Interval Neutrosophic Multi-Attribute Decision-Making Based on Grey Relational Analysis

Surapati Pramanik[1*], Kalyan Mondal[2]

[1*]Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, PO-Narayanpur, and District: North 24 Parganas, Pin Code: 743126, West Bengal, India. Mobile: +919477035544, Email: sura_pati@yahoo.co.in

[2]Birnagar High School (HS), Birnagar, Ranaghat, and District: Nadia, Pin Code: 741127, West Bengal, India. Mobile: +918926855025, E mail: kalyanmathematic@gmail.com

[1*]Corresponding Author: email address: sura_pati@yahoo.co.in

Abstract

This purpose of this paper is to introduce multi-attribute decision making based on the concept of interval neutrosophic sets. While the concept of neutrosophic sets is a powerful tool to deal with indeterminate and inconsistent data, the interval neutrosophic sets is also a powerful mathematical tool as well as more flexible to deal with incompleteness. The rating of all alternatives is expressed in terms of interval neutrosophic values characterized by interval truth-membership degree, interval indeterminacy-membership degree, and interval falsity-membership degree. Weight of each attribute is partially known to the decision maker. The authors have extended the single valued neutrosophic grey relational analysis method to interval neutrosophic environment and applied it to multi-attribute decision making problem. Information entropy method is used to obtain the unknown attribute weights. Accumulated arithmetic operator is defined to transform interval neutrosophic set into single value neutrosophic set. Neutrosophic grey relational coefficient is determined by using Hamming distance between each alternative to ideal interval neutrosophic estimates reliability solution and the ideal interval neutrosophic estimates un-reliability solution. Then interval neutrosophic relational degree is defined to determine the ranking order of all alternatives. Finally, an example is provided to illustrate the applicability and effectiveness of the proposed approach.

Keywords: Neutrosophic set, Single-valued neutrosophic set, Interval neutrosophic set, Grey relational analysis, Information Entropy, Multi-attribute decision making.

1. Introduction:

The neutrosophic sets (NS) was introduced by one of the greatest mathematician and philosopher Smarandache [1, 2] in 1995. The root of neutrosophic set is the neutrosophy, a new branch of philosophy. Neutrosophy penetrates all branches of sciences, social sciences and humanities. The thrust of the study of neutrosophy creates the new concept of sets known as neutrosophic sets. It has caught the great attention of the researchers for its capability of handling uncertainty and incomplete information. Neutrosophic set [3, 4] generalizes the classical set initiated by Smith [5] in 1874 and popularized by German mathematician Cantor [6] in 1883, fuzzy set introduced by Zadeh [7], interval valued fuzzy sets studied independently by several researchers namely, Zadeh [8], Grattan-Guiness [9], Jahn [10], Sambuc [11], L-fuzzy sets studied by Goguen [12], intuitionistic fuzzy set proposed by Atanassov [13], interval valued intuitionistic fuzzy sets studied by Atanassov and Gargov [14], vague sets proposed by Gau, and Buehrer [15], grey sets proposed by Deng [16], paraconsistent set proposed by Brady [17], faillibilist set [2], paradoxist set [2], pseudoparadoxist set [2], tautological set [2] based on the philosophical point of view. To use neutrosophic sets in practical fields such as real scientific and engineering applications, Wang et al.[18] extended the concept of neutrosophic set to single valued neutrosophic sets (SVNSs) and studied the set theoretic operators and various properties of SVNSs. Neutrosophic sets and its various extensions have been studied and applied in different fields such as medical diagnosis [19, 20, 21, 22, 23], decision making problems [24, 25, 26, 27,28, 29, 30, 31, 32, 33], decision making in hybrid system [34, 35], social problems [36, 37], educational problem [38, 39], conflict resolution [40, 41], etc.

However, it is recognized by the researchers that in many real world problems, the decision information may be suitably presented by interval form instead of real numbers. In order to deal with this type of situations the concept of interval neutrosophic set (INS) [42] is originated by Wang et al. INS is characterized by a membership function, non-membership function and an indeterminacy function, whose values are interval forms.

Broumi and Smarandache [43] studied correlation coefficient of interval neutrosophic sets and applied it in medical diagnosis. Broumi and Smarandache [44] studied cosine similarity measure in interval neutrosophic environment. Zhang et al. [45] studied interval neutrosophic sets and its application in multi attribute decision making. Ye [46] studied similarity measures between interval neutrosophic sets and their applications in multi criteria decision making. In this paper, we extend single valued neutrosophic multi attribute decision making based on grey relational analysis to interval neutrosophic environment.

Rest of the paper is organized in the following way. Section 2 presents preliminaries of neutrosophic sets and interval neutrosophic sets. Section 3 is devoted to present grey relational analysis method for multi attribute decision-making in interval neutrosophic environment. Section 4 presents a numerical example of the proposed method. Finally section 5 presents concluding remarks.

