INTERVAL NEUTROSOPHIC MST CLUSTERING ALGORITHM AND ITS AN APPLICATION TO TAXONOMY

RIDVAN SAHIN*

Abstract. Interval neutrosophic sets (INSs) are a generalization of interval valued intuitionistic fuzzy sets (IVIFSs) whose the membership and non-membership values of elements consist of interval values. In this paper, we extend the clustering techniques based graph theory given for IVIFSs to INSs. Firstly, we propose a minimal spaning tree (MST) algorithm based on distance measure under interval neutrosophic environment and then use the interval neutrosophic MST (INMST) clustering algorithm to classify any dataset in here. Finally, we present some numerical examples in order to demonstrate the availability and effectiveness of the developed clustering algorithm.

1. INTRODUCTION

Clustering process is a procedure which divides a given dataset into groups such that similar objects are collect in a group whereas objects that is not similar are in different groups, and has a significant place in many fields including medicine, computational biology, economics, image processing and mobile communication. The need of gathering the objects with similar characteristic in same group causes to development of many methods. In the literature, a wide variety of clustering algorithms such as hierarchical, partitional, graph-based model-based and density-based have been proposed. Since similarity measure between sets is often can expressed by a graph, an interesting and important variant of data clustering is graph clustering. An application of fuzzy set theory in cluster analysis has proposed in the work of Ruspini [4]. Dong et al. [23] proposed a hierarchical clustering algorithm using the connectedness property of fuzzy graphs. A graph based clustering is actually a minimum spanning tree (MST) clustering. MST is a significant structure used to design many clustering algorithms and to connect all the data points either by a direct edge or by a path. To overcome many of the problems encountered by the classical clustering algorithms, it has been commonly studied by many authors in biological data analysis [13], image processing [11, 14] and pattern recognition [15]. The best known graph-based clustering algorithm, which starts by finding a minimum spanning tree in the graph and then removes inconsistent edges from the MST to create clusters, is Zahn's Minimum Spanning Tree (MST) clustering [1]. Recently,

Key words and phrases. Neutrosophic set, interval neutrosophic set, minumum spannig tree, graph based clustering algorithm, interval neutrosophic distance.

Wang et al. [5] have presented a fast minimum spanning tree-inspired clustering algorithm by using an efficient implementation of the cut and the cycle property of the MTSs. Gryorash et al. [17] have proposed two minimum spanning tree based clustering algorithms and applied the algorithms to image color clustering. There are several well known MST algorithms to solve minimum spanning tree problem [7, 16, 19]. By constructing the fuzzy similarity relation matrix, Chen et al. [3] have defined the concept of maximum spanning tree and used the threshold of this matrix to cut maximum spanning tree and obtained the classification on the respective level. Zhao et al. [24] have introduced an intuitionistic fuzzy clustering method which is called intuitionistic fuzzy MST clustering algorithm based on the graph theoretic techniques and the intuitionistic fuzzy distance measure to cluster intuitionistic fuzzy information, and extended them to clustering interval valued intuitionistic fuzzy information. Based on the idea of the conventional MST clustering algorithm and the hesitant fuzzy distance matrix, Zhang et al. [22] have proposed a hesitant fuzzy MST clustering algorithm.

Neutrosophic set theory [25] is defined by Smarandache for modelling uncertaintly in real word. However, since neutrosophic sets is be difficult to use in real scientific or engineering applications, Wang et al. [6] defined the concept of interval neutrosophic set (INS) which is a more general platform than the classic set, fuzzy set [12], interval valued fuzzy set [20], intuitionistic fuzzy set [9] and interval valued intuitionistic fuzzy set [10]. Because of this generalization, the existing MST clustering algorithms cannot cluster the interval neutrosophic data. So we need to develop the clustering techniques to cluster such data. This paper focuses on a clustering method based graph to handle the data represented by interval neutrosophic information.

The organization of this paper is as follows: Section 2 presents a short summary on the interval neutrosophic sets and its distance measures, graph theory and MST clustering algorithm. The proposed approach for the interval neutrosophic clustering algorithm based on MST as well as the experimental results have been shown in Section 3. The last section summarizes the conclusions.

2. Preliminaries

In the section, we first give some basic concepts of a graph and its minimum spanning tree (MST). Then we recall the definition of interval neutrosophic set and its some relevant relations.

2.1. **The graph and the minimum spanning tree (MST).** Graphs are mathematical structures used to represent pairwise relations between objects from a certain collection. A graph G consists of a set V and a set E, where the elements of V is called nodes and the elements of E are called edges. An undirected graph is a graph, in which each edge is an unordered pair $\{v_1, v_2\}$, where the nodes v_1 and v_2 are called the endpoints of an edge, while a directed graph is a graph, in which each edge is ordered pair. A subgraph H of G has an edge set $E^{'} \subseteq E$, and a node set induced by $E^{'}$. A path in G is a sequence of nodes $v_0, v_1, ..., v_k$ such that there is an edge between any two adjacent nodes v_i, v_{i+1} in the sequence. A cycle in G is a path whose endpoint is the same as its start point. A graph is connected if there is a path between any two nodes in the graph. A tree is a graph that is minimally connected, that does not contain any cycle. A spanning tree of G is a subgraph of G that is a tree and that covers every node in G. A weighted graph is a graph that a weight function $w : E \longrightarrow R$ is defined on the edge set E of a graph G. Weight of a graph is sum of the weights of all edges denoted by number $w(e)$ for an edge $e \in E$. In Fig. 1 we present an example of weighted graph.

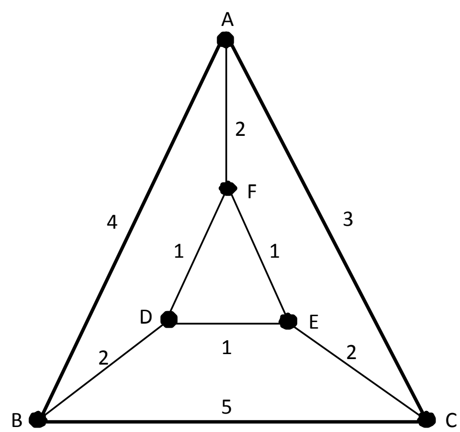


FIGURE 1. A weighted graph

Since a spanning tree H of G is connected, there is a path involving only edges in H between any two nodes in G, and since it is a tree, this path is unique. Figure 2 shows two different spanning trees of weighted graph given in Fig. 1.

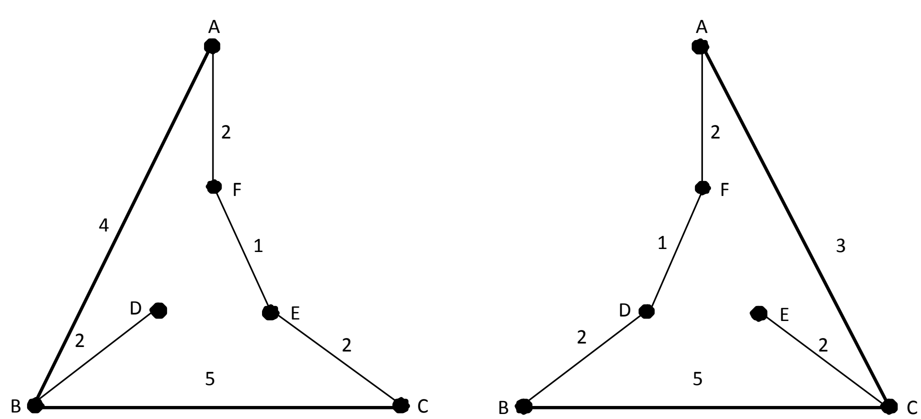


FIGURE 2. Two spanning trees

A minimum spanning tree (MST) in a connected and weighted graph is a spanning tree with minimum weight among all spanning trees. That is, a MST is a tree obtained from a subset of the edges in an undirected graph and has the following two properties:

(1) it is a minimum, i.e., the total weight of all the edges is as low as possible.
(2) it spans the graph, i.e., it includes every node in the graph.

