Applications of Logic Functions

Abstract. By using logic functions we can express sign function, rectangular function, box car function and Heaviside function. It is also possible to express the functions such as square wave, triangular wave, sawtooth wave and rectangular wave.

Consider the function $Y=[[(|x|+|d|+x)/(|x|+|d|+d)].(1/e)]$, where $-\infty<d<\infty$.

Now, $Y1=|Y/\log Y|$ and $Y2=|Y1/\log Y1|$, which for n number of terms becomes

$Yn=|Yn-1/\log Yn-1|$.

Taking the limit as $n\to\infty$ yields, for $x>d$ then $Yn=e$ and $Z=f$.

For $x=d$ then $Yn=1/e$ and $Z=g$, for $x<d$ then $Yn=0$ and $Z=h$.

This result can be proven, but it is related to mathematics. We call Yn as a fundamental logic function and consider Yn as standard math library function. The function Z is given by

$Z=f[(Yn-1/e)Yn/(e-1/e)e]+g[(Yn-e)Yn/(1/e-e)(1/e)]+h[(Yn-e)(Yn-1/e)]$,

Which can be written as $Z=af+bg+ch$,

where $a=[(Yn-1/e)Yn/(e-1/e)e]$, $b=[(Yn-e)Yn/(1/e-e)(1/e)]$, and

$c=[(Yn-e)(Yn-1/e)]$.

Here f, g, and h are functions of x. This formula can be used to replace a decision algorithm by a simple algorithm, which does not use decision trees. The fundamental logic function Yn(x, d), which depends on variable x and constant d, has the functions a, b, and c. Therefore, a, b, and c are also logic functions, i.e., they can only assume two values: either 0 or 1. Logic functions a, b, and c are considered standard math library functions and their use enables us to replace logical trees by a single mathematical equation.

If $x>d$ then $a=1$, $b=0$, $c=0$ and Z becomes $Z=f$

If $x=d$ then $b=1$, $c=0$, $a=0$ and Z becomes $Z=g$

If $x<d$ then $c=1$, $a=0$, $b=0$ and Z becomes $Z=h$

1.The expression for square wave of amplitude A , frequency f and instantaneous time t, angular frequency w, instantaneous angle v can be obtained by putting

$f=g=A$ and $h=-A$, also $x=2\sin v$ and $d=\cos v$ we get square wave, where $v=wt$

$Z=(a+b-c)A$

2. We can obtain the expression for triangular wave, sawtooth wave and rectangular wave by using the reduction formula Z.

3.We can express the Heaviside step function, rectangular function, boxcar function and step function using reduction formula Z.

I) Heaviside step function:

We can express Heaviside step function, using half maximum convention as a function of a discrete variable n:

$H[n]=1$ for $n>0$, $H=1/2$ for $n=0$, and $H=0$ for $n<0$

Putting $d=0$ and $f=1$, $g=1/2$, $h=0$ in the reduction formula and logic function

We get the expression for Heaviside step function $Z=a+(b/2)$

ii)Sign function:

The signum function of a real number x is defined as

$\mathrm{sgn}(x)=1$ for $x>0$, $\mathrm{sgn}(x)=0$ for $x=0$ and $\mathrm{sgn}(x)=-1$ for $x<0$

Putting $d=0$ and $f=1$, $g=0$, $h=-1$ in the logic function and reduction formula we get the expression for signum function as

$Z=a-c$

iii) Rectangular function:

$\mathrm{rect}(t)= 0$ for $|t|>1/2$, $\mathrm{rect}(t)=1/2$ for $|t|=1/2$, and $\mathrm{rect}(t)=1$ for $|t|<1/2$

The logic functions for $d=1/2$ is a, b, c and the functions are $f=0$, $g=1/2$ and $h=a1f1+b1g1+c1h1$

The logic functions for $d1=-1/2$ is a1, b1, c1 and $f1=1$, $g1=1/2$, $h1=0$

Putting in the reduction formula and logic functions yields

$Z=af+bg+ch$ where $h=a1f1+b1g1+c1h1$

Putting the values of f, g, h, f1, g1 and h1

$Z=b/2+c(a1+(b1/2))$

These applications are useful in digital signal processing. The formula for square wave, triangular wave, sawtooth wave and rectangular wave are most useful than using Fourier transform.

References.

1) Wikipedia