Gedankenexperiment for refining the Unruh metric tensor uncertainty principle via Schwartzshield Geometry and Planckian Space-time with initial non zero entropy and applying the Riemannian-Penrose inequality and the initial K.E. for a lower bound to the Graviton mass

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Abstract. First of all, we restate a proof of a highly localized special case of a metric tensor uncertainty principle first written up by Unruh. Unruh did not use the Roberson-Walker geometry which we do, and it so happens that the dominant metric tensor we will be examining, is variation in δg_{tt}. The metric tensor variations given by δg_{rr}, $\delta g_{\theta\theta}$, and $\delta g_{\phi\phi}$ are nonexistent, as compared to the δg_{tt}. Afterwards, what is referred to by Barbour as emergent duration of time δt is from the HUP applied to δg_{tt} in such a way as to give, in the Planckian space-time regime a nonzero minimum non zero lower ground to a massive graviton, $m_{graviton}$. The lower bound to the massive graviton, is influenced by δg_{tt} and kinetic energy which is in the Planckian emergent duration of time δt as $(E-V)$. We find from δg_{tt} version of the HUP, that the quantum value of the $\Delta t \cdot \Delta E$ HUP is likely not recoverable due to $\delta g_{tt} \neq \mathrm{O}(1) \sim g_{tt} \equiv 1$. I.e. $\delta g_{tt} \neq \mathrm{O}(1)$ is consistent with non curved space, so $\Delta t \cdot \Delta E \geq \hbar$ no longer holds. This even if $T_{ii} = diag(\rho, -p, -p, -p)$ fluid approximation is used. Our treatment of the inflaton is via Handley et al, where we consider the lower mass limits of the graviton as due to when the inflaton is many times larger than a Potential energy, with KE proportional to $\rho_w \propto a^{-3(1-w)} \sim g^* T^4$, with g^* initial degrees of freedom, and T initial temperature .Leading to non zero initial entropy as stated in **Appendix A**. In addition we also examine a Ricci scalar value at the boundary between Pre Planckian to Planckian regime of space-time, setting the magnitude of k as approaching flat space conditions right after the Planck regime. Furthermore, we have an approximation as to initial entropy production. N ~ $S_{initial(graviton)} \sim 10^{37}$ Finally, this entropy is N, and we get an initial version of the cosmological "constant" as **Appendix D** which is linked to initial value of a graviton mass. **Appendix E**, is for the Riemannian- Penrose inequality, which is either a nonzero NLED scale factor or quantum bounce as of LQG. Finally, **Appendix F** gives conditions so that a pre Planckian kinetic energy(inflaton) value greater than Potential energy occurs, which is foundational to the lower bound to Graviton mass. We will in the future add more structure to this calculation so as to confirm via a precise calculation that the lower bound to the graviton mass, is about 10^-70 grams. Our lower bound is a dimensional approximation so far. We will make it exact.

i. Introduction

The first matter of business will be to introduce a framework of the speed of gravitons in "heavy gravity", and this is important since eventually, as illustrated by Will [1,2] it could possibly be observed.

Secondly, it also will involve an upper bound to the rest mass of a graviton. The second aspect of the inquiry of our manuscript will be to come up with a variant of the HUP, involving a metric tensor, as well as the Stress energy tensor, which will in time allow us to establish a lower bound to the mass of a graviton, preferably at the start of cosmological evolution**.** The article concludes in its last section as to why a statement by Mukhanov in Marcel Grossman 14, 2015, Rome, that a multiverse contribution to a new universe would have a causal barrier averaging of time contributions even if there were contributions from a multiverse, so there was only one space-time contribution is possibly indefensible.

We reference what was done by Will in his living reviews of relativity article as to the 'Confrontation between GR and experiment". Specifically we make use of his experimentally based formula of [1,2] , with $v_{graviton}$ the speed of a graviton, and $m_{graviton}$ the rest mass of a graviton, and $E_{graviton}$ in the inertial rest frame given as:

$$\left(\frac{v_{graviton}}{c}\right)^2 = 1 - \frac{m_{graviton}^2 c^4}{E_{graviton}^2} \tag{1}$$

Furthermore, using [2], if the rest mass of a graviton is very small we can make a clear statement of

$$\begin{aligned} \frac{v_{graviton}}{c} &= 1 - 5\times10^{-17}\cdot\left(\frac{200Mpc}{D}\right)\cdot\left(\frac{\Delta t}{1\sec}\right) \\ &\doteq 1 - 5\times10^{-17}\cdot\left(\frac{200Mpc}{D}\right)\cdot\left(\frac{\Delta t = \Delta t_a - (1+z)\cdot\Delta t_b}{1\sec}\right) \\ \Leftrightarrow \frac{2m_{graviton}c^2}{E_{graviton}} &\approx 5\times10^{-17}\cdot\left(\frac{200Mpc}{D}\right)\cdot\left(\frac{\Delta t_a - (1+z)\cdot\Delta t_b}{1\sec}\right) \end{aligned} \tag{2}$$

Here, Δt_a is the difference in arrival time, and Δt_e is the difference in emission time/in the case of the early Universe, i.e. near the big bang, then if in the beginning of time, one has, if we assume that there is an average $E_{graviton} \approx \hbar\cdot\omega_{graviton}$, and

$$\begin{aligned} &\Delta t_a \sim 4.3\times10^{17}\sec \\ &\Delta t_e \sim 10^{-33}\sec \\ &z \sim 10^{50} \end{aligned} \tag{3}$$

Then, $\left(\frac{\Delta t_a - (1+z)\cdot\Delta t_b}{1\sec}\right) \sim 1$, and if $D \sim 4.6\times10^{26} meters = radii(universe)$, so one can set

$$\left(\frac{200Mpc}{D}\right) \sim 10^{-2} \tag{4}$$

And if one sets the mass of a graviton [3] into Eq. (1) , then we have in the present era, that if we look at primordial time generated gravitons, that if one uses the

$$\begin{aligned}
&\Delta t_a \sim 4.3\times10^{17}\,\mathrm{sec} \\
&\Delta t_e \sim 10^{-33}\,\mathrm{sec} \qquad (5)\\
&z \sim 10^{55}
\end{aligned}$$

Note that the above frequency, for the graviton is for the present era, but that it starts assuming genesis from an initial inflationary starting point which is not a space – time singularity.