2 Preliminaries of neutrosophic sets

2.1 Definitions on neutrosophic Set

Definition 2.1.1: Let E be a space of points (objects) with generic element in E denoted by x. Then a neutrosophic set P in E is characterized by a truth membership function T_P, an indeterminacy membership function I_P and a falsity membership function F_P. The functions T_P, I_P and F_P are real standard or non-standard subsets of $]^-0,1^+[$ that is T_P: $E \to]^-0,1^+[$; I_P: $E \to]^-0,1^+[$; F_P: $E \to]^-0,1^+[$.

It should be noted that there is no restriction on the sum of $T_P(x), I_P(x), F_P(x)$ i.e.

$^-0 \le \sup T_P(x) + \sup I_P(x) + \sup F_P(x) \le 3^+$

Definition 2.1.2 (complement): The complement of a neutrosophic set P is denoted by P^c and is defined as follows:

$T_{P^c}(x) = \{1^+\} - T_P(x)$; $I_{P^c}(x) = \{1^+\} - I_P(x)$, and $F_{P^c}(x) = \{1^+\} - F_P(x)$

Definition 2.1.3 (Containment): A neutrosophic set P is contained in the other neutrosophic set Q, $P \subseteq Q$ if and only if the following result holds.

$\inf T_P(x) \le \inf T_Q(x)$, $\sup T_P(x) \le \sup T_Q(x)$

$\inf I_P(x) \ge \inf I_Q(x)$, $\sup I_P(x) \ge \sup I_Q(x)$

$\inf F_P(x) \ge \inf F_Q(x)$, $\sup F_P(x) \ge \sup F_Q(x)$

for all x in E.

Definition 2.1.4 (Single-valued neutrosophic set):

Let E be a universal space of points (objects) with a generic element of E denoted by x.

A single valued neutrosophic set [18] S is characterized by a truth membership function $T_S(x)$, a falsity membership function $F_S(x)$ and indeterminacy function $I_S(x)$ with $T_S(x)$, $F_S(x)$, $I_S(x) \in [0,1]$ for all x in E.

When E is continuous, a SNVS S can be written as follows:

$S = \int_x \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in E$

and when E is discrete, a SVNS S can be written as follows:

$S = \sum \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in E$

It should be observed that for a SVNS S,

$0 \le \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \le 3, \forall x \in E$

Definition 2.1.5: The complement of a single valued neutrosophic set S is denoted by S^c and is defined by

$T_S^{\,c}(x) = F_S(x)$; $I_S^{\,c}(x) = 1 - I_S(x)$; $F_S^{\,c}(x) = T_S(x)$

Definition 2.1.6: A SVNS S_P is contained in the other SVNS S_Q, denoted as $S_P \subseteq S_Q$ iff, $T_{S_P}(x) \le T_{S_Q}(x)$; $I_{S_P}(x) \ge I_{S_Q}(x)$; $F_{S_P}(x) \ge F_{S_Q}(x)$, $\forall x \in E$.

Definition 2.1.7: Two single valued neutrosophic sets S_P and S_Q are equal, i.e. $S_P = S_Q$, iff, $S_P \subseteq S_Q$ and $S_P \supseteq S_Q$

Definition 2.1.8: (Union) The union of two SVNSs S_P and S_Q is a SVNS S_R, written as $S_R = S_P \cup S_Q$.

Its truth membership, indeterminacy-membership and falsity membership functions are related to S_P and S_Q by the relations as follows:

$T_{S_R}(x) = \max\left(T_{S_P}(x), T_{S_Q}(x)\right)$;

$I_{S_R}(x) = \max\left(I_{S_P}(x), I_{S_Q}(x)\right)$;

$F_{S_R}(x) = \min\left(F_{S_P}(x), F_{S_Q}(x)\right)$ for all x in E

Definition 2.1.9 (Intersection): The intersection of two SVNSs P and Q is a SVNS V, written as $V = P \cap Q$. Its truth membership, indeterminacy membership and falsity membership functions are related to P an Q by the relations as follows:

$T_{S_V}(x) = \min\left(T_{S_P}(x), T_{S_Q}(x)\right)$;

$I_{S_V}(x) = \max\left(I_{S_P}(x), I_{S_Q}(x)\right)$;

$F_{S_V}(x) = \max\left(F_{S_P}(x), F_{S_Q}(x)\right), \forall x \in E$

Distance between two neutrosophic sets.

