on the circle is 2 ,* (when n = 4) , then exist the following :

1 . **The circumscribed square** ◙ = 2 . □ the inscribed square □
2 . **The circumscribed circle** ☻ = 2 . © the circle ………. ©
3 . **The circle** © = 2 . ● the inscribed circle ●
4 . **That square of area equal to the circle ,** ■ = © >> ■ = © = 2 . ●

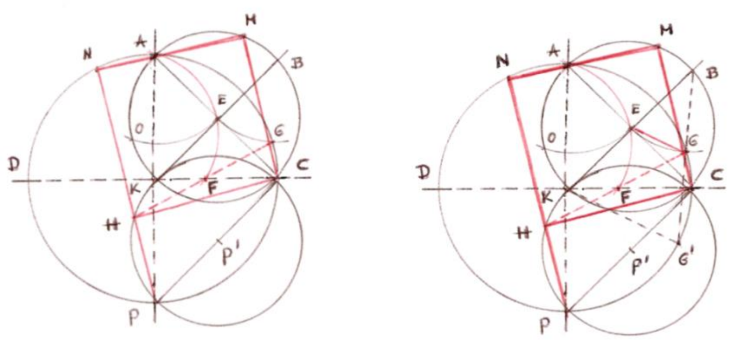
The property , of the Resemblance Ratio be equal to 2 *on that square* ■ , *is transferred simultaneously by the equality of the two areas , **when*** ■ = © *, and then that square is twice the area of the inscribed circle . Exists also the opposite logic .*

F38. [1- 4] > [F.41]

1 Draw the circle (E , EB) , with point E as center and radius EB , and the perpendicular diameters BEK , C E A forming the inscribed square CBAK .
2 The circumscribed circle (K , KA = KC = EB .√2) intersects circle (B , BE = BG) at point G .
3 Draw diameters AP , CD and with P ′ as center draw the circle (P ′, P′K = P ′C = EB) .
4 Draw Circle (A , AE = EB) to intersect circle (E , E B) at the point O , and with point O as centre draw the equal circle (O ,O E = OA) intersecting CD at the point F.
5 Produce line GF to the point H lying on the circle (P ′, P′K) .

For chord CH exists : CH ² = π . EB ² = π . EC ² = π . OE ² = π . P′K ²

Proof :



F.41. *The moving Square* (CMNH=Space) *and Anti-square* (Idle) *on a circle .*

1 Since CEA ⊥ BEK and also AC = BK , **therefore shape ABCK is square ,** This square is the inscribed to the circle.

2 On any diameter **KB** exists KA = KE √ 2 = AB , **therefore** circle (K , KA) is the circumscribed to the circle (E , E B) .
Since on diameter **KB** , at the edge point K is drawn the circumscribed circle (K , KA = KC) and at the other edge point B is drawn circle (B , BE = EB) , **then** the intersecting **point G** lies on the inscribed circle (E , EG = KA / 2) , which is the constant locus of ***Resemblance Ratio be equal to 2 of the circle (E , EB) .***
This Proposal may be valid as a theorem and it is as follows :

Theorem : **[F38.1 - 41]**

On each diameter **KEB** of a circle (**E , E B**) we draw :

1 . the circumscribed circle (K , KA = KE .√2) at the edge point K as center ,
2 . the inscribed circle (E , EB / √2 = KA / 2) at the midpoint E as center ,
*3 . the circle (B , BE) = (E , EB) at the edge point B as center **,***

Then the three circles ***pass through point G , and the symmetrical to KB point G1 forming an axis perpendicular to KB*** **, which has the Properties of the circles ,** ***Radical axis*** **, and** ***the tangent from point B*** **to the circle (K , KA = KC) is constant and equal to 2 . EB ^{2} .**

proof :

Since BC ⊥ CK , the tangent from point B to the circle (K , KA) is equal to :
BC ^{2} = BK ^{2} - KC ^{2} = (2. EB) ^{2} - (EB .√ 2) ^{2} = 2 EB ^{2} = (2 EB) . EB **= (2.BG) . BG**
and since 2.BG = BG′ then **BC² = BG . BG ′,** where G′ lies on the circumscribed circle.
and this means that BG produced , intersects circle (K , KA) at a point G′ twice as much as BG . Since E is the midpoint of BK and also G midpoint of BG′, **so EG** is the diamesus of the two sides BK,BG′ of the triangle BKG′ and equal to 1/ 2 of radius KG′ = KC *,the base,* and since the radius of the inscribed circle is ½ of the circumscribed radius , **then the circle (E , EB / √ 2 = KA / 2) passes through point G .** As BC is perpendicular to the radius KC of the circumscribed circle , ***so BC is tangent and*** **equal to BC ^{2} = 2 . EB ^{2} . (o.ε.δ)**
Since the three circles , *the circumscribed (K , KA = KC = EB . √2* **,** *the inscribed (E , EB / √2)* **,** *the circle (B , BE)* **,** are intersected at point G , therefore point G is common to the three circles ,lies on the inscribed circle and has the three Resemblance Ratio equal to 2 , in other words ***Point G for the diameter KB and for any other diameter KB of any circle (E , EB) , is the Geometrical Expression of Resemblance Ratio equal to 2 .***

3 Since CK ⊥ KA and also CK = KA then angle KAC = 45° . Angle ACP = 90 ° because exists on diameter AP , so the triangle ACP is isosceles and site CA = CP , and is also the circle (P ′,P ′C) equal to the circle (E , E B) . *In this way , the two equal and perpendicular line Sectors CA ,CP with the three circles (E,EB),(P',P'C),(K,KA)constituent the Plane Procedure .*

In > [F.41]

Since BC ⊥ CK , BC is tangent from point B to the circumscribed circle (K ,KC) and the tangent is equal to (EB.√ 2) ^{2} = 2 . EB ^{2} . The equal circles (E , EB) , (P′, P′C) are intersected on chord C K which is the radius of circle (K , KA) and aslo the common chord for the three circles .
Because edge point C of the perpendicular diameters CA, CP lies on ***the radical axis* CA , CP** of circles (E , EB) , (K , KA) and (P' , P' C) and because the two circles alternate at the two edges of the diameter CA , CP the resultancy is that **tangent from point C** to the two couples of circles is the same and equal to CB ^{2} = 2 . EB ^{2} , so
The tangent from point C to circle (O,OA) is equal to CO^2 – OA^2 = [(2EC) ^{2} - AO ^{2}] – AO ^{2} = 4 EC ^{2} - 2 AO ^{2} = 2 EC ^{2} = 2 EB ^{2} , so circle (O ,OA = OE) is the circle of Resemblance Ratio equal to 2 for the two circles (K , KC) and (E , EB) .
Since chord CK is common to the two equal circles (E , EB) , (P′, P′C) , ***therefore ,***

Point F is the constant point of Resemblance Ratio equal to 2 for the three circles of this Geometrical formation . Point F can also be found as the common point of intersection of circles (O , OA) , (O′,O′P ′) , representing the two systems of circles (K , KC) , (E , EB) and (K , KC) , (P′, P′C) with R.R = 2 respectively .

In 38.3-4 > [F.41] The geometrical Machine AB ⊥ AC = AC

1 Let **H** be any point on the circle (P′, P′K) , **N** the point of intersection of line PH produced to the circumscribed circle (K , KA) , **M** the point of intersection of line NA produced to the circle (E , EA) , **C** the common point of intersection of the three circles .
Show that shape **CMNH** is square.

Proof :

Angle < CHP = 90° because is inscribed on the diameter CP of the circle (P′, P′K) . The supplementary angle < CHN =180 – 90 = 90° . Angle < PNA = 90° because is inscribed on the diameter

AP of the circle (K , KA) . Angle < CMA = 90 ° because is inscribed on the diameter AC of the circle (E , EA) .
The upper three angles of the quadrilateral CHMN are 90+90+90 = 270 , and from the total of 360° , angle < MCH = 360 – 270 = 90° . Therefore shape CMNH is **rightangled** and so CM ⊥ CH .
Since CM ⊥ CH and CA ⊥ CP therefore angle < MCA = HCP .
The rightangled triangles CAM , CPH are equal because Hypotynousa CA = CP and angles < CMA = CHP = 90° , < MCA = HCP , **therefore side CH = CM** .

Because CH = CM , the rechtangle CMNH is Square .

Namely the two equal and perpendicular line sectors CA , CP construct the Isosceles rightangled triangle APC and the three circles on the sides as diameters . From any point H on the first circle is conscructed the square CHNM with vertices on the three circles . This Geometrical Formation is a mooving Machine and is called << **Plane Formation of Constructing Squares** >> .

2 As points G and F are of Resemblance Ratio equal to 2 , as regard circle (E , EB) and also for the three circles (E ,EB) , (P′,P′K) , (K ,KC) and as **similarity** exists on the triangle of sides the three diameters of the circles , ***therefore ,***

the direction GF **is a line of Resemblance Ratio equal to 2** and point H on the circle (P′, P′K) has the three Resemblance Ratio equal to 2 and then exists :

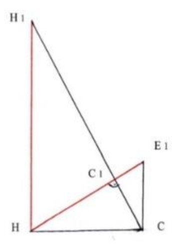
$2 = \pi .(EB \sqrt{2})^2 / \pi . (EB)^2 = \pi . (EB)^2 / \pi . (EB / \sqrt{2})^2 = \mathbf{CH^2 / \pi. (P'H / \sqrt{2})^2}$

or the same as : $\mathbf{CH^2 = \pi . (P' H)^2 = \pi . (EB)^2 = \pi . CE^2}$ and $\mathbf{\pi = CH^2 / CE^2}$ **,**

which means that ,π, is an algebraic and constructable number as follows ,
Draw the sector $CE_1 = CE$ on line CH , where $CE_1 \perp HC$, and form the rightangled triangle CHE_1 . By drawing $CC1 \perp HE1$ then we have from Pythagorian theorem ,

$CH^2 : CE_1^2 = C_1H^2 : C_1E_1^2$ and $CH^2 = \pi . CE^2$ and since $CE = CE_1$ then :
$\pi = CH^2 / CE_1^2 = (HC_1)/(E_1 C_1)$ **= [C_1H / C_1E_1] an algebraic number . .(a)**
Let H_1 be the point of intersection of line CC_1 produced and the line $HH_1 \perp HC$.
From the similar rightangled triangles C_1E_1C , C_1H_1H we have :
$HH_1 / CE_1 = C_1H / C_1E_1$ and from (a) above = π
also $HH_1 / CE = C_1H / C_1E = \pi$ **or $HH_1 = \pi . CE$ = the Semicircle .**

2.π.CE / 2



F.42. Trial 1 of Squaring the circle .

Euclid logic on Unity is now extended (*using De Moivre's Formula and the Roots of Unity*) to the Properties of the **n = 1 …∞ roots of unity** (*The Uknown Geometrical Elements*) , always under to the set restrictions to solve this problem (*using a ruler and a compass*) .

Harmonic mean , Golden ratio [(√5-1) / 2] and other known geometrical constructions exist on *the steady Formation* , and the roots of unity **AB** on the *Changable Formation* , ***of The Plane Formation of Constructing Squares < The Method > .***

The Geometrical Controlling of , *Resemblance Ratio of Areas on Plane Formation* , gives the solution to the Unsolved Problems .

THE APROXIMATE NUMBER π IN ALGEBRAIC FORM

Using the referred procession it is easy to find π magnitude as follows :

$$A = \frac{4 + \sqrt{3} - \sqrt{6 . \sqrt{3} - 4}}{\sqrt{3} + \sqrt{7} - 1 - \sqrt{6 . \sqrt{3} - 4}}$$

$$B = 8 . \sqrt{7} . A^2 - 7 . A^2 + 10 . \sqrt{7} . A - 40 . A - 9$$ then

$$\pi = \left| \frac{[\, 4 . A^2 - A^2\sqrt{7} + 5 . A + \sqrt{B}\,]^2 + [\, 4 . A^2 + A . \sqrt{7} - 4 . A - 1 + A . \sqrt{B}\,]^2}{[\, 4 . \{ A^2 + 1 \} \,]^2} \right| \quad (m)$$

$= 3,141\ 030\ 312$

1. Remarks :

Geometrical Elements	Geometry Method -	Symbolic Method	Definitions
Point **C** on a straight line .	A B ------ **C** --------	CA + CB = AB	The whole is equal to the Parts Equality > > Equation
Point **C** not on a S.line	A C B -……•…...-	CA + CB > AB	The whole is less than the Parts >> Inequality .
Line **CD** parallel to **AB**	__A________B__ **d** | **d** | ---C-----------D---	d + 0 = d d . 0 = 0	The geometrical Properties of Point 0 . The Empty space is Un-dimensional , no Position .
The Hypotenuse of an Isosceles right angled Triangle ABC	AC= √ 2 C __| 1 A 1 B	$AC = AB.\sqrt{2}$	Incommensurable with side AB .
Number π	**A = C** ----------- H	Equation (m)	Algebraic number .

The exact Numeric Magnitude of $\sqrt{\mathbf{2}}$ can be found only with the ∞ number of decimals after 1 , and number 2 is only = $\sqrt{2} . \sqrt{2}$ and not in other way .

The exact Numeric Magnitude of **Equation** (**m**) can be found only with the ∞ numbers of decimals of units $\sqrt{3}$, $\sqrt{7}$, $\sqrt{A}$, $\sqrt{B}$ and not differently .