Note this comes from a scale factor, if $z \sim 10^{55} \Leftrightarrow a_{scale-factor} \sim 10^{-55}$, i.e. 55 orders of magnitude smaller than what would normally consider, but here note that the scale factor is not zero, so we do not have a space – time singularity.

We will next discuss the implications of this point in the next section, of a non zero smallest scale factor . Secondly the fact we are working with a massive graviton , as given will be given some credence as to when we obtain a lower bound, as will come up in our derivation of modification of the values[3]

$$\begin{aligned}
&\left\langle \left(\delta g_{uv}\right)^2 \left(\hat{T}_{uv}\right)^2 \right\rangle \ge \frac{\hbar^2}{V_{Volume}^2} \\
&\xrightarrow[uv\to tt]{} \left\langle \left(\delta g_{tt}\right)^2 \left(\hat{T}_{tt}\right)^2 \right\rangle \ge \frac{\hbar^2}{V_{Volume}^2} \qquad (6)\\
&\& \quad \delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+
\end{aligned}$$

The reasons for saying this set of values for the variation of the non g_{tt} metric will be in the 3rd section and it is due to the smallness of the square of the scale factor in the vicinity of Planck time interval.

2.Non zero scale factor, initially and what this is telling us physically. Starting with a configuration from Unruth.

Begin with the starting point of[4,5]

$$\Delta l \cdot \Delta p \ge \frac{\hbar}{2} \qquad (7)$$

We will be using the approximation given by Unruth [4,5], of a generalization we will write as

$$\begin{aligned}
&(\Delta l)_{ij} = \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2} \\
&(\Delta p)_{ij} = \Delta T_{ij} \cdot \delta t \cdot \Delta A
\end{aligned} \qquad (8)$$

If we use the following, from the Roberson-Walker metric[6].

$$
\begin{aligned}
g_{tt} &= 1 \\
g_{rr} &= \frac{-a^2(t)}{1-k\cdot r^2} \\
g_{\theta\theta} &= -a^2(t)\cdot r^2 \\
g_{\phi\phi} &= -a^2(t)\cdot \sin^2\theta\cdot d\phi^2
\end{aligned}
\tag{9}
$$

Following Unruth [4,5] , write then, an uncertainty of metric tensor as, with the following inputs

$$
a^2(t) \sim 10^{-110}, r \equiv l_P \sim 10^{-35} meters \tag{10}
$$

Then, the surviving version of Eq. (7) and Eq. (8) is, then, if $\Delta T_{tt} \sim \Delta\rho$

$$
\begin{aligned}
& V^{(4)} = \delta t\cdot \Delta A\cdot r \\
& \delta g_{tt}\cdot \Delta T_{tt}\cdot \delta t\cdot \Delta A\cdot \frac{r}{2} \geq \frac{\hbar}{2} \\
& \Leftrightarrow \delta g_{tt}\cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}
\end{aligned}
\tag{11}
$$

This Eq. (11) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time[6]

$$
T_{ii} = diag(\rho, -p, -p, -p) \tag{12}
$$

Then

$$
\Delta T_{tt} \sim \Delta\rho \sim \frac{\Delta E}{V^{(3)}} \tag{13}
$$

Then, Eq.(11) and Eq. (12) and Eq. (13) together yield

$$
\begin{aligned}
& \delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \\
& Unless \quad \delta g_{tt} \sim O(1)
\end{aligned}
\tag{14}
$$

How likely is $\delta g_{tt} \sim O(1)$? Not going to happen. Why ? The homogeneity of the early universe will keep

$$
\delta g_{tt} \neq g_{tt} = 1 \tag{15}
$$

In fact, we have that from Giovannini [6] , that if ϕ is a scalar function, and $a^2(t) \sim 10^{-110}$, then if

$$\delta g_{tt} \sim a^2(t)\cdot\phi << 1 \tag{16}$$

Then, there is no way that Eq. (14) is going to come close to $\delta t \Delta E \geq \frac{\hbar}{2}$. Hence, the Mukhanov suggestion as will be discussed toward the end of this article, is not feasible. Finally, we will discuss a lower bound to the mass of the graviton.

3 . How we can justifying writing very small $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^{+}$ values.

To begin this process, we will break it down into the following co ordinates

In the rr, $\theta\theta$, and $\phi\phi$ co ordinates, we will use the Fluid approximation, $T_{ii} = diag(\rho,-p,-p,-p)$ [7] with

$$\begin{aligned}
\delta g_{rr}T_{rr} &\geq -\left|\frac{\hbar\cdot a^2(t)\cdot r^2}{V^{(4)}}\right| \xrightarrow[a\to 0]{} 0 \\
\delta g_{\theta\theta}T_{\theta\theta} &\geq -\left|\frac{\hbar\cdot a^2(t)}{V^{(4)}\left(1-k\cdot r^2\right)}\right| \xrightarrow[a\to 0]{} 0 \\
\delta g_{\phi\phi}T_{\phi\phi} &\geq -\left|\frac{\hbar\cdot a^2(t)\cdot \sin^2\theta\cdot d\phi^2}{V^{(4)}}\right| \xrightarrow[a\to 0]{} 0
\end{aligned} \tag{17}$$

If as an example, we have negative pressure, with T_{rr} , $T_{\theta\theta}$, and $T_{\phi\phi}$ < 0, and $p=-\rho$, then the only choice we have, then is to set $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^{+}$**, since there is no way that** $p=-\rho$ is zero valued.

