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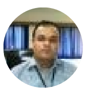

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Unit Graph of Some Finite Group Z_n, C_n and D_n

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Abstract

We represent finite group in the form of a graph, these graphs are called unit graph. Since the main role in obtaining the graph is played by the unit element of the group, this study is innovative. Also study of different properties like the subgroups of a group, normal subgroups of a group are carried out using the unit graph of the group.

Mathematics Subject Classification: 20F65, 05C25

Keywords: Finite Group, Commutative Ring, Graphs.

1. INTRODUCTION:

The phenomenon of representing Groups using Graphs has been studied theoretically by number of researchers [3-8]. In a series of investigations Akbari, S and Mohammadian [9, 10] discussed the zero divisor graphs of finite rings. In [12] maximum finite Groups are represented as graphs with examples, diagrams and theorems. In this article we give unit graph of finite groups such as Z_n, C_n and D_n.

2. UNIT GRAPH OF GROUP:

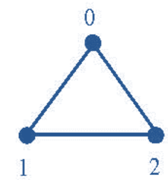
2.1 Definition: *A graph G (V, E) is Unit Graph of a Group of group (G,.) if*

i. *Elements v_i and v_j are adjacent in graph G (V, E) if $v_i . v_j = e$ in group (G, .).*
ii. *Every element of group (G,.) is adjoined with the unity of group (G,.).*

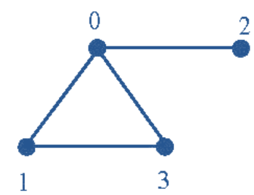
2.2 UNIT GRAPH OF GROUP Z_n:

***2.2.1** (**Unit graph of group Z_2 :**) Let $Z_2 = \{0, 1\}$ be the group under addition modulo 2. The unit graph of Z_2 is*

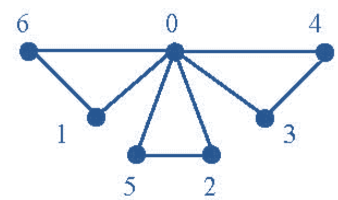
2.2.2 (Unit graph of group Z_3 :) *Let $Z_3 = \{0, 1, 2\}$ be the group under addition modulo three. The unit graph of Z_3 is*



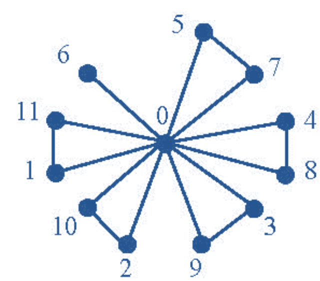
2.2.3 (Unit graph of group Z_4 :) *Let $Z_4 = \{0, 1, 2, 3\}$ be the group under addition modulo four. The unit graph of the group Z_4 is*



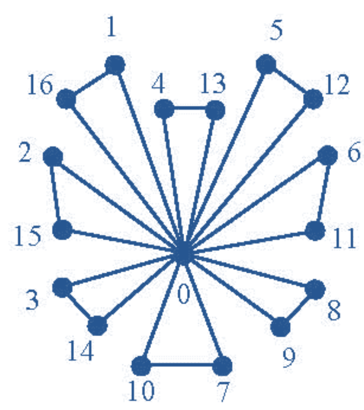
2.2.4 ***(Unit graph of group Z_7 :)*** *Let $Z_7 = \{0, 1, 2\ldots 6\}$, the group of integers modulo 7 under addition. The unit graph of Z_7 is*