Here, the total weight is the sum of the weights of all the edges of the spanning tree. In Fig. 3 we present two different minimum spanning trees of weighted graph given in Fig. 3.

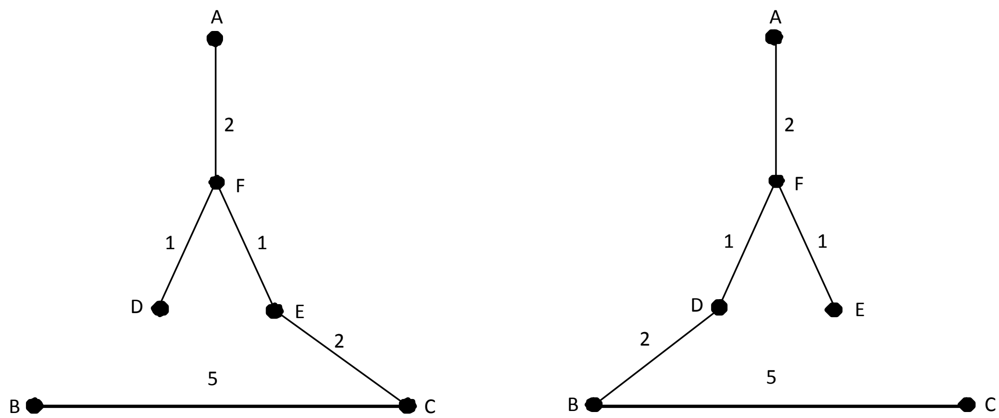


FIGURE 3. Two minimum spanning trees

The set E in a simple graph defines a crisp relation over $V \times V$. In other words, if there exists an edge between the nodes $v_1, v_2 \in V$, then membership degree of the pair equals 1 ($\mu_E(v_1, v_2) = 1$), if there not, it follows that $\mu_E(v_1, v_2) = 0$. If R is defined as a fuzzy relation over $V \times V$, then such a graph is referred to as fuzzy graph and the membership function $\mu_E(v_1, v_2)$ takes values from 0 to 1. If R is an interval valued intuitionistic fuzzy relation over $V \times V$, then $G = (V, R)$ is referred to as interval valued intuitionistic fuzzy graph [24]. Similarity, we define the interval neutrosophic graph by interval neutrosophic relation over $V \times V$.

In clustering analysis based MST, the distance between the end points forming an edge is commonly considered as the weight for this edge. So a MST algorithm can identify potentially significant edges or path in the graph. Well known algorithms for finding MST are Kruskal's algorithm [7], Boruvka's algorithm [16], Prim's algorithm [19] and Karger et al.'s faster randomized MST algorithm [2]. Algorithms considered in the rest of the article are Kruskal's and Prim's algorithms.

Kruskal's algorithm based on the edge selection starts by creating disjoint subsets of V containing only that node and for each node. It then controls the edges according to non-decreasing weight. If an edge connects two nodes in disjoint subsets, the edge is added and the subsets are merged into one set. The algorithm finishes when all the subsets are merged into one set.

Prim's algorithm based on the node selection grows by starting from an arbitrary node. At each stage, a new node and edge are added to the tree that is already constructed, and the algorithm finishes when all the nodes have been chosen.

2.2. The concept of INS.

Definition 2.1. [25] *Let X be a space of points (objects) and $x \in X$. A neutrosophic set N in X is defined by a truth-membership function $\mathcal{T}_N$, an indeterminacy-membership function $\mathcal{I}_N$ and a falsity-membership function $\mathcal{F}_N$, where $\mathcal{T}_N(x)$, $\mathcal{I}_N(x)$ and $\mathcal{F}_N(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$. That is, $\mathcal{T}_N : X \longrightarrow]0^-, 1^+[$, $\mathcal{I}_N : X \longrightarrow]0^-, 1^+[$ and $\mathcal{F}_N : X \longrightarrow]0^-, 1^+[$.*

There is no restriction on the sum of $\mathcal{T}_N(x)$, $\mathcal{I}_N(x)$ and $\mathcal{F}_N(x)$, so $0^- \leq sup\mathcal{T}_N(x) + sup\mathcal{I}_N(x) + sup\mathcal{F}_N(x) \leq 3^+$. Here, for practical purposes and to keep the discussion relatively simpler we are assuming the range of $[0, 1]$.

Definition 2.2. [6] *Let X be a set and $Int[0, 1]$ be the set of all closed subsets of $[0, 1]$. An INS A in X is defined with the form $A = \{(x, u_A(x), w_A(x), v_A(x)) : x \in X\}$, where $u_A : X \longrightarrow Int[0, 1]$, $w_A : X \longrightarrow Int[0, 1]$ and $v_A : X \longrightarrow Int[0, 1]$ with the condition $0 \leq \sup u_A(x), \sup w_A(x), \sup v_A(x) \leq 3$, for all $x \in X$.*

The intervals $u_A(x), w_A(x)$ and $v_A(x)$ denote the truth-membership degree, indeterminacy-membership degree and falsity-membership degree of x to A, respectively.

For convenience, if let $u_A(x) = \left[u_A^+(x), u_A^-(x)\right]$, $w_A(x) = \left[w_A^+(x), w_A^-(x)\right]$ and $v_A(x) = \left[v_A^+(x), v_A^-(x)\right]$, then $A = \left\{\left(x, \left[u_A^-(x), u_A^+(x)\right], \left[w_A^-(x), w_A^+(x)\right], \left[v_A^-(x), v_A^+(x)\right]\right) : x \in X\right\}$ with the condition $0 \leq \sup(x) + \sup w_A^+(x) + \sup v_A^+(x) \leq 3$, for all $x \in X$. If $w_A(x) = [0, 0]$ and $u_A^+(x) + v_A^+(x) \leq 1$, then A reduces to an interval valued intuitionistic fuzzy set [9] and if $w_A(x) = [0, 0]$ and $v_A(x) = [0, 0]$, then A reduces to a interval valued fuzzy set [20]. The relationship of interval neutrosophic set and other sets is presented in Table 1.

Interval neutrosophic set
$\Uparrow$
Interval valued intuitionistic fuzzy set
$\Uparrow$
Intuitionistic fuzzy set (Interval valued fuzzy set)
$\Uparrow$
Fuzzy set
$\Uparrow$
Classic set

TABLE 1. Relationships between sets

2.3. Relations of INS.

Let INS(X) denote the set of all interval neutrosophic sets over X.