Having said this, the value of δg_{tt} being non zero, will be part of how we will be looking at a lower bound to the graviton mass which is not zero.

4. Lower bound to the graviton mass using Barbour's emergent time

In order to start this approximation, we will be using Barbour's value of emergent time [8,9] restricted to the Plank spatial interval and massive gravitons, with a massive graviton [10]

$$\left(\delta t\right)^2_{emergent} = \frac{\sum_i m_i l_i \cdot l_i}{2\cdot\left(E-V\right)} \to \frac{m_{graviton} l_P \cdot l_P}{2\cdot\left(E-V\right)} \tag{18}$$

Initially, as postulated by Babour [8,9], this set of masses, given in the emergent time structure could be for say the planetary masses of each contribution of the solar system. Our identification is to have an initial mass value, at the start of creation, for an individual graviton.

If $\left(\delta t\right)^2_{emergent} = \delta t^2$ in Eq.(11), using Eq.(11) and Eq. (18) we can arrive at the identification of

$$m_{graviton} \geq \frac{2\hbar^2}{\left(\delta g_{tt}\right)^2 l_P^2} \cdot \frac{(E-V)}{\Delta T_{tt}^2} \tag{19}$$

Key to Eq. (19) will be identification of the kinetic energy which is written as $E-V$. This identification will be the key point raised in this manuscript. Note that [11 raises the distinct possibility of an initial state, just before the 'big bang' of a kinetic energy dominated 'pre inflationary' universe. I.e. in terms of an inflaton $\dot{\phi}^2 >> (P.E \sim V)$ [7] . The key finding which is in [11] is, that, if the kinetic energy is dominated by the 'inflaton' that

$$K.E. \sim (E-V) \sim \dot{\phi}^2 \propto a^{-6} \tag{20}$$

This is done with the proviso that w <-1, in effect, what we are saying is that during the period of the 'Planckian regime' we can seriously consider an initial density proportional to Kinetic energy, and call this K.E. as proportional to [7]

$$\rho_w \propto a^{-3(1-w)} \tag{21}$$

If we are where we are in a very small Planckian regime of space-time, we could, then say write Eq.

(21) as proportional to $g^* T^4$ [7] , with g^* initial degrees of freedom, and T the initial temperature as low

Just before the onset of inflation. The question to ask, then is, what is the value of the initial degrees

of freedom, and what is the temperature, T, at the start of expansion? For what it is worth, the starting

supposition, is that there would then be a likelihood for an initial low temperature regime

5. Multiverse, and answering the Mukhanov hypothesis. Influence of the Einstein spaces

Here, the initial $a_0 \sim a_{initial} \sim 10^{-55}$, or so and so the density in Eq. (21) at Planck time would, be proportional to the Planck Frequency[7]

$$\omega_P = \frac{1}{t_P} = \sqrt{\frac{c^5}{\hbar G}} = 1.85487 \times 10^{43} s^{-1} \sim 1.85 \times 10^{43} Hz \tag{22}$$

This is at the instant of Planck time. We can then ask what would be an initial time contribution before the onset of Planck time. i.e. does Eq. (22) represent the initial value of graviton frequency?

This value of the frequency of a graviton, which would be red shifted enormously would be in tandem with an initial time step of as given by[12]

$$t_{initial} \approx \frac{1}{\sqrt{6\pi\rho_{initial}}} \sim \frac{a_0^2}{\sqrt{6\pi}} \tag{23}$$

This value for the initial time step would be probably lead to Pre Planckian time , i.e. smaller than 10^ -43 seconds, which then leads us to consider, what would happen if a multi verse contributed to initial space-time conditions as seen in Eq. (11) above. If the cosmic fluid approximation as given by Eq. (12) were legitimate, and one could also look at Eq. (13), then

$$m_{graviton} \geq \frac{2\hbar^2}{\left(\delta g_{tt}\right)^2 l_P^2} \cdot \frac{(E-V)}{\Delta T_{tt}^2} \Rightarrow \left(\delta g_{tt}\right)^2 \geq \frac{2\hbar^2}{m_{graviton} \cdot l_P^2} \cdot \frac{(E-V)}{\Delta T_{tt}^2} \tag{24}$$

But, then if one is looking at a multiverse, we first will start at the Penrose hypothesis for a cyclic conformal universe, starting with[13]

$$\begin{aligned}
&\hat{g}^{uv} = \Omega_{uv} g^{uv} \\
&\Omega_{uv}(new-universe) = (\Omega_{uv}{}^{-1} old-universe) \\
&i.e. \\
&\Omega_{uv} \to \Omega_{uv}{}^{-1}(inversion)
\end{aligned} \tag{25}$$

However, in the multiverse contribution to Eq.(12) above, we would have, that

$$\Omega_{uv}{}^{-1} old-universe \to \frac{1}{N}\sum_{j=1}^{N}\left[\Omega_{uv}{}^{-1}(inversion)\right]_j \tag{26}$$

So, does something like this hold ? In a general sense ?

$$\begin{aligned}
&\left(\delta g_{uv}\right)^2\Big|_{initial} \hat{g}^{uv} = \Omega_{uv} g^{uv} \\
&\to \frac{1}{N}\sum_{j=1}^{N}\left[\Omega_{uv}{}^{-1}(inversion)\right]_j g^{uv} \\
&\sim ?? \sim 1/\left[\beta M_{Planck}^2\right] + \varepsilon^{+}
\end{aligned} \tag{27}$$

If the fluid approximation as given in Eq. (12) and Eq. (13) hold, then Eq. (27) conceivably could be identifiable as linkable to.