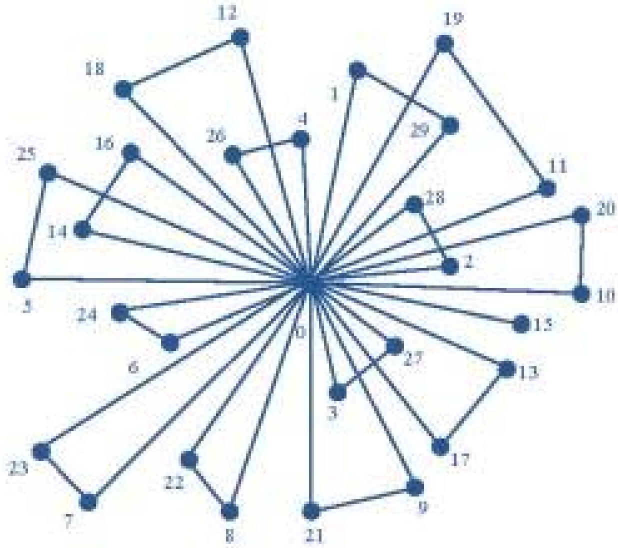
2.2.5 ***(Unit graph of group Z_{12} :)*** *Let $Z_{12} = \{0, 1, 2\ldots 11\}$ be the group under addition modulo 12. The unit graph of Z_{12} is*



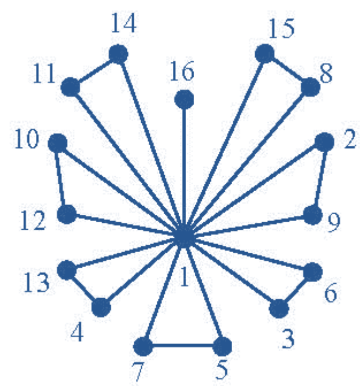
2.2.6 ***(Unit graph of group Z_{17} :)*** *Let $Z_{17} = \{0, 1, 2\ldots 16\}$ be the group under addition modulo 17. The unit graph of Z_{17} is*



2.2.7 ***(Unit graph of group Z_{17} :)*** *Let G = Z_{17}\ {0} = {1, 2… 16} be the group under multiplication modulo 17. The unit graph associated with G is*

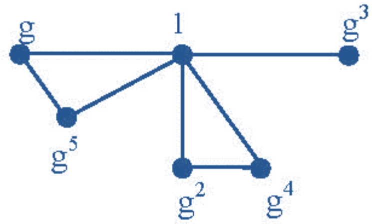


2.2.8 ***(Unit graph of group Z_{30} :)*** *Let G = Z_{30} = {0, 1, 2… 29} be the group under addition modulo 30. The unit graph associated with G is as follows.*

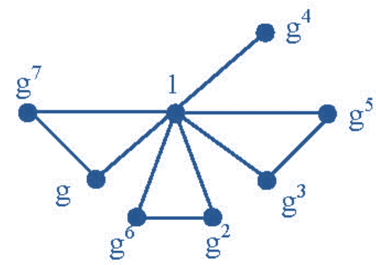


2.3 UNIT GRAPH OF CYCLIC GROUP C_n:

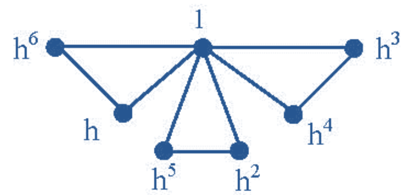
2.3.1 ***(Unit graph of cyclic group C_6 :)*** *Let G = C_6 = {g | g^6 = 1} the cyclic group of order 6 under multiplication. The unit graph of G is*



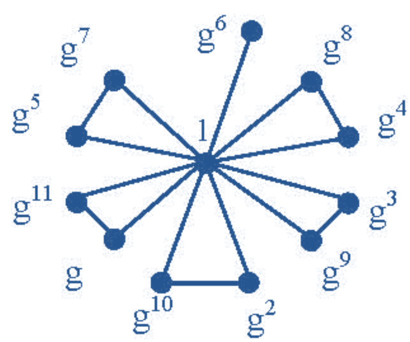
2.3.2 *(**Unit graph of cyclic group C_8 :**) Let $G = C_8 = \{g \mid g^8 = 1\}$ the cyclic group of order 8 under multiplication. The unit graph of G is*



