Gedankenexperiment for degree of flatness, or lack of, in early Universe conditions.

Andrew Walcott Beckwith

Physics Department, Chongqing University, College of Physics, Chongqing University Huxi Campus, No. 44 Daxuechen Nanlu, Shapinba District, Chongqing 401331, People's Republic of China

Rwill9955b@gmail.com; abeckwith@uh.edu

Abstract. This document will from first principles delineate the degree of flatness, or deviations from, in early universe models. We will, afterwards, make comparison with recent results we have looked at concerning metric tensor fluctuations and comment upon the role of what early universe gravitational energy may play a role in the presumed deviation from flat space results. Note that N ~ $S_{initial(graviton)} \sim 10^{37}$ will be tied into the presumed results for initial state density, in ways we will comment upon. Leading to observations as to leading to Eq.(25) of this document as to GW, from relic conditions.

Key words; HUP, stress energy tensor, quantum bounce, Infinite quantum statistics, heavy Gravity

i. Introduction

We will start off first, with a description of the following equation which we will derive in the next section. We discuss the implications of a deviation from flat space, with a description of what $S_{initial(graviton)} \sim 10^{37}$ in the aftermath of a quantum bounce implies [1] , and what we should be looking forward in terms of structure formation afterwards. The relevant equation we will be working with is from the time component of the Stress energy Tensor which we will write up as, if $V^{(3)}$ is a statement of volume and if $\Lambda_{initial-value} = \Lambda_{today}$ for the cosmological "constant"

$$
\begin{aligned}
T_{00} = \rho_{Energy-density} &= \frac{-(g_{00}=1)}{16\cdot\pi}\cdot\left(\frac{3\tilde{k}_{Curvature-measure}}{a^2_{initial-scale-factor}} + \Lambda_{initial-value}\right) \\
\Leftrightarrow \tilde{k}_{Curvature-measure} &= -\frac{a^2_{initial-scale-factor}}{3}\times\left[\left(16\cdot\pi\cdot\left(\rho_{Energy-density} = \left[\frac{S_{initial-Entropy}\cdot m_{graviton}}{V^{(3)}}\right]\right)\right) + \Lambda_{initial-value}\right]
\end{aligned}
\tag{1}
$$

In picking this, we are using Ng infinite quantum statistics[1] as a counting factor and likely for $V^{(3)}$ have a Planck length cubed, volume as a starting point, if so then, the mass of the graviton, will be important as well as some considerations given if $\Lambda_{initial-value}$ stays the same, to the present era, or if it has quintessence [2,3], a topic we will bring up. In delineating Eq.(1) above, we will examining the following from first principle, while keeping in mind that [4]

$$
\Re = Riemann-scalar = -\frac{6\cdot\tilde{k}_{Curvature-measure}}{\left(a_{initial-value}\right)^2}
\tag{2}
$$

Then

$$
\begin{aligned}
& T_{uv} = \frac{1}{\sqrt{-|g|}} \cdot \frac{\delta S_{action}}{\delta g_{uv}} = \left(\frac{\delta L_{GR}}{\delta g_{uv}} - \frac{g_{uv}}{2} \cdot L_{GR} \right) \\
& \& \\
& L_{GR} = -\frac{\Re}{16\pi} + \frac{\Lambda_{initial-value}}{8\pi} \\
& \Leftrightarrow T_{uv} = -\frac{g_{uv}}{2} \cdot L_{GR} = \frac{g_{uv}}{2} \cdot \left(\frac{\Re}{16\pi} - \frac{\Lambda_{initial-value}}{8\pi} \right) \\
& \xrightarrow[g_{uv} \to g_{00} = 1]{(u,v) \to (0,0)} \rho_{Energy-density} = T_{00} = \left(\frac{\Re}{32\pi} - \frac{\Lambda_{initial-value}}{16\pi} \right)
\end{aligned}
\tag{3}
$$

If we make the substitution of $\rho_{Energy-density} = \left[\frac{S_{initial-Entropy} \cdot m_{graviton}}{V^{(3)}} \right]$ Then the results, above follow for Eq.(1)

Our supposition is, that if $S_{initial(graviton)} \sim 10^{37}$ [1,5] is used, as well as $a^2_{initial-scale-factor} \sim 10^{-110}$ [5], and $V^{(3)} \sim l_P = Planck-length, cubed$, and $m_{graviton}$ ~ 10 ^ - 62 grams [6] then we have an almost but not zero negative value for the $\tilde{k}_{Curvature-measure}$ value, we will from here discuss its implications and what it says physically.