The general SVNS can be presented in the following form:

$S = \{(x/(T_S(x), I_S(x), F_S(x))) : x \in E\}$

Finite SVNSs can be represented as follows:

$$S = \{(x_1/(T_S(x_1), I_S(x_1), F_S(x_1))), \cdots, (x_m/(T_S(x_m), I_S(x_m), F_S(x_m)))\}, \forall x \in E \tag{1}$$

Definition 2.1.10: Let

$$S_P = \{(x_1/(T_{S_P}(x_1), I_{S_P}(x_1), F_{S_P}(x_1))), \cdots, (x_n/(T_{S_P}(x_n), I_{S_P}(x_n), F_{S_P}(x_n)))\} \tag{2}$$

$$S_Q = \{(x_1/(T_{S_Q}(x_1), I_{S_Q}(x_1), F_{S_Q}(x_1))), \cdots, (x_n/(T_{S_Q}(x_n), I_{S_Q}(x_n), F_{S_Q}(x_n)))\} \tag{3}$$

be two single-valued neutrosophic sets, then the Hamming distance between two SNVS P and Q is defined as follows:

$$d_S(S_P, S_Q) = \sum_{i=1}^{n} \left\langle \left| T_{S_P}(x) - T_{S_Q}(x) \right| + \left| I_{S_P}(x) - I_{S_Q}(x) \right| + \left| F_{S_P}(x) - F_{S_Q}(x) \right| \right\rangle \tag{4}$$

and normalized Hamming distance between two SNVS S_P and S_Q is defined as follows:

$$^{N}d_S(S_P, S_Q) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \left| T_{S_P}(x) - T_{S_Q}(x) \right| + \left| I_{S_P}(x) - I_{S_Q}(x) \right| + \left| F_{S_P}(x) - F_{S_Q}(x) \right| \right\rangle \tag{5}$$

with the following properties

1. $0 \le d_S(S_P, S_Q) \le 3n$ (6)
2. $0 \le {}^{N}d_S(S_P, S_Q) \le 1$ (7)

Definition 2.1.11:

Based on the concept of the neutrosophic cube [48], the ideal interval neutrosophic estimates reliability solution (IINERS)

$R_S^+ = \left[r_{S_1}^+, r_{S_2}^+, \cdots, r_{S_n}^+ \right]$ is defined as a solution in which every component $r_{S_j}^+ = \left\langle T_j^+, I_j^+, F_j^+ \right\rangle$ is characterized by $T_j^+ = \max_i \{T_{ij}\}$, $I_j^+ = \min_i \{I_{ij}\}$ *and* $F_j^+ = \min_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_S = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{m \times n}$ (see equation 8)for i = 1, 2, …, n and j = 1, 2, …, m.

Definition 2.1.12:

Based on the concept of the neutrosophic cube [48], maximum un-reliability happens when the indeterminacy membership grade and the degree of falsity membership reach maximum simultaneously. So, the ideal interval neutrosophic estimates un-reliability solution (IINEURS) $R_S^- = \left[r_{S_1}^-, r_{S_2}^-, \cdots, r_{S_n}^- \right]$is a solution in which every component $r_{S_j}^- = \left\langle T_j^-, I_j^-, F_j^- \right\rangle$ is characterized as follows: $T_j^- = \max_i \{T_{ij}\}$, $I_j^- = \min_i \{I_{ij}\}$ *and* $F_j^- = \min_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_S = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{m \times n}$ (see equation 8) for i = 1, 2, …, n and j = 1, 2, …, m.

2.2 Interval Neutrosophic Sets

Definition 2.2 [42]

Let X be a space of points (objects) with generic elements in X denoted by x. An interval neutrosophic set (INS) M in X is characterized by truth-membership function $T_M(x)$, indeterminacy-membership $I_M(x)$, function and falsity-membership function $F_M(x)$. For each point x in X, we have, $T_M(x), I_M(x), F_M(x) \in [0, 1]$.

For two IVNS,

$M_{INS} = \{< x, \left(\left[T_M^L(x), T_M^U(x) \right], \left[I_M^L(x), I_M^U(x) \right], \left[F_M^L(x), F_M^U(x) \right] \right) > \mid x \in X\}$ and

$N_{INS} = \{< x, \left(\left[T_N^L(x), T_N^U(x) \right], \left[I_N^L(x), I_N^U(x) \right], \left[F_N^L(x), F_N^U(x) \right] \right) > \mid x \in X\}$ the two relations are defined as follows:

(1) $M_{INS} \subseteq N_{INS}$ if and only if $T_M^L \le T_N^L, T_M^U \le T_N^U$; $I_M^L \le I_N^L$, $F_M^L \le F_N^L$; $F_M^L \le F_N^L$, $F_M^L \le F_N^L$

(2) $M_{INS} = N_{INS}$ if and only if $T_M^L = T_N^L, T_M^U = T_N^U$; $I_M^L = I_N^L$, $F_M^L = F_N^L$; $F_M^L = F_N^L$, $F_M^L = F_N^L$

for all $x \in X$

3. Grey relational analysis method for multi attributes decision-making in interval neutrosophic environment.

Consider a multi-attribute decision making problem with m alternatives and n attributes. Let $A_1, A_2, ..., A_m$ and $C_1, C_2, ..., C_n$ denote the alternatives and attributes respectively.

