Gravity is not geometry

Mirosław J. Kubiak

Grudziądz, Poland

Abstract: *In this paper was presented description of gravity without the geometry of the space-time.*

Introduction

General Relativity (GR) is a theory which a since about 100 years describes the gravitational phenomena as a geometric properties of the four-dimensional space-time. Although GR is widely accepted as a fundamental theory of gravitation for the many physicists still this is not a perfect theory.

In GR the space-time plays a very important role. The space-time continuum is a mathematical model that joins three-dimensional space and one dimension time into a single idea, the four-dimensional space-time. Under influence outer gravitational field the four-dimensional space-time is active and is curved.

Can we describe the gravitational phenomena without active the space-time?

The answer is in the affirmative, if we will put in to the place of the space-time *the material medium*. This medium is the four-dimensional space-time filled with a continuously mass. In other words, this space-time gained the mass about the certain density. Under influence outer gravitational field this medium is stressed and strained, while the four-dimensional space-time is not active. As we will see later, the new theory of the gravity is kept in the spirit of Mach, in opposite to GR theory.

The new theory of gravity we will call *m(GR)* theory.

Alternative description of the gravitational phenomena

Suppose that there is an alternative description of the gravitational phenomena. The arena, where gravitational phenomena take place is the continuous material medium, immersed in the Minkowski four-dimensional space-time, which is only a passive background.

The material medium is infinite collection of material bodies filling the whole space-time about a certain mass density ρ, which has the capacity to propagate the gravitational interactions.

Let us assume that in the absence of the outer gravitational field this medium becomes *the bare medium*. This medium with *the bare mass density* ρ^{bare} is homogeneous, isotropic, independent of the time and is defined as follows

$$\rho_{\mu\nu}^{bare} \overset{def}{=} \rho^{bare} \cdot \eta_{\mu\nu} = \mathrm{diag}(-\rho^{bare}, \rho^{bare}, \rho^{bare}, \rho^{bare}) \tag{1}$$

where: $\rho_{\mu\nu}^{bare}$ is *the bare mass density tensor*, $\eta_{\mu\nu}$ is *the Minkowski tensor*, μ, ν = 0, 1, 2, 3.

The bare medium is equivalent, in a some sense, with the Minkowski space-time, where $\rho^{bare} = 0$.

For example, the bare mass density tensor in the spherical coordinates has form

$$\rho_{\mu\nu}(r) = \rho^{bare} \cdot \mathrm{diag}(-1,\ 1,\ r^2,\ r^2 \cdot \sin^2\theta)$$

components: ρ_{00} and ρ_{rr} has dimension [kg m^{-3}], but components $\rho_{\phi\phi}$ and $\rho_{\theta\theta}$ [kg m^{-1}].

Under influence outer gravitational field the bare medium with the bare mass density ρ^{bare} is stressed and strained and becomes *the effective medium* with *the effective mass density tensor* $\rho_{\mu\nu}$. In the *m(GR)* theory the four-dimensional space-time is not active and is only the background.

The metric

The metric is defined as

$$ds^2(\rho_{\mu\nu}) \overset{def}{=} \frac{\rho_{\mu\nu}(x)}{\rho^{bare}} dx^\mu dx^\nu \tag{2}$$

where: $\rho_{\mu\nu}(x)$ is a symmetric, position dependent, the effective mass density tensor[1]. Tensor $\rho_{\mu\nu}(x)$ describes the relationship between the effective medium and the bare medium and is equivalent, in a some sense, to *the metric tensor* $g_{\mu\nu}(x)$.

If we will assume $\rho(x) = 0$ then effective medium is equivalent, in a some sense, with the pseudo-Riemann four-dimensional space-time. In absence of the outer gravitational field the metric (2) has form of the Minkowski metric.

Is the effective mass density does change during the motion along the geodesic?