$$\delta t\Delta E \geq \frac{\hbar}{(\delta g_{tt})} \equiv \frac{N\cdot\hbar}{\left(\left(\sum_j \left[\Omega_{tt}^{-1}\right]_j\right)\cdot\delta g_{tt}\right)}$$
$$\Leftrightarrow \delta t \geq \frac{1}{\Delta E}\cdot\frac{N\cdot\hbar}{\left(\left(\sum_j \left[\Omega_{tt}^{-1}\right]_j\right)\cdot\delta g_{tt}\right)} \approx \frac{m_{graviton}l_P\cdot l_P}{2\cdot(E-V)} \tag{28}$$

If we could write, say

$$\frac{1}{\Delta E}\cdot\frac{N\cdot\hbar}{\left(\left(\sum_j \left[\Omega_{tt}^{-1}\right]_j\right)\cdot\delta g_{tt}\right)} = \frac{N\cdot\hbar/\Delta E}{\left(\sum_j \left[\Omega_{tt}^{-1}\cdot(\delta g_{tt})\right]_j\right)} \sim \frac{m_{graviton}l_P\cdot l_P}{2\cdot(E-V)}$$
$$\Leftrightarrow \Delta E \sim 2\cdot(E-V)$$
$$\&$$
$$m_{graviton}l_P\cdot l_P \sim \frac{N\cdot\hbar}{\left(\sum_j \left[\Omega_{tt}^{-1}\cdot(\delta g_{tt})\right]_j\right)} \tag{29}$$

Then, if each j is the jth contribution of N “multiverse” contributions to a new single universe being nucleated, one could say that there was, indeed, likely an “averaging” and that the causal barrier which Mukhanov spoke of, as to each δt, and actually to each graviton entering into the present universe , one could mathematically average out the results of a sum up of each of the contributions from each prior to a present universe, according to

$$\frac{N\cdot\hbar}{\left(\sum_j \left[\Omega_{tt}^{-1}\cdot(\delta g_{tt})\right]_j\right)} \equiv \frac{\hbar}{\frac{1}{N}\cdot\sum_j \left[\Omega_{tt}^{-1}\cdot(\delta g_{tt})\right]_j} \tag{30}$$

If Eq. (30) held, then we could then write

$$\delta t \geq \frac{1}{\Delta E}\cdot\frac{\hbar}{\left(\frac{1}{N}\sum_j \left[\Omega_{tt}^{-1}\cdot\delta g_{tt}\right]_j\right)} \approx \frac{m_{graviton}l_P\cdot l_P}{2\cdot(E-V)} \tag{31}$$

Instead, we have, Eq. (28), and that it is safe to say that for each collapsing universe which might contribute to a re cycled universe that the following inequality is significant.

$$\frac{1}{N}\cdot\sum_j \left[\Omega_{tt}^{-1}\cdot(\delta g_{tt})\right]_j \neq \frac{1}{N}\cdot\sum_j \left[\Omega_{tt}^{-1}\right]_j\cdot\delta g_{tt} \tag{32}$$

Hence, the absence of an averaging procedure, due to a multiverse, would then rule against a causal barrier, as was maintained by Mukhanov, in his discussion with the author, in Marcel Grossman 14, in Italy. Then the possible approximation say of

$$\left(T_{tt}\right)^2 \sim \omega_{graviton}^2 \propto \beta M_{Planck}^2 \sim \left(t_{initial} \approx \frac{1}{\sqrt{6\pi\rho_{initial}}} \sim \frac{a_0^2}{\sqrt{6\pi}} \right)^{-2} \tag{33}$$

Would not hold, and that in itself may lead to a break down of the Causal barrier hypothesis of Mukhanov, which the author emphatically disagreed with.

6. Conclusion. Considering Eq. (6) and Eq. (11) in lieu of Einstein space, and further research questions

A way of solidifying the approach given here, in terms of early universe GR theory is to refer to Einstein spaces, via [14] as well as to make certain of the Stress energy tensor [15] as we can write it as a modified Einstein field equation. With, then $\aleph$ as a constant.

$$R_{ij} = \aleph g_{ij} \tag{34}$$

Here, the term in the Left hand side of the metric tensor is a constant, so then if we write, with R also a constant [15]

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{ij}} = -\frac{1}{8\pi}\cdot\left[\aleph - R + \Lambda\right]\cdot g_{ij} \tag{35}$$

The terms, if we use the fluid approximation given by Eq. (12) as well as the metric given in Eq. (9) will then tend to a constant energy term on the RHS of Eq. (35) as well as restricting i, and j, to t and t

So as to recover ,via the Einstein spaces, the seemingly heuristic argument given above. Furthermore when we refer to the Kinetic energy space as an inflaton $\dot{\phi}^2 >> (P.E \sim V)$ [7] , we can also then utilize the following operator equation for the generation of an 'inflaton field' given by the following set of equations

$$\begin{aligned}
&\phi(t,\cdot) = \cos(t\sqrt{K}) f + \frac{\sin(t\sqrt{K})}{\sqrt{K}} g \\
&f(x) = \phi(0,x) \\
&g(x) = \frac{\partial\phi(0,x)}{\partial t} \\
&-\frac{\partial^2\phi}{\partial t^2} = K\phi
\end{aligned} \tag{36}$$

In the case of the general elliptic operator K if we are using the Fulling reference, [16] in the case of the above Roberson-Walker metric, with the results that the elliptic operator, in this case become,

$$
\begin{aligned}
K &= -\nabla^2 + (m^2 + \xi R) \\
&= -\sum_{i,j} \frac{\partial_i \left(g^{ij} \sqrt{|\det g|} \partial_j \right)}{\sqrt{|\det g|}} + (m^2 + \xi R) \\
&\xrightarrow[i,j \to t,t]{} -\frac{\partial^2}{\partial t^2} + (m^2 + \xi R)
\end{aligned}
\tag{37}
$$

Then, according to [16] , if R above, in Eq. (37) is initially a constant, we will see then, if m is the inflation mass, that

$$
\begin{aligned}
&\phi(t,\cdot) = \cos(t\sqrt{K}) f \\
&-\frac{\partial^2}{\partial t^2} \to \omega^2 \\
&\Leftrightarrow \phi(t,\cdot) = \cos(t\sqrt{\omega^2 + (m^2 + \xi R)})
\end{aligned}
\tag{38}
$$