All these Magnitudes exist on the < **Plane Formation of the first dimentional unit AB** > as geometrical elements consisted of , ***the Steady Formulation ,*** (The Plane System of Triangle ABC with the three Circles on the sides) and ***the moving Changeable Formulation of the twin , System - Image*** (The Plane System of the Squares – Antisquares) .

A marvellous Presentation of the Method can be seen on Dr. Geo Machine Macro - constructions .

Starting from this logic of correlation upon the Unit , we can control *Resemblance Ratio* and construct all Regular Polygons on the unit Circle as this is shown in the case of squares .
On this **System** of these three circles (The Plane Procedure which is a Constant System) is created a continues and also a not continues Symmetrical Formation (The changeable System of the Regular Polygons) and the ***Image*** (The Changeable System of the Regular anti-Polygons) ***Idol*** , as much this in **Space** and also this in **Time** and it is proved that , in this Constant System , the Rectilinear motion of the Changeable Formation is Transformed into a twin Symmetrically axial-centrifugal rotation (the motion) on this Constant System . [43]

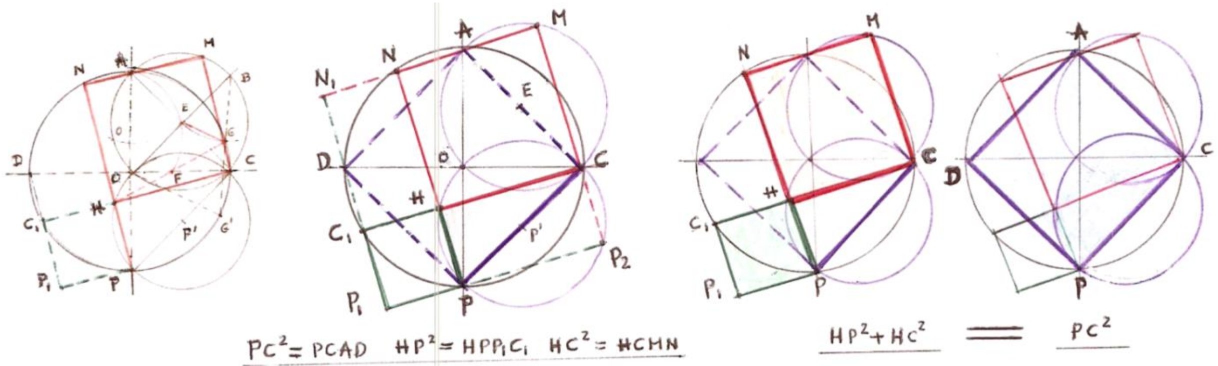
The conservation of the Total Impulse and Momentum , as well as the conservation of the Total Energy in this Constant System with all properties included , exists in this Empty Space of the un-dimensional point Units .

All the forgoing referred can be shown (maybe presented) with a Ruler and a Compass , or can be seen , live , on any Personal Computer .

The theorem of *Hermit-Lindeman* that number , pi , is not algebraic , is based on the theory of Constructible numbers and number fields (*number analysis*) and not on the
< *Pure Geometrical Logic , unit elements and the derivation of the origin basis* >

The mathematical reasoning (*the Method*) is based on the restrictions imposed to seek the solution < *with a ruler and a compass* > . By extending Euclid logic of Units on the Unit circle *to unknown and now proved Geometrical unit elements* , and the settled age-old question for the unsolved problems is now approached and continuous standing . Mathematical interpretation and all relative Philosophical reflection based on the theory of non-solvability must properly revised .

7.8-2 ***A Simplified Approach of Squaring the circle using the Plane Procedure Method*** **→ (Trial -2-) [8]**



(1) (2) (3) (4)

F.43. *The moving Squares* (CMNH = Space) *and Supplementary anti-squares* (PHC1P1=Idle) *on circle .*

The Open Problem ? Which logic exists on this moving machine CH ⊥ PN such that the area of the changeable Square CMNH *or changeable Cube* , to be equal to that of the circle , *or Sphere* ?

1. It has been proved [8] – F43.(1-3) that the two equal and perpendicular Units CA , CP , *in plane ACP,* construct the Isosceles rightangled triangle ACP and the three circles on the sides as diameters . From any point M on the first circle is conscructed the square CMNH with vertices on the three circles . This Geometrical Formation is a **mooving Machine (a Geometry of motion)** and is called < **The Plane Formation of Constructing Squares** > . Since point M is on circle (E , EC) , then square CMNH is a ***Rotational Square*** on the three circles .
On geometrical machine CA ⊥ CP, CA = CP of the two equal circles (E , EA) ,(P′, P′C) and on the

circumcircle (O, OA) of the triangle ACP , are drawn two Changeable Squares HNMC , HPP_1C_1 which Sum is equal to the inscribed one CADP , or $PC^2 = HC^2 + HP^2$, which is a Plane System where → ***The Total Area CADP is conserved on the Changable System of the two squares CMNH and Antisquares*** $PHC_1 P_1$ **.**

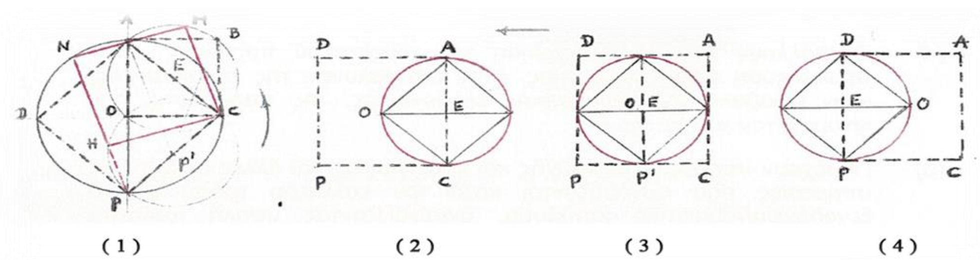
2. The sliding System [*cirle E,EC) and inscribed square ABCO*] in circumscribed square CADP . In F43.(2 - 4) , half area of circle (E , EA) is in circumscribed square CADP and in (3) the Total area of the inscribed circle is in Total square CADP . The rest is the four sectors .

In case that square CMNH F43.(3) ***is equal to the circle (E, EA) then it is holding : square HNMC = circle (E , EA)*** and the ***square*** HPP_1C_1 ***= square CADP - circle (E, EA) = Sectors which means that this happens only when the System (circle and inscribed square) is in the center of square ACDP , i.e. the diameter AC of the circle to be at point P` .***

3. The Central and parallel System [*cirle E,EC) and inscribed square ABCO*] from central point O in the circumscribed square CADP . (F.44)

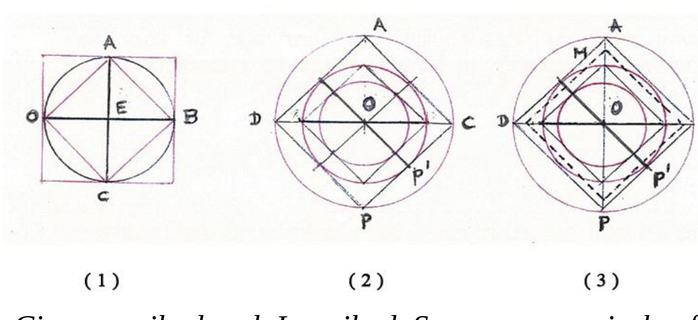
It has been also prooved that in any circle of diameter AC exists at points A , C one inscribed and one circumscribed square and a circle such that the circumscribed square or circle is twice the area of the inscribed .

In Fig 44.(1-4) ABCO is the inscribed square and ACPD the circumscribed of circle (E , EA). Any point M *(on line OA)* between points O , A of circle (O ,OP` = EA) formulates a square CMNH which is between them and parallel to the inscribed .
Simultaneously (since AC ⊥ CP) , the two Systems [circle (E, EC) and square ABCO] move from point C to point P` and to P ( *Sliding formation* ) which means again that the square equal to the circle is that which passes only from point P` . (diameter AC is always at point P`) .



F.44. *The Inscribed square ACPD on O,OA Circle of Rotating Resemblance ratio 2 .*

F.45. *The Circumscribed and Inscribed Square on a circle of Drawing Resemblance ratio 2 .*



F.45. *The Circumscribed and Inscribed Square on a circle of Drawing Resemblance ratio 2 .*

Conclutions :

1.. When point M moves (*is rotated*) on circle (E, EC) from point B to point A , is then constructed the inscribed square CBAO and the circumscribed square CADP .

One square say, CMNH between them is equal to the area of circle (E, EC) . [43-44]

2.. Simultaneousley diameter AC of circle (E, EC) is sliding (*linearly*) on CP from point C to point P and at point P`, the circle is totally in the circumscribed square CADP .

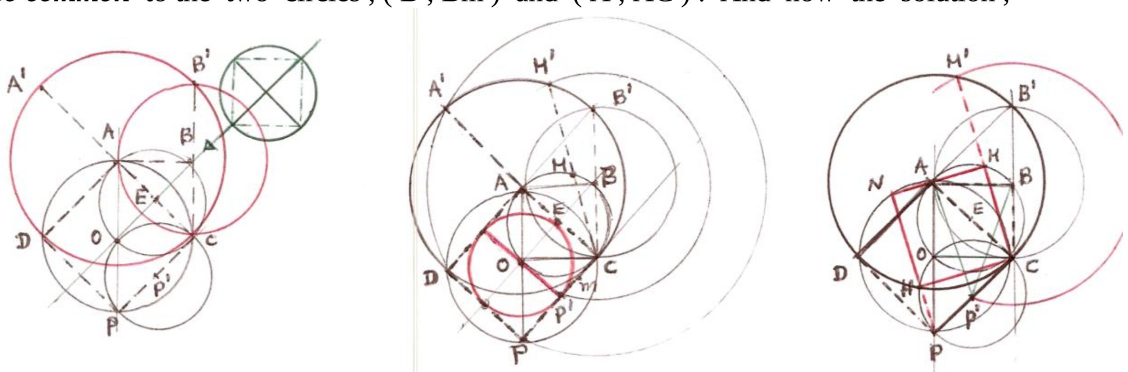
3.. On this Plane Formation is simultaneously produced *Rotational* (point M on arc BA) and *Linear displacement* (diameter AC is perpendicularly moving on CP) .

4.. In both cases the common Position (*of rotation and displacement*) is that when the moving circle is at point P` .
Furthermore we know that points on circle (E , EA = EC) , is the locus of mid-points of circle (A , AC) which is twice the radius of EA and then exists ,

a.. ***At the point C*** → ***Exists*** Square CBAO , BB` = BC , AB` = AC . F.46(1)
The inscribed square CBAO is on CA , and since BC = BA then circle (B, BC) with centre point B passes through the three points C , A , B`. Since also AC $\perp$ CP then circle ( E , EC) is sliding in the circumscribed square ACPD at point C . Also since the rightangled triangles BB`A , BCA are equal therefore hypotenuse AB` = AC and ***Point B` is also common*** to the two circles , (B , BC) and (A , AC) .

b.. ***At the point P*** → ***Exists*** Square PCAD , BA` = BP , AA` = AC . F.46(1)
Since rightangled triangles BB`A`, BAP are equal therefore hypotenuse BA` = BP .
The inscribed square CBAO is on segment CA = PD , and since BP = BA` then circle ( B, BP ) with centre point B and radius BP = BA` passes through the three points P , A` , D. Since also AC $\perp$ CP then circle (E , EC) is sliding in the circumscribed square ACPD at point P .
Since hypotenuse BA` = BP of the rightangled triangles BB`A` , BAP are equal , therefore ***Point A` is also common*** to the two circles , (B , BP) and (A , AC) .

c.. ***At any point* m *on CP*** → ***Exists*** Square CMNH , BM ` = Bm , AM ` = AC . F.46(2)
At any point **m** on line CP is constructed circle (B, B m) that intersects circle (A , AC) at point M ` and the line CM ` intersects circle (E , EC) at point M . On system CA $\perp$ CP square CMNH is constructed . Inscribed square CBAO is at point **m** , in the circumscribed square ACPD . Since Bm = BM ` then circle ( B, Bm ) with centre point B and radius Bm = BM ` passes through points **m** ,**M `** .Since also AC $\perp$ CP then circle ( E ,EC ) is sliding in the circumscribed square ACPD at point **m** . Since BM `= Bm , therefore ***Point M ` is also common*** to the two circles , (B , Bm) and (A , AC) . And now the solution ,



(1) (2) (3)

F.46. *The Geometrical Machine making all Squares on a circle .*

3.. ***Question .***

At what Position Line Segment CP formulates Square CMNH (on geometrical machine CA $\perp$ *CP) equal to the circle . ?? F.46-3*

In Fig (46).(1) CBAO is the inscribed and CADP is the circumscribed square of the circle with diameter AC . Inscribed square CBAO is at points C , m , P` , P respectively .
At point C the **System** [*Circle (E ,EC) - Inscribed square CBAO – Circumscribed square CADP*] is of half area of circle (E , EC) in Total square CADP . The same at point P .
At point **m** the **System** [*Circle (E ,EC) - Inscribed square CBAO – Circumscribed square CADP*] is of more than half area of circle (E , EC) in Total square CADP .