We suppose that yes. Based solely on the intuition, we will write the equation, without a mathematical proof

$$\frac{d^2}{d\tau^2}\delta\rho \sim -\frac{G}{c^4}\sigma_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\delta\rho \tag{3}$$

where: $\sigma_{\mu\nu}$ *the effective medium strain tensor* with the physical dimension [Nm^{-2}], $\delta\rho$ the change of the effective mass density between two points that are located within a very short distance, *G* is the Newtonian constant, *c* is the speed of light. Note that $\frac{G}{c^4}$ has physical dimension [N^{-1}]. The equation (3) we will call *the effective mass density deviation*.

The field equation

In GR the pseudo-Riemann four-dimensional space-time is active and is curved. Einstein's field equation has form

[1] For the simplicity we will assume, that $\rho^{bare} = 1$.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \cdot R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

where: $R_{\mu\nu}$ is *the Ricci tensor*, R is *the Ricci scalar*, $T_{\mu\nu}$ is *the stress–energy tensor*.

John Archibald Wheeler expressed Einstein's field equation with the words: *Space-time tells matter how to move; matter tells space-time how to curve.*

In our theory, the gravitational field interacts with the bare medium causes that this medium is stressed and becomes the effective medium. The stress–energy tensor $T_{\mu\nu}$ mathematically describes the magnitude of these stresses. Under the influence of the stress the effective medium is strain. The effective medium strain tensor $\sigma_{\mu\nu}$ mathematically describes the magnitude of these strain.

That is the difference between the two theories. We suppose also that the effective medium strain tensor $\sigma_{\mu\nu}$ is equivalent, in a some sense, the Ricci curvature tensor $R_{\mu\nu}$, known from GR.

We looking for the field equation of the second order in the effective medium variables $\rho_{\mu\nu}$, which to be consistent with the observations, and in the weak field and slow motion limit, must reduce to the classical Poisson equation for the gravitation.

We write the total action over an arbitrary medium region as

$$S = \int_{\Omega} \left(L_{eff} + L_m \right) \sqrt{-\rho} \cdot d^4 x$$

where: L_{eff} is the Lagrangian densities for the effective medium, L_m is the Lagrangian densities for the matter and fields, $\rho = \det(\rho_{\mu\nu})$ is the determinant of the effective mass density tensor. The term $\sqrt{-\rho} \cdot d^4 x$ describes the strain volume in the effective medium, $d^4 x = dx^0 \cdot dx^1 \cdot dx^2 \cdot dx^3$.

After substituting the total action for the *m(GR)* theory has form

$$S = \int \left(\frac{1}{2} \sigma + L_m \right) \sqrt{-\rho} \cdot d^4 x \tag{4}$$

where: $\sigma \equiv \rho^{\mu\nu} \cdot \sigma_{\mu\nu}$ is *the effective medium strain scalar* with the physically dimension [N m^{-2}].

If we will use the following substitution $\rho_{\mu\nu}(x) \to g_{\mu\nu}(x)$, after well-known, from GR, mathematical calculations, the field equation will take the form

$$\sigma_{\mu\nu} - \frac{1}{2} \rho_{\mu\nu} \cdot \sigma = 8\pi \cdot T_{\mu\nu} \tag{5}$$

where

$$T_{\mu\nu} \equiv -2 \frac{\partial L_m}{\partial \rho^{\mu\nu}} + L_m \rho_{\mu\nu} .$$

The left side of the equation (5) represents the strain of the effective medium with the physically dimension [Nm^{-2}]. The right side represents distribution of the matter and energy, expressed by on the stress–energy tensor, with the physically dimension [Nm^{-2}].

Paraphrasing the words of the John Archibald Wheeler, we can say that*: matter tells the effective medium how to strain, the effective medium under strain tells matter how to move.*

The field equation (5) has the same form as the Einstein's field equation if we will use the following substitution

$$\begin{aligned} \sigma_{\mu\nu}(\rho_{\mu\nu}) &\to \frac{c^4}{G}\cdot R_{\mu\nu}(g_{\mu\nu}) \\ \sigma(\rho_{\mu\nu}) &\to \frac{c^4}{G}\cdot R(g_{\mu\nu}) \end{aligned} \tag{6}$$

Note that $\frac{c^4}{G}$ has physical dimension [N] and is the greatest possible force in the Nature.