Then c_1 as an unspecified, for now constant will lead to a first approximation of a Kinetic energy dominated initial configuration, with details to be gleaned from [16,17,18] to give more details to the following equation, **R** here is linked to curvature of spacetime, and **m** is an inflaton mass, connected with the field $\phi(t,\cdot) = \cos(t\sqrt{K}) f$ with the result that

$$
\dot{\phi}^2(t,\cdot) \approx \left[\omega^2 + (m^2 + \xi R) \right] \cdot c_1 >> V(\phi) \tag{39}
$$

If the frequency, of say, Gravitons is of the order of Planck frequency as in Eq.(22) , then this term, would likely dominate Eq.(39). More of the details of this will be worked out, and also candidates for the $V(\phi)$ will be ascertained, most likely, we will be looking the Rindler Vacuum as specified in [19] as well as also details of what is relevant to maintain local covariance in the initial space-time fields as given in [20]

Why is a refinement of Eq. (39) necessary?

The details of the elliptic operator K will be gleaned from [16,17,18] whereas the details of inflaton $\dot{\phi}^2 >> (P.E \sim V)$ [7] are important to get a refinement on the lower mass of the graviton as given by the left hand side of Eq. (24) . We hope to do this in the coming year. The mass, **m**, in Eq.(37) for the inflaton, not the Graviton, so as to have links to the beginning of the expansion of the universe. We look to what Corda did, in [21] for guidance as to picking values of **m** relevant to early universe conditions.

Finally, as far as Eq. (39) is concerned, there is one serious linkage issue to classical and quantum mechanics, which should be the bridge between classical and quantum regimes, as far as space time applicability. Namely, from Wald(19), if we look at first of all arbitrary operators, A and B

$$\left(\Delta A\right)^{2} \cdot \left(\Delta B\right)^{2} \geq \left(\frac{1}{2i}\left\langle\left[A,B\right]\right\rangle\right) \tag{40}$$

As we can anticipate, the Pre Planckian regime may the place to use classical mechanics, and then to bridge that to the Planckian regime, which would be quantum mechanical. Taking [19] again, this would lead to a sympletic structure via the following modification of the Hamilton equations of motion, namely we will from (19) get the following re write,

$$\begin{aligned}
&\frac{dq_{\mu}}{dt} = \frac{\partial H}{\partial p_{\mu}}, \quad \frac{dp_{\mu}}{dt} = -\frac{\partial H}{\partial q_{\mu}} \\
&H = H(q_{1,\ldots\ldots,}q_{n}; p_{1,\ldots\ldots,}p_{n}) \\
&y = (q_{1,\ldots\ldots,}q_{n}; p_{1,\ldots\ldots,}p_{n}) \\
&\Omega^{\mu\nu} = 1, if\, \nu = \mu + n \\
&\Omega^{\mu\nu} = 0, otherwise \\
&\frac{dy^{\mu}}{dt} = \sum_{\nu=1}^{n} \Omega^{\mu\nu} \frac{\partial H}{\partial y^{\nu}}
\end{aligned} \tag{41}$$

Then there exists a re formulation of the Poisson brackets, as seen by

$$\left\{f,g\right\} = \Omega^{\mu\nu}\nabla_{\mu} f \nabla_{\nu} g \tag{42}$$

So, then the following , for classical observables, f, and g, we could write, by [19]

$$\begin{aligned}
&\wedge : \Theta \rightarrow \hat{\Theta} \\
&\Theta = classical - observable \\
&\hat{\Theta} = quantum - observable \\
&\hbar = 1 \\
&\left[\hat{f}, \hat{g}\right] = i \cdot \left(\widehat{\{f, g\}}\right)
\end{aligned} \tag{43}$$

Then, we could write, say Eq. (40) and Eq.(43) as

$$\begin{aligned}
&\left[\hat{f}, \hat{g}\right] = i \cdot \left(\widehat{\{f, g\}}\right) \\
&f = classical - observable \\
&\hat{f} = quantum - observable \\
&\left(\Delta\hat{f}\right)^{2} \cdot \left(\Delta\hat{g}\right)^{2} \geq \left(\frac{1}{2i}\left(\left\langle\left[\hat{f}, \hat{g}\right]\right\rangle\right)\right) = \left(\frac{1}{2}\left(\left(\widehat{\{f, g\}}\right)\right)\right)
\end{aligned} \tag{44}$$

If so, then we can set, in the interconnection between the Planck regime, and just before the Planck regime, say, by setting classical variables, as given by

$$f = -\frac{\left[\aleph - R + \Lambda\right]\cdot g_{tt}}{8\pi} \qquad (45)$$
$$g = \delta g_{tt}$$

Then by utilization of Eq.(44) we may be able to effect more precision in our early universe derivation, especially making use of derivational work, in addition as to what is given here, as to understand how to construct a very early universe partition function Z based upon the inter relationship between Eq.(44) and Eq.(45) so as to write up an entropy based upon, as given in [19]