2. Implications as to choosing $S_{initial(graviton)} \sim 10^{37}$ for our problem: Where it comes from

First of all, this non zero initial value of the entropy is consistent with a quantum bounce, as can be postulated through LQG, as by [5,7] but it says more than that. In reality the very small value for the $\tilde{k}_{Curvature-measure}$ in the aftermath of the quantum bounce, with $a^2_{initial-scale-factor} \sim 10^{-110}$, has some very interesting implications for information transfer from a prior to a present universe which we will be brought up next. We start with what Turok [8] wrote up as to the initial starting point of analysis, as to where he described the cosmological evolution to describe a perfect bounce," in which the universe passes smoothly through the initial singularity. In what we analyze four our purposes, we have that the 2nd order perturbative term of $h^{T(n)}$ for cosmological perturbations obey, here with a 2nd order contribution we can set as

$$
\psi^{(2)}(\eta, x) \sim \frac{A^2}{12} \cdot \left[\exp\left(-\frac{2}{\sqrt{3}} i\tilde{k}_0 \eta \right) \right] \cdot (1 + 2\cos(2k_0 x)) + \ldots \tag{4}
$$

Which is a 2nd order perturbative term for the equation for the evolution of h , if $J^n(\eta, x)$ is nonlinear[8]

$$
\frac{\partial^2 h^{T(n)}}{\partial \eta^2} + \frac{2}{\eta} \cdot \frac{\partial h^{T(n)}}{\partial \eta} - \frac{\partial^2 h^{T(n)}}{\partial x^2} = -J^n(\eta, x) \tag{5}
$$

Then setting a conformal time as approaching early universe conditions requires that

$$\eta \xrightarrow[a \to a_{INITIAL} \sim 10^{-55}]{} -10^{\xi}; \xi \approx very \quad big \neq \infty \tag{6}$$

Our supposition is, then that we have the following for well behaved GW and early cosmological perturbations being viable, in the face of cosmological evolution with modifying the formalism of Turok [8] to obtain

$$\tilde{k}_0 |\eta| \sim \tilde{k}_0 \times 10^{\xi} < 1/\varepsilon \Leftrightarrow \tilde{k}_0 < 10^{-\xi} / \varepsilon \tag{7}$$

In practical terms near the initial expansion point it would mean that near the beginning of cosmological expansion we would have an initial energy density of the order of

$$\rho(initial - energy - density) \sim \hbar \cdot 10^{-\xi} / l_P^3 \varepsilon \tag{8}$$

If so then , if we assume that gravitons, of initial mass about 10^-62 grams, i.e. and that we have Planck mass of about 10^-5 grams, if gravitons were the only 'information' passed into a new universe, making use of the following expression for the initiation of quantum effects, i.e. by Haggard and Rovelli [7]

$$r \sim \frac{7}{3} m \tag{9}$$

Then, we would have, the initiation of quantum effects as of about[8]

$$r_{entropy-gravitons.contribution} \sim \frac{7}{3} \times S(entropy - count) \times 10^{-57} \times l_P \tag{10}$$

Then by making use of Eq.(10) we could, by dimensional analysis, start the comparison by setting values from Eq. (7) and Eq. (10) to obtain

$$10^{-\xi} / \varepsilon \sim \frac{7}{3} \times S(entropy - count) \times 10^{-57} \tag{11}$$

So that to first order, a graviton count, for a radii of about the order of l_P would be

$$S\left(entropy-count\right) \sim 10^{57} \times \frac{3}{7} \times 10^{-\xi} / \varepsilon \tag{12}$$