The rating describes the performance of alternative A_i against attribute C_j. Weight vector $W = \{w_1, w_2,...,w_n\}$ is assigned to the attributes. The weight w_j (j = 1, 2, ..., n) reflects the relative importance of attributes C_j (j = 1, 2, ..., m) to the decision makers. The values associated with the alternatives for MADM problems presented in the following table.

Table1: Interval neutrosophic decision matrix

$$D_s = \langle d_{ij} \rangle_{m\times n} =$$

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \langle d_{11} \rangle & \langle d_{12} \rangle & \cdots & \langle d_{1n} \rangle \\ A_2 & \langle d_{12} \rangle & \langle d_{22} \rangle & \cdots & \langle d_{2n} \rangle \\ . & \cdots & \cdots & \cdots & \cdots \\ . & \cdots & \cdots & \cdots & \cdots \\ A_m & \langle d_{m1} \rangle & \langle d_{m2} \rangle & \cdots & \langle d_{mn} \rangle \end{array} \quad (8)$$

where $\langle d_{ij} \rangle$ is interval neutrosophic number according to the i-th alternative and the j-th attribute.

Grey relational analysis (GRA) is one of the adoptive methods for MADM. The steps of GRA under interval neutrosophic environments are described below.

Step1: Determination the criteria

There are many attributes in decision making problems. Some of them are important and others may be less important. So it is necessary to select the proper criteria for decision making situations. The most important criteria may be fixed with help of experts' opinions.

Step 2: Data pre-processing and construction of the decision matrix with interval neutrosophic form

A multiple attribute decision making problem having m alternatives and n attributes, the general form of decision matrix can be presented as shown in Table-1. It may be mentioned here that the original GRA method can deal mainly with quantitative attributes. There exists some complexity in the case of qualitative attributes. In the case of a qualitative attribute (quantitative value is not available), an assessment value is taken as interval neutrosophic environment.

For multiple attribute decision making problem, the rating of alternative A_i (i = 1, 2,…m) with respect to attribute C_j (j = 1, 2,…n) is assumed as interval neutrosophic sets. It can be represented with the following forms:

$$A_i=\left[\begin{array}{l} {}^{C_1}\!\!\diagup\!\left\langle N_1\left(\left[T_1^L,T_1^U\right],\left[I_1^L,I_1^U\right],\left[F_1^L,F_1^U\right]\right)\right\rangle, {}^{C_2}\!\!\diagup\!\left\langle N_2\left(\left[T_2^L,T_2^U\right],\left[I_2^L,I_2^U\right],\left[F_2^L,F_2^U\right]\right)\right\rangle,\cdots, \\ {}^{C_n}\!\!\diagup\!\left\langle N_n\left(\left[T_n^L,T_n^U\right],\left[I_n^L,I_n^U\right],\left[F_n^L,F_n^U\right]\right)\right\rangle : C_j\in C \end{array}\right]$$

$$=\left[{}^{C_j}\!\!\diagup\!\left\langle N_j\left(\left[T_j^L,T_j^U\right],\left[I_j^L,I_j^U\right],\left[F_j^L,F_j^U\right]\right)\right\rangle : C_j\in C\right] for \quad j=1,2,\cdots,n \tag{9}$$

Here $\left\langle N_j\left(\left[T_j^L,T_j^U\right],\left[I_j^L,I_j^U\right],\left[F_j^L,F_j^U\right]\right)\right\rangle$, (j = 1, 2, ..., n) is the interval neutrosophic set with the degrees of interval truth membership $\left[T_j^L,T_j^U\right]$, the degrees of interval indeterminacy membership $\left[I_j^L,I_j^U\right]$ and the degrees of interval falsity membership $\left[F_j^L,F_j^U\right]$ of the alternative A_i satisfying the attribute C_j

The interval neutrosophic decision matrix can be represented in the following form (see the Table 2):

$$d_N=\left\langle\left[T_{ij}^L,T_{ij}^U\right],\left[I_{ij}^L,I_{ij}^U\right],\left[F_{ij}^L,F_{ij}^U\right]\right\rangle_{m\times n}=$$