$$S(entropy) = \ln Z + \beta E \qquad (46)$$

If this program were affected, with a first principle construction of a partition function , we may be able to answer if Entropy were zero in the Planck regime, or something else, which would give us more motivation to examine the sort of partition functions as stated in [22, 23].See **appendix A** as to possible scenarios. Here keep in mind that in the Planck regime we have non standard physics. Appendix A indicate that due to the variation we have worked out in the Planckian regime of space-time that the initial entropy is not zero. The consequences of this show up in this paper's **Appendix B**, as to a specific formulation of the Ricci scalar. The consequences of Appendix A and **Appendix B** may be for a small cosmological constant, and large " Hubble expansion" that there would be an initially large magnitude of cosmological pressure, even if negative, which would give credence to a non zero cosmological entropy, that if large negative pressure, even in the Pre Planckian regime will lead to a large ΔT_{tt} terms which would show up in Eq. (1A), even if we used a partition function based upon Lattice Hamiltonians, as on page 135 of [26] which would usually in a lattice gauge arrangement would have considerably smaller contributions than ΔT_{tt}. Note the conditions of flat space, are that Eq.(B9) almost vanishes due to the behavior of the numerator, no matter how small $a_{initial}^2$ is. The supposition is that the numerator becomes far smaller than $a_{initial}^2$ The initiation of conditions of flat space, is also the regime in which we think that non zero entropy is started, and **Appendix C** gives an initial estimate of what we think Entropy would be in the aftermath of the uncertainty relationship we have outlined in this article. I.e. to first order, $S_{initial(graviton)} \sim 10^{37}$. We finalize our treatment as of space-time fluctuations and geometry by considering the applications of **Appendix D** to graviton mass, and **Appendix E** to the Riemann-Penrose inequality for conditions as to a minimum frequency, as a consequence of cosmological evolution, and what it portrays as consequences for Electromagnetic fields. Appendix D and E give varying initial graviton masses as a starting point, with Appendix D giving a higher initial graviton mass than what is assumed as of today. Finally, **Appendix F** states a pre Planckian kinetic energy so the inflaton $\dot{\phi}^2 >> (P.E \sim V)$ [7]. This last step, so important to our development will be considerably refined in a future document.

Appendix A, scenarios as to the value of entropy in the beginning of space-time nucleation

We will be looking at inputs from page 290 of [23] so that if $E \sim M \sim \Delta T_{tt} \cdot \delta t_{time} \cdot \Delta A \cdot l_P$

$$S(entropy) = \ln Z + \frac{\left(E \sim \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot l_P\right)}{k_B T_{temperature}} \tag{1A}$$

And using Ng's infinite quantum statistics, we have to first approximation [24, 25]

$$\begin{aligned} S(entropy) &\sim \ln Z + \frac{\left(\left(E \sim \Delta T_{tt}\right) \cdot \delta t \cdot \Delta A \cdot l_P\right)}{k_B T_{temperature}} \\ &\sim \ln Z + \left(\frac{\hbar}{k_B T_{temperature} \delta g_{tt}}\right) \\ &\xrightarrow[T_{temperature} \to \# anything]{} \left[S(entropy) \sim n_{count}\right] \neq 0 \end{aligned} \tag{2A}$$

This is due to a very small but non vanishing δg_{tt} with the partition functions covered by [23], and also due to [24,25] with n_{count} a non zero number of initial 'particle' or information states, about the Planck regime of space-time, so that the initial entropy is non zero.

Appendix B, calculation of the Ricci Tensor for a Roberson-Walker space-time, with its effect upon the measurement of if or not a space time, is open, closed or flat.

We begin with Kolb and Turner [7] discussion of the Roberson-Walker metric, say page 49 with, if R is the Ricci scalar, and k the measurement of if we have a close, open, or flat universe, that if

$$a = a_{initial} \cdot \exp(H \cdot t) \tag{B1}$$

Then by [7]

$$H^2 = -\frac{k}{a^2} + \frac{8\pi G\rho}{3} \tag{B2}$$

$$3H^2 + \left[\frac{2k}{a^2} + \frac{R}{6}\right] = 0 \tag{B3}$$

Leading to

$$a^2 = \frac{1}{k} \cdot \left[\frac{R}{6} + 8\pi G\rho\right] \tag{B4}$$

If $\rho = -p$ [7], then with a bit of algebra

$$|p| = \frac{1}{8\pi G} \cdot \left[\frac{R}{6} + \left(a_{initial}\right)^2 \cdot \exp\left[\sqrt{\frac{4\Lambda}{3}} \cdot t_{time}\right]\right] \tag{B5}$$

Next, using [27] , on page 47, at the boundary between Pre Planckian to Planckian space-time we will find

$$R = 8\pi \cdot \left(T_0^0 + T_1^1 + T_2^2 + T_3^3 \right) + 4\Lambda \xrightarrow[\mathrm{Pr}e-Planckian-Conditions]{} 8\pi \cdot \left(T_0^0 \right) + 4\Lambda \tag{B6}$$

Then, we can obtain

Right at the start of the Planckian era,

$$|p|_{Planckian} \sim \frac{1}{8\pi G} \cdot \left[\frac{8\pi \cdot \left(T_0^0 + T_1^1 + T_2^2 + T_3^3 \right) + 4\Lambda}{6} \right] \tag{B7}$$

The consequences of this would be that right after the entry into Planckian space time, that there would be the following change of pressure

$$\begin{aligned}
&|p|_{\mathrm{Pr}e-Planckian} = \frac{1}{8\pi G} \cdot \left[\frac{8\pi \cdot \left(T_0^0 \right) + 4\Lambda}{6} + \left(a_{initial} \right)^2 \cdot \exp\left[\sqrt{\frac{4\Lambda}{3}} \cdot t_{time} \right] \right] \\
&\Rightarrow |p|_{\mathrm{Pr}e-Planckian} \sim \frac{1}{8\pi G} \cdot \left| \frac{8\pi \cdot \left(T_0^0 \right) + 4\Lambda}{6} + 0^+ \right| \\
&|p|_{Planckian} \sim \frac{1}{8\pi G} \cdot \left| \frac{8\pi \cdot \left(T_0^0 + T_1^1 + T_2^2 + T_3^3 \right) + 4\Lambda}{6} \right| \\
&\Delta P = |p|_{Planckian} - |p|_{\mathrm{Pr}e-Planckian} \sim \left[\frac{\left(T_1^1 + T_2^2 + T_3^3 \right)}{6G} \right]
\end{aligned} \tag{B8}$$

Then, the change in the k term would be like, say, from Pre Planckian to Planckian space time

$$\Delta k = \frac{1}{a_{initial}^2} \cdot \left[8\pi G(\rho - \Delta P) \right] \tag{B9}$$

This goes almost to zero if the numerator shrinks far more than the denominator, even if the initial scale factor is of the order of 10 ^ - 110 or so.