Correspondence with Newtonian theory of the gravitation

In Newtonian approximation we can decompose $\rho_{\mu\nu}$ to following simple form

$$\rho_{\mu\nu}(x) = \rho_{\mu\nu}^{bare} + \rho_{\mu\nu}^{*}(x)$$

where $\rho_{\mu\nu}^{*}(x) << 1$ is a small perturbation in the effective mass density tensor.

The field equation (5) in the Newtonian approximation takes the form of the Poisson equation

$$\nabla^2 \rho_{00}^{*}(r) = -\frac{8\pi G}{c^2}\rho(r) \tag{7}$$

with solution (in the particular case)

$$\rho_{00}^{*}(r) = \frac{2G}{c^2}\int \frac{\rho dV}{r} = \frac{2GM}{c^2 r} = \frac{2V(r)}{c^2} \tag{8}$$

where $V(r)$is the well-known gravitational potential. The equation (8) shows also the relationship between $\rho_{00}^{*}(r)$ and $V(r)$.

The metric in the static and the spherically symmetric gravitational field

In the static and the spherically symmetric gravitational field the metric, expressed by component $\rho_{00}^{*}(r)$, takes the form

$$ds^2 \approx -\left(1-\rho_{00}^{*}(r)\right)\cdot c^2 dt^2 + \frac{dr^2}{1-\rho_{00}^{*}(r)} + r^2\left(d\theta^2 + \sin^2\theta \cdot d\phi^2\right) \tag{9}$$

This metric is diagonal, time-independent and does not have a singularity. **Does this mean that the material medium eliminates the singularity?**

When the equation (8) is satisfied, then metric (9) becomes the Schwarzschild metric and appear a singularity of the space-time. **Does this mean that such the singularity is an artifact only in GR?**

Comparing with the metric (2) we get nonzero components of the effective mass density tensor

$$\begin{aligned}
&\rho_{tt}(r) = -\left(1-\rho^{*}_{00}(r)\right),\\
&\rho_{rr}(r) = \left(1-\rho^{*}_{00}(r)\right)^{-1},\\
&\rho_{\theta\theta}(r) = r^2,\\
&\rho_{\phi\phi}(r) = r^2 \cdot \sin^2\theta.
\end{aligned}$$

According to our theory, the effective mass density tensor of the body is change in the gravitational field. Because $\rho_{tt}(r)$ depends on the mass *M* of the star and the planet orbit radius *r*, we should observe fluctuations in the effective mass density tensor for the elliptical orbit. Annual the biggest relative fluctuations, from perihelion to perihelion, for $\rho_{tt}(r)$ and for $\rho_{rr}(r)$ should be measured in *the Solar System* and the estimated value for the planet Earth is equal to $6.6\cdot 10^{-10}$.

This small change (the small anisotropy) in the effective mass density tensor, for example, disturbs Newtonian orbit, causing the anomalous motion in the planet's perihelion.

Clocks and rods

Let's go back to the equation (9). If we assume that $t = const$, $\theta = const$ and $\phi = const$ then we have the infinitesimal radial distance *dR* in the form

$$dR = \frac{dr}{\sqrt{1-\rho^{*}_{00}(r)}}$$

If we assume that $\rho^{*}_{00}(r)\uparrow$ then rods with the effective mass density will measure different length than the rods with the bare mass density and $dR > dr$. But if we assume that $\rho^{*}_{00}(r)\to 0$ then rods with the effective mass density and the rods with the bare mass density will measure the same length $dR = dr$.

Let us now turn our attention to the time. According to the eq. (9) (where now we assumed that $dr = d\theta = d\phi = 0$) we have

$$d\tau = dt\sqrt{1-\rho^{*}_{00}(r)}$$

If we assume that $\rho^{*}_{00}(r)\uparrow$ then clocks with the effective mass density will measure different time than the clocks with the bare mass density and $d\tau < dt$. But if we assume that $\rho^{*}_{00}(r)\to 0$ then clocks with the effective mass density and the clocks with the bare mass density will measure the same time $d\tau = dt$.