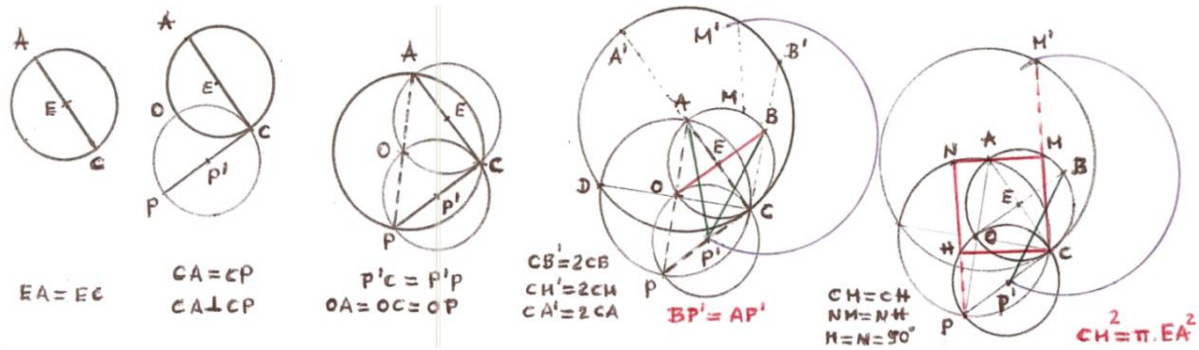
At point **P `** the **System** [*Circle (E ,EC)* - *Inscribed square CBAO – Circumscribed square CADP*] is of Total area of circle (E , EC) in circumscribed square CADP .
Since (*machine CA* ⊥ *CP*) constructs squares from point C to CA (from side CC , CB ,CM to CA) **and since** all these points , is the locus of midpoints of chords , CC , CB` , CM ` , CA` , in the constant circle ( A , AC ) , which points are common to the circles ( B , BC , Bm , BP ` , BP) with point B as circle, ***therefore , the common point M ` of circles ( B , BP`), (A , AC) defines point M on circle (E , EC) such that the constructed square to be equal to that of this circle .***

Proof :

In Fig.(47).(1) are the steps , where the rightangled triangle POC , point P` is in the middle of hypotenuse PC ,and therefore P`O = P`C . Since point P` is on the midperpendicular of AB (AB // OC) therefore P`A = P`B and that means point P` is the only point which constantly equidistances the three Systems 1,2,3 , the

Sliding System 1 [*Circle (E ,EC) - Inscribed square CBAO – Circumscribed square CADP*]
Central & // System 2 [*Circle (A , AC)*]
Rotational System 3 [*Circle (B , BC, Bm , BP` , BP)*]

Circle (B ,BP`= AP`) , intersects circle (A, AC) at point M `, determines line CM ` which intersects arc BA at point M such that side MN = MC of square CMNH equidistance Squares which are formed at point P` and has also the properties of the inscribed circle , which one is the area of this circle , therefore this square CMNH is equal to the circle with diameter AC .



(1) (2) (3) (4) (5)

F.47. *The circumscribed Circle and Square on a circle .*

The geometrical construction :

***The steps for Squaring any circle* E , EA = EC** (F.47)

1.. Let E be the center and CA is the diameter of any circle (E, EA = EC)
2.. Draw CP = CA perpendicular at point C and also the equal circle (P` , P`C = P`P)
3.. From midpoint O of hypotynuse AP as center , Draw the circle (O , OA = OP) and from point A as center draw circle (A , AC) .
4.. Draw diameter OEB on circle (E , EC) and from point B as center the circle (B , BP` = AP`) intersecting circle (A , AC) at point M ` .
5.. Draw line CM ` intersecting circle ( E , EA ) at point M and line MA produced intersecting circle ( O ,OA ) at point N and line PN intersecting circle ( P`,P`C) at point H
6.. Square CMNH is equal to that of the circle (E , EA) , or $CM^2 = \pi . EA^2$

Analysis gives the followings : **$CM^2 = \pi . EA^2$ where π**

$$\pi = \frac{48 - \sqrt{31} + 8.\sqrt{2} . \sqrt{16 - \sqrt{31}}}{16 + 2.\sqrt{2} . \sqrt{16 - \sqrt{31}}} = 3,141\,941\,071 \ldots.$$

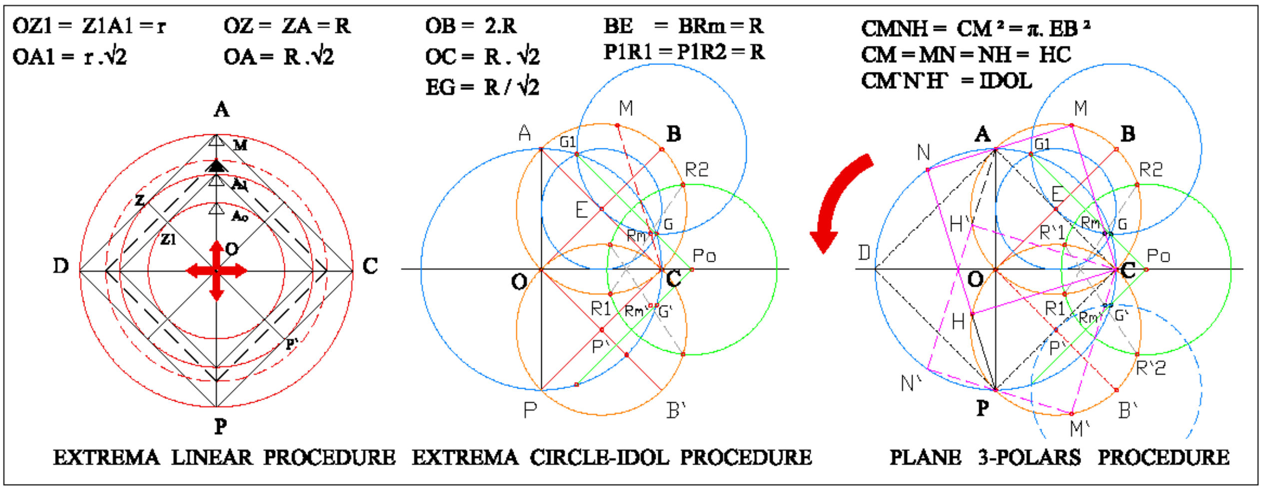
Trial 1 → π = 3, 141 030 312

Trial 2 → π = 3, 141 941 071

Trial 3 → π = 3, 141 ………

7.8-3 ***A Recent method** (Approach) **of Squaring the circle using the Plane Procedure Method*** **→ (Trial -3-).**

Figure 48 : The Extrema Linear Expanding Squares , interchange on Extrema Plane Rotating Squares through the four Conjugate circles of monad AC to square CMNH equal to that of the circle.



(1) (2) (3)

F.48. *The three Circles, [Inscribed –Circle – Circumscribed],on any diameter OB of a Circle, The common Radical axis of the system and Euler-Savary Plane Kinematic of Mechanics.*

Remarks :

1.. In Fig.48-1 was proved that any circle (O,OZ = R) is of area A = π.R² and the inscribed circle of radius ,r, Ai = π.r ² and since R ² = r² + r² = 2r² then A/ Ai = 2 , meaning that the area of a circle is twice the inscribed and half of the cirmscribed . The same also for the squares . This constant proportionality for squares happens because these exist as ***Expanding Squares*** , *Extrema Squares* . This procedure is linearly happening for concentric Circles and Squares and in Extrema case is altered to a , ***Plane – of two Polars - Rotation from one constant pole*** , and called Plane -Polar Procedure .The only Sign of the existence of the ***Expanding Squares*** is their common points on the circle which these belong and carry it as identity card , so in Extrema case this property must be investigated by the common points of circles only .

2.. In Fig.48-2 was proved as a theorem that → On each diameter **OEB** of a **circle** (**E , E B**) when we draw :

1 . the circumscribed circle (O , OA = OE .√2 = R.√2) at the edge point O as center ,

2 . the inscribed circle (E , EB / √2 = R / √2 = OA / 2) at the midpoint E as center ,

3 . the circle (B , BE) = (E , EB) at the edge point B as center **,**

Then the three circles *pass through points G , G1* (axis GG1) which have the Properties of the three circles, *The tangents from point B* to the circle (O , OA = OC) is constant and equal to 2 .EB ² , and *Occupy* the minimum position , *η Αρχή = Base* , in the moving system and line G1G (*axis GG1*) intersects line OC (*axis OC*) at point Po extended to → ∞ . This geometrical property of circles is the same also to the Idol square symmetric to OC axis (line) .

3.. In Fig.48-3 was proved that the two equal and perpendicular Units CA , CP , *in Plane ACP* , construct the isosceles rightangled triangle ACP and the three circles as diameters . From any point M on the first circle is constructed the square CMNH with vertices on the three circles . This Geometrical Formation is a **mooving Machine (A Mechanic, Cinematic Geometry motion)** and called < **The Plane (** ***three Polars*** **) Formation of Constructing Squares** > . Since point M is on circle (E , EC) , then square CMNH is a changable ***Rotational Square*** on the three circles. On this geometrical machine CA ⊥ CP where CA = CP of the two equal circles (E , EA),(P′, P′C) and on the circumcircle (O, OA) of the triangle ACP , are drawn , *formulated by rotation ,* the Changeable Squares CMNH and its equal and symmetrical Idol CM`N`H`.

Squares and Idol have a common Pole of rotation (point Po) , and the two constant Poles (Point C and P) for each system , i.e. a Plane - 3.Polar System of rotation , with two Symmetrical to OC axis , the line PoGG` and its symmetrical. Because of constancy of poles C, P, the linear expansion of squares happens to the Plane System as rotation on circle (B,BE) and its Idol .

Question ? Which logic exists on this moving machine BC ⊥ BA , M .., MC ⊥ MN , such that the two Conjugate circles (B,BE),(M,MH/2) or (Po,BE) have the properties of the common points on circles only , and the equal tangency on line G1G of the Initial circles and which tangency is $T \rightarrow R = OC/2.\sqrt{2} = r.\sqrt{2}$ and $T \rightarrow OC = R.\sqrt{2}$. ???

Drawing conjugate circle (Po,BE = R) at point Po , the Radical axis R1-R2 of the two equal circles occupy the same initial tangential properties . Point ,**Pm,** on Radical axis which is common to the initial circle (B ,BE) is is such that , ***The Tangency on Inscribed circle exists from this point*** ,Pm, ***of circle (B,BE) .***

In terms of Mechanics , a).Exist two constant systems of Square COAB and the symmetrical Idle COPB` . b). Circles (E,EC),(P`,P`C) is the constant System-Idol. c) The two Systems rotate on three constant Poles Po,C,P where Po is the base of the whole rotation and PoG , PoG` the two Radical axis controlling the two Systems-Idols d). Conjugate circle (Po ,EC) is checking the rotation of Expanding Square COAB on R1R2 axis while circle (B ,BG) is checking the local motion of the system ,and which is signed on R1R2 axis. The same to the Idle . e). The Rectilinear motion of the Changeable Formation , *the Squares CMNH* , is Transformed into a twin Symmetrically axial-centrifugal rotation (the motion) on this Constant System . This mechanical motion passing from the local extrema formulates points Pm , P`m on ,control axis, such that formulate the two Squares, *System and Idol* , equal to the area of the circle . An extend analysis will be soon prepared .

4.. In F.43 was proved that on the geometrical machine CA ⊥ CP , CA = CP of the two equal circles (E, EA), (P′, P′C) and on the circumcircle (O, OA) of the triangle ACP , are drawn two Changeable Squares HNMC , HPP1C1 which Sum is equal to the inscribed one CADP , or $PC^2 = HC^2 + HP^2$, which is a Plane System

where → ***The Total Area CADP is conserved on the Changable System of the two squares CMNH and Antisquares PHC1 P1 meaning that it is the Space and Anti-space in Sub-space and in terms of Mechanics , it is the Energy Conservation*** .

5.. The construction in Fig.48 .