Appendix C. Initial entropy, from first principles.

We are making use of the Padmanabhan publication of [28, 29] where we will make use of

$$\rho_\Lambda \approx \frac{G E_{system}{}^6}{c^8 \hbar^4} \Leftrightarrow \Lambda \approx \frac{1}{l_{Planck}^2} \cdot \left(E_{system} / E_{Planck} \right)^6 \tag{C1}$$

Then, if E_{system} is for the energy of the Universe after the initiation of Eq.(11) as a bridge between Pre Planckian, to Planckian physics regimes we could write, then

$$E_{system} \propto n_{gravitons} \cdot m_{graviton}$$
$$\Lambda \approx \frac{1}{l^2_{Radius-Universe-today}} \tag{C2}$$
$$\Leftrightarrow m_{graviton} \sim 10^{-62}\, grams \Rightarrow n_{gravitons} \sim 10^{37}$$
$$\Rightarrow S_{initial(graviton)} \sim 10^{37}\, at-Planck-time$$

The value of initial entropy, $S_{initial(graviton)} \sim 10^{37}$ should be contrasted with the entropy for the entire Universe as given in [30] below.

Appendix D. : Information flow, Gravitons, and also upper bounds to Graviton mass

Here we can view the possibility of considering the following, namely [31] is extended by [32] so we can we make the following identification?

$$N = N_{graviton}\Big|_{r_H} = \frac{c^3}{G\cdot\hbar}\cdot\frac{1}{\Lambda} \approx \frac{1}{\Lambda} \tag{D1}$$

Should the N above, be related to entropy, and Eq. (8) This supposition has to be balanced against the following identification, namely, as given by T. Padmanabhan[28, 29]

$$\Lambda_{Einstein-Const.Padmanabhan} = 1/l^2_{Planck}\cdot\left(E/E_{Planck}\right)^6 \tag{D2}$$

But should the energy in the numerator in Eq. (D2) be given as say by (C2) , of Appendix C, we have defacto quinessence. then there would have been defacto quintessence, i.e. variation in the "Einstein constant" , which would have a large impact upon mass of the graviton, with a sharp decrease in g_* being consistent with an evolution to the ultra light value of the Graviton , with initial frequencies of the order of say for wavelength values initially the size of an atom,

$$\omega_{initial}\Big|_{r_H \sim atomic-size} \sim 10^{21}\, Hz \tag{D3}$$

The final value of the frequency would be of a magnitude smaller than one Hertz, so as to have value of the mass of the graviton would be then of the order of 10^-62 grams [10] , due to Eq.(D2) approaching [31] below, namely

$$\Lambda_{Einstein-Const.} = 1/l^2_{Radius-Universe} \tag{D4}$$

Leading to the upper bound of the Graviton mass of about 10^-62 grams [31, 32]in the present era

$$m_{graviton} = \frac{\hbar}{c} \cdot \sqrt{\frac{(2\Lambda)}{3}} \approx \sqrt{\frac{(2\Lambda)}{3}} \qquad \text{(D5)}$$

Eq. (D5) has a different value if the entropy / particle count is lower , as has been postulated in this note. But the value of Eq.(D5) becomes the Graviton mass of about 10^-62 grams [10] in the present era which is in line with the entropy being far larger in the present era [30]

Appendix E. : Applying the Riemannian Penrose Inequality with applications in our fluctuation.

If from Giovannini [33] we can write

$$\delta g_{tt} \sim a^2(t) \cdot \phi << 1 \qquad \text{(E1)}$$

Refining the inputs from Eq.(E1) means more study as to the possibility of a non zero minimum scale factor [34] , as well as the nature of ϕ as specified by Giovannini [33] . We hope that this can be done as to give quantifiable estimates and may link the non zero initial entropy to either Loop quantum gravity "quantum bounce" considerations [35] and/or other models which may presage modification of the sort of initial singularities of the sort given in [1]. Furthermore if the non zero scale factor is correct, it may give us opportunities as to fine tune the parameters given in [34] below.

$$\begin{gathered} \alpha_0 = \sqrt{\frac{4\pi G}{3\mu_0 c}} B_0 \\ \hat{\lambda}(defined) = \Lambda c^2 / 3 \end{gathered} \qquad \text{(E2)}$$

$$a_{\min} = a_0 \cdot \left[\frac{\alpha_0}{2\hat{\lambda}(defined)} \left(\sqrt{\alpha_0^2 + 32\hat{\lambda}(defined) \cdot \mu_0 \omega \cdot B_0^2} - \alpha_0 \right) \right]^{1/4}$$

Where the following is possibly linkable to minimum frequencies linked to E and M fields [34] , and possibly relic Gravitons

$$B > \frac{1}{2 \cdot \sqrt{10\mu_0 \cdot \omega}} \qquad \text{(E3)}$$

So, now we investigate the question of applicability of the Riemann Penrose inequality which is [36] , p431, which is stated as

Riemann Penrose Inequality: Let (**M, g**) be a complete , asymptotically flat 3- manifold with Non negative-scalar curvature, and total mass m, whose outermost horizon Σ has total surface area A. Then

$$m_{total-mass} \geq \sqrt{\frac{A_{surface-Area}}{16\pi}} \tag{E4}$$

And the equality holds, iff (**M, g**) is isometric to the spatial isometric spatial Schwartzshield manifold **M** of mass **m** outside their respective horizons.