Definition 2.3. [6] *Let A and B be two interval neutrosophic sets,*

$$A = \left\{\left(x, \left[u_A^-(x), u_A^+(x)\right], \left[w_A^-(x), w_A^+(x)\right], \left[v_A^-(x), v_A^+(x)\right]\right) : x \in X\right\},$$
$$B = \left\{\left(x, \left[u_B^-(x), u_B^+(x)\right], \left[w_B^-(x), w_B^+(x)\right], \left[v_B^-(x), v_B^+(x)\right]\right) : x \in X\right\}.$$

Then some operations can be defined as follows:

(1) $A \subseteq B$ *if and only if* $u_A^-(x) \leq u_B^-(x)$, $u_A^+(x) \leq u_B^+(x)$, $w_A^-(x) \geq w_B^-(x)$, $w_A^+(x) \geq w_B^+(x)$, $v_A^-(x) \geq v_B^-(x)$ *and* $v_A^+(x) \geq v_B^+(x)$ $\forall x \in X$;
(2) $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$;
(3) $A^c = \left\{\left(x, \left[v_A^-(x), v_A^+(x)\right], \left[1 - \sup w_A(x), 1 - \inf w_A(x)\right], \left[u_A^-(x), u_A^+(x)\right]\right) : x \in X\right\}.$

2.4. **The distance measures of INSs.** Suppose that $X = \{x_1, x_2, ..., x_n\}$ is an universe of discourse. Consider that the elements x_i $(i = 1, 2, ..., n)$ in the universe X may have different importance, let $\omega = \{\omega_1, \omega_2, ...\omega_n\}^T$ be the weight vector of x_i $(i = 1, 2, ..., n)$, with $\omega_i \geq 0$, $i = 1, 2, ..., n$, $\sum_{i=1}^{n} \omega_i = 1$. Suppose that A and B are two interval neutrosophic sets over X.

Ye [8] has defined the generalised distance as follows:

$$\begin{aligned} d_1(A, B) &= \left(\frac{1}{6n}\sum_{i=1}^{n}(\left|u_A^-(x) - u_B^-(x)\right|^p + \left|u_A^+(x) - u_B^+(x)\right|^p\right. \\ &\quad \left. + \left|w_A^-(x) - w_B^-(x)\right|^p + \left|w_A^+(x) - w_B^+(x)\right|^p + \left|v_A^-(x) - v_B^-(x)\right|^p + \left|v_A^+(x) - v_B^+(x)\right|^p)\right)^{\frac{1}{p}}. \end{aligned} \tag{1}$$

If it is taken as $p = 1$ and $p = 2$, then it is obtained the normalized Hamming distance and the normalized Euclidean distance, respectively:

(1) The normalized Hamming distance

$$\begin{aligned} d_2(A, B) &= \left(\frac{1}{6n}\sum_{i=1}^{n}(\left|u_A^-(x) - u_B^-(x)\right| + \left|u_A^+(x) - u_B^+(x)\right|\right. \\ &\quad \left. + \left|w_A^-(x) - w_B^-(x)\right| + \left|w_A^+(x) - w_B^+(x)\right| + \left|v_A^-(x) - v_B^-(x)\right| + \left|v_A^+(x) - v_B^+(x)\right|)\right) \end{aligned} \tag{2}$$

(2) The normalized Euclidean distance

$$\begin{aligned} d_3(A, B) &= \left(\frac{1}{6n}\sum_{i=1}^{n}(\left|u_A^-(x) - u_B^-(x)\right|^2 + \left|u_A^+(x) - u_B^+(x)\right|^2\right. \\ &\quad \left. + \left|w_A^-(x) - w_B^-(x)\right|^2 + \left|w_A^+(x) - w_B^+(x)\right|^2 + \left|v_A^-(x) - v_B^-(x)\right|^2 + \left|v_A^+(x) - v_B^+(x)\right|^2)\right)^{\frac{1}{2}} \end{aligned} \tag{3}$$

Ye [8] has extended the weighted distance to generalised distance as follows:

$$d_4(A,B) = \Bigg(\frac{1}{6}\sum_{i=1}^{n}\omega_i\left(\left|u_A^-(x)-u_B^-(x)\right|^p+\left|u_A^+(x)-u_B^+(x)\right|^p\right. \quad (4)$$
$$\left.+\left|w_A^-(x)-w_B^-(x)\right|^p+\left|w_A^+(x)-w_B^+(x)\right|^p+\left|v_A^-(x)-v_B^-(x)\right|^p+\left|v_A^+(x)-v_B^+(x)\right|^p\right)\Bigg)^{\frac{1}{p}}$$

If it is taken as $p=1$ and $p=2$, then it is obtain the normalized Hamming distance and the normalized Euclidean distance, respectively:

(1) The normalized Hamming distance

$$d_5(A,B) = \Bigg(\frac{1}{6}\sum_{i=1}^{n}\omega_i\left(\left|u_A^-(x)-u_B^-(x)\right|+\left|u_A^+(x)-u_B^+(x)\right|\right. \quad (5)$$
$$\left.+\left|w_A^-(x)-w_B^-(x)\right|+\left|w_A^+(x)-w_B^+(x)\right|+\left|v_A^-(x)-v_B^-(x)\right|+\left|v_A^+(x)-v_B^+(x)\right|\right)\Bigg)$$

(2) The normalized Euclidean distance

$$d_6(A,B) = \Bigg(\frac{1}{6}\sum_{i=1}^{k}\omega_i\Big(\left|u_A^-(x)-u_B^-(x)\right|^2+\left|u_A^+(x)-u_B^+(x)\right|^2 \quad (6)$$
$$+\left|w_A^-(x)-w_B^-(x)\right|^2+\left|w_A^+(x)-w_B^+(x)\right|^2+\left|v_A^-(x)-v_B^-(x)\right|^2+\left|v_A^+(x)-v_B^+(x)\right|^2\Big)\Bigg)^{\frac{1}{2}}.$$

Moreover,

Xu [21] has defined the generalised distance measure of IVIFSs as follows:

$$d_1(A,B) = \Bigg(\frac{1}{4}\sum_{i=1}^{n}\omega_i(\left|u_A^-(x_i)-u_B^-(x_i)\right|^2+\left|u_A^+(x_i)-u_B^+(x_i)\right|^2+ \quad (7)$$
$$+\left|v_A^-(x_i)-v_B^-(x_i)\right|^2+\left|v_A^+(x_i)-v_B^+(x_i)\right|^2)\Bigg)^{\frac{1}{2}}$$

Burillo et al. [18] has defined the generalised distance measure of IVFSs as follows:

$$(8) \qquad d_1(A,B) = \left(\frac{1}{2}\sum_{i=1}^{n}\omega_i(\left|u_A^-(x_i)-u_B^-(x_i)\right|^2+\left|u_A^+(x_i)-u_B^+(x_i)\right|^2)\right)^{\frac{1}{2}}$$

3. Clustering algorithm for INSs

Now, we define the concept of interval neutrosophic distance matrix.

Definition 3.1. *Let a_j $(j=1,2,...,m)$ be a m INSs, then $D=(d_{ij})_{m\times m}$ is called an interval neutrosophic distance matrix, where $d_{ij}=d(a_i,a_j)$ is the distance between a_i and a_j, which has the following properties:*

(1) $d_{ij}=0$ *iff* $a_i=a_j$;

(2) $0 \leq d_{ij} \leq 1$ *for all* $i, j = 1, 2, ..., m$;

(3) $d_{ij} = d_{ji}$ *for all* $i, j = 1, 2, ..., m$.

