Apparent Space and Time Alterations under the Classical Theories of Light

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ABSTRACT

Applying basic classical physics concepts of time and space, while considering event information was ultimately communicated through light—or electromagnetic—signals, it was shown that proper time interval measured in an inertial reference frame (labeled as the "traveling frame") between two co-local events occurring at the frame origin underwent apparent alteration and perceived as an altered interval in another inertial frame (labeled as the "stationary frame"), when the two frames were in relative motion. It was shown through obtained modified Galilean transformations that "apparent" length contraction and expansion were associated with "apparent" time dilation and contraction, respectively. In the case the Emission Theory of light was considered, symmetry in regard to the time and space alteration factors between the frames was shown. The known classical Doppler Effect was readily derived from the established alteration factors. For all classical approaches, and in the case of light, the wave length was deemed invariant.

In the case the Special Relativity approach was considered, i.e. when the speed of light was assumed constant with respect to all inertial reference frames, inconsistent time "alterations" were perceived, so an *ad hoc* assumption was required, imposing an artificial conversion factor, leading to the Lorentz transformation, applicable under special conditions of the space coordinates in the direction of motion. Misconceptions in the Special Relativity interpretation of the Lorentz transformation were systematically revealed. Time alteration was perceived dilated for receding frames, and contracted for approaching ones.

When the frames receded and then approached during equal proper time intervals, the net time interval is perceived dilated by γ for the Special Relativity approach, as opposed to γ^2 for the Light Emission theory. For the Ether Theory assumption, either time dilation or time invariance was obtained depending on whether the traveling frame was taken to be the ether frame.

The known relativistic Doppler Effect was readily derived. For the case of light, the perceived frequency exhibiting a blue shift in the case of approaching frames was in line with the established time contraction in this study, contradicting the Special Relativity prediction of time dilation. In addition, the wavelength exhibited an increase in the case of receding frames, whereas it decreased when the source was approaching.

CONTENTS

1 BACKGROUND

Let the time associated with an event occurring at a fixed point in a reference frame (i.e., at rest with respect to the frame), and measured in the same frame, be called the event proper [occurrence] time; and the frame time interval between two events occurring at the same or separate fixed point(s) in the frame be called the events proper time interval.

An event that has occurred at a certain proper time with respect to a reference frame—let's label it as the traveling frame— will be observed in a different frame—labelled as the stationary frame— at a different perceived time depending on the distance between the two frames, the instantaneous variation of this distance, and the speed of information propagation from the "traveling" to the "stationary" frame. If the event can be communicated thru signals that travel at an ultimate finite speed, then the information cannot possibly be perceived by any means in the stationary frame before a time equal to the event proper time in the traveling frame incremented by the signal travel time at the ultimate speed to reach the second frame.

If two events, separated by a certain proper time interval, have occurred in the traveling frame, the formerly event will be perceived in the stationary frame after a certain time increment, and if the frames are moving relative to each other, the next event will be perceived after a time interval larger or smaller than the proper interval, depending on whether the frames are receding from or approaching each other.

An observer in the stationary frame should then correct the perceived time interval, so as the absolute time can be calculated, using corrected equations relating the frames space coordinates.

If the apparent time variations between the reference frames were to be taken into consideration and incorporated in the classical Galilean transformation converting space coordinates from one reference frame to another, this transformation shall take different corrected forms, depending on the assumptions made on the conveyance of events from one frame to another. Ultimately, events are transmitted via light or electromagnetic signals; there is no known faster means of communicating information. So, if event signals are assumed to propagate at no faster than the speed of light, considered as the ultimate communication speed, then the perceived event time shall depend on the proper time, and the speed of light and its traveled distance as it travels from the "traveling" to the "stationary" frame.

The self-posed question would then be; what if there's some other means to communicate events at a speed faster than the speed of light? Or, what if there were no means to detect light signals in the stationary frame? For instance, suppose the frames are traveling in a medium where communication could be carried out through sound signals only; then time will be perceived totally differently. The traveling frame proper time is fixed and independent of the communication signal speed. Whereas, the corresponding measured time in the stationary frame is variable, and depends on the communication signal speed, as well as the relative speed between the frames. Thus, the available communication means will dictate the time transformation equations, and time will become malleable, taking different shapes according to the imposed communication means. Hence, the idea of absolute time is definitely essential to save the sanity in the concept of time! This should justify why the time variable perceived as a function of the event communication speed is only apparent. In this paper, apparent time based on events communication at the speed of light is considered.

In classical physics, there are two principal theories governing the nature and behavior of light; the Emission Theory and the Ether Theory.