Under the influence of the gravitational field **the physical properties of the rods and clocks are change** but not properties of the space and time.

The measurement results are consistent with GR but their physical meaning was changed.

The equations of motion

The Lagrangian function for the particle with the effective mass density tensor $\rho_{\mu\nu}(x)$ has form

$$L = \frac{1}{2}\rho_{\mu\nu}(x)\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau} \tag{10}$$

The Euler–Lagrange equation gives the equations of motion

$$\frac{dp_{\gamma}(x)}{d\tau} - \frac{1}{2}\frac{\partial \rho_{\mu\nu}(x)}{\partial x^{\gamma}}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau} = 0 \tag{11}$$

It is the geodesic equation, where: $p_{\gamma}(x) = \rho_{\gamma\nu}(x)\frac{dx^{\nu}}{d\tau}$ is the density of the four-momentum, τ is the proper time. In the particular case, for the bare medium, $\frac{dp_{\gamma}}{d\tau} = 0$ and the four-momentum p_{γ} is constant along the world line.

The equation (11) has the also different equivalent form (if $\rho_{\gamma\nu}(x)$ does not depends *explicitly* on τ)

$$\rho_{\gamma\nu}(x)\frac{d^2x^{\nu}}{d\tau^2} + \Gamma_{\gamma\mu\nu}\left(\rho_{\mu\nu}(x)\right)\cdot\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau} = 0 \tag{12}$$

where:

$$\Gamma_{\gamma\mu\nu}\left(\rho_{\mu\nu}(x)\right) \overset{def}{=} \frac{1}{2}\left(\frac{\partial \rho_{\gamma\mu}(x)}{\partial x^{\nu}} + \frac{\partial \rho_{\gamma\nu}(x)}{\partial x^{\mu}} - \frac{\partial \rho_{\mu\nu}(x)}{\partial x^{\gamma}}\right)$$

is *the Christoffel symbols of the first kind*, now is expressed by the effective mass density tensor $\rho_{\mu\nu}(x)$. Dimension of the Christoffel symbols of the first kind is [kg/m^{-4}].

Analyzing the equation of motion (12), we can see that the distribution and motion of the surrounding masses, expressed by the term

$$\Gamma_{\gamma\mu\nu}\left(\rho_{\mu\nu}(x)\right)\cdot\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}$$

is the source of the inertial forces, expressed by the term

$$\rho_{\gamma\nu}(x)\frac{d^2x^{\nu}}{d\tau^2}$$

The motion of the body does not depend on the properties of the space-time but the presence and motion of the surrounding masses and their distribution. These surrounding masses and their distribution are the source of the inertial forces.

All phenomena of gravity are expressed in terms of the relationship between bodies, and not between the body and surrounding the space-time. The new quality of the understanding, kept in Mach's spirit, has been reached[2].

The bare medium as the reference frame

In particular case, when the surrounding masses are the bare masses, i.e.

$$\Gamma_{\gamma\mu\nu}\left(\rho_{\gamma\nu}^{bare}\right)=0$$

then the inertial forces will disappear

$$\rho^{bare}\frac{d^2x^{\nu}}{d\tau^2}=0\,, \tag{13}$$

note that is always $\rho^{bare}\neq 0$.

According to the equation (13) we can say that the body with the bare mass density ρ^{bare} is in the rest or moves in a straight line with the constant speed in respect to **the surrounding bare masses and their distribution** (not respect to the space-time).

The equation (13) determines the new reference frame – *the bare medium reference frame*.

Reference

[1]. A. Einstein, *The Meaning of Relativity*, Princeton University Press, published 1922, p. 59.

[2] *This is the reason why E. Mach was led to make the attempt to eliminate space as an active cause in the system of mechanics. According to him, a material particle does not move in not accelerated motion relatively to space, but relatively to the centre of all the other masses in the Universe; in this way the series of causes of mechanical phenomena was closed, in contrast to the mechanics of Newton and Galileo* [1].