Based on the interval neutrosophic distance matrix, we extend the interval-valued intuitionistic MST clustering algorithm to interval neurosophic MST clustering algorithm, which is given by following steps:

Step 1 : Construct the interval neutrosophic distance matrix and interval neutrosophic graph.

(1) Calculate the distance $d_{ij} = d(a_i, a_j)$ by Eq (6) to obtain the interval neutrosophic distance matrix $D = (d_{ij})_{m\times m}$.

(2) Build the interval neutrosophic graph $G = (E, V)$, which has m nodes related to the samples a_j (for $j = 1, 2, ..., m$) represented by INSs and has $m(m-1)/2$ edge that every edge between a_i and a_j having the weight d_{ij}, which is an element of the interval neutrosophic distance matrix $D = (d_{ij})_{m\times m}$ and demonstrates the dissimilarity degree between the samples a_i and a_j.

Step 2: Compute the MST of the interval neutrosophic graph $G = (E, V)$ by Kruskal method [7] or Prim method [19]:

(1) Sort the edges of G in increasing range from the smallest weight to the largest one and choose the edge with the smallest weight.

(2) Choose the edge with the smallest weight from the each rest edges such that do not form a cycle with edge previously added.

(3) Repeat the process (2) until $(m-1)$ edges have been chosen. Hence, we obtain the MST of the interval neutrosophic graph $G = (V, E)$.

Step 3: Cluster the nodes into groups by disconnecting all the edges of the MST with weights greater than a threshold λ. Hence, we can obtain a certain number of sub-trees (clusters). The clustering results obtained by the sub-trees is not connected with some particular MST.

3.1. **Numerical examples.** In the section, we give some examples to demonstrate the real applications and effectiveness of the clustering algorithm proposed for INSs.

In the world, there is no a precise information about number of living species but scientists have estimate that there are species between 10 and 30 million. Because of this number of species, scientific studies become more difficult and this situation makes classification imperative. Our aim is to give a method based on graph that allows this classification.

A biologist group want to make a classification of eight rediscovered living species based on two attributes: x_1-physical structure and x_2-anatomical structure. According to the two attributes, they analyze the eight living species and report the interval neutrosophic date as

$$
\begin{aligned}
a_1 &= \{(x_1,[0.3,0.8],[0.2,0.4],[0.4,0.5]),(x_2,[0.3,0.5],[0.1,0.4],[0.6,0.7])\},\\
a_2 &= \{(x_1,[0.1,0.2],[0.8,0.9],[0.1,0.9]),(x_2,[0.1,0.8],[0.5,0.9],[0.2,0.5])\},\\
a_3 &= \{(x_1,[0.1,0.4],[0.5,0.7],[0.3,0.5]),(x_2,[0.1,0.4],[0.3,0.6],[0.5,0.6])\},\\
a_4 &= \{(x_1,[0.2,0.3],[0.4,0.5],[0.5,0.7]),(x_2,[0.5,0.6],[0.2,0.3],[0.3,0.4])\},\\
a_5 &= \{(x_1,[0.1,0.7],[0.1,0.7],[0.2,0.4]),(x_2,[0.3,0.5],[0.5,0.8],[0.4,0.5])\},\\
a_6 &= \{(x_1,[0.5,0.6],[0.3,0.5],[0.4,0.6]),(x_2,[0.3,0.7],[0.1,0.2],[0.1,0.2])\},\\
a_7 &= \{(x_1,[0.5,0.6],[0.5,0.7],[0.2,0.4]),(x_2,[0.1,0.2],[0.5,0.7],[0.2,0.8])\},\\
a_8 &= \{(x_1,[0.8,0.9],[0.7,0.8],[0.3,0.5]),(x_2,[0.1,0.5],[0.3,0.4],[0.3,0.9])\}.
\end{aligned}
$$

Let the weight vector of the attributes x_j $(j=1,2)$ be $\omega=(0.45,0.55)^T$. We use the INMST clustering algorithm to group these species $a_j(j=1,2,...,8)$:

Step 1: Construct the interval neutrosophic distance matrix and interval neutrosophic graph:

(1) Calculate the distance $d_{ij}=d(a_i,a_j)$ by Eq (6) to obtain the interval neutrosophic distance matrix $D=(d_{ij})_{8\times 8}$. Then we can obtain the interval neutrosophic distance matrix $D=(d_{ij})_{8\times 8}$ as follows:

$$
D=\begin{bmatrix}
0,0000 & 0,4029 & 0,2073 & 0,2234 & 0,2273 & 0,2478 & 0,2714 & 0,2650\\
0,4029 & 0,0000 & 0,2519 & 0,3192 & 0,3074 & 0,3762 & 0,3133 & 0,3747\\
0,2073 & 0,2519 & 0,0000 & 0,2146 & 0,1843 & 0,2865 & 0,1914 & 0,2752\\
0,2234 & 0,3192 & 0,2146 & 0,0000 & 0,2712 & 0,1717 & 0,3139 & 0,3373\\
0,2273 & 0,3074 & 0,1843 & 0,2712 & 0,0000 & 0,3042 & 0,2224 & 0,3264\\
0,2478 & 0,3762 & 0,2865 & 0,1717 & 0,3042 & 0,0000 & 0,3317 & 0,3114\\
0,2714 & 0,3133 & 0,1914 & 0,3139 & 0,2224 & 0,3317 & 0,0000 & 0,2018\\
0,2650 & 0,3747 & 0,2752 & 0,3373 & 0,3264 & 0,3114 & 0,2018 & 0,0000
\end{bmatrix}
$$

(2) Build the interval neutrosophic graph $G=(E,V)$, which has 8 nodes related to the samples a_j (for $i=1,2,...,8$) represented by INSs and has 28 edge that every edge between a_i and a_j having the weight d_{ij}, which is an element of the interval neutrosophic distance matrix $D=(d_{ij})_{8\times 8}$ and demonstrates the dissimilarity degree between the samples a_i and a_j. Then we can give the interval neutrosophic graph

$G = (E, V)$ in Fig. 4.

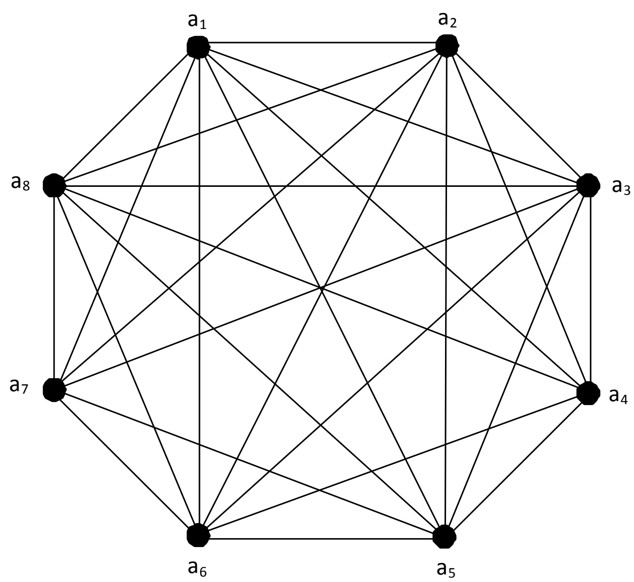


FIGURE 4. The interval neutrosophic graph $G = (V, E)$

Step 2: Compute the MST of the interval neutrosophic graph $G = (E, V)$ by Kruskal method [7] or Prim method [19]:

(1) Sort the edges of G in increasing range from the smallest weight to the largest one and choose the edge with the smallest weight, that is the edge e_{46} between a_4 and a_6.

$$\begin{aligned} d_{46} &< d_{35} < d_{37} < d_{78} < d_{13} < d_{34} < d_{57} < d_{14} < d_{15} < d_{16} \\ &< d_{23} < d_{18} < d_{45} < d_{17} < d_{38} < d_{36} < d_{56} < d_{25} < d_{68} \\ &< d_{27} < d_{47} < d_{24} < d_{58} < d_{67} < d_{48} < d_{28} < d_{26} < d_{12}. \end{aligned}$$

(2) Choose the edge with the smallest weight from the each rest edges such that do not form a cycle with edge previously added, that is the edge e_{35} between a_3 and a_5

(3) Repeat the process (2) until seven edges have been chosen. Thus, we obtain the MST of the interval neutrosophic graph $G = (V, E)$ as shown in Fig. 5.

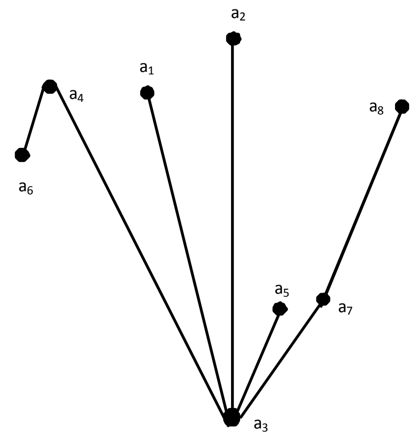


FIGURE 5. The MST of interval neutrosophic graph $G = (V, E)$

Step 3: Cluster the nodes into groups: by choosing a threshold λ and by disconnecting all the edges of the MST with weights greater than λ, we can obtain a certain number of sub-trees (clusters) as listed in Table 2.

Table 2. INMST clustering results

λ	Corresponding to clustering result
$\lambda = d_{23} = 0,2519$	$\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$
$\lambda = d_{34} = 0,2146$	$\{a_2\}, \{a_1, a_3, a_4, a_5, a_6, a_7, a_8\}$
$\lambda = d_{13} = 0,2073$	$\{a_2\}, \{a_4, a_6\} \{a_1, a_3, a_5, a_7, a_8\}$
$\lambda = d_{78} = 0,2018$	$\{a_1\}, \{a_2\}, \{a_4, a_6\} \{a_3, a_5, a_7, a_8\}$
$\lambda = d_{37} = 0,1914$	$\{a_1\}, \{a_2\}, \{a_4, a_6\} \{a_3, a_5, a_7\}, \{a_8\}$
$\lambda = d_{35} = 0,1843$	$\{a_1\}, \{a_2\}, \{a_3, a_5\}, \{a_4, a_6\}, \{a_7\}, \{a_8\}$
$\lambda = d_{46} = 0,1717$	$\{a_1\}, \{a_2\}, \{a_3\}, \{a_5\}, \{a_4, a_6\}, \{a_7\}, \{a_8\}$
$\lambda = 0$	$\{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}, \{a_5\}, \{a_6\}, \{a_7\}, \{a_8\}$

To compare the interval neutrosophic MST (INMST) clustering algorithm and the interval valued intuitionistic fuzzy MST (IVIFMST) clustering algorithm, we extend the example of Zhao [24] by adding the indeterminacy-membership degree to each attributes.

Example 3.1. *The six sets of operational plans are made to complete an operational mission. The basic idea is to cluster these operational plans according to their comprehensive functions. For this purpose, a military committee has been established to evaluate the information on them. However, it is required that the evaluation is made with respect to two following considerations:* x_1= *the effectiveness of operational organization and* x_2= *the effectiveness of operational command. After* *the performance of the six operational plans based to the attributes x_j $(j = 1, 2)$ is evaluated by the military committee, it have been reported the data INS as follows:*

$$
\begin{aligned}
a_1 &= \{(x_1, [0.60, 0.80], [0.25, 0.35], [0.10, 0.20]), (x_2, [0.50, 0.70], [0.15, 0.20], [0.10, 0.30])\}, \\
a_2 &= \{(x_1, [0.30, 0.50], [0.45, 0.60], [0.25, 0.45]), (x_2, [0.70, 0.85], [0.30, 0.45], [0.00, 0.15])\}, \\
a_3 &= \{(x_1, [0.45, 0.65], [0.15, 0.75], [0.15, 0.35]), (x_2, [0.60, 0.80], [0.35, 0.45], [0.05, 0.20])\}, \\
a_4 &= \{(x_1, [0.34, 0.54], [0.05, 0.10], [0.25, 0.45]), (x_2, [0.50, 0.70], [0.25, 0.55], [0.10, 0.30])\}, \\
a_5 &= \{(x_1, [0.40, 0.60], [0.00, 0.50], [0.25, 0.40]), (x_2, [0.65, 0.80], [0.15, 0.25], [0.10, 0.20])\}, \\
a_6 &= \{(x_1, [0.45, 0.65], [0.55, 0.75], [0.15, 0.35]), (x_2, [0.47, 0.67], [0.12, 0.20], [0.05, 0.25])\}.
\end{aligned}
$$

Let the weight vector of the attributes x_j $(j = 1, 2)$ be $\omega = (0.45, 0.55)^T$. We use the INMST clustering algorithm to group these plans $a_j(j = 1, 2, ..., 6)$:

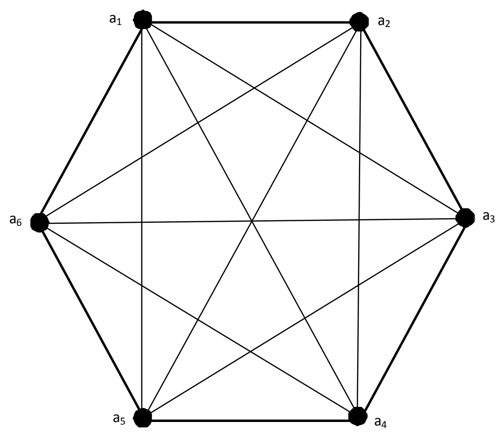
Step 1: Construct the interval neutrosophic distance matrix and interval neutrosophic graph:

(1) Calculate the distance $d_{ij} = d(a_i, a_j)$ by Eq (6) to obtain the interval neutrosophic distance matrix $D = (d_{ij})_{6\times6}$. Then we can obtain the interval neutrosophic distance matrix $D = (d_{ij})_{6\times6}$ as follows:

$$
D = \begin{bmatrix}
0,0000 & 0,2099 & 0,1742 & 0,1906 & 0,1455 & 0,1571 \\
0,2099 & 0,0000 & 0,1230 & 0,2020 & 0,1579 & 0,1580 \\
0,1742 & 0,1230 & 0,0000 & 0,2014 & 0,1244 & 0,1609 \\
0,1906 & 0,2020 & 0,2014 & 0,0000 & 0,1611 & 0,2591 \\
0,1455 & 0,1579 & 0,1244 & 0,1611 & 0,0000 & 0,1843 \\
0,1571 & 0,1580 & 0,1609 & 0,2591 & 0,1843 & 0,0000
\end{bmatrix}
$$

(2) Build the interval neutrosophic graph $G = (E, V)$, which has 6 nodes related to the samples a_j $(j = 1, 2, ..., 6)$ represented by INSs and has 15 edge that every edge between a_i and a_j having the weight d_{ij}, which is an element of the interval neutrosophic distance matrix $D = (d_{ij})_{6\times6}$ and demonstrates the dissimilarity degree between the samples a_i and a_j. Then we can give the interval neutrosophic graph $G = (E, V)$ in

Fig. 6.