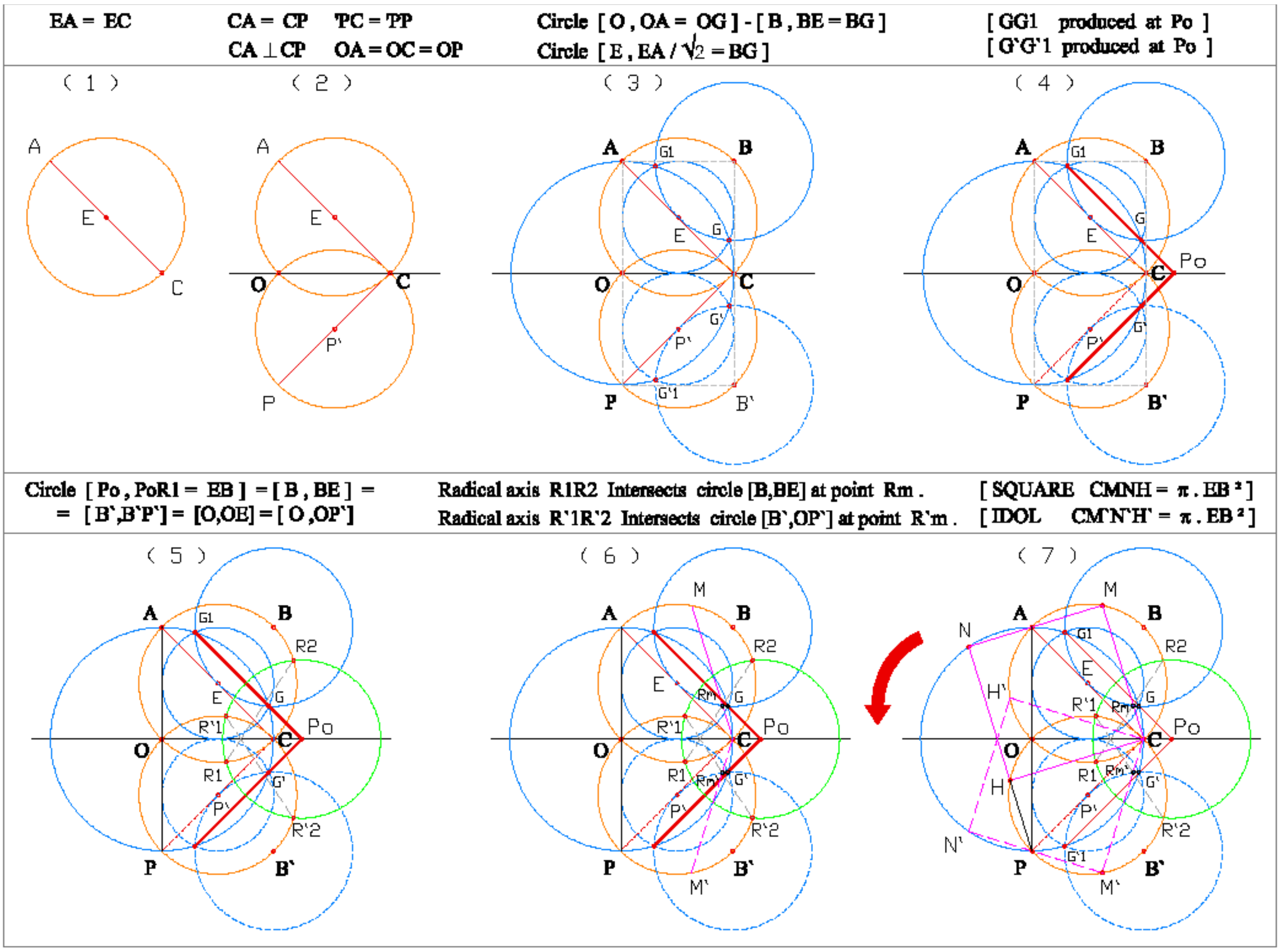
1.. Let E be the center and CA is the diameter of any circle (E, EA = EC)
2.. Draw CP = CA perpendicular at point C and on it the equal circle (P` , P`C = P`P)
3.. From midpoint O of hypotynuse AP as center , Draw the circle (O , OC = OA = OP) .
4.. Draw the perpendicular diameter OEB on CA of circle (E , EC) and from point B as center the circle (B , BE) intersecting circle (O , OC) at points G1 , G respectively .
5.. G1G line produced , intersects axis OC at point Po and Draw the equal and conjugate circle (Po,BE) .
6.. Point ,Pm , is the point of intersecting of circle (B ,BE = BG) and radical axis R1-R2 of the two circles .
7.. Line CPm produced intersects circle (E,EB) at point M .
8.. Complete Square CMNH on mechanism CA ⊥ CP .
9.. Square CMNH is equal to that of the circle (E , EA) , or $CM^2 = \pi . EA^2$

Proof :

a.. Points G1 , G are the two points having property of Tangency $T \rightarrow R = r.\sqrt{2}$ and $T \rightarrow OC = R.\sqrt{2}$
b.. Circles (Po,BE) , (E,EB) are the equal and Initial Conjugate circles , with their Radical axis R1 - R2 of this moving system of squares and point ,Pm , is the only , maxima existing common point of the three circles on this system , and common to the expanding and initial inscribed squares .*Point Pm is the common point of the Square and the Circle.*
c.. Segment CM passing through this common point ,Pm , intersects circle (E , EB) at point M formulating square CMNH with tangency $T = r.\sqrt{2}$ and $T = R.\sqrt{2}$.
d.. Segment CM is such that , $CM^2 = \pi . EA^2$ where π number is given very soon .

An extend analysis of the Energy Space , Anti-space Universe in [39] , markos 30/8/2015
I considered that it is better to give the solution of the Unsolved ancient problems this moment than later , so it is analytically presented in pages 34 as Extrema cases [L] and below . 11/9/2015 markos .

7.8-4 *The Extrema method of Squaring the circle in Fig - 49* .



In F.49 → *The steps for Squaring any circle* (E , EA = EC) *on diameter* CA .

On the geometrical Mechanism CA = CP where CA⊥CP , exist the Four Conjugate circles and the Fifth circle on OC axis controlling the Plane Mechanism of the Changable squares .

Geometrical construction : F.49

1.. Let E be the center and CA is the diameter of any circle (E, EA = EC)
2.. Draw CP = CA perpendicular at point C and also the equal diameter circle (P` , P`C = P`P)
3.. From midpoint O of hypotynuse AP as center , Draw the circle (O , OA = OP = OC) and complete squares OCBA , OCB`P . On perpendicular diameters OB , OB` and from points B , B` draw circles (B , BE) , (B`,B`P`) intersecting (O,OA) = (O,OP) circle at double points [G ,G1] , [G`G`1] respectively .
4.. Draw on the symmetrical to OC axis , lines GG1 and G`G`1 intersecting OC axis at point Po .
5.. On point Po , draw the conjugate circle (Po, EO = P`O) intersecting circle ( E , EC ) at points R1 , R2 and draw Radical axis R1R2 . The same also for circles (Po,EO),(P`,P`P = P`O) , draw Radical axis R`1R`2 .
6.. Circle (B , BE) intersects Radical axis R1 , R2 at point Rm and Circle (B`,B`P`) intersects Radical axis R`1,R`2 at point R`m . Draw lines CRm ,CR`m intersecting (E,EC) , (P`,P`C) circles at points M , M` respectively.
7.. Draw line CM and liine MA produced intersecting circle (O ,OA) at point N and line PN intersecting circle (P`,P`C) at point H and complete square CMNH , and call it the Space = the System . Draw line CM` and line M`P produced intersecting circle (O ,OA) at point N` and line AN` intersecting circle (E , EA) at point H` and complete square CM`N`H`, and call the Anti-space = Idol = Anti-System .

Show that this is an Extrema Mechanism on where ***the Two dimensional Space is Quantized to CMNH square of side CM*** *where* $CM^2 = \pi . EA^2$.

Proof :

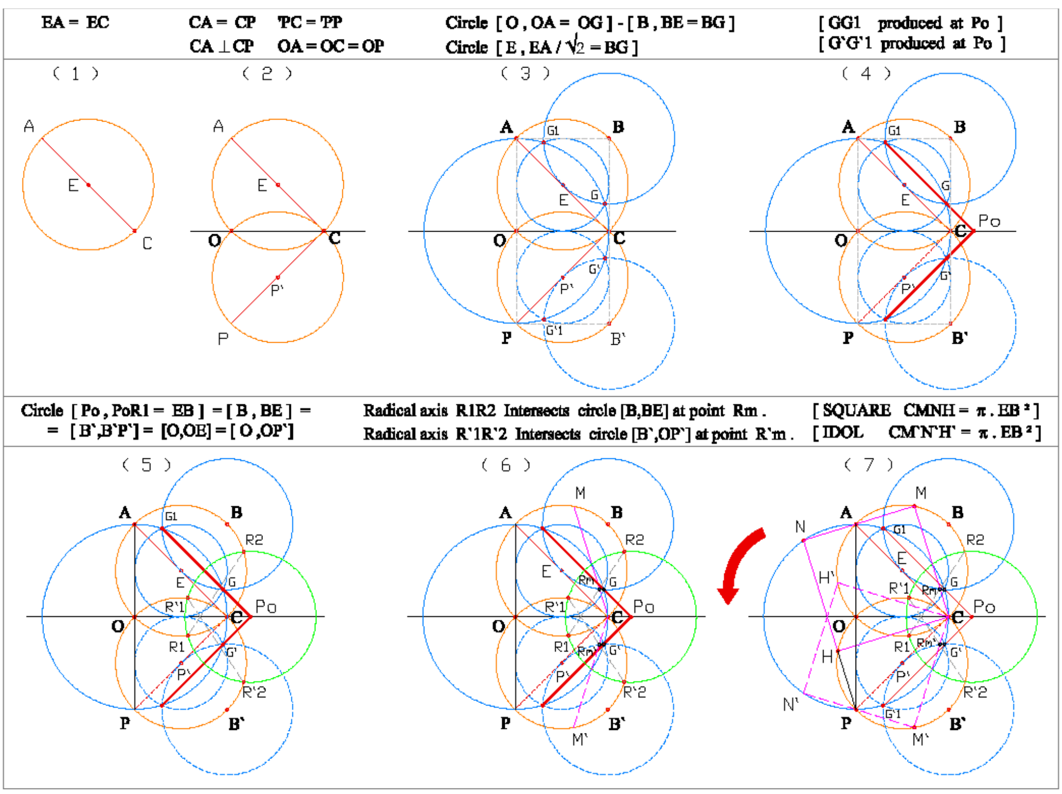
In (1) EA = EC and the unique circle (E , EA) of Segment AC .
In (2) Since circles (E , EA) , (P` , P`P) are symmetrical to OC axis (line) then are equal (conjugate) and since they are Perpendicular , follow Linear Quantization .
In (3) The circles (E,EO) , (P`,P`O) on diameters OB , OB` follow , *My Theorem of the Diameters on a circle* , where the pair of points G , G1 and G`,G`1 consist a Fix and Constant system of lines GG1 and G`G`1.
In (4) Lines GG1 and G`G`1 intersect each other on their bisector OC at a point Po and lines PoGG1,PoG`G`1 consist a Constant and Parallel to CA , CP System and this because CA = CP and CA ⊥CP and from symmetry then PoG1 = PoG`1 and PoG1 ⊥ PoG`1 .
In (5) Circle (Po, PoR1=PoR`1) is conjugate to the four equal circles (B,BE),(E,EO),(P`,P`O),(B`,B`P`) and this because all diameters are equal and perpendicular each other .
In (6) The Radical axis R1R2 , R`1R`2 of the Conjugate circles is perpendicular and Constant to GG1,G`G`1 System of lines . Since also Radical axis and System of Conjugate circles is a constant Formation then their intersection which are points Pm , P`m are unique and included in *my Theorem of the Diameters on a circle* , i.e. *The Circumscribed* , *The Circle* , *The Inscribed circle of the conjugate system of the five circles meet at a common point Pm and to the Symmetrical P`m* .
In (7) Squares CMNH ,CM`N`H` rotate on the three Poles A,P,C controlled from the fourth conjugate Pole Po. The unique axis (*lines*) CPmM , CP`mM` identify on O,OA circle , the points M , M` such that when completing the System of , *Squares CMNH and Anti-squares CM`N`H`* , formulates the two squares which have tangency T = EC.√2 from common points Pm , P`m on radical axis ,i.e. Squares CMNH , CM`N`H` are equal to the circles (E , EA ) ,(B`B`P`) , or $CM^2 = CM`^2 = \pi . EA^2$

Segments CM = CM` are the Plane Quantization of Euclidean Geometry through this Mould (→ The Plane Procedure Method is a Geometrical machine constructing Squares) of Squaring of the circle . [This is the Plane Quantization of E-Geometry i.e. The Area of square CMNH is equal to that of one of the five conjugate circles , or $CM^2 = \pi . CE^2$] . Since System is constant then all magnitudes are constant and number π also .

Remarks :

Since Monads AC = ds = 0 → ∞ are simultaneously (*actual infinity*) and (potential infinity) in Complex number form , and this defines , infinity exists between all points which are not coinciding , and because **ds** comprises any two edge points with imaginary part then this property differs between the infinite points . **This is the Vector relation of Monads , ds , (or , as Complex Numbers in their general form w = a + b. i) , which is the Dual Nature of lines (discrete and continuous .** Algebraic number ,π, is given later because of short time .

7.8-5 ***A trial with*** **Extrema-Kinematic** ***method for Squaring the circle in Fig - 50*** .



In F.50 → *The steps for Squaring any circle* (E , EA = EC) *on diameter* CA .

On the geometrical Mechanism CA = CP where CA⊥CP , exist the Four Conjugate circles and the Fifth circle on OC axis controlling the Plane Mechanism of the Changable squares .