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \left\langle\begin{array}{c}\left[T_{11}^L,T_{11}^U\right],\\ \left[I_{11}^L,I_{11}^U\right],\\ \left[F_{11}^L,F_{11}^U\right]\end{array}\right\rangle & \left\langle\begin{array}{c}\left[T_{12}^L,T_{12}^U\right],\\ \left[I_{12}^L,I_{12}^U\right],\\ \left[F_{12}^L,F_{12}^U\right]\end{array}\right\rangle & \cdots & \left\langle\begin{array}{c}\left[T_{1n}^L,T_{1n}^U\right],\\ \left[I_{1n}^L,I_{1n}^U\right],\\ \left[F_{1n}^L,F_{1n}^U\right]\end{array}\right\rangle \\ A_2 & \left\langle\begin{array}{c}\left[T_{21}^L,T_{21}^U\right],\\ \left[I_{21}^L,I_{21}^U\right],\\ \left[F_{21}^L,F_{21}^U\right]\end{array}\right\rangle & \left\langle\begin{array}{c}\left[T_{22}^L,T_{22}^U\right],\\ \left[I_{22}^L,I_{22}^U\right],\\ \left[F_{22}^L,F_{22}^U\right]\end{array}\right\rangle & \cdots & \left\langle\begin{array}{c}\left[T_{2n}^L,T_{2n}^U\right],\\ \left[I_{2n}^L,I_{2n}^U\right],\\ \left[F_{2n}^L,F_{2n}^U\right]\end{array}\right\rangle \\ . & \cdots & \cdots & \cdots & \cdots \\ . & \cdots & \cdots & \cdots & \cdots \\ A_m & \left\langle\begin{array}{c}\left[T_{m1}^L,T_{m1}^U\right],\\ \left[I_{m1}^L,I_{m1}^U\right],\\ \left[F_{m1}^L,F_{m1}^U\right]\end{array}\right\rangle & \left\langle\begin{array}{c}\left[T_{m2}^L,T_{m2}^U\right],\\ \left[I_{m2}^L,I_{m2}^U\right],\\ \left[F_{m2}^L,F_{m2}^U\right]\end{array}\right\rangle & \cdots & \left\langle\begin{array}{c}\left[T_{mn}^L,T_{mn}^U\right],\\ \left[I_{mn}^L,I_{mn}^U\right],\\ \left[F_{mn}^L,F_{mn}^U\right]\end{array}\right\rangle \end{array} \tag{10}$$

Step 3: Determination of the accumulated arithmetic operator (AAO)

Let us consider an interval neutrosophic set as

$$\left\langle N_j\left(\left[T_j^L,T_j^U\right],\left[I_j^L,I_j^U\right],\left[F_j^L,F_j^U\right]\right)\right\rangle.$$

We transform the interval neutrosophic number to SVNSs by the following operator. The accumulated arithmetic operator (AAO) is defined in the following form.

$$N_{ij}\left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle =$$

$$N_{ij}\left\langle \left[\frac{T_{ij}^{L}+T_{ij}^{U}}{2}\right], \left[\frac{I_{ij}^{L}+I_{ij}^{U}}{2}\right], \left[\frac{F_{ij}^{L}+F_{ij}^{U}}{2}\right]\right\rangle \quad (11)$$

The decision matrix is transformed in the form of SVNSs as follows:

$$d_S = \left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle_{m\times n} =$$

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{12}, I_{12}, F_{12}\rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n}\rangle \\ A_2 & \langle T_{21}, I_{21}, F_{21}\rangle & \langle T_{22}, I_{22}, F_{22}\rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n}\rangle \\ . & \cdots & \cdots & \cdots & \cdots \\ . & \cdots & \cdots & \cdots & \cdots \\ A_m & \langle T_{m1}, I_{m1}, F_{m1}\rangle & \langle T_{m2}, I_{m2}, F_{m2}\rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn}\rangle \end{array} \quad (12)$$

Step 4: Determination of the weights of criteria

During decision-making process, decision makers may often encounter unknown or partial attribute weights. In many cases, the importance of attributes to the decision maker is not equal. So, it is necessary to determine attribute weight for decision making.

4.1 Method of Entropy:

Entropy has an important usefulness for measuring uncertain information. Majumder and Samanta [48] developed some similarity and entropy measures for SVNSs. The entropy measure can be used to determine the attributes weights when these are unequal and completely unknown to decision maker.

Now, using AAO operator we transform all interval neutrosophic numbers to single valued neutrosophic numbers. In this paper we use an entropy method for determining attribute weight. For entropy measure of an INS, we consider the following as:

$$T_{S_P}(x_i) = \left[\frac{T_{ij}^{L}+T_{ij}^{U}}{2}\right], \; I_{S_P}(x_i) = \left[\frac{I_{ij}^{L}+I_{ij}^{U}}{2}\right], \; F_{S_P}(x_i) = \left[\frac{F_{ij}^{L}+F_{ij}^{U}}{2}\right]$$

we write, $S_P = \left\langle T_{S_P}(x_i), I_{S_P}(x_i), F_{S_P}(x_i)\right\rangle$. Then entropy value is defined as follows:

$$E_i(S_P) = 1 - \frac{1}{n}\sum_{i=1}^{m}(T_{S_P}(x_i) + F_{S_P}(x_i))\left|I_{S_P}(x_i) - I^{c}{}_{S_P}(x_i)\right| \quad (13)$$

which has the following properties:

1. $E_i(S_P) = 0 \Rightarrow S_P$ *is a crisp set and* $I_{S_P}(x_i) = 0\ \forall x \in E.$

2. $E_i(S_P) = 1 \Rightarrow \left\langle T_{S_P}(x_1), I_{S_P}(x_1), F_{S_P}(x_1)\right\rangle = \left\langle 0.5, 0.5, 0.5\right\rangle \forall x \in E.$

3. $E_i(S_P) \ge E_i(S_P) \Rightarrow$

$(T_{S_P}(x_1) + F_{S_P}(x_1) \le (T_{S_P}(x_1) + F_{S_P}(x_1)\ and \left|I_{S_P}(x_1) - I^c{}_{S_P}(x_1)\right| \le \left|I_{S_P}(x_1) - I^c{}_{S_P}(x_1)\right|$

4. $E_i(S_P) = E_i(S_{P^c})\ \forall x \in E.$

In order to obtain the entropy value E_j of the j-th attribute C_j (j = 1, 2,…, n), equation (13) can be written as:

$$E_j = 1 - \frac{1}{n}\sum_{i=1}^{m}(T_{ij}(x_i) + F_{ij}(x_i))\left|I_{ij}(x_i) - I_{ij}^C(x_i)\right|$$

For i = 1, 2, …, n; j = 1, 2, …, m (14)

It is observed that $E_j \in [0,1]$. Due to Hwang and Yoon [49], the entropy weight of the j-th attribute C_j is presented as follows:

$$W_j = \frac{1-E_j}{\sum_{j=1}^{n}\left(1-E_j\right)} \tag{15}$$

We have weight vector W = $(w_1, w_2,…,w_n)^T$ of attributes C_j (j = 1, 2, …, n) with $w_j \ge 0$ and $\sum_{l=1}^{n} w_j = 1$.

Step 5: Determination of the ideal interval neutrosophic estimates reliability solution (IINERS) and the ideal interval neutrosophic estimates un-reliability solution (IINEURS) for interval neutrosophic decision matrix

For a interval neutrosophic decision making matrix $D_S = \langle T_{ij}, I_{ij}, F_{ij}\rangle_{m\times n}$, T_{ij}, I_{ij}, F_{ij} are the degrees of membership, degree of indeterminacy and degree of non membership of the alternative A_i satisfying the attribute C_j. The interval neutrosophic estimate reliability solution (see definition 2.1.11, and 2.1.12) can be determined from the concept of SVNS cube.

Step 6: Calculation of the interval neutrosophic grey relational coefficient of each alternative from IINERS and IINEURS

Grey relational coefficient of each alternative from IINERS is:

$$G_{ij}^{+} = \frac{\min\limits_i \min\limits_j \Delta_{ij}^{+} + \rho \max\limits_i \max\limits_j \Delta_{ij}^{+}}{\Delta_{ij}^{+} + \rho \max\limits_i \max\limits_j \Delta_{ij}^{+}}$$, where

$\Delta_{ij}^{+} = d\left(q_{S_j}^{+}, q_{S_{ij}}\right)$, i = 1, 2, …,m and j = 1, 2, ….,n (16)

Grey relational coefficient of each alternative from IINEURS is:

$$G_{ij}^{-} = \frac{\min\limits_i \min\limits_j \Delta_{ij}^{-} + \rho \max\limits_i \max\limits_j \Delta_{ij}^{-}}{\Delta_{ij}^{-} + \rho \max\limits_i \max\limits_j \Delta_{ij}^{-}}$$, where

$$\Delta_{ij}^{-} = d\left(q_{S_{ij}}, q_{S_{ij}}^{-}\right),\ i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n \tag{17}$$

$\rho \in [0,1]$ is the distinguishable coefficient or the identification coefficient. It is used to adjust the range of the comparison environment, and to control level of differences of the relation coefficients. When $\rho = 1$, the comparison environment is unchanged. When $\rho = 0$, the comparison environment disappears. Smaller value of distinguishing coefficient will reflect the large range of grey relational coefficient. Generally, $\rho = 0.5$ is fixed for decision making.

Step 7: Calculation of the interval neutrosophic grey relational coefficient

Calculate the degree of interval neutrosophic grey relational coefficient of each alternative from IINERS and IINEURS using the following two equations respectively:

$$G_i^{+} = \sum_{j=1}^{n} w_j G_{ij}^{+} \quad \text{for } i = 1, 2, \ldots, m \tag{18}$$

$$G_i^{-} = \sum_{j=1}^{n} w_j G_{ij}^{-} \quad \text{for } i = 1, 2, \ldots, m \tag{19}$$

Step 8: Calculation of the interval neutrosophic relative relational degree

Calculate the interval neutrosophic relative relational degree of each alternative from ITFPIS (indeterminacy truthfulness falsity positive ideal solution) with the help of following two equations:

$$R_i = \frac{G_i^{+}}{G_i^{-} + G_i^{+}}, \text{ for } i = 1, 2, \ldots, m \tag{20}$$

Step 9: Rank the alternatives

The ranking order of alternatives can be determined based on the interval relative relational degree. The highest value of R_i indicates the most desirable alternative.