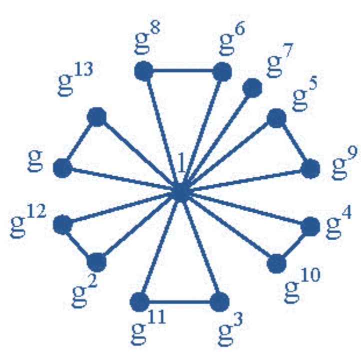
2.3.3 *(**Unit graph of cyclic group C_7 :**) Let $G = C_7 = \{g \mid g^7 = 1\}$ the cyclic group of order 7 under multiplication. The unit graph of G is*



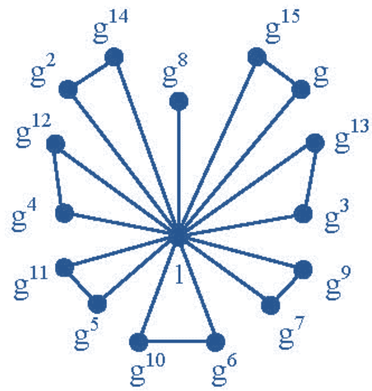
2.3.4 *(**Unit graph of cyclic group C_{12} :**) Let $G = C_{12} = \{g \mid g^{12} = 1\}$ the cyclic group of order 12 under multiplication. The unit graph of G is*



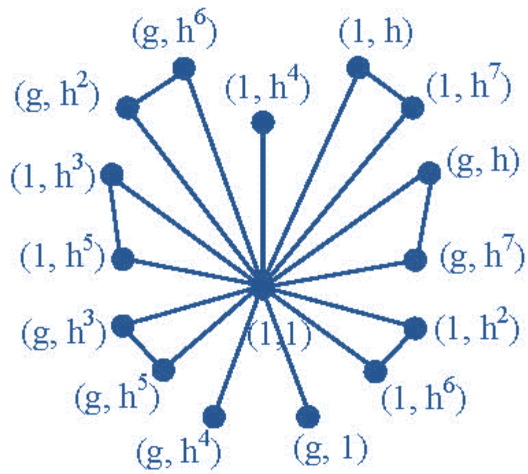
2.3.5 *(**Unit graph of cyclic group C_{14} :**) Let $G = C_{14} = \{g \mid g^{14} = 1\}$ the cyclic group of order 14 under multiplication. The unit graph of G is*



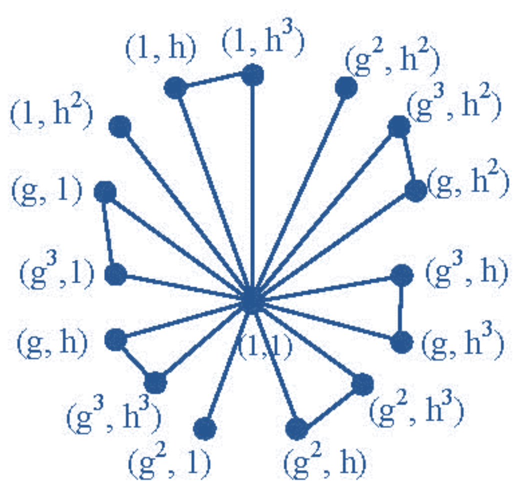
2.3.6 (***Unit graph of cyclic group C_{16} :***) *Let G = C_{16} = {g | g^{16} = 1} the cyclic group of order 16 under multiplication. The unit graph of G is*



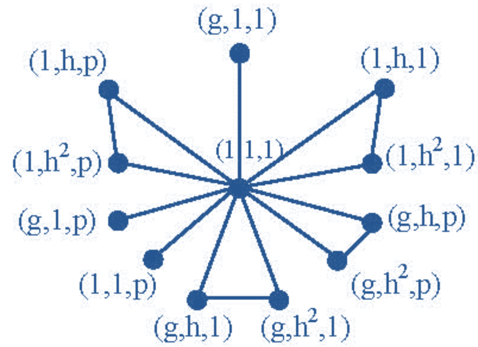
2.3.7 (***Unit graph of cyclic group C_2 x C_8 :***) *Let G = C_2 x C_8 = {g | g^2 = 1} x {g | g^8 = 1} = {(1, 1) (1, h), (1, h^2), (1, h^3), (1, h^4), (1, h^5), (1, h^6), (1, h^7), (g, h), (g, h^2), (g, h^3), (g, h^4), (g, h^5), (g, h^6), (g, 1), (g, h^7)}. The cyclic group of order 16 under multiplication. The unit graph of G is*