FUGURE 6. The interval neutrosophic graph $G = (V, E)$

Step 2: Compute the MST of the interval neutrosophic graph $G = (E, V)$ by Kruskal method [7] or Prim method [19]:

(1) Sort the edges of G in increasing range from the smallest weight to the largest one and choose the edge with the smallest weight, that is the edge e_{23} between a_2 and a_3.

$$\begin{aligned} d_{23} &< d_{35} < d_{15} < d_{16} < d_{25} < d_{26} < d_{36} < d_{45} \\ &< d_{13} < d_{56} < d_{14} < d_{34} < d_{24} < d_{12} < d_{46}. \end{aligned}$$

(2) Choose the edge with the smallest weight from the each rest edges such that do not form a cycle with edge previously added, that is the edge e_{35} between a_3 and a_5.

(3) Repeat the process (2) until seven edges have been chosen. Thus, we obtain the MST of the interval neutrosophic graph $G = (V, E)$ as shown in Fig. 7.

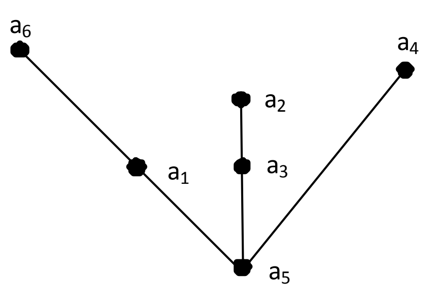


FIGURE 7. The interval neutrosophic graph $G = (V, E)$

Step 3: Cluster the nodes into groups: by choosing a threshold λ and by disconnecting all the edges of the MST with weights greater than λ, we can obtain a certain number of sub-trees (clusters) as listed in Table 3.

Table 3. INMST clustering results

λ	Corresponding to clustering result
$\lambda = d_{45} = 0,1611$	$\{a_1, a_2, a_3, a_4, a_5, a_6\}$
$\lambda = d_{16} = 0,1571$	$\{a_4\}, \{a_1, a_2, a_3, a_5, a_6\}$
$\lambda = d_{15} = 0,1455$	$\{a_4\}, \{a_1, a_2, a_3, a_5\}, \{a_6\}$
$\lambda = d_{35} = 0,1244$	$\{a_1\}, \{a_4\}, \{a_2, a_3, a_5\}, \{a_6\}$
$\lambda = d_{23} = 0,1230$	$\{a_1\}, \{a_2, a_3\}, \{a_4\}, \{a_5\}, \{a_6\}$
$\lambda = 0$	$\{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}, \{a_5\}, \{a_6\}$

Example 3.2. *Suppose that the performance of a group of six operational plans based on the attributes x_j $(j = 1, 2)$, whose weight vector is $\omega = (0.45, 0.55)^T$, is asked the military committee for evaluation according to data expressed by IVIFS, then it can be transformed the interval neutrosophic dataset of Example 3.1 into interval valued intuitionistic fuzzy dataset by removing the indeterminacy-membership degree to each attributes as follows:*

$$\begin{aligned}
a_1 &= \{(x_1, [0.60, 0.80], [0.10, 0.20]), (x_2, [0.50, 0.70], [0.10, 0.30])\}, \\
a_2 &= \{(x_1, [0.30, 0.50], [0.25, 0.45]), (x_2, [0.70, 0.85], [0.00, 0.15])\}, \\
a_3 &= \{(x_1, [0.45, 0.65], [0.15, 0.35]), (x_2, [0.60, 0.80], [0.05, 0.20])\}, \\
a_4 &= \{(x_1, [0.34, 0.54], [0.25, 0.45]), (x_2, [0.50, 0.70], [0.10, 0.30])\}, \\
a_5 &= \{(x_1, [0.40, 0.60], [0.25, 0.40]), (x_2, [0.65, 0.80], [0.10, 0.20])\}, \\
a_6 &= \{(x_1, [0.45, 0.65], [0.15, 0.35]), (x_2, [0.47, 0.67], [0.05, 0.25])\},
\end{aligned}$$

Let the weight vector of the attributes x_j $(j = 1, 2)$ be $\omega = (0.45, 0.55)^T$. We use the IVIFMST clustering algorithm to group these plans $a_j (j = 1, 2, ..., 6)$:

Step 1: Construct the interval valued intuitionistic fuzzy distance matrix and interval valued intuitionistic fuzzy graph:

(1) Calculate the distance $d_{ij} = d(a_i, a_j)$ by (7) to obtain the interval valued intuitionistic fuzzy distance matrix $D = (d_{ij})_{6\times6}$.
Then we can obtain the interval valued intuitionistic fuzzy distance matrix $D = (d_{ij})_{6\times6}$ as follows:

$$D = \begin{bmatrix} 0,0000 & 0,2070 & 0,1111 & 0,1573 & 0,1479 & 0,0938 \\ 0,2070 & 0,0000 & 0,0985 & 0,1158 & 0,0702 & 0,1440 \\ 0,1111 & 0,0985 & 0,0000 & 0,0971 & 0,0515 & 0,0706 \\ 0,1573 & 0,1158 & 0,0971 & 0,0000 & 0,0832 & 0,0768 \\ 0,1479 & 0,0702 & 0,0515 & 0,0832 & 0,0000 & 0,0971 \\ 0,0938 & 0,1440 & 0,0706 & 0,0768 & 0,0971 & 0,0000 \end{bmatrix}$$

(2) Build the interval valued intuitionistic fuzzy graph $G = (E, V)$, which has 6 nodes related to the samples a_j $(j = 1, 2, ..., 6)$ represented by IVIFSs and has 15 edge that every edge between a_i and a_j having the weight d_{ij}, which is an element of the interval valued intuitionistic fuzzy distance matrix $D = (d_{ij})_{6\times 6}$ and demonstrates the dissimilarity degree between the samples a_i and a_j. Then the interval valued intuitionistic fuzzy graph $G = (E, V)$ is identical to Fig. 4.

Step 2: Compute the MST of the interval valued intuitionistic fuzzy graph $G = (E, V)$ by Kruskal method [7] or Prim method [19]:

(1) Sort the edges of G in increasing range from the smallest weight to the largest one and choose the edge with the smallest weight, that is the edge e_{35} between a_3 and a_5.

$$\begin{aligned} d_{35} &< d_{25} < d_{36} < d_{46} < d_{45} < d_{16} < d_{56} = d_{34} \\ &< d_{23} < d_{13} < d_{24} < d_{26} < d_{15} < d_{14} < d_{12}. \end{aligned}$$

(2) Choose the edge with the smallest weight from the each rest edges such that do not form a cycle with edge previously added, that is the edge e_{25} between a_2 and a_5.

(3) Repeat the process (2) until seven edges have been chosen. Thus, we obtain the MST of the interval valued intuitionisitc fuzzy graph $G = (V, E)$ as shown in Fig. 8.