Geometrical construction : F.50

1.. Let E be the center and CA is the diameter of any circle (E, EA = EC)
2.. Draw CP = CA perpendicular at point C and also the equal diameter circle (P\` , P\`C = P\`P)
3.. From midpoint O of hypotynuse AP as center , Draw the circle (O , OA = OP = OC) and complete squares OCBA , OCB\`P . On perpendicular diameters OB , OB\` and from points B , B\` draw circles (B , BE) , (B\`,B\`P\`) intersecting (O,OA) = (O,OP) circle at double points [G ,G1] , [G\`G\`1] respectively .
4.. Draw on the symmetrical to OC axis , lines GG1 and G\`G\`1 intersecting OC axis at point Po .
5.. On point Po , draw the conjugate circle (Po, EO = P\`O) intersecting circle ( E , EC ) at points R1 , R2 and draw Radical axis R1R2 . The same also for circles (Po,EO),(P\`,P\`P = P\`O) , draw Radical axis R\`1R\`2 .
6.. Circle (B , BE) intersects Radical axis R1 , R2 at point Rm and Circle (B\`,B\`P\`) intersects Radical axis R\`1,R\`2 at point R\`m . Draw lines CRm ,CR\`m intersecting (E,EC) , (P\`,P\`C) circles at points M , M\` respectively.
7.. Draw line CM and liine MA produced intersecting circle (O ,OA) at point N and line PN intersecting circle (P\`,P\`C) at point H and complete square CMNH , and call it the Space = the System . Draw line CM\` and line M\`P produced intersecting circle (O ,OA) at point N\` and line AN\` intersecting circle (E , EA) at point H\` and complete square CM\`N\`H\`, and call the Anti-space = Idol = Anti-System .

Show that this is an Extrema Mechanism on where ***the Two dimensional Space is Quantized to CMNH square of side CM*** *where* $CM^2 = \pi . EA^2$.

Proof :

In (1) EA = EC and the unique circle (E , EA) of Segment AC .
In (2) Since circles (E , EA) , (P\`, P\`P) are symmetrical to OC axis (line) then are equal (conjugate) and since they are Perpendicular , follow Linear Quantization .
In (3) The circles (E,EO) , (P\`,P\`O) on diameters OB , OB\` follow , *My Theorem of the Diameters on a circle* , where the pair of points G , G1 and G\`,G\`1 consist a Fix and Constant system of lines GG1 and G\`G\`1.
In (4) Lines GG1 and G\`G\`1 intersect each other on their bisector OC at a point Po and lines PoGG1,PoG\`G\`1 consist a Constant and Parallel to CA , CP System and this because CA = CP and CA ⊥CP and from symmetry then PoG1 = PoG\`1 and PoG1 ⊥ PoG\`1 .
In (5) Circle (Po, PoR1=PoR\`1) is conjugate to the four equal circles (B,BE),(E,EO),(P\`,P\`O),(B\`,B\`P\`) and this because all diameters are equal and perpendicular each other .
In (6) The Radical axis R1R2 , R\`1R\`2 of the Conjugate circles is perpendicular and Constant to GG1,G\`G\`1 System of lines . Since also Radical axis and System of Conjugate circles is a constant Formation then their intersection which are points Pm , P\`m are unique and included in *my Theorem of the Diameters on a circle* , i.e. *The Circumscribed , The Circle , The Inscribed circle of the conjugate system of the five circles meet at a common point Pm and to the Symmetrical P\`m* .
In (7) Squares CMNH ,CM\`N\`H\` rotate on the three Poles A,P,C controlled from the fourth conjugate Pole Po. The unique axis (*lines*) CPmM , CP\`mM\` identify on O,OA circle , the points M , M\` such that when completing the System of , *Squares CMNH and Anti-squares CM\`N\`H\`* , formulates the two squares which have tangency $T = EC.\sqrt{2}$ from common points Pm , P\`m on radical axis ,i.e. Squares CMNH , CM\`N\`H\` are equal to the circles (E , EA ) ,(B\`B\`P\`) , or $CM^2 = CM`^2 = \pi . EA^2$

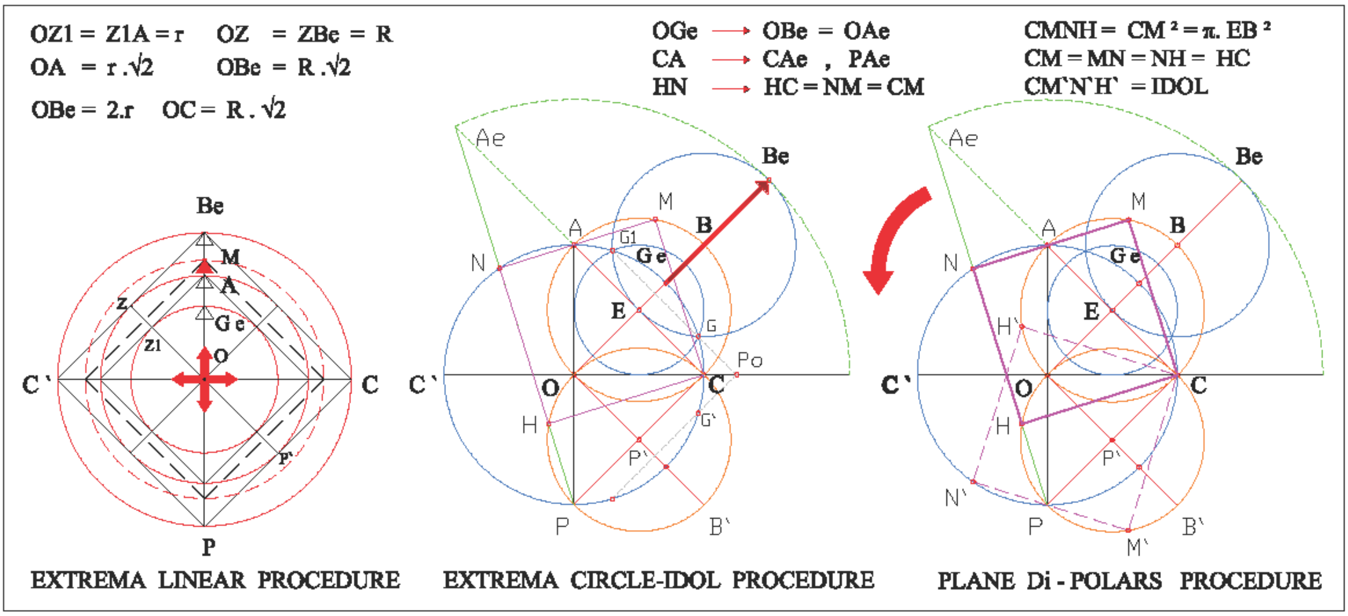
Segments CM = CM\` are the Plane Quantization of Euclidean Geometry through this Mould (→ The Plane Procedure Method is a Geometrical machine constructing Squares) of Squaring of the circle . [This is the Plane Quantization of E-Geometry i.e.* The Area of square CMNH is equal to that of one of the five conjugate circles , or** $CM^2 = \pi . CE^2$ **] .** ***Since System is constant then all magnitudes are constant and number π also .

Remarks :

Since Monads AC = ds = 0 → ∞ are simultaneously (*actual infinity*) and (potential infinity) in Complex number form , and this defines , infinity exists between all points which are not coinciding , and because **ds** comprises any two edge points with imaginary part then this property differs between the infinite points .

This is the Vector relation of Monads , ds , (or , as Complex Numbers in their general form w = a + b. i) , which is the Dual Nature of lines (discrete and continuous . Algebrical number ,*π*, is given later because of short time . .

7.8-6 .. The Plane , Extrema - Procedure *method for Squaring the circle* Fig 51 . **15/10/2015** .



(1) (2) (3)

F.51 → *The steps for Squaring any circle* (E , EA = EC) *on diameter* CA *through Linear – Di-Polar Procedure* .

The Plane Procedure method is consisted of two equal and perpendicular vectors CA , CP , *the Mechanism* , where CA = CP and CA⊥CP , *such that the Work produced is zero* , formulating Four Conjugate circles and the Fifth circle on OC axis, controlling the Plane Mechanism of the Changable Squares through Four constant Poles of rotation , and thus converting the Rectilinear motion to ***Four - Polar*** Expanding motion.

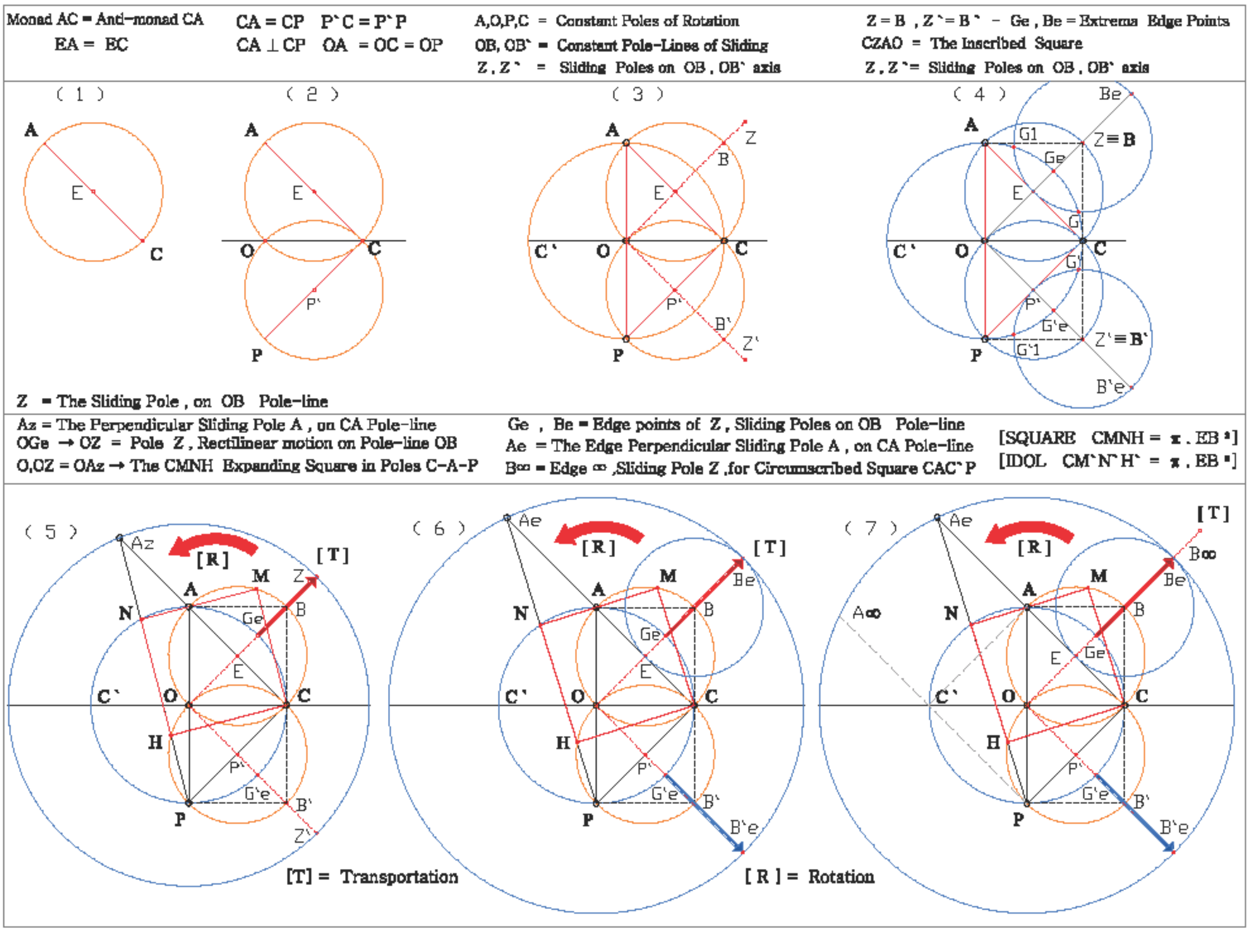
The Geometrical construction : F.51

1.. Let E be the center and CA is the diameter of any circle (E, EA = EC)
2.. Draw CP = CA perpendicular at point C and also the equal diameter circle (P\` , P\`C = P\`O)
3.. From midpoint O of hypotynuse AP as center , Draw the circle (O , OA = OP = OC) and complete squares OCBA , OCB\`P . On perpendicular diameters OB , OB\` and from points B,B\` draw circles (B, BE=Be) , (B\`,B\`P\`) intersecting O,OA = O,OP circle at double points [G ,G1] , [G\`G\`1] respectively , and OB , OB\` produced at points Be , B\`e respectively.
4.. Draw on the symmetrical to OC axis , lines GG1 and G\`G\`1 intersecting OC axis at point Po .
5.. Draw the circle (O,OBe) intersecting CA produced at point Ae and draw PAe intersecting (O, OA) , (P\`,P\`P) circles at points N-H ,N\`-H\` respectively.

6.. Draw line NA produced intersecting the circle (E ,EA) at point M and Segments CM , CH and complete quatrilateral CMNH , calling it the *Space = the System* .Draw line CM\` and line M\`P produced intersecting circle (O,OA) at point N\` and line AN\` intersecting circle (E, EA) at point H\`, and complete quatrilateral CM\`N\`H\` , calling it *the Anti-space = Idol = Anti-System.*

A.. Show that CMNH , CM\`N\`H\` are Squares.

B.. Show that it is an Extrema Mechanism ,on Four Poles where , ***The Two dimensional Space (the Plane) is Quantized to CMNH square of side CM = HN*** , *where holds* $CM^2 = \pi . EA^2$



F.52 → *The Steps for Squaring the circle* (E , EA = EC) *on diameter* CA *through Plane Procedure Mechanism*

In (1) EA = EC and the unique circle (E , EA) of Segment AC. AC is monad and CA the Anti-monad .

In (2) Since circles (E , EA) , (P`, P`P) are symmetrical to OC axis (line) then are equal (*conjugate*) and since they are Perpendicular so , No work is executed for any motion.