In the light Emission Theory, also known as the ballistic theory, often attributed to Isaac Newton for his Corpuscular theory, light is assumed to exhibit a nature incorporating a corpuscular behavior. Under this conjecture, light is emitted at a constant speed c relative to its emission source. So, if the source (emitter) is moving at a speed v relative to an observer, light will travel towards the observer at a speed of $c \pm v$. Hence, there is no preferred reference frame for light propagation.

On the other hand, the Ether Theory was prevailed in the 19th century when the electromagnetic wave nature of light had been established and described by the Maxwell's equations. The corpuscular nature of light conjecture had been dropped. Since waves must propagate through a medium, then the ether, an assumed medium required to carry light waves, was supposed to fill the entire interstitial space. Therefore, light must propagate at a constant speed c relative to the ether rest frame. Hence, the speed of light with respect to a certain reference frame would depend on the speed of that frame with respect to the ether rest frame. However, doubts about the Ether Theory had been raised following the negative results of the famous Michelson-Morley experiment[1] designed to verify the Ether Theory by attempting to detect directional variations in the light speed relative to the earth, supposedly moving with respect to the ether. The Special Relativity theory came later on to replace the Ether Theory, introducing new concepts of space and time.

In the theory of Special Relativity,[2] Einstein postulated that there was no preferred reference frame for light propagation (first postulate: physics laws are the same in all inertial reference frames), and that the speed of light was independent of the source state of motion. Hence, the speed of light would be always the same and equal to a universal constant c in all inertial reference frames (second postulate). Consequently, since an observer measures the same speed of light in his rest frame and in another traveling inertial frame, space and time in the latter must be deformed with respect to the observer in order for this speed invariance to be maintained.

Based on the Special Relativity postulates, the Lorentz transformation, a set of space-time equations to convert coordinates between two inertial frames of reference in relative motion, predicting time dilation and length contraction under particular interpretations, was derived.[2]

In this paper, it is demonstrated that apparent time, as well as length, alterations (dilation or contraction) are the natural consequence of assuming that events are ultimately communicated via light —or electromagnetic wave— signals. Each of the above light theories would give different extents of time and space alterations between inertial reference frames in relative motion. Modified versions of the Galelian transformation are obtained. Although the approach used leads to the Lorentz transformation in the case of the Special Relativity assumption for the light speed, several misconceptions with the Special Relativity interpretations are revealed.

2   TIME INTERVAL ALTERATION

Consider two inertial frames of reference, $K\left(x,y,z,\right)$ and $K'\left(x',y',z'\right)$, in relative translational motion with parallel corresponding axes, and let their origins be aligned along the overlapped x- and x'-axes. Let v be the relative motion velocity in the direction of the x-x' axis oriented in such a way that with respect to an observer in K (i.e., K is "stationary" relative to this observer), the relative travel direction of K' ("traveling" frame) is in the positive x-x' direction when the frames are receding; this arrangement shall be referred to as the "forward scenario". For an observer in K' (i.e., K' is "stationary" relative to this observer), the relative travel direction of K ("traveling" frame) would be in the negative x-x' direction when the frames are receding; this arrangement shall be referred to as the "backward scenario". The objective is to determine how a proper time interval measured in the traveling frame would be perceived in the stationary frame.

2.1   GENERAL TIME ALTERATION FACTOR DERIVATION FOR RECEDING FRAMES

Suppose that K and K' are overlapping at the time $t = t' = 0$. The event coordinates can then be considered as space and time intervals measured from the initial zero coordinates of the overlapped-frames event.

2.1.1   Perception in K (stationary frame) of a proper time interval for events in K' (traveling frame)—receding frames "forward scenario"

Suppose a signal of an event $E'(0,0,0)$ is emitted from K' origin at time t' with respect to K', which will be perceived at time t in K.

Let the speed of the light signal traveling from K' to K be $c'_{K' \to K}$ with respect to K', and $c_{K' \to K}$ with respect to K.