Depending upon $\varepsilon < \tilde{k}_0 \cdot \eta < 1/\varepsilon$, this will then lead to a condition for which Eq. (4) vanishes, which is in turn due to

$$10^{-\xi}/\varepsilon \sim 10^{-20} \tag{13}$$

Eq.(13) would put restrictions upon the following, namely

3. Considerations of what could lead to Eq.(4), i.e. 2nd order perturbation to cosmological evolution, vanishing

The simple short course as to the radius achieving its starting point to being quantum mechanical in its effects, from the big bang initiating from a quantum bounce is to have the following threshold for quantum effects to be in action, to the vanishing of Eq.(1). Here the quantum effects start with a value of

$$r(quantum-effects) \sim \left(10^{-\xi}/\varepsilon\right) \times l_P \tag{14}$$

If Eq.(4) is zero due to $x = r(quantum-effects)$ and we want Eq.(4) to vanish, it leads to the following for the vanishing of the 2nd order perturbative effect, with λ the critical value of wavelength for which Eq.(4) vanishes, i.e. hence ,

$$\begin{aligned}
&\cos(k_0 \cdot r(quantum-effects)) = -1/2 \\
&\Leftrightarrow k_0 \cdot r(quantum-effects) = \frac{2\pi}{3} \\
&\Leftrightarrow k_0 = \left(\frac{2\pi}{3} \times \frac{\varepsilon}{l_P} \times 10^{\xi}\right) \sim \frac{2\pi}{\lambda} \\
&\Leftrightarrow \lambda \sim \frac{3 \cdot l_P}{\varepsilon} \times 10^{-\xi}
\end{aligned} \tag{15}$$

It means that there is the following interval may be our best Quantum Mechanical perturbative indicator in terms of Eq.(4) , that is

$$\frac{l_P}{\varepsilon} \times 10^{-\xi} < x < \frac{3 \cdot l_P}{\varepsilon} \times 10^{-\xi} \tag{16}$$

4. Comparing the variance in position given in Eq.(16) with modified HUP

Note this very small value of x comes from a scale factor, if [9] $z \sim 10^{55} \Leftrightarrow a_{scale-factor} \sim 10^{-55}$, i.e. 55 orders of magnitude smaller than what would normally consider, but here note that the scale factor is not zero, so we do not have a space – time singularity.Then

$$\begin{aligned} &\frac{l_P}{\varepsilon} \times 10^{-\xi} < \left(x = \Delta l \right) < \frac{3 \cdot l_P}{\varepsilon} \times 10^{-\xi} \\ &\Delta l \cdot \Delta p \geq \frac{\hbar}{2} \end{aligned} \tag{17}$$

We will next discuss the implications of this point in the next section, of a non zero smallest scale factor

We will be using the approximation given by Unruth [10,11], of a generalization we will write as

$$\begin{aligned} &(\Delta l)_{ij} = \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2} \\ &(\Delta p)_{ij} = \Delta T_{ij} \cdot \delta t \cdot \Delta A \end{aligned} \tag{18}$$

If we use the following, from the Roberson-Walker metric[3,4].

$$\begin{aligned} &g_{tt} = 1 \\ &g_{rr} = \frac{-a^2(t)}{1 - k \cdot r^2} \\ &g_{\theta\theta} = -a^2(t) \cdot r^2 \\ &g_{\phi\phi} = -a^2(t) \cdot \sin^2\theta \cdot d\phi^2 \end{aligned} \tag{19}$$

Following Unruth [5] , write then, an uncertainty of metric tensor as, with the following inputs

$$a^2(t) \sim 10^{-110}, r \equiv l_P \sim 10^{-35} meters \tag{20}$$

Then, if $\Delta T_{tt} \sim \Delta \rho$ [3,4,5]

$$\begin{aligned} &V^{(4)} = \delta t \cdot \Delta A \cdot r \\ &\delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} \geq \frac{\hbar}{2} \\ &\Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}} \end{aligned} \tag{21}$$

5. Conclusion. Eq. (21) may, with refinements of r=x , in the four dimensional Volume delineate the new HUP, in our problem