In (3) Points A,O,P and C are the constant ***Poles*** of Rotation and OB , OB` the two constant ***Pole-lines*** of the Sliding points Z , Z` while CA,CP are the constant Pole-lines of the Sliding point A and of Rotation P .

In (4) Circles (E, EO) , (P`, P`O) on diameters OB, OB` follow , *My Theorem of the Diameters on a circle* where the pair of points G , G1 and G`,G`1 consist a Fix and Constant system of lines GG1 and G`G`1. Points Z,Z` coincide with the Fix points B,B` and thus forming the inscribed Square CBAO or CZAO , *(this is because point* Z *is at point* A . PA , *Pole-line,* rotates through pole P where Ge , Be are the Edge points of the Sliding poles on the Rectilinear-Rotating System.

In (5) Sliding poles Z,Z` are forming Squares CMNH , CM`N`H` *and this is because Proof is as* , A-B Proof , *where* PN , AN` are *Pole-lines* rotating through poles P,A and diamesus HM passes through O . The circles (E, EO) , (P`, P`O) on diameters OB , OB` follow, *my Theorem of the Diameters on a circle*.

In (6) , Sliding poles Z,Z` being at Edge point Ge ≡ Z *formulates* CBAO *Inscribed square*, at Edge point Be Be ≡ Z *formulates* CMNH *equal square to that of circle* and , at Edge point B∞ , *formulates* CAC`P square, which is the *Circumscribed square*.

In (7) , CBAO *Inscribed square* , CMNH *The equal to the circle square* , CAC`P *Circumscribed square* .

A-B. Proof :

Theorem **:** [F.51-2] , [F.52-5]

On each diameter **OEB** of a circle (**E , E B**) we draw,

1 . ***the circumscribed circle*** *(O , OA = OE .√2) at the edge point* ***O*** *as center ,*
2 . ***the inscribed circle*** *(E ,OE/√2 = OA/2 = EG) at the mid-point* ***E*** *as center ,*
3 . ***the circle*** *(B , BE = BBe) = (E , EO) at the edge point* ***B*** *as center* **,**

(1). Then the three circles ***pass through the common points G ,G1 , and the symmetrical to OB point G1 forming an axis perpendicular to OB ,*** **which has the Properties of the circles , where** ***the tangent from point B*** **to the circle (O , OA = OC) is constant and equal to $2EB^2$.**
(2). Any point Z , which moves on diameter OEB produced , creates the ***Changeable moving Squares*** **, on the three circles which have point Z as Pole .** ***Circumscribed*** **circle OZ is expanded through center O to all** ***Inscribed*** **and** ***Circle`s*** **limit points.**
(3). Through the four constant Poles A,O,P - C of the ***Plane Procedure Mechanism*** **, pass (*rotate*) Sides and Diamesus of Squares , Anti-Squares.**
(4). Pole Z moves from edge points Ge (*forming inscribed square CBAO* **) , in-between points Ge-Be (***forming any square CMNH* **) , at point Be (** *forming that square CMNH equal to the circle* **) , between points Be - ∞ , (***forming circumscribed square CAC`P)*

B - Proof (1) :

Since BC ⊥ CO , the tangent from point B to the circle (O , OA) is equal to :
$\mathbf{BC^2} = BO^2 - OC^2 = (2. EB)^2 - (EB .\sqrt{2})^2 = 2 EB^2 = (2 EB).EB$ **= (2.BG) . BG** and since 2.BG = BG1 then **BC² = BG . BG1 ,** where point G1 lies on the circumscribed circle , and this means that BG produced intersects circle (O , OA) at a point G1 twice as much as BG . Since E is the mid-point of BO and also G midpoint of BG1, **so EG** is the diamesus of the two sides BO,BG1 of the triangle BOG1 and equal to 1/2 of radius OG1 = OC , *the base* , and since the radius of the inscribed circle is ½ of the circumscribed radius **then the circle (E , EB/√ 2 = OA/2) passes through point G .** Because BC is perpendicular to the radius OC of the circumscribed circle , ***so BC is tangent and*** **equal to $BC^2 = 2 . EB^2$. (o.ε.δ).(q.e.d).**
B - Proof (2-3) :

A point **Z** moving on **OB** Pole-line , defines point Az as that , of intersection of circle (O,OZ) and line CA. Polar-line PAz defines N,H points such that CHNM rightangled is completed as Square without any more assumptions . As in prior A-B proof ,
Angle < CHP = 90° because is inscribed on the diameter CP of the circle (P′, P′P) . The supplementary angle < CHN =180 – 90 = 90° . Angle < PNA = PNM = 90° because is inscr ibed on the diameter AP of the circle (O , OA) and Angle < CMA = 90 ° because is inscribed on the diameter CA of the circle (E , EA = EC) . The upper three angles of the quadrilateral CHMN are of a sum of 90+90+90 = 270 , and from the total of 360° , the angle < MCH = 360 – 270 = 90°, therefore shape CMNH is rightangled and exists CM ⊥ CH . Since also CM ⊥ CH and CA ⊥ CP therefore angle < MCA = HCP .
The rightangled triangles CAM , CPH are equal because have hypotynousa CA = CP and also angles < CMA=CHP = 90°, <MCA= HCP and side CH = CM therefore , rechtangle CMNH is Square on **CA,CP** Mechanism , through the three constant Poles C,A,P of rotation . The same for square CM`N`H` . (o.ε.δ) .

From the equal triangles COH , CBM angle < CHO = CHM = 45◦ and so points H,O,M lie on line HM .i.e. Diagonal **HM** of squares CMNH on Mechanism passes through central Pole O . (o.ε.δ).(q.e.d) .

The two equal and perpendicular vectors CA , CP , *the Plane Mechanism* , of the Changable Squares through the two constant Poles C , P of rotation , is converting *the Circular motion to Four-Polar Rotational motion.* Transferring the above property to [F.16-5] where is,

(O,OGe) → (O,OBe) Expanding circumscribed circle,
(O,OA) → (O,OAz) The expanding circle ,
(O,OZ1) → (E,EG) Is the inscribed circle ,

All are Concentrical at O point , and then the , *Changable Squares through the two constant Poles C , P of rotation* ,are converting *the →* ***Linear Expansion*** *OGe → OB → OBe to the →* ***Four - Polar Expansion,*** *PA → PN → PC`* , of the above squares and circles.

i.e. It was found a Mechanism where the Linearly Expanding Squares → **CBAO – CMNH – CAC`P** ***, and circles →*** **(E,EG) – (B , BE) – (O,OA)** *, which are between the Inscribed and Circumscribed ones* ***, are Polarly - Expanded as Four – Polar Squares .***

One square is equal to the area of the circle but , which one ?? , ***Answer*** → That square which is formed on the Extrema Procedure Mechanism of the ***Edge point*** **Be ,** *on Expanding circles* [O ,OGe → O ,OB → O, OBe] , and which is the point Ae .

The why circle (O, OGe) is Expanded from point Ge to point Be , is *because the* **Polarly-linear motion** *valids in monad`s boundaries (edges)* , while circle`s (O, OBe) rotation from point Be , A` to Ge , A , N is *a* ***Polarly - Expanded in Four – Polar motion .***

This point *is the Point of intersection of ,circle* O,OBe *and line* CA *produced* .
Line PA` is the *common and extrema axis* position of the Linear and Four-Polar Mehanism , and so defines that extrema square equal to the circle .
A quick measure for radius r = 2694 m gives side of square 4775 m and $\pi = 3,1416048 \rightarrow$ 11/10/2015

Segments* CM = CM ` *is the Plane Procedure Quantization of Euclidean Geometry through this Mould* (*The Plane Procedure Method* → CA⊥CP) *which is a Geometrical machine constructing Squares and Anti-squares and that equal to the circle . This is the Plane Quantization of E-Geometry i.e. The Area of square CMNH is equal to that of one of the five conjugate circles , or* $CM^2 = \pi . CE^2$ *, and System with number* π *is constant . More analysis in [49] .

B - Proof (4) :

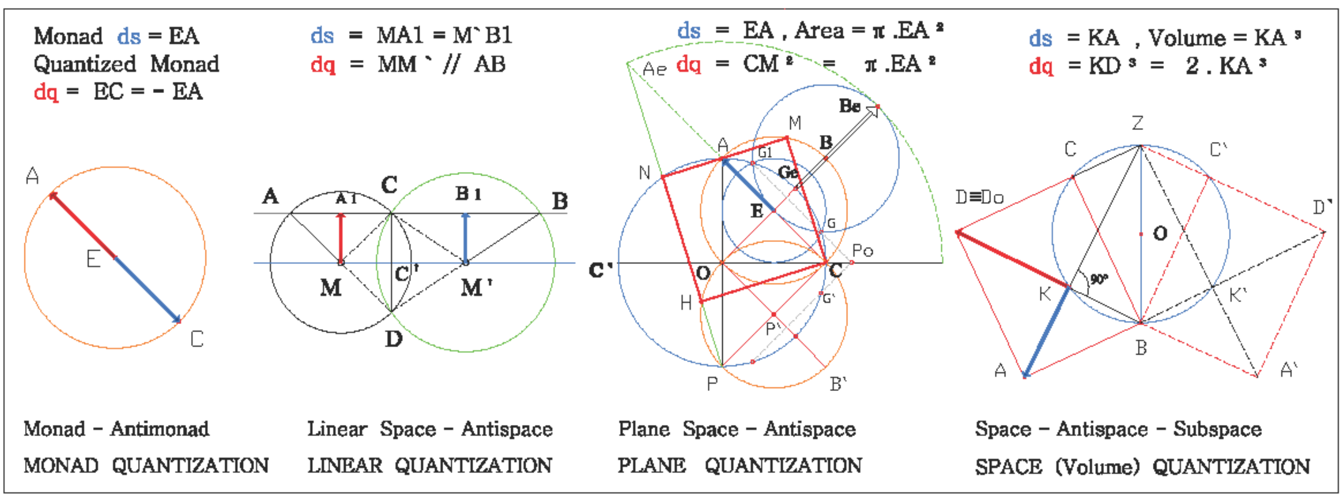
Circle (O,OGe) intersects CA vector at point A forming the inscribed square CBAO , circle (O,OGz) intersecting CA at point Az forming square CMzNzHz while circle (O,OBe) intersecting CA at point Ae forming square CMNH equal to the circle and circle (O,OB∞) intersecting CA at point A∞ (*parallel*) forming the circumscribed square CAC`P .

Remarks :

Since Monads AC = ds = 0 → ∞ are simultaneously (*actual infinity*) and (potential infinity) in Complex number form , *and this defines that the infinity exists between all points which are not coinciding* , and because **ds** comprises any two edge points with imaginary part , then this property differs between the infinite points between edges .

This is the Vector relation of Monads , ds = EA ,
(or , as Complex Numbers in their general form w = a + b. i = *discrete and continuous* **) , and which is the Dual Nature of Segments = monads in Plane , to be discrete and continuous) . Their monad-meter in Plane is above Mechanism of squaring the circle with monad as the diameter of the circle .**

8. The Quantization of E-Geometry to its moulds , F-53 .



(1) (2) (3) (4)

F.53 → *The Point , Linear , Plane , Space (volume) Mould for E-geometry Quantization ,* ***of*** *monad* EA *to Antimonad* EC – ***of*** AB *line to Parallel line* MM` - ***of*** CE *Radius to* CM Square *Segment* ***of*** KA *Segment to* KD Cube *Segment* - .

Quantization of E-geometry is the way of Points to become → (Segments , Anti-segments = Monads) , (Segments , Parallel-segments = Equal monads) , (Equal Segments Perpendicular-segments = Plane Vectors) , (Un-equal Segments twice-Perpendicular-segments = The Space Vectors = Quaternion)

The METERS of Quantization of monad **ds = AB** ***are as ,***

In any point A , ***happens through*** *Mould in itself* (The material point as a → ± dipole in [43]) .