4 Numerical example

In this section, interval neutrosophic MADM is considered to demonstrate the application and the effectiveness of the proposed approach. Let us consider a decision-making problem (adopted from Mondal and Pramanik [27]) taking all data in interval neutrosophic form is stated as follows. Suppose there is a conscious guardian, who wants to admit his/her child to a suitable school for proper basic education. There is a panel with three possible alternatives (schools) to admit his/her child: (1) A_1 is a Christ missionary school; (2) A_2 is a Basic English medium school; (3) A_3 is a Bengali medium kinder garden. The proposed decision making method can be arranged in the following steps.

Step 1: Determination the most important criteria

The guardian must take a decision based on the following four criteria: (1) C_1 is the distance and transport; (2) C_2 is the cost; (3) C_3 is stuff and curriculum; and (4) C_4 is the administration and other facilities.

Step 2: Data pre-processing and Construction of the decision matrix with interval neutrosophic form

We obtain the following interval neutrosophic decision matrix based on the experts' assessment:

Table3. Decision matrix with interval neutrosophic number

$d_S = \left\langle [T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U] \right\rangle_{3\times 4} =$

$$\begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline A_1 & \left\langle \begin{array}{l}[0.6,0.8],\\ [0.2,0.4],\\ [0.3,0.5]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.6,0.8],\\ [0.2,0.4],\\ [0.1,0.3]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.6,0.8],\\ [0.1,0.3],\\ [0.3,0.5]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.7,0.9],\\ [0.2,0.4],\\ [0.2,0.4]\end{array}\right\rangle \\ A_2 & \left\langle \begin{array}{l}[0.5,0.7],\\ [0.3,0.5],\\ [0.1,0.3]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.7,0.9],\\ [0.4,0.6],\\ [0.3,0.5]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.6,0.8],\\ [0.2,0.4],\\ [0.1,0.3]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.7,0.9],\\ [0.3,0.5],\\ [0.4,0.6]\end{array}\right\rangle \\ A_3 & \left\langle \begin{array}{l}[0.5,0.7],\\ [0.2,0.4],\\ [0.4,0.6]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.6,0.8],\\ [0.5,0.7],\\ [0.1,0.3]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.6,0.8],\\ [0.4,0.6],\\ [0.4,0.6]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.7,0.9],\\ [0.3,0.5],\\ [0.3,0.5]\end{array}\right\rangle \end{array} \tag{21}$$

Step 3: Determination of the accumulated arithmetic operator (AAO)

Using accumulated arithmetic operator (AAO) from equation (11) we have the decision matrix in SVNS form is presented as follows:

$$\begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline A_1 & \langle 0.7,0.3,0.4\rangle & \langle 0.7,0.3,0.2\rangle & \langle 0.7,0.2,0.4\rangle & \langle 0.8,0.3,0.3\rangle \\ A_2 & \langle 0.6,0.4,0.2\rangle & \langle 0.8,0.5,0.4\rangle & \langle 0.7,0.3,0.2\rangle & \langle 0.8,0.4,0.5\rangle \\ A_3 & \langle 0.6,0.3,0.5\rangle & \langle 0.7,0.6,0.2\rangle & \langle 0.7,0.5,0.5\rangle & \langle 0.8,0.4,0.4\rangle \end{array} \tag{22}$$

Step 4: Determination of the weights of attribute

Entropy value E_j of the j-th (j = 1, 2, 3, 4) attributes can be determined from the decision matrix d_S (12) and equation (14) as: E_1= 0.6533, E_2 = 0.8200, E_3 = 0.6600, E_4 = 0.6867.

Then the corresponding entropy weights w_1, w_2, w_3, w_4 of all attributes according to equation (15) are obtained by w_1 = 0.2938, w_2 = 0.1568, w_3 = 0.2836, w_4 = 0.2658 such that $\sum_{j=1}^{n} w_j = 1$

Step 5: Determination of the ideal interval neutrosophic estimates reliability solution (IINERS) and the ideal interval neutrosophic estimates un-reliability solution (IINEURS)

The ideal interval neutrosophic estimates reliability solution (IINERS) is presented as follows.