If from Giovannini [12] we can write

$$\delta g_{tt} \sim a^2(t)\cdot\phi << 1 \tag{22}$$

Refining the inputs from Eq.(22) means more study as to the possibility of a non zero minimum scale factor [34] , as well as the nature of ϕ as specified by Giovannini [12] . Then we will assert that if **r=x** then if we use $\frac{l_P}{\varepsilon}\times 10^{-\xi} < (x=r) < \frac{3\cdot l_P}{\varepsilon}\times 10^{-\xi}$ and then the volume $V^{(4)} = \delta t\cdot\Delta A\cdot r$, as used in [3,4,5]

$$(\delta t\cdot\Delta A)\times\frac{l_P}{\varepsilon}\times 10^{-\xi} < V^{(4)} < (\delta t\cdot\Delta A)\times\frac{3\cdot l_P}{\varepsilon}\times 10^{-\xi} \tag{23}$$

This Eq. (19) will be put into $\delta g_{tt}\cdot\Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}$, if $\Delta T_{tt} \sim \Delta\rho$, it means that $\delta g_{tt}\cdot\Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}$ that this is defined for all x as to where and when $\frac{l_P}{\varepsilon}\times 10^{-\xi} < (x=r) < \frac{3\cdot l_P}{\varepsilon}\times 10^{-\xi}$ holds, with the lower value for x signifying the spatial range of x for which quantum mechanics is valid, with three times that value connected as to when the perturbative methods break down. Thereby influencing the range of values for $V^{(4)} = \delta t\cdot\Delta A\cdot r$ in $\delta g_{tt}\cdot\Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}$. Furthermore we have, if there is an eventual weak field approximation according to Katti [4] gravitational spin off according to $g_{ij} = \eta_{ij} + h_{ij}$, with a gravitational wave signal according to, if $V^{(3)} = \Delta A\cdot r$ [4]

$$h_{kl}\left(x^i\right) = -\frac{4G}{c^2}\cdot\int_{V^{(3)}}\frac{T^*_{kl}\left(x'^i\right)}{R}\cdot d^3x' \equiv -\frac{4G}{c^2}\cdot\int_{V^{(3)}}\frac{T^*_{kl}\left(x'^i\right)}{\left[\eta_{ij}\cdot\left(x^i - x'^i\right)\cdot\left(x^j - x'^j\right)\right]^{1/2}}\cdot d^3x' \tag{24}$$

If the contribution from Pre-Planckian to Planckian is due to the stress energy tensor as given in $\Delta T_{tt} \sim \Delta\rho$ form [5], it means that the relevant relic GW signal will be of the form, with D^{ij} a small quadrupole tensor. This with space-time which is almost flat according to Eq. (1) initially as the genesis of the GW which may be analyzed with a dominant contribution coming from [4]

$$\begin{aligned} h_{00}\left(x^i\right) &= -\frac{4G}{c^2}\cdot\int_{V^{(3)}}\frac{T^*_{00}\left(x'^i\right)}{R}\cdot d^3x' \\ &\sim -\frac{4G}{c^2}\cdot\int_{V^{(3)}}\frac{\rho_{Energy-density}\left(x'^i\right)}{R}\cdot d^3x' \\ &\sim -\frac{4G}{c^2}\cdot\int_{V^{(3)}}\frac{\left(\frac{\Re}{32\pi} - \frac{\Lambda_{initial-value}}{16\pi}\right)}{R}\cdot d^3x' \end{aligned} \tag{25}$$

This value of Eq.(25) would have as its origins the near flat space physics given by Eq.(1) as its genesis with this to consider, as the start.[5]

$$
\begin{aligned}
&\delta g_{rr} T_{rr} \geq -\left|\frac{\hbar \cdot a^2(t) \cdot r^2}{V^{(4)}}\right| \xrightarrow[a \to 0]{} 0 \\
&\delta g_{\theta\theta} T_{\theta\theta} \geq -\left|\frac{\hbar \cdot a^2(t)}{V^{(4)}\left(1-k \cdot r^2\right)}\right| \xrightarrow[a \to 0]{} 0 \\
&\delta g_{\phi\phi} T_{\phi\phi} \geq -\left|\frac{\hbar \cdot a^2(t) \cdot \sin^2\theta \cdot d\phi^2}{V^{(4)}}\right| \xrightarrow[a \to 0]{} 0
\end{aligned}
\tag{26}
$$

Further refinements may be due to [5,13] where we consider as given in [13] details of a quantum bounce which may give more information, as well as additional investigations into what Turok brought up [3] . The issues as to Eq.(26) and what they imply are quite different from [14] for reasons we will go into in a future publication.

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