K' perspective

From the perspective of K', the origin of K' at the event occurring time is at a distance of vt' from that of K (Fig. 1).
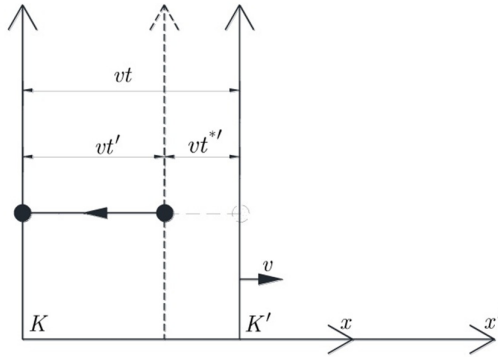
Fig. 1 Signal propagation—receding frames

Let $t^{*'}$ be the time interval it takes the event signal to reach the origin of K, from K' perspective. By that time, K' will have moved a further distance of $vt^{*'}$ away from K, bringing the distance traveled by the signal to $vt' + vt^{*'}$ with respect to K'. The time interval $t^{*'}$ can then be expressed as

$$t^{*'} = \frac{vt' + vt^{*'}}{c'_{K' \to K}},$$

leading to

$$t^{*'} = \frac{vt'}{c'_{K' \to K} - v}.$$

It follows that the event perception time t in K with respect to K' will be given by

$$t = t' + t^{*'};$$

$$t = t'\left(\frac{c'_{K'\to K}}{c'_{K'\to K} - v}\right);$$

$$t = \frac{t'}{1 - \dfrac{v}{c'_{K'\to K}}}, \tag{1}$$

exhibiting time dilation.

K perspective

Now, from the perspective of K, the origin of K' at the event occurring time is at a distance of vt' from that of K. The signal will have traveled a distance of vt', at the speed of $c_{K'\to K}$ with respect to K, when it reaches K origin. Therefore, the event will be perceived at time t in K, given by

$$t = t' + \frac{vt'}{c_{K'\to K}};$$

$$t = t'\left(1 + \frac{v}{c_{K'\to K}}\right), \tag{2}$$

dilated with respect to t'.

2.1.2   Perception in K' (stationary frame) of a proper time interval for events in K (traveling frame)—receding frames "backward scenario"

Let's now consider the case of the signal of an event $E(0,0,0)$ being emitted from K origin at time τ with respect to K, which will be perceived at time τ' in K'. — τ and τ' are used to denote "proper time" and "perceived time", respectively, as opposed to t and t' denoting "perceived time" and "proper time", respectively, in the "forward scenario".

Let the speed of a light signal traveling from K to K' would be $c_{K\to K'}$ with respect to K, and $c'_{K\to K'}$ with respect to K'.

K' perspective

From the perspective of K', the origin of K at the event occurring time is at a distance of $v\tau$ from that of K'. The signal will have traveled a distance of $v\tau$, at a speed of $c'_{K\to K'}$ with respect to K', when it reaches K' origin. Therefore, the event will be perceived at time τ' in K', given by

$$\tau' = \tau + \frac{v\tau}{c'_{K\to K'}};$$

$$\tau' = \tau\left(1 + \frac{v}{c'_{K\to K'}}\right). \tag{3}$$

exhibiting time dilation.

K perspective

From the perspective of K, the origin of K at the event occurring time is at a distance of $v\tau$ from that of K'. Let τ^* be the time interval it takes the event signal to reach the origin of K', from K perspective. By that time, K will have moved a further distance of $v\tau^*$ away from K', bringing the distance traveled by the signal to $v\tau + v\tau^*$ with respect to K. The time interval τ^* can then be expressed as

$$\tau^* = \frac{v\tau + v\tau^*}{c_{K\to K'}},$$

leading to

$$\tau^* = \frac{v\tau}{c_{K\to K'} - v}.$$

It follows that the event perception time τ' in K' with respect to K will be given by

$$\tau' = \tau + \tau^*;$$

$$\tau' = \tau + \frac{v\tau}{c_{K\to K'} - v};$$

$$\tau' = \frac{\tau}{1 - \dfrac{v}{c_{K\to K'}}}, \tag{4}$$

exhibiting time dilation.

2.2   GENERAL TIME ALTERATION FACTOR DERIVATION FOR APPROACHING FRAMES

Suppose that the time is set to $t = t' = 0$ when K and K' are at a distance of d from each other.

2.2.1   Perception in K (stationary frame) of a proper time interval for events in K' (traveling frame)—approaching frames "forward scenario"

Suppose a signal of an event $E'_o(0,0,0)$ is emitted from K' origin at time $t'_o = 0$ with respect to K', which will be perceived at time t_o in K, and another signal of an event $E'(0,0,0)$ is emitted at time t' from K' origin, which will be perceived at time t in K.

Let the speed of a light signal traveling from K' to K be $c'_{K'\to K}$ with respect to K', and $c_{K'\to K}$ with respect to K.