$$Q_S^+ = \left\langle q_{S_1}^+,\ q_{S_2}^+,\ q_{S_3}^+, q_{S_4}^+ \right\rangle =$$
$$\begin{bmatrix} \left\langle \max_i\{T_{i1}\}, \min_i\{I_{i1}\}, \min_i\{F_{i1}\}\right\rangle, \left\langle \max_i\{T_{i2}\}, \min_i\{I_{i2}\}, \min_i\{F_{i2}\}\right\rangle, \\ \left\langle \max_i\{T_{i3}\}, \min_i\{I_{i3}\}, \min_i\{F_{i3}\}\right\rangle, \left\langle \max_i\{T_{i4}\}, \min_i\{I_{i4}\}, \min_i\{F_{i4}\}\right\rangle \end{bmatrix}$$

$$= \left[\langle 0.7, 0.3, 0.2\rangle, \langle 0.8, 0.3, 0.2\rangle, \langle 0.7, 0.2, 0.2\rangle, \langle 0.8, 0.3, 0.3\rangle\right]$$

The ideal interval neutrosophic estimates un-reliability solution (IINEURS) is presented as follows.

$$Q_S^- = \left\langle q_{S_1}^-,\ q_{S_2}^-,\ q_{S_3}^-, q_{S4}^- \right\rangle =$$
$$\begin{bmatrix} \left\langle \min_i\{T_{i1}\}, \max_i\{I_{i1}\}, \max_i\{F_{i1}\}\right\rangle, \left\langle \min_i\{T_{i2}\}, \max_i\{I_{i2}\}, \max_i\{F_{i2}\}\right\rangle, \\ \left\langle \min_i\{T_{i3}\}, \max_i\{I_{i3}\}, \max_i\{F_{i3}\}\right\rangle, \left\langle \min_i\{T_{i4}\}, \max_i\{I_{i4}\}, \max_i\{F_{i4}\}\right\rangle \end{bmatrix}$$

$$= \left[\langle 0.6, 0.4, 0.5\rangle, \langle 0.7, 0.6, 0.4\rangle, \langle 0.7, 0.5, 0.5\rangle, \langle 0.8, 0.4, 0.5\rangle\right]$$

Step 6: Calculation of the interval neutrosophic grey relational coefficient of each alternative from IINERS and IINEURS

By using Equation (16) the interval neutrosophic grey relational coefficient of each alternative from IINERS can be obtained as the following matrix.

$$\left[G_{ij}^+\right]_{3\times 4} = \begin{bmatrix} 0.8000 & 1.0000 & 0.8000 & 0.6667 \\ 0.8000 & 0.5714 & 1.0000 & 0.4444 \\ 0.5714 & 0.5714 & 0.4444 & 0.5000 \end{bmatrix} \tag{23}$$

Similarly, from Equation (17) the interval neutrosophic grey relational coefficient of each alternative from IINEURS is presented as the following matrix.

$$\left[G_{ij}^-\right]_{3\times 4} = \begin{bmatrix} 0.4545 & 0.3333 & 0.3846 & 0.4545 \\ 0.4545 & 0.5556 & 0.3333 & 1.0000 \\ 0.7143 & 0.5556 & 1.0000 & 0.7143 \end{bmatrix} \tag{24}$$

Step 7: Determine the degree of interval neutrosophic grey relational co-efficient of each alternative from IINERS and IINEURS. The required interval neutrosophic grey relational co-efficient corresponding to IINERS is obtained by using equations (18) as follows:

$$G_1^+ = 0.7961,\ G_2^+ = 0.7264,\ G_3^+ = 0.5164 \tag{25}$$

and corresponding to IINEURS is obtained with the help of equation (19) as follows:

$$G_1^- = 0.4156,\ G_2^- = 0.5810,\ G_3^- = 0.7704 \tag{26}$$

Step 8: Thus interval neutrosophic relative degree of each alternative from IINERS can be obtained with the help of equation (20) as follows:

$R_1 = 0.6570$, $R_2 = 0.5556$, $R_3 = 0.4013$ (27)

Step 9: The ranking order of all alternatives can be determined according to the decreasing order of the value of interval neutrosophic relational degree i.e. $R_1 > R_2 > R_3$. It is seen that the highest value of interval neutrosophic relational degree is R_1 therefore A_1 (Christ missionary school) is the best alternative (school) to admit the child.

Conclusion

In this paper, the authors have introduced interval neutrosophic multi-attribute decision-making problem with completely unknown attribute weight information based on modified GRA. The authors have introduced the operator AAO (accumulated arithmetic operator) to transform interval neutrosophic matrix into SVNS. Here all the attribute weights information are unknown. Entropy based modified GRA analysis method has been introduced to solve this MADM problem. Interval neutrosophic grey relation coefficient has been proposed for solving multiple attribute decision-making problems. Finally, the effectiveness of the proposed approach is illustrated by solving a numerical example.

However, the authors hope that the concept presented here will open new avenue of research in current neutrosophic decision-making arena. The main thrusts of the paper will be in the field of practical decision-making, medical diagnosis, pattern recognition, data mining, clustering analysis, etc.

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