Assume that the frequency, say using the frequency of Eq.(E3) , and $A \approx A_{\min}$ of Eq.(E4) is employed. So then say we have , if we use dimensional analysis appropriately, that

$$\begin{aligned}&\left(v = velocity \equiv c\right) = f\,(frequency) \times \lambda(wavelength)\\ &\Rightarrow \omega \approx \omega_{initial} \sim \frac{c}{d_{\min}} \sim \left.\frac{1}{d_{\min}}\right|_{c\equiv 1} \quad \& \, d_{\min} \sim A^{1/3} \propto a_{\min}\end{aligned} \tag{E5}$$

Assume that we also set the input frequency as to Eq. (E3) as according to $10 < \zeta \leq 37$ i.e. does

$$\begin{aligned}&\left(m_{total-mass} \sim 10^{\zeta} \cdot m_{graviton}\right)^2 \propto a_{\min}^3 / 16\pi\\ &\Leftrightarrow \omega \approx \omega_{initial} \sim \frac{1}{d_{\min}} \sim \left(16\pi \times 10^{\zeta} \cdot m_{graviton}\right)^{-2/3}\end{aligned} \tag{E6}$$

Our supposition is that Eq.(E6) should give the same frequency as of Eq. (D3) above. So if we have in

In doing this, this is a frequency input into Eq. (E3) above where we are safely assuming a graviton mass of about [10]

$$\begin{aligned}&m_{total-mass} \sim 10^{37} \cdot m_{graviton}\\ &m_{graviton} \sim 10^{-62}\, grams\end{aligned} \tag{E7}$$

Does the following make sense ? I.e. look at , when $10 < \zeta \leq 37$

$$\begin{aligned}&\left(m_{total-mass} \sim 10^{\zeta} \cdot m_{graviton}\right)^2 \propto a_{\min}^3 / 16\pi\\ &\Leftrightarrow \omega \approx \omega_{initial} \sim \frac{1}{d_{\min}} \sim \left(16\pi \times 10^{\zeta} \cdot m_{graviton}\right)^{-2/3}\end{aligned} \tag{E8}$$

We claim that if this is an initial frequency and that it is connected with relic graviton production, that the minimum frequency would be relevant to Eq. (E3), and may play a part as to admissible B fields

Note, if Appendix D is used, this makes a re do of Eq. (E8) which is a way of saying that the graviton mass given by [10] no longer holds.

In either case, Eq. (E8) and Eq. (E3) in some configuration may argue for implementation of work the author did in reference [37] as to relic cylindrical GW, i.e. their allowed frequency and magnitude, so considered.

Appendix F : First principle treatment of pre Planckian kinetic energy so the Inflaton $\dot{\phi}^2 >> (P.E \sim V)$ [7]

We give this as a plausibility argument which undoubtedly will be considerably refined, but its importance cannot be overstated. I.e. this is for Pre inflationary , Pre Planckian physics, so as to get a lower bound to the Graviton mass. To do this, we look at what [7] is saying and also we will be enlisting a new reference, [38] , by Bojowald, and also T. Padmanbhan [39] as to details to put in, so as to confirm a dominance of Kinetic energy. Start with a Friedman equation of

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k_{curvature}}{a^2} = \frac{4\pi G}{3} \cdot \frac{p_\phi^2}{a^6} + \Lambda \tag{F1}$$

We will treat, then the Hubble parameter, as

$$\left(\frac{\dot{a}}{a}\right) = H_{initial} \equiv \frac{2}{t \cdot \left(1 + \frac{P}{\rho}\right)} \xrightarrow{P = -\rho + \varepsilon^+} \frac{2}{t \cdot \left(\frac{\varepsilon^+}{\rho}\right)} \xrightarrow{t \to t_P} \frac{2\rho}{t_P \cdot \varepsilon^+} \tag{F2}$$

Now from Padmanabhan, [39], we can write density, in terms of flux according to

$$\frac{d\rho}{dt} = \frac{1}{V^{(3)} = Volume} \cdot (A = Area) \cdot (\Im = Flux) \sim \frac{(\Im = Flux)}{l_P} \tag{F3}$$

Then using 463 of [39], if T is temperature, here, then if N is the particle count in the flux region per unit time (say Planck time), as well as using the 'ideal gas law' approximation, for superhot conditions

$$\begin{aligned}
&\frac{d\rho}{dt} = \frac{1}{V^{(3)} = Volume} \cdot (A = Area) \cdot (\Im = Flux) \sim \frac{(\Im = Flux)}{l_P} \\
&\rho \sim \frac{(\Im = Flux)}{c} \\
&\Rightarrow H = \frac{N}{\varepsilon^+} \cdot \frac{1}{V^{(4)} = 4 - Dim \quad Volume} \cdot \sqrt{\frac{8}{\pi}} \sqrt{\frac{k_B T}{m_{flux-particle}}}
\end{aligned} \tag{F4}$$

Next, according to [38], we can make the following substitution.

$$p_{\phi} = a^3 \cdot \dot{\phi} \quad \text{(F5)}$$

Therefore, if

$$\dot{\phi}^2 \approx a^{-6} \cdot (12\pi G) \cdot V^{(4)} \cdot \left(H^2 + |\Lambda|\right)$$
$$\approx a^{-6} \cdot (12\pi G) \cdot V^{(4)} \left(\left[\frac{N}{\varepsilon^+} \cdot \frac{1}{V^{(4)} = 4 - Dim \quad Volume} \cdot \sqrt{\frac{8}{\pi}} \sqrt{\frac{k_B T}{m_{flux-particle}}} \right]^2 + |\Lambda| \right) \quad \text{(F6)}$$

If the scale factor is very small, say of the order of $a = a_{initial} \sim 10^{-55}$, then no matter how fall the initial volume is, in four space (it cancels out in the first part of the brackets), its easy to see then that $\dot{\phi}^2 >> (P.E \sim V)$ **[7]**

We will in the future add more structure to this calculation so as to confirm via a precise calculation that the lower bound to the graviton mass, is about 10^-70 grams. This value of 10^-70 grams is an approximation, via dimensional analysis of

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