K' perspective

From the perspective of K', the origin of K' at the event E'_o occurring time is at a distance of d from that of K. Let $t^{*\prime}_o$ be the time interval it takes the event E'_o signal to reach the origin of K, from K' perspective. By that time, K' will have moved a distance of $vt^{*\prime}_o$ closer to K, bringing the distance traveled by the signal to $d - vt^{*\prime}_o$ with respect to K'. The time interval $t^{*\prime}_o$ can then be expressed as

$$t^{*\prime}_o = \frac{d - vt^{*\prime}_o}{c'_{K'\to K}},$$

leading to

$$t^{*\prime}_o = \frac{d}{c'_{K'\to K} + v},$$

and

$$t_o = t'_0 + t^{*\prime}_o;$$

$$t_o = \frac{d}{c'_{K'\to K} + v}. \tag{5}$$

Similarly, with respect to K', the origin of K' at the event E' occurring time is at a distance of $d - vt'$ from that of K (Fig. 2).
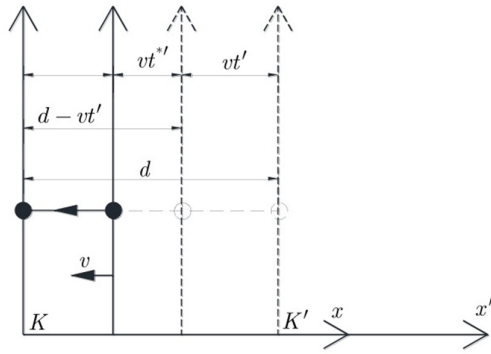
Fig. 2 Signal propagation—approaching frames

Let $t^{*\prime}$ be the time interval it takes the event E' signal to reach the origin of K, from K' perspective. By that time, K' will have moved a distance of $vt^{*\prime}$ closer to K, bringing the distance traveled by the signal to $d - vt' - vt^{*\prime}$ with respect to K'. The time interval $t^{*\prime}$ can then be expressed as

$$t^{*\prime} = \frac{d - vt' - vt^{*\prime}}{c'_{K'\to K}},$$

leading to

$$t^{*\prime} = \frac{d - vt'}{c'_{K'\to K} + v} = \frac{d}{c'_{K'\to K} + v} - \frac{vt'}{c'_{K'\to K} + v}.$$

It follows that the event perception time t of E' in K with respect to K' will be given by

$$t = t' + t^{*\prime},$$

$$t = t' + \frac{d}{c'_{K'\to K} + v} - \frac{vt'}{c'_{K'\to K} + v}.$$

Therefore,

$$\Delta t = t - t_o;$$

$$\Delta t = \frac{t'}{1 + \dfrac{v}{c'_{K'\to K}}}.$$

Since $t'_o = 0$, then

$$\Delta t = \frac{\Delta t'}{1 + \dfrac{v}{c'_{K' \to K}}}. \tag{6}$$

exhibiting time contraction.

K perspective

Now, from the perspective of K, the origin of K' at the event E'_o occurring time is at a distance of d from that of K. The signal will have traveled a distance d, at the speed $c_{K' \to K}$ with respect to K, when it reaches K origin. Therefore, the event will be perceived at time t_o in K, given by

$$t_o = \frac{d}{c_{K' \to K}}.$$

Similarly, with respect to K, the origin of K' at the event E' occurring time is at a distance of $d - vt'$ from that of K. The signal will have traveled this distance at the speed $c_{K' \to K}$ with respect to K when it reaches K origin. Therefore, the event will be perceived at time t in K, given by

$$t = t' + \frac{d}{c_{K' \to K}} - \frac{vt'}{c_{K' \to K}}.$$

Therefore,

$$\Delta t = t - t_o;$$

$$\Delta t = t'\left(1 - \frac{v}{c_{K' \to K}}\right).$$

Since $t'_o = 0$, then

$$\Delta t = \Delta t'\left(1 - \frac{v}{c_{K' \to K}}\right), \tag{7}$$

exhibiting time contraction.

2.2.2   Perception in K' (stationary frame) of a proper time interval for events in K (traveling frame)—approaching frames "backward scenario"

The frame K is considered to be the "traveling" frame in this case. Suppose a signal of an event $E_o(0,0,0)$ is emitted from K origin at time $\tau_o = 0$ with respect to K, which will be perceived at

time τ'_o in K', and another signal of an event $E(0,0,0)$ is emitted at time τ from K origin, which will be perceived at time τ' in K'.

Let the speed of a light signal traveling from K to K' be $c_{K \to K'}$ with respect to K, and $c'_{K \to K'}$ with respect to K'.