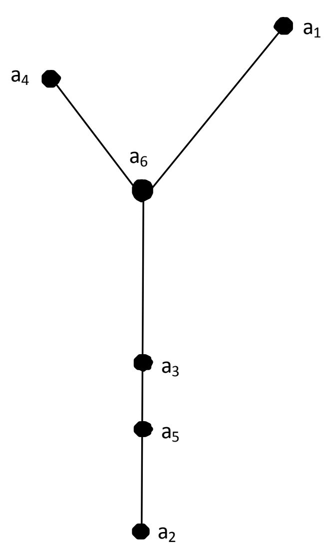


FIGURE 8. The interval valued intuitionistic fuzzy graph $G = (V, E)$

Step 3: Cluster the nodes into groups: by choosing a threshold λ and by disconnecting all the edges of the MST with weights greater than λ, we can obtain a certain number of sub-trees (clusters) as listed in Table 4.

Table 4. IVIFMST clustering results

λ	Corresponding to clustering result
$\lambda = d_{16} = 0,0938$	$\{a_1, a_2, a_3, a_4, a_5, a_6\}$
$\lambda = d_{46} = 0,0768$	$\{a_1\}, \{a_2, a_3, a_4, a_5, a_6\}$
$\lambda = d_{36} = 0,0706$	$\{a_1\}, \{a_4\}, \{a_2, a_3, a_5, a_6\}$
$\lambda = d_{25} = 0,0702$	$\{a_1\}, \{a_4\}, \{a_2, a_3, a_5\}, \{a_6\}$
$\lambda = d_{35} = 0,0515$	$\{a_1\}, \{a_2\}, \{a_4\}, \{a_3, a_5\}, \{a_6\}$
$\lambda = 0$	$\{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}, \{a_5\}, \{a_6\}$

Example 3.3. *Suppose that the performance of a group of six operational plans based on the attributes x_j $(j = 1, 2)$, whose weight vector is $\omega = (0.45, 0.55)^T$, is asked the military committee for evaluation according to data expressed by IVFS, then it can be transformed the interval neutrosophic dataset of Example 3.1 into interval valued fuzzy dataset by removing the indeterminacy-membership and falsity membership degrees to each attributes as follows:*

$$
\begin{aligned}
a_1 &= \{(x_1, [0.60, 0.80]), (x_2, [0.50, 0.70])\}, \\
a_2 &= \{(x_1, [0.30, 0.50]), (x_2, [0.70, 0.85])\}, \\
a_3 &= \{(x_1, [0.45, 0.65]), (x_2, [0.60, 0.80])\}, \\
a_4 &= \{(x_1, [0.34, 0.54]), (x_2, [0.50, 0.70])\}, \\
a_5 &= \{(x_1, [0.40, 0.60]), (x_2, [0.65, 0.80])\}, \\
a_6 &= \{(x_1, [0.45, 0.65]), (x_2, [0.47, 0.67])\},
\end{aligned}
$$

Let the weight vector of the attributes x_j $(j = 1, 2)$ be $\omega = (0.45, 0.55)^T$. We use the IVMST clustering algorithm to group six operational plans $a_j (j = 1, 2, ..., 6)$:

Step 1: Construct the interval valued fuzzy distance matrix and interval valued fuzzy graph:

(1) Calculate the distance $d_{ij} = d(a_i, a_j)$ by (8) to obtain the interval valued fuzzy distance matrix $D = (d_{ij})_{6\times 6}$.

Then we can obtain the interval valued fuzzy distance matrix $D = (d_{ij})_{m\times m}$ as follows:

$$
D = \begin{bmatrix}
0,0000 & 0,2401 & 0,1250 & 0,1744 & 0,1641 & 0,1030 \\
0,2401 & 0,0000 & 0,1164 & 0,1338 & 0,0766 & 0,1832 \\
0,1250 & 0,1164 & 0,0000 & 0,1046 & 0,0425 & 0,0964 \\
0,1744 & 0,1338 & 0,1046 & 0,0000 & 0,1027 & 0,0770 \\
0,1641 & 0,0766 & 0,0425 & 0,1027 & 0,0000 & 0,1211 \\
0,1030 & 0,1832 & 0,0964 & 0,0770 & 0,1211 & 0,0000
\end{bmatrix}
$$

(2) Build the interval valued fuzzy graph $G = (E, V)$, which has 6 nodes related to the samples a_j $(j = 1, 2, ..., 6)$ represented by IVFS and has 15 edge that every edge between a_i and a_j having the weight d_{ij}, which is an element of the interval valued

fuzzy distance matrix $D = (d_{ij})_{6\times 6}$ and demonstrates the dissimilarity degree between the samples a_i and a_j. Then the interval valued fuzzy graph $G = (E, V)$ is identical to Fig. 4.

Step 2: Compute the MST of the interval valued fuzzy graph $G = (E, V)$ by Kruskal method [7] or Prim method [19]:

(1) Sort the edges of G in increasing range from the smallest weight to the largest one and choose the edge with the smallest weight, that is the edge e_{35} between a_3 and a_5.

$$\begin{aligned} d_{35} &< d_{25} < d_{46} < d_{36} < d_{45} < d_{16} < d_{34} < d_{23} \\ &< d_{56} < d_{13} < d_{24} < d_{15} < d_{14} < d_{26} < d_{12}. \end{aligned}$$

(2) Choose the edge with the smallest weight from the each rest edges such that do not form a cycle with edge previously added, that is the edge e_{25} between a_2 and a_5

(3) Repeat the process (2) until seven edges have been chosen. Thus, we obtain the MST of the interval valued fuzzy graph $G = (V, E)$ as shown in Fig. 9.

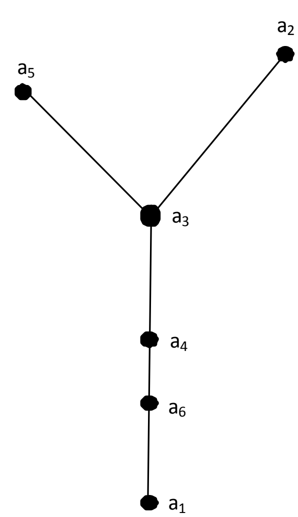


FIGURE 9. The interval valued fuzzy graph $G = (V, E)$

Step 3: Select a threshold λ and disconnect all the edges of the MST with weights greater than λ to obtain a certain number of sub-trees (clusters) automatically, as listed in Table 5.

Table 5. IVFMST clustering results of the eight different countries

λ	Corresponding to clustering result
$\lambda = d_{16} = 0,1030$	$\{a_1, a_2, a_3, a_4, a_5, a_6\}$
$\lambda = d_{36} = 0,0964$	$\{a_1\}, \{a_2, a_3, a_4, a_5, a_6\}$
$\lambda = d_{46} = 0,0770$	$\{a_1\}, \{a_2, a_3, a_5\}, \{a_4, a_6\}$
$\lambda = d_{25} = 0,0766$	$\{a_1\}, \{a_4\}, \{a_2, a_3, a_5\}, \{a_6\}$
$\lambda = d_{35} = 0,0425$	$\{a_1\}, \{a_2\}, \{a_4\}, \{a_3, a_5\}, \{a_6\}$
$\lambda = 0$	$\{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}, \{a_5\}, \{a_6\}$

An INS is a generalization of an IVIFS while an IVIFS is a generalization of an IVFS (also fuzzy set). In Table 6, we can say that the clustering results obtained from clustering algorithms are rather different. The main reason of this situation is form of functions that characterize to set. That is, an IVIFS is only characterized with membership and non-membership function consisting of interval values while an IVFS is only characterized with membership function consisting of interval values. So it can be said that the information loss in IVFSs is more than IVIFSs on the same universe of discourse. However, an INS has basic three functions with independent of each other such that truth-membership, indeterminacy-membership and falsity- membership, it allows less information loss in the environment. Then it is more reasonable to use the INSs for clustering process.