In monad ds = AC , ***happens through*** *Mould in itself for two points* (The material dipole in inner monad Structure as the Electromagnetic Cycloidal field which equilibrium in dipole by the Anti-Cycloidal field as in [43]). ***For monad ds = EA the quantized Anti-monad is dq = EC = ± EA***
Remark : *The two opposite signs of monads EA , EC represent the two Symmetrical monads of Space-Antispace Geometrical dipole AC on points A,C which consist space AC* . F53 - (1)

Linearly , ***happens through*** *Mould of Parallel Theorem* ***, where for any point M not on ds = ± AB , the Segment MA1 = Segment M`B1 = Constant .*** F53 - (1-2)

Remark : *The two opposite signs of monads represent the two Symmetrical monads in the Geometrical machine of the equal and Parallel monads [MM` // AB where MA1⊥ AB , M`B1⊥ AB and MA1 = M`B1 ] which are → The Monad MA1 – Antimonad M`B1 , or → The Inner monad MA1 structure – The Inner Anti monad structure M`B1 = - MA1 = Idle and* **{ *Space = line AB , Anti-space = Parallel line MM`* }**. [41-43]

Plainly , ***happens through*** *Mould of Squaring of the circle* ***, where for any monad ds = CA = CP , the Area of square CMNH is equal to that of One of the five conjugate circles and π = constant , or*** **C*M² = π* . CE ².** ***On monad ds = EA = EC , the Area = π.EC² and the quantized Anti-monad dq = CM ² = ± π. EC² .*** F53 - (3)

Remark : *The two opposite signs represent the two Symmetrical squares in Geometrical machine of the equal and perpendicular monads* [CA⊥ CP *, and CA = CP*] *, which are → The Square CMNH – Antisquare CM`N`H`, or → The Space – Idle = Anti-space .* ***In Mechanics this propety of monads is very useful in Work area , where two perpendicular vectors produce Zero Work . { Space = square CMNH ,Anti-space = Anti-square CM`N`H`}.***

In Space , ***happens through*** *Mould Doubling of the Cube* ***, where for any monad ds = KA , the Volume or , The cube of a segment KD is the double the volume of KA cube , or monad KD ³ = 2 . KA ³ . On monad ds = KA the Volume = KA³ and the quantized Anti-monad dq = KD ³ = ± 2. KA ³ .*** F53 - (4)

Remark : *The two opposite signs represent the two Symmetrical Volumes in Geometrical machine of triangles* [Δ ADZ ⊥ Δ ADB] *, which are → The cube of a segment KD is the double the volume of KA cube – The Anti-cube of a segment K`D` is the double the Anti-volume of K`A` cube* **,** *Monad ds = KA , the Volume = KA³ and the quantized Anti-monad dq = KD ³ = ± 2. KA³* **. {** ***Space = cube KA³ , Anti-space = Anti-cube KD³*** **}.**

In Mechanics this property of Material monads is very useful in the Interactions of Electromagnetic Systems where Work of two perpendicular vectors is Zero .{ Space =Volume of KA , Anti-space =Anti-Volume of KD, and this in applied to Dark-matter , Energy in Physics }. [43]

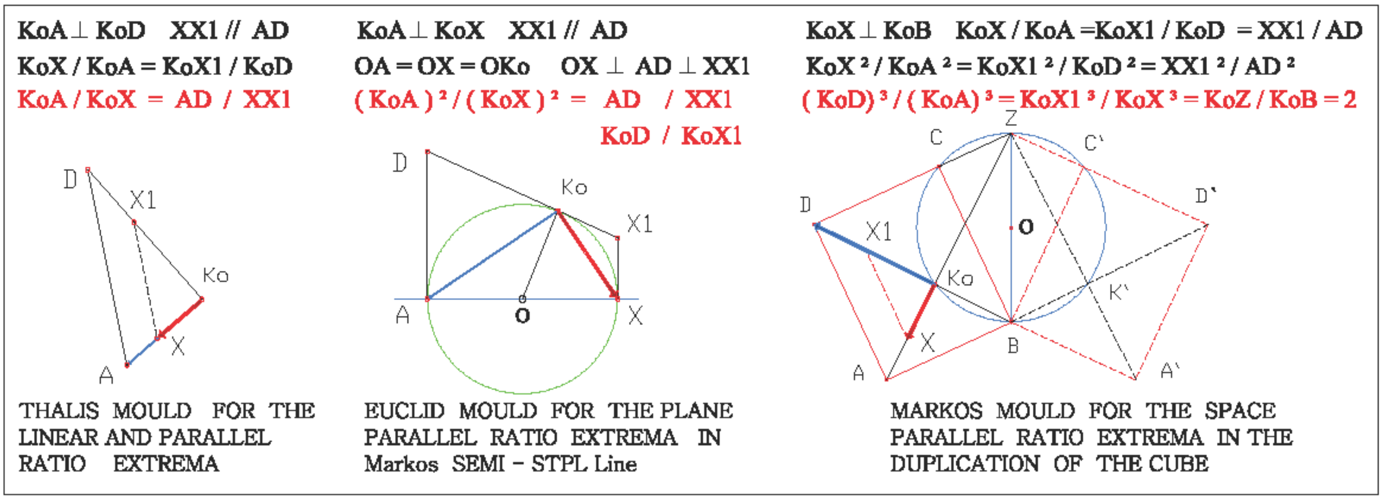
Radiation of Energy is enclosed in a cavity of the tiny energy volume λ ,(the cycloidal wavelength) with perfect and reflecting boundaries and this cavity may become infinite in every direction and thus getting in maxima cases (the limits) the properties of radiation in free space . The electromagnetic vibrations in this volume is analogous to vibrations of an Elastic body (Photo-elastic stresses in an elastic material) in this volume and thus Fringes are a superposition of these standing (*stationary*) vibrations . [41]

Above are analytically shown , the Moulds (The three basic Geometrical Machines) of Euclidean Geometry which create the METERS of monads → Linearly is the Segment MA1 , In Plane the square CMNH , and in Space is volume KD ³ , in all Spaces , Anti-spaces and Sub-spaces .

This is the Euclidean Geometry Quantization to its constituents , i.e. the

METER of Point A is the Material Point A , the
METER of line is the Segment ds = AB = monad = constant , the
METER of Plane is that of circle on Segment = monad , which is the Square equal to the circle , and the
METER of Volume is that of Cube on Segment = monad , which is the Double Cube of Segment and
Thus measuring the Spaces , Anti-spaces and Sub-spaces in this cosmos . markos 11/9/2015 **.**

The Three Master -Meters in One , for E-geometry Quantization , F-54



(1) (2) (3

F.54 → *The Thales ,Euclid ,Markos Mould , for the* ***Linear – Plane - Space*** *, Extrema Ratio Meters.*

Saying **master-meters** , we mean That the Ratio of two or three geometrical magnitudes , is such that they have a linear relation (*continuous analogy*) in all Spaces , as this happens to the Compatible Coordinate Systems as it is the Rectangular [x,y,z],[i,j,k] , the Cylindrical and Spherical-Polar . The position and the distance of points can be calculated between the points , and thus to ***perform independent Operations*** (Divergence , Gradient , Curl , Laplacian) on points .

Remarks :

In (54-1) ,The Linear Ratio begins from the same Common point Ko , of the two Un-equal , Concentrical and Co-parallel Direction monads .

In (54-2) ,The Linear Ratio begins from the same Common point Ko , of the two Un-equal , Consentrical and Co-perpendicular Direction monads .

In (54-3) ,The Linear Ratio begins from the same Common point Ko , of the two Un-equal , Consentrical and Co-perpendicular Direction monads .

In (1) → Segment KoA ⊥KoD , Ratio KoX / KoA = KoX1 / KoD , and Linearly (*in one dimension*) the Ratio of KoA / KoX = AD / XX1 , i.e. in Thales linear mould [XX1 //AD] , ***Linear Ratio of Segments* XX1 , AD *is , constant and Linear , and it is the Master key Analogy of the Two Segments, monads .***

In (2) → Segment KoA ⊥KoX , OKo = OA = OX , and since OX1 , OD are diameters of the two circles then KoD = AD, KoX1 = XX1 , and Linearly (*in one dimension*) the Ratio of KoA / KoX = AD / XX1 , in Plane (*in two dimensions*) the Ratio $[KoA]^2 / [KoX]^2$ = AD / XX1 , i.e. in Euclid`s Plane mould [KoA⊥KoX] , ***Plane Ratio square of Segments* – KoA , KoX - *is , constant and Linear , and for any Segment KoX on circle (O,OKo) exists KoA such that* → KoA^2 / KoX^2 =AD / XX1 = KoD/KoX1← *i.e. the Square Analogy of the sides in any rectangle triangle AKoX is linear to Extrema Semi-segments AD , XX1 or KoD , KoX1 .***

In (3) → Segment KoB ⊥KoX , OKo = OB = OZ , and since XX1 // AD , then KoA / KoD = KoX / KoX1 = AD / XX1 , and Linearly (*in one dimension*) the Ratio of KoA / KoX = AD / XX1 , and in Space (*Volume*) (*in three dimensions*) the Ratio $[KoA]^3 / [KoD]^3 = [KoX / KoX1]^3$ = 1 / 2 , i.e. in Euclid`s Plane mould [KoA // KoX , KoD // KoX1] , ***Volume Ratio of volume Segments* – KoA , KoD - *, is constant and Linear , and for any Segment KoX exists KoX1 such that* → $KoX1^3 / KoX^3 = 2$ ← *i.e. the Duplication of the cube.***

In F-54 , The ***three*** dimensional Space [KoA ⊥ KoD ⊥ Ko…] , where XX1 // AD , The ***two*** dimensional Space [KoA ⊥ KoX] , where XX1 // AD , The ***one*** dimensional Space [XX1 // AD] , where XX1 // AD , is constant and Linearly Quantized in each dimension . i.e. All dimensions of Monads coexist linearly in *Segments –monads* separately (*they are the units of the three dimensional axis* x,y,z - i , j, k -) and consequently in Volumes , Planes, Lines , Segments , and Points of Euclidean geometry, which are all the one point only and which is nothing. For more in [46] . 25/9/2015

At the beginning of the article it was referred to Geometers scarcity from which instigated to republish this article and to locate the weakness of prooving these Axioms which created the Non-Euclid geometries and which deviated GR in Space-time confinement . Now is more referred ,

a). There is not any Paradoxes of the infinite because is clearly defined what is a Point and what is a Segment.
b). ***The Algebra of constructible numbers and number Fiels is an Absurd theory*** based on groundless Axioms as the fields are , with directed non-Euclid orientations and must be properly revised.
c). ***The Algebra of Transcental numbers has been devised to postpone the Pure geometrical thought*** , which is the base of all sciences , by changing the base-field of solutions to Algebra as base .
d). All theories concerning ***the Unsolvability of the Special Greek problems are based on Cantor`s shady proof***, < *that the totality of all algebraic numbers is denumerable* > and not edifyed on the geometrical logic.
The problem of Doubling the cube as that of the Trisection of angle , is a Mechanical problem and could not be seen differently and the proposed Geometrical solution is clearly exposed to the critic of all readers.
All trials for Squaring the circle are shown and the set questions will be answerd on the Changeable System of the two Expanding squares , *Translation and Rotation* . The solution of the squaring the circle using the Plane Procedure method is now presented in F.51-52 .
e). Geometry is the base of all sciences and it is the reflective logic from the objective reality and which is nature.

9. Criticism to Non-Euclid Geometries

The essential difference between Euclidean and non-Euclidean geometries is the nature of parallel lines.

Euclid's fifth postulate, the parallel postulate, states that, within a two-dimensional plane ABM for a given line AB and a point M, which is not on AB .F5(3), i.e. MA+MB > AB, there is exactly one line through M that does not intersect AB because if MA+MB = AB then point M is on line AB and then lines MA, MB coincide each one passing from two points only and thus is answered the why any line contains at least two points. In Euclid geometry, in case of two straight lines that are both perpendicular to a third line, the lines remain at a constant distance from each other and are known as parallels. Now is proved that, a point M on the Nth Space, of any first dimensional Unit AB = 0 → ∞, jointly exists, with all Sub-Spaces of higher than N Spaces, and with all Spaces of lower than N Subspaces . [F-6] .

Linearly the constant Segments = monads , ***exist and happen through*** *the Mould of Parallel Theorem*

This is the Structure of Euclidean geometry. F5(2) As in fundamental theorem of Algebra Equations of Nth degree can be reduced to all N-a or N+a degree, by using the roots of the equations, in the same way Multi –Spaces are formed on AB. Nano-scale-Spaces, inorganic and organic, Cosmic-scale-Spaces are now unified in our world scale. Euclidean Empty Space is Homogenously Continues, but all first dimensional Unit-Spaces Heterogeneous and this because all Spaces constitute another Unit (the Nth Space Tensor is the boundaries of N points). All above referred and many others are springing from the first acceptance for point, and the approaching of Points. By multiplication is created another one very important logical notion for the laws concerning Continuation or not Continues Transformations in Space and in Time for Mechanics , Physics Chemistry and motions generally. From this logic yields that a limited and not an unlimited Universe can Spring anywhere.. Since Non-existence is found everywhere then Existence is found and is Done everywhere. [43] *Infinity exists between all points which are not coinciding, and because Monads ,ds, comprises any two edge points with Imaginary part, then this property differs between the ,i, infinite points and it is a Quaternion* $d\bar{s} = \lambda i + \nabla i$.

Wisdom tetrad problems, Quadrature (three trial in [45]) , Doubling the cube (a trial in [44]) , Trisection (The solution in [11]) , The Parallel Postulate (The solution in [9-32-38]), predispose the right direction for acquiring the Euclidean logic which exists in geometry and in all nature .

If Universe follows Euclidean geometry, then this is not expanded indefinitely at escape velocity, but is moving in Changeable Spaces with all types of motions, < a twin symmetrically axial –centrifugal rotation > into a Steady Space (This is the machine System AB ⊥AB = 0 → AB → ∞), with all types of curvatures. (It is a Moving and Changeable Universe into a Steady Formation) [8]. It was proved that on every point in Euclid Spaces exist infinite Impulse P = 0 → P → ∞, and so is growing the idea that Matter was never concentrated at a point and also Energy was never high < very high energy > [18], i.e. Bing Bang has never been existed, but it is a Space - Energy conservation State → $W = \int_{A-B} [P.ds] = \Sigma P.\delta = 0$. [23-25]

Gravity is particle also, in Space-Energy level which is beyond Plank's length level which needs a new type of light to see, with wave length smaller than that of our known visible light and thus can enter our wave length of light, and thus the Euclidean geometry describes this physical Space.