K' perspective

from the perspective of K', the origin of K at the event E_o occurring time is at a distance of d from that of K'. The signal will have traveled a distance d, at the speed $c'_{K \to K'}$ with respect to K', when it reaches K' origin. Therefore, the event will be perceived at time τ'_o in K', given by

$$\tau'_o = \frac{d}{c'_{K \to K'}}.$$

Similarly, with respect to K', the origin of K at the event E occurring time is at a distance of $d - v\tau$ from that of K. The signal will have traveled this distance, at the speed of $c'_{K \to K'}$ with respect to K', when it reaches K' origin. Therefore, the event will be perceived at time τ' in K', given by

$$\tau' = \tau + \frac{d}{c'_{K \to K'}} - \frac{v\tau}{c'_{K \to K'}}.$$

Therefore,

$$\Delta\tau' = \tau' - \tau'_o;$$

$$\Delta\tau' = \tau\left(1 - \frac{v}{c'_{K \to K'}}\right).$$

Since $\tau_o = 0$, then

$$\Delta\tau' = \Delta\tau\left(1 - \frac{v}{c'_{K \to K'}}\right), \tag{8}$$

exhibiting time contraction.

K perspective

It can be shown, using the same methodology for the case of the time interval perception in K from the perspective of K', that from the perspective of K, the time $\Delta\tau'$ would be obtained as

$$\Delta \tau' = \frac{\Delta \tau}{1 + \dfrac{v}{c_{K \to K'}}}, \tag{9}$$

contracted with respect to $\Delta \tau$.

3   TIME INTERVAL ALTERATION APPLICATIONS

The obtained general equations for the apparent alteration of a proper time interval measured in the traveling frame relative to the stationary frame shall be applied to the different light conjectures, from the Emission theory to Special Relativity, going through the Ether theory, each allowing to specify the speed of light relative to the involved reference frames.

Appendix A gives a tabulated summary of the results.

3.1   EMISSION THEORY

In this conjecture, the speed of light is constant, say c, with respect to the source rest frame. The speed of light relative to the other frame becomes $c \pm v$, according to the classical addition of velocities.

3.1.1   Case of Receding Reference Frames—Emission Theory Approach

3.1.1.1   *Change of duration for events occurring at K' (traveling frame) origin—Emission Theory—receding frames "forward scenario"*

The speed $c'_{K' \to K}$ and $c_{K' \to K}$ of a light signal traveling from K' to K with respect to K' and K, would be c and $c - v$, respectively.

Applying Eq. (1) for the perceived time interval in K from the K' perspective, we get

$$t = \frac{t'}{1 - \dfrac{v}{c'_{K' \to K}}} = \frac{t'}{1 - \dfrac{v}{c}}.$$

Whereas, the same perceived time interval in K from the perspective of K, is given by Eq. (2) as

$$t = t'\left(1 + \frac{v}{c_{K' \to K}}\right) = t'\left(1 + \frac{v}{c - v}\right);$$

$$t = \frac{t'}{1 - \dfrac{v}{c}}.$$ (10)

Therefore, the perceived time interval in K is the same from the perspective of both frames.

It follows that the time interval measured at the origin of the traveling frame K' between two events will be perceived as a time interval in the stationary frame K, dilated by a factor of $(1 - v / c)^{-1}$.

3.1.1.1.1 *Space alteration*

If the time t' measured at the traveling frame K' origin was for an event that has taken place at a point of coordinate $x' > 0$ $(x > 0)$ on the x-x' axis, then t' could be replaced by x' / c and t by $x / (c - v)$ in Eq. (10), yielding

$$\frac{x_+}{c - v} = \frac{x'_+}{c} + \frac{vt}{c};$$

$$x_+ = \left(1 - \frac{v}{c}\right)x'_+ + vt';$$ (11)

$$x_+ = \left(1 - \frac{v}{c}\right)\left(x'_+ + vt\right);$$ (12)

Equation (12) shows the x-coordinate contracted by the factor $(1 - v / c)$ with respect to its value $(x' + vt)$ given by the classical Galilean transformation. In fact, Eq. (11) can be physically deduced, since by the time the light signal, emitted from a distance x' with respect to K', reaches the origin of K', K' would have moved closer to the starting point, making the distance x' shorter with respect to K. Let x'^* be the perceived x' in K. Then, we can write

$$x'^* = x' - \frac{vx'^*}{c - v};$$

$$x'^* = x'\left(1 - \frac{v}{c}\right),$$

and, since $x = x'^* + vt'$, Eqs. (11) and (12) would follow.