Table 6. Comparison of clustering results

classes	INMST clustering Alg.	IVIFMST clustering Alg.	IVFMST clustering Alg.
1	$\{a_1, a_2, a_3, a_4, a_5, a_6\}$	$\{a_1, a_2, a_3, a_4, a_5, a_6\}$	$\{a_1, a_2, a_3, a_4, a_5, a_6\}$
2	$\{a_4\}$, $\{a_1, a_2, a_3, a_5, a_6\}$	$\{a_1\}$, $\{a_2, a_3, a_4, a_5, a_6\}$	$\{a_1\}$, $\{a_2, a_3, a_4, a_5, a_6\}$
3	$\{a_4\}$, $\{a_1, a_2, a_3, a_5\}, \{a_6\}$	$\{a_1\}, \{a_4\}$, $\{a_2, a_3, a_5, a_6\}$	$\{a_1\}$, $\{a_2, a_3, a_5\}, \{a_4, a_6\}$
4	$\{a_1\}, \{a_4\}$, $\{a_2, a_3, a_5\}, \{a_6\}$	$\{a_1\}, \{a_4\}$, $\{a_2, a_3, a_5\}, \{a_6\}$	$\{a_1\}, \{a_4\}$, $\{a_2, a_3, a_5\}, \{a_6\}$
5	$\{a_1\}, \{a_2, a_3\}$, $\{a_4\}, \{a_5\}, \{a_6\}$	$\{a_1\}, \{a_2\}, \{a_4\}$, $\{a_3, a_5\}, \{a_6\}$	$\{a_1\}, \{a_2\}, \{a_4\}$, $\{a_3, a_5\}, \{a_6\}$
6	$\{a_1\}, \{a_2\}, \{a_3\}$, $\{a_4\}, \{a_5\}, \{a_6\}$	$\{a_1\}, \{a_2\}, \{a_3\}$, $\{a_4\}, \{a_5\}, \{a_6\}$	$\{a_1\}, \{a_2\}, \{a_3\}$, $\{a_4\}, \{a_5\}, \{a_6\}$

4. Conclusion

Interval neutrosophic set theory present a more general platform for modeling uncertainty and recently been studied by many authors in a wide range of applications. However, in the literature, until now there is no any work on clustering of dataset in interval neutrosophic environment despite the large number of clustering algorithm. In this article, we focused our attention on the clustering analysis for interval neutrosophic environment. Based on the graph theory, we proposed an interval neutrosophic MST clustering algorithm which is more general than the existing algorithms. The effectiveness of the algorithm via some numerical examples is presented.

References

[1] C.T. Zahn, Graph-theoretical methods for detecting and describing gestalt clusters. IEEE Transactions on Computers, 20 (1971) 68–86.

[2] D.R. Karger, P.N. Klein and R.E. Tarjan, A Randomized Linear-Time Algorithm to Find Minimum Spanning Trees, Journal of the ACM, Vol. 42 (1995) 321-328.

[3] D. S. Chen, K. X. Li and L. B. Zhao, Fuzzy graph maximal tree clustering method and its application. Operations Research and Management Science, 16 (2007) 69–73.

[4] E.H. Ruspini, A new approach to clustering, Information and Control, 15 (1969) 22–32.

[5] I. L. Wang, Y. C. J. and F. Li, A network flow model for clustering segments and minimizing total maintenance and rehabilitation cost. Computers & Industrial Engineering, 60 (2011) 593–601.

[6] H. Wang, F. Smarandache, Y.Q. Zhang, and R. Sunderraman, Interval neutrosophic sets and logic: Theory and applications in computing, Hexis, Phoenix, AZ, (2005).

[7] J.B. Kruskal, On the shortest spanning subtree of a graph and the traveling salesman problem. Proceedings of the American Mathematical Society, 7 (1956) 48–50.

[8] J. Ye, Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making, Journal of Intelligent and Fuzzy Systems doi: 10.3233/IFS-120724 (2013).

[9] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87-96.

[10] K. Atanassov and G. Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 31 (1989) 343-349.

[11] L. Peihua, A clustering-based color model and integral images for fast object tracking, Signal Processing: Image Communication, 21 (2006) 676-687.

[12] L.A. Zadeh, Fuzzy Sets, Information and Control 8 (1965) 338-353.

[13] Y. Xu, V. Olman and D. Xu, Clustering gene expression data using a graph-Theriotic approach: An application of minimum spanning trees, Bioinformatics, 18 (2002) 536-545.

[14] Z. M. Wang, Yeng Chai Soh, Qing Song, Kang Sim, Adaptive spatial information-theoretic clustering for image segmentation, Pattern Recognition, 42 (2009) 2029-2044.

[15] N. Paivinen, Clustering with a minimum spanning tree of scale-freelike structure, Pattern Recognition Letters, 26 (2005) 921-930.

[16] O. Boruvka, O jistém problému minimálním (About a certain minimal problem), Prácemor. prírodoved. spol. v Brne III, 3 (1926) 37–58, .

[17] O. Gryorash , Y. Zhou ands Z, Jorgenssn, Minimum spanning tree-based clustering algorithms, Proc. IEEE Inn Conf. Tools with Artificial Intelligence, (2006) 73-81.

[18] P. Burillo and H. Bustince, Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets , Fuzzy Sets and Systems, 78 (1996) 305–016.

[19] R.C. Prim, Shortest connection networks and some generalizations. Bell System Technology Journal, 36 (1957) 1389–1401.

[20] R. Sambuc, Fonctions $\phi-$floues. Application l'aide au diagnostic en pathologie thyroidienne, Ph. D. Thesis Univ. Marseille, France, 1975.

[21] Z.S. Xu, J. Chen, An overview of distance and similarity measures of intuitionistic fuzzy sets, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 16 (2008) 529–555

[22] X. Zhang, X. Zeshui, An MST cluster analysis method under hesitant fuzzy environment, Control and Cybernetics, 41 (2012) 645-665.

[23] Y. H. Dong, Y. T. Zhuang, K. Chen and X. Y. Tai , A hierarchical clustering algorithm based on fuzzy graph connectedness. Fuzzy Sets and Systems, 157 (2006) 1760–1774.

[24] Z. Hau, X. Zeshui, L. Shousheng and W. Zhong, Intuitionistic fuzzy MST clustering algorithm, Computer&Industrial Engineering, 62 (2012) 1130-1140.

[25] F. Smarandache, A unifying field in logic neutrosophy: Neutrosophic probability, set and logic, American Research Pres, Rehoboth, (1999).

Department of Mathematics, Faculty of Science, Ataturk University, Erzurum, 25240, Turkey.

E-mail address: mat.ridone@gmail.com (*Corresponding author)