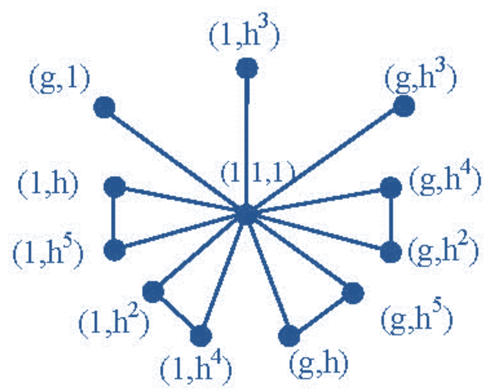
2.3.8 (***Unit graph of cyclic group C_4 x C_4 :***) *Let G = C_4 x C_4 = {g | g^4 = 1} x {g | g^4 = 1} = {(1, 1), (1, h), (1, h^2), (1, h^3), (g, h) (g, 1) (g, h^2), (g, h^3), (g^2, 1) (g^2, h^2), (g^2, h) (g^2, h^3) (g^3, 1) (g^3, h) (g^3, h^2) (g^3, h^3)}.The cyclic group of order 16 under multiplication. The unit graph of G is*



2.3.9 ***(Unit graph of cyclic group C_2 x C_3 x C_2 :)*** *Let G = C_2 x C_3 x C_2 = {g | g^2 = 1} x {g | g^3 = 1} x {g | g^2 = 1} = {(1 1 1), (g 1 1), (1 h 1), (1 h^2 1), (g h 1), (g h^2 1), (1 1 p), (g h p), (g 1 p), (g h^2 p), (1 h p), (1 h^2 p)}.The cyclic group of order 12 under multiplication. The unit graph of G is*



2.3.9 ***(Unit graph of cyclic group C_2 x C_6 :)*** *Let G = C_2 x C_6 = {g | g^2 = 1} x {g | g^6 = 1} = {(1 1), (g 1), (1 h), (1 h^2), (1 h^3), (1 h^4), (1 h^5), (g h), (g h^2), (g h^3), (g h^4), (g h^5)}.The cyclic group of order 12 under multiplication. The unit graph of G is*

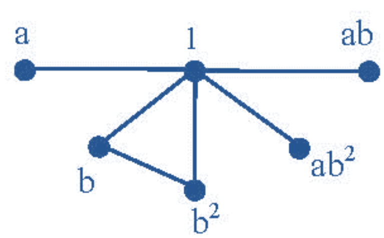


THEOREM 2.3.1: *If G =C_p = {g | g^p = 1} be a cyclic group of order p, p a prime. Then the unit graph formed by G has only triangles infact (p – 1) / 2 triangles.*

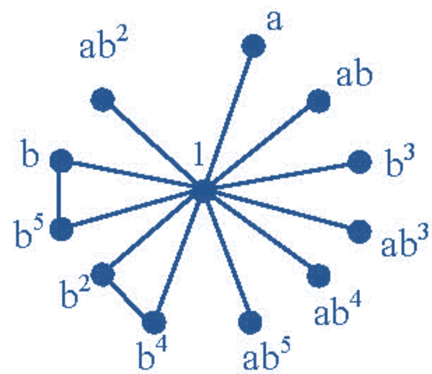
COROLLARY 2.4: *If G is a cyclic group of odd order then also G has the unit graph G_i which is formed only by triangles with no lines.*