3.1.1.2 *Change of duration for events occurring at K (traveling frame) origin—Emission Theory—receding frames "backward scenario"*

If K was considered to be the traveling frame, the speed $c'_{K \to K'}$ and $c_{K \to K'}$ of a light signal traveling from K to K' with respect to K' and K, would be $c - v$ and c, respectively.

Applying Eq. (3) for the perceived time interval τ' in K' from the K' perspective, we get

$$\tau' = \tau \left(1 + \frac{v}{c'_{K \to K'}} \right) = \tau \left(1 + \frac{v}{c - v} \right).$$

$$\tau' = \frac{\tau}{1 - \dfrac{v}{c}}. \tag{13}$$

Whereas, the same perceived time interval in K' from the perspective of K, is given by Eq. (4) as

$$\tau' = \frac{\tau}{1 - \dfrac{v}{c_{K \to K'}}};$$

$$\tau' = \frac{\tau}{1 - \dfrac{v}{c}}. \tag{14}$$

Therefore, the perceived time interval in K' is the same from the perspective of both frames.

It follows that the proper time interval measured at the origin of the traveling frame K ("backward scenario") between two events will be perceived as a time interval in the stationary frame K' dilated by a factor of $(1 - v / c)^{-1}$, the same as the time dilation factor for the "forward scenario" (Eq. (10)). Hence the Emission Theory results in symmetry with respect to the reference frames as to the extent of the time dilation between the receding reference frames.

3.1.1.2.1 *Space alteration*

Let's suppose the time τ measured at the traveling frame K origin was for an event that has taken place initially at a point of coordinate $x < 0$ $(x' < 0)$ on the x-x' axis. Then, τ could be replaced by $-x / c$ and τ' by $-x / (c - v)$ in Eq. (14), yielding

$$\tau = \tau' - \frac{v\tau'}{c}$$

$$\frac{-x}{c} = \frac{-x'}{c - v} + \frac{-v\tau'}{c};$$

$$x'_- = x_- \left(1 - \frac{v}{c}\right) - v\tau. \tag{15}$$

$$x'_- = x_- \left(1 - \frac{v}{c}\right) - v\tau' \left(1 - \frac{v}{c}\right).$$

$$x'_- = \left(1 - \frac{v}{c}\right)(x_- - v\tau'). \tag{16}$$

Equation (16) shows the x'- coordinate contracted by the factor $(1 - v / c)$ with respect to its value $(x - v\tau')$ given by the classical Galilean transformation. In fact, Eq. (15) can be physically deducted, since by the time the light signal, emitted from a distance x with respect to K, reaches the origin of K, K would have moved closer to the starting point, making the distance x shorter with respect to K'. Let x^* be the perceived x in K. Then, we can write

$$x^* = x - \frac{vx^*}{c - v};$$

$$x^* = x\left(1 - \frac{v}{c}\right)$$

and, since $x' = x^* - v\tau$, Eqs. (15) and (16) would follow.

Comparing Eqs. (11) & (12) with Eqs. (15) & (16), it is observed that the Emission Theory results in symmetry with respect to the reference frames as to the extent of the length contraction perceived between the receding reference frames.

3.1.1.3 Doppler Effect

3.1.1.3.1 Forward Scenario

If the proper time interval in K' represents the period of a periodic event (e.g., wave, vibration or rotation period), then the relation between the actual and perceived frequency of the event can be determined from Eq. (10) as

$$f = f'\left(1 - \frac{v}{c}\right), \tag{17}$$

where, f and f' are the perceived and actual frequency with respect to an observer in K and K', respectively. Hence, the perceived frequency is lower than the proper frequency in the receding source frame.

Equation (17) expresses the Doppler effect for the case of a receding source from the observer at a uniform velocity v, under the assumption that the speed of light is c with respect to the receding source, $c - v$ with respect to the observer, when light travels towards the observer.

For the case of a light wave, assume λ and λ' are the perceived and actual wave length with respect to K and K'. Then, Eq.(17) leads to

$$\frac{c - v}{\lambda} = \frac{c}{\lambda'}\left(1 - \frac{v}{c}\right);$$

$$\lambda = \lambda'.$$

Therefore, the perceived wave length of a light wave in the observer frame is the same as the emitted light wave in the moving source frame.

3.1.1.3.2   *Backward Scenario*

If the proper time interval in K represents the period of a periodic event, then the relation between the actual and perceived frequency of the event can be determined from Eq. (14) as

$$\phi' = \phi\left(1 - \frac{v}{c}\right). \tag{18}$$

where, ϕ' and ϕ are the perceived and actual frequency with respect to an observer in K' and K, respectively, for a source in K.

Equation (18) expresses the Doppler effect for the case of a receding source from the observer at a uniform velocity v, under the assumption that the speed of light is $c - v$ with respect to the observer, c with respect to the receding source, when light travels towards the observer.

It follows that the Doppler Effect is symmetrical relative to the receding reference frames under the Emission Theory assumption.

3.1.2   Case of Approaching Reference Frames—Emission Theory

3.1.2.1   *Change of duration for events occurring at K'(traveling frame) origin—Emission Theory—approaching frames "forward scenario"*

In this case, $c'_{K' \to K} = c$, and $c_{K' \to K} = c + v$. Applying Eq. (6) for the perceived time interval in K from the K' perspective, we get

$$\Delta t = \frac{\Delta t'}{1 + \dfrac{v}{c'_{K' \to K}}} = \frac{\Delta t'}{1 + \dfrac{v}{c}}; \tag{19}$$

Whereas, the same perceived time interval in K from the perspective of $K,$ is given by Eq. (7) as

$$\Delta t = \Delta t' \left(1 - \frac{v}{c_{K' \to K}} \right) = \Delta t' \left(1 - \frac{v}{c + v} \right).$$

$$\Delta t = \frac{\Delta t'}{1 + \dfrac{v}{c}}. \tag{20}$$

Therefore, the perceived time interval in K is the same from the perspective of both frames.

3.1.2.1.1 Space alteration

If the time t' measured at the traveling frame K' origin was for an event that has taken place at a point of coordinate $x' > 0 \ (x > 0)$ on the x-x' axis, then $\Delta t'$ could be replaced by $\Delta x' / c$, and Δt by $\Delta x / (c + v)$ in Eq. (20), yielding

$$\Delta x_+ = \left(1 + \frac{v}{c} \right) \Delta x'_+ - v \Delta t'; \tag{21}$$

$$\Delta x_+ = \left(1 + \frac{v}{c} \right) \left(\Delta x'_+ - v \Delta t \right); \tag{22}$$

Equation (22) shows the x- coordinate expanded by the factor $(1 + v / c)$ with respect to its value $(\Delta x' - v \Delta \tau)$ given by the classical Galilean transformation. In fact, Eq. (21) can be physically deduced, since by the time the light signal, emitted from a distance x' with respect to K', reaches the origin of K', K' would have moved farther from the starting point, making the distance x' longer with respect to K. Let x'^* be the perceived x' in K. Then, we can write

$$x'^* = x' + \frac{v x'^*}{c + v};$$

$$x'^* = x' \left(1 + \frac{v}{c} \right),$$

and, since $\Delta x = \Delta x'^* - v \Delta t'$ for approaching frames, Eqs. (21) and (22) would follow.

3.1.2.1.2 Receding–approaching frames

We note from Eqs. (10) and (20) that if the frames receded for a certain proper time interval $\Delta t'_o$ and then approached for an equal proper time interval, then the total perceived time interval in K would become

$$\Delta t = \frac{\Delta t_o'}{1 - \dfrac{v}{c}} + \frac{\Delta t_o'}{1 + \dfrac{v}{c}},$$

$$\Delta t = \frac{2\Delta t_o'}{1 - \dfrac{v^2}{c^2}} = \frac{\Delta t'}{1 - \dfrac{v^2}{c^2}},$$

with a net dilation factor of $\left(1 - v^2 / c^2\right)^{-1}$.

3.1.2.2   Change of duration for events occurring at K (traveling frame) origin—Emission Theory—approaching frames "backward scenario"

In this case, $c_{K \to K'} = c$, and $c'_{K \to K'} = c + v$. Applying Eq. (8) for the perceived time interval $\Delta \tau'$ in K' from the K' perspective, we get

$$\Delta \tau' = \Delta \tau \left(1 - \frac{v}{c'_{K \to K'}}\right) = \Delta \tau \left(1 - \frac{v}{c + v}\right);$$

$$\Delta \tau' = \frac{\Delta \tau}{1 + \dfrac{v}{c}}. \tag{23}$$

Whereas, the same perceived time interval in K' from the perspective of K, is given by Eq. (9) as

$$\Delta \tau' = \frac{\Delta \tau}{1 + \dfrac{v}{c_{K \to K'}}};$$

$$\Delta \tau' = \frac{\Delta \tau}{1 + \dfrac{v}{c}}. \tag{24}$$

Therefore, the perceived time interval in K' is the same from the perspective of both frames.

It follows that the proper time interval measured at the origin of the traveling frame K ("backward scenario") between two events will be perceived as a time interval contracted in K' by a factor of $(1 + v / c)^{-1}$, the same as the time contraction factor in K for the "forward scenario" (Eq. (20)). Hence, the time contracted for approaching frames is symmetrical with respect to the reference frames in the case when the Emission Theory is adopted.

3.1.2.2.1 Space alteration

Let's suppose the time τ measured at K origin was for an event that has taken place initially at a point of coordinate $x < 0$ $(x' < 0)$ on the x-x' axis. Then, τ could be replaced by $-x / c$ and τ' by $-x / (c + v)$ in Eq. (44), yielding

$$\Delta x'_- = \left(1 + \frac{v}{c}\right)\Delta x_- + v\Delta\tau; \tag{25}$$

$$\Delta x'_- = \left(1 + \frac{v}{c}\right)\left(\Delta x_- + v\Delta\tau'\right). \tag{26}$$

Equation (26) shows the x'- coordinate expanded by the factor $(1 + v / c)$ with respect to its value $(\Delta x + v\Delta\tau')$ given by the classical Galilean transformation for approaching frames. In fact, Eq. (25) can be physically deducted, since by the time the light signal, emitted from a distance x with respect to K, reaches the origin of K, K would have moved farther from the starting point, making the distance x longer with respect to K'. Let x^* be the perceived x in K'. Then, we can write

$$x^* = x + \frac{vx^*}{c + v};$$

$$x^* = x\left(1 + \frac{v}{c}\right),$$

and, since $\Delta x' = \Delta x^* + v\Delta\tau$ for approaching frames, Eqs. (25) and (26) would follow.

Comparing Eqs. (21) & (22) with Eqs. (25) & (26), it is observed that the Emission Theory results in symmetry with respect to the reference frames as to the extent of the length expansion perceived between the approaching reference frames.

3.1.2.2.2 Receding–approaching frames

We note from Eqs.(14) and (24) that if the frames receded for a certain proper time interval $\Delta\tau_o$ and then approached for an equal proper time interval, then the total perceived time interval in K' would become

$$\Delta\tau' = \frac{2\Delta\tau_o}{1 - \dfrac{v^2}{c^2}} = \frac{\Delta\tau}{1 - \dfrac{v^2}{c^2}},$$

with a net dilation factor of $\left(1 - v^2 / c^2\right)^{-1}$,

symmetrical with respect to the "forward scenario", where K' is the traveling frame.

3.1.2.3 Doppler Effect

3.1.2.3.1 Forward Scenario

If the proper time interval in K' represents the period of a periodic event (e.g., wave, vibration or rotation period), then the relation between the actual and perceived frequency of the event can be determined from Eq. (20) as

$$f = f'\left(1 + \frac{v}{c}\right),\tag{27}$$

where, f and f' are the perceived and actual frequency with respect to an observer in K and K', respectively. Hence, the perceived frequency is higher than the proper frequency in the approaching frame.

Equation (27) expresses the Doppler effect for the case of a source approaching the observer at a uniform velocity v, under the assumption that the speed of light is c with respect to the source, $c + v$ with respect to the observer, when light travels towards the observer.

For the case of a light wave, assume λ and λ' are the perceived and actual wave length with respect to K and K'. Then, Eq.(27) leads to

$$\frac{c + v}{\lambda} = \frac{c}{\lambda'}\left(1 + \frac{v}{c}\right);$$

$$\lambda = \lambda'.$$

Therefore, the perceived wave length of a light wave in the observer frame is the same as the emitted light wave in the moving source frame.

3.1.2.3.2 Backward Scenario

If the proper time interval in K represents the period of a periodic event, then the relation between the actual and perceived frequency of the event can be determined from Eq. (24) as

$$\phi' = \phi\left(1 + \frac{v}{c}\right).\tag{28}$$

where, ϕ' and ϕ are the perceived and actual frequency with respect to an observer in K' and K, respectively, for a source in K.

Equation (28) expresses the Doppler effect for the case of an approaching source to the observer at a uniform velocity v, under the assumption that the speed of light is $c + v$ with respect to the observer, c with respect to the approaching source, when light travels towards the observer.

It follows that the Doppler Effect is symmetrical relative to the approaching reference frames under the Emission Theory assumption.

3.2 ETHER THEORY

In this conjecture, the speed of light is constant with respect to the rest frame of the ether, an assumed propagation medium for light. Let c be the speed of light with respect to K, considered to be the ether rest frame.

3.2.1 Case between the Ether and a Receding Frame

3.2.1.1 *Change of duration for events occurring at K' (traveling frame) origin—Ether Theory— receding