An extend analysis in [23].→ [25 – 37--39]

Hyperbolic geometry, by contrast, states that there are infinitely many lines through M , not intersecting AB. In Hyperbolic geometry, the two lines " curve away " from each other, increasing in distance as one moves further from the points of intersection with the common perpendicular, which have been called ultra-parallels. The simplest model for Hyperbolic geometry is the pseudo-sphere of Beltrami-Klein, which is a portion of the appropriate curvature of Hyperbolic Space, and the Klein model, by contrast, calls a segment as line and the disk as Plane.??? In hyperbolic geometry the three angles of a triangle add less than 180 ▫, without referring that triangle is not in Plane but on Sphere < Spherical triangle Fig -7 > This omission created the wrong hyperbolic geometry. Mobius strip and Klein bottle (complete one-sided objects of three and four dimensions) transfers the parallel Postulate to a problem of one point M and a Plane, because all curves and other curve lines are not lines (For any point on a straight line exists < the whole is equal to the parts which is an equality > and not the inequality of the three points) because contradict to the three points only and anywhere. Einstein's theory of general relativity is bounded in deviation Plank's length level ,where exists Space-time. Euclid geometry is extended to zero length level where Gravity exists as particle with wavelength near zero and infinite Energy, a different phenomenon than Space-time. In this way is proved that propositions are true only then, they follow objective logic of nature which is the meter of all logics and answers also to those who compromise incompatibility by addition or mixture.

If our Universe follows Hyperbolic geometry then this is expanded indefinitely, which contradicts to the homogenous and isotropic Empty Spaces. [37] .

This guides to a concentrated at a point matter and Energy < quaternion with very high energy in a tiny space > , Bing Bang event .Elliptic geometry, by contrast, states that, all lines through point M , intersect AB. In Elliptic geometry the two lines "curve toward" each other and eventually intersect. The simplest model for Elliptic geometry is a sphere, where lines are "great circles"??? For any great circle (which is not a straight line ???) and a point M which is not on the circle all circles (not lines ???) through point M will intersect the circle. In elliptic geometry the three angles of a triangle add greater than 180▫, without referring that triangle is not in Plane, but in the Sphere, < it is a Spherical triangle as F-7 >. This omission created the wrong elliptic geometry.

If Universe follows Elliptic geometry then this is expanded to a halt and then this will stark to shrink possibly not to explode as is said, but to change the axial-centrifugal motion to the initial Rectilinear.

G-R of Einstein assimilates gravity as the curvature in space-time , *i.e. ties Time with space* , and not as Force and this based on Elliptic geometry, by contrast, stating that, all lines through a point M and parallel to a line AB intersect line. This is for me one of the enormous faults of Relativity because has not conceived the essence and the bond to geometry [39-40-41] . Now in [46]

In Elliptic geometry the two lines "curve toward" each other and eventually intersect. The simplest model for Elliptic geometry is a sphere, where lines are "great circles". For any great circle (which is not a straight line) and a point M which is not on the circle all circles through point M will intersect the circle. In elliptic geometry the three angles of a triangle add greater than 180▫, without referring that triangle is not in Plane, but in the Sphere (spherical triangle). *This omission created the wrong elliptic geometry and all others*.

Assuming the postulate of Relativity, c = constant, was valid without restrictions, this would imply that all forces of nature must be invariant under Lorentz transformations in order that principle be rigorously and universally true.

In [40-41] has been proved the why velocity of light is constant and where is this holding.

Also what is said for an object flying-pass a massive object , then space time is curved by the massive object is wrong because Gravity is the medium causing attraction.[41].

It is proved [9] that from any point, M, not on line AB can be drawn one and only one parallel to AB, which parallel doesn`t intersect line . GR assimilating gravity as the curvature in space-time and not as Force , and this based on Elliptic geometry , by contrast , which states that , all lines through a point M and parallel to a line AB intersect line is failing ????. Since also in [34-36] is proved that Gravity is force [$\nabla i=2(wr)^2$] in the Medium-Field Material-Fragment $|\pm s^2|=(wr)^2$ =[MFMF] so this is the base for all motions , so

Elliptic Geometry must be properly revised.

Appealing space-time a Priori accepts the two elements, *Space and Time*, as the fundamental elements of universe without any proof for it, so anybody can say that *this stay on air*. It has been proofed [24-28] that any space AB is composed of points A,B which are nothing and equilibrium by the opposite forces $P_A = -P_B$ following *Principle of Virtual Displacement* .Time is the conversion factor between the conventional units (*second*) and length units (*meter).*

By considering the moving monads (*particles etc. in space*) at the speed of light, pass also through Time, this is an widely agreeable illusion and not reality.

The Parallel postulate is proved to be dependent on the other four therefore is a theorem , and *was one of the unsolved from ancient times Greek problems* and because *, this age-old question was faulty considered settled with the Non-Euclidean geometries*, part of modern Physics and mathematics from Astrophysics to Quantum mechanics have been so progressively developed on these geometries, resulting to Relativity`s space-time confinement and thus weak to conceive the beyond Planck`s existence and explaine universe. Now is given the quite new frame of thinking following and completing the anciant one , which is that of Euclidean logic only and the binding of Euclidean geometry with the Physical world .

It was referred [43] that Fragments $s^2 = \pm|(\overline{w}.r)^2|$ occupying the minimum quantized space $|s^2|$ are deported and fill all [STPL] cylinder which is the Rest Quantized Field $\pm [(\overline{w}.r)^2]$ or it is, the material point in mechanics as the base of all motions, where force $[(\overline{w}.r)^2\nabla i] = 2.[(\overline{w}.r)^2]$ is linearly vibrating on length $2.|[(\overline{w}.r)^2]| = \lambda$ as a Stationary Wave, and creates the curl Electromagnetic Field $E \perp P$, on which is the Universal Quantized force called Gravity. This Gravity-Field $Gf=[E+\bar{v}xP]$ is the unmovable, forced welded spinning dipole,$\rightarrow [|+(\overline{w}.r)^2|\leftrightarrow|-(\overline{w}.r)^2| = |\lambda|] \equiv [\{A(PA)\leftarrow 0\rightarrow (PB)B\}] \leftarrow$ causing attraction and because jointed with force, means that Newton`s laws issue in both the Absolute System [S] and Relative System [R] . The Gravity - Force is equal to $Fg = |\bar{q}|.[E+\bar{v}xP]$ and is exerted on any movable particle with charge $\bar{q}$. Because elements $[\pm \bar{c}.s^2]$ of Dark-matter are heavier then Gravity-Force is equal to $F_D = |\bar{q}|.[E_D+\bar{v}.P_D]$. For Dark matter in [43]

It is a provocation to all scarce today Geometers and to mathematicians to give an answer to this article . All Geometrical solutions of the Unsolved Problems are clearly exposed , and revealing the faults of Relativity .

A wide analysis of Energy-Space nature is in [43,46].

10. Conclusions

A line is not a great circle, so anything is built on this logic is a mislead false.

The fact that the sum of angles on any triangle is 180▫ is springing for the first time, in this article (Rational Figured numbers or Figures).

This admission of two or more than two parallel lines, instead of one of Euclid's, does not proof the truth of the admission. The same to Euclid's also, until the present proved method. Euclidean geometry does not distinguish, Space from time because time exists only in its deviation - Plank's length level-, neither Space from Energy because – Energy exists as quanta on any first dimensional Unit AB - which connects the only two fundamental elements of Universe, that of points or sector = monad = quaternion , and that of energy. [23]-[39].

The proposed Method in this article, based on the prior four axioms only, proofs, (not using any admission but a pure geometric logic under the restrictions imposed to seek the solution) that, through point M on any Plane ABM (three points only that are not coinciding and which consist the Plane) , passes only one line of which all points equidistant from AB as point M , i.e. the right is to Euclid Geometry.

The what is needed for conceiving the alterations from Points which are nothing , to segments , i.e. quantization of points as , *the discreteting = monads = quaternion* , to lines , plane and volume , is the acquiring and having the Extrema knowledge . In Euclidean geometry the inner transformations exist as ***pure*** Points , segments, lines , Planes , Volumes, etc. as the Absolute geometry is (*The Continuity of Points*) , automatically transformed through the three basic Moulds *(the three Master and Linear transformations exist as one Quantization*) to the Relative external transformations which exist as the , ***material*** , Physical world of matter and energy *(Discrete of Monads)* . [43]

The new Perception connecting the Relativistic time and Einstein`s Energy is Refining Time and Darm –matter Force clearly proves that Gig-Bang have never been existed .

In [17] is shown the most important ***Extrema Geometrical Mechanism*** , the STPL lines , that produces and composite all opposite space Points from Spaces to Anti-Spaces and Sub - Spaces in a Common Circle , *it is the Sub-Space* , to lines or Cylinder . ***This extrema mould is the transformation***, i.e. the Quantization of Euclidean geometry to the Physical world ***, to Physics ,*** and is based on the following geometrical logic ,

Since Primary point ,A, is the only Space and this point to exist , *to b*e , at any other point ,B, which is not coinciding with point ,A, then on this couple exists the Principle of Virtual Displacements $W = \int_A^B P.\,ds = 0$ or $[ds.(P_A + P_B) = 0]$, i.e. for any $ds > 0$ Impulse $P = (P_A + P_B) = 0$ and $[\, ds.\,(P_A + P_B) = 0\,]$, Therefore ,Each Unit $AB = ds > 0$, exists by this Inner Impulse (P) where $P_A + P_B = 0$, →

i.e. The Position and Dimension of all Points which are connected across the entire Universe and that of Spaces , exists , because of this equilibrium Static Inner Impulse and thus show the Energy-Space continuum . Applying the above logic on any monad = *quaternion* $(s + \bar{v}.\nabla i)$,where s = the real part and $\bar{v}.\nabla i)$ the imaginary part of the quaternion then ,

Thrust of two equal and opposite quaternion is the , Action of these quaternions $(s +\bar{v}.\nabla i) . (s + \bar{v}.\nabla i) = [\, s+\bar{v}.\nabla i\,]^2 =$

$s^2+|\bar{v}|^2.\nabla i^2 + 2|s|x|\bar{v}|.\nabla i = s^2 - |\bar{v}|^2 + 2|s|x|\bar{w}.\bar{r}|.\nabla i =$
$[\, s^2\,] – [\, |\bar{v}|^2\,] + [\, 2\bar{w}.|s|\, |\bar{r}|.\nabla i\,]$ where,

$[+s^2] \rightarrow s^2 = (w.r)^2$, is the real part of the new quaternion which is , the positive Scalar product , of Space from the same scalar product ,s,s with ½ ,3/2,,, spin and this because of ,w, and which represents the massive , Space , part of quaternion .

$[-s^2] \rightarrow - |\bar{v}|^2 = - |\bar{w}.\bar{r}|^2 = - [|\bar{w}|.|\bar{r}|]^2 = - (w.r)^2 \rightarrow$ is the always , the negative Scalar product , of Anti-space from the dot product of ,$\bar{w},\bar{r}$ vectors , with -½ ,-3/2, spin and this because of , - w , and which represents the massive , Anti-Space , part of quaternion.

$[\nabla i] \rightarrow 2.|s|\ x\ |\ \bar{w}.\bar{r}\,|.\nabla i =2|wr|.|(wr)|.\nabla i = 2.(w.r)^2 \rightarrow$ is a vector of , the velocity vector product , from the cross product of $\bar{w}$,$\bar{r}$ vectors with double angular velocity term giving 1,3,5, spin and this because of , ± w, in inner structure of monads , and represents the , Energy Quanta , of the Unification of the Space and Anti-Space through the Energy (*Work*) part of quaternion . A wider analysis is given in articles [40-43] .

When a point ,A, is quantized to point ,B, then becomes the line segment AB = vector AB = quaternion [AB] and is the closed system ,A B, ***and since*** also from the law of conservation of energy , *it is the first law of thermodynamics* , which states that the energy of a closed system remains constant , therefore *neither increases nor decreases without interference from outside* , and so the total amount of energy in this closed system , AB , in existence has always been the same , ***Then*** the Forms that this energy takes are constantly changing . This is the unification of this Physical world of , *Matter and Energy* , and that of Euclidean Geometry which are , *Points , Segments , Planes and Volumes* .

The three Moulds (i.e. The three Geometrical Machines) of Euclidean Geometry which create the METERS of monads and which are , *Linear* for a perpendicular Segment , *Plane* for the Square equal to the circle on Segment ,*Space* for the Double Volume of initial volume of the Segment ,and exist on Segment in Spaces , Anti-spaces and Sub-spaces .

This is the Euclidean Geometry Quantization to its constituents (Geometry in its moulds) , i.e. the
METER of Points A is the Point A , the
METER of line is the Segment ds = AB = monad = constant and equal to monad , or to the perpendicular distance of this segment to the set of two parallel lines between points A,B , the
METER of Plane is that of circle on Segment = monad and which is that Square equal to the circle , the
METER of Volume is that of Cube , on Segment = monad which is equal to the Double Cube of the Segment and Measures all the Spaces , the Anti-spaces and the Subspaces in this cosmos .

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The essence of ideas contained in the article were formulated many years ago after a pedant continuous conceptual understandable to assimilation in Euclidean logic this particular problem which is connected to the physical world. Many questions by mathematicians gave me the chance for a better critical understanding.

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