2.4 UNIT GRAPH OF DIHEDRAL GROUP $D_{m, n}$:

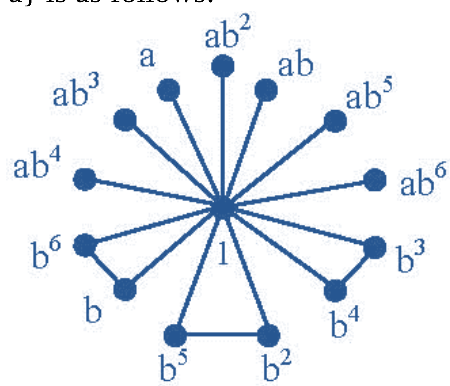
2.4.1 (Unit graph of Dihedral group $D_{2, 3}$:) *Let $D_{2.3}$ = {a, b | a^2= b^3 = 1; b a b = a} be the dihedral group. O ($D_{2.3}$) = 6. The identity graph associated with $D_{2.3}$ is given below.*



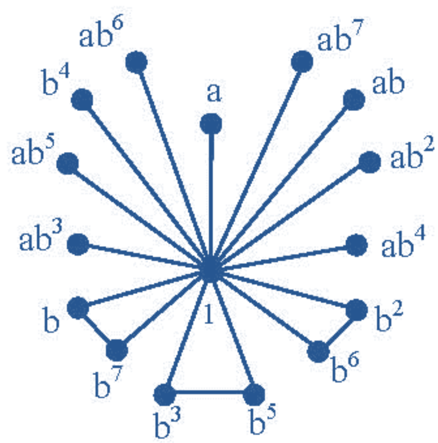
2.4.2 (Unit graph of Dihedral group $D_{2,6}$:) *The dihedral group $D_{2.6} = \{a, b \mid a^2 = b^6 = 1, bab = a\} = \{1, a, b, ab, ab^2, ab^3, ab^4, ab^5, b^2, b^3, b^4, b^5\}$. The identity graph of $D_{2.6}$ is*



2.4.3 (Unit graph of Dihedral group $D_{2,7}$:) The identity graph of the dihedral group $D_{2.7} = \{a, b \mid a^2 = b^7 = 1, bab = a\}$ is as follows:



2.4.4 (Unit graph of Dihedral group $D_{2,8}$:) *Let $D_{2.8} = \{a, b \mid a^2 = b^8 = 1, bab = a\} = \{1, a, b, b^2, b^3, b^4, b^5, b^6, b^7, ab, ab^2, ab^3, ab^4, ab^5, ab^6, ab^7\}$ be the dihedral group of order 16. The identity graph of $D_{2.8}$ is*



CONCLUSION:

The first time we represent finite group in the form of a graphs so this is very exciting area of research & relatively new in mathematics with a range of applications including coding & digital communications.

REFERENCES:

1. Akbari, S and Mohammadian, A., *on zero divisor graphs of finite rings*, J. Algebra, 314, 168-184 (2007).
2. Anderson, D.F. and Livingston, P.S., *The zero divisor graph of a commutative ring*, J. Algebra, 217, 434-447 (1999).
3. Beck I., *Colouring of a commutative ring*, J. Algebra, 116, 208-226 (1988).
4. Birkhoff, G. and Bartee, T.C. *Modern Applied Algebra*, Mc- Graw Hill, New York, (1970).
5. Bollobas, B., *Modern Graph Theory*, Springer-Verlag, New York (1998).
6. Castillo J., *The Smarandache Semigroup*, International Conference on Combinatorial Methods in Mathematics, II Meeting of the project 'Algebra, Geometria eCombinatoria',Faculdade deCiencias da Universidade do Porto, Portugal, 9-11 July 1998.
7. DeMeyer, F.R. and DeMeyer, L., *Zero divisor graphs of Semigroup*, J. Algebra. 283,190 - 198 (2005).
8. Hall, Marshall, *Theory of Groups*. The Macmillan Company, New York, (1961).163
9. Herrnstein I.N., *Topics in Algebra*, Wiley Eastern Limited, (1975).
10. Lang, S., *Algebra*, Addison Wesley, (1967).
11. Smarandache, Florentine, *Special Algebraic Structures*, in Collected Papers, Abaddaba, Oradea, 3, 78-81 (2000).
12. Vasantha Kandasamy, W. B. and Singh S.V., *Loops and their applications to proper edge colouring of the graph K2n*, Algebra and its applications, edited by Tariq et al, Narosa Pub., 273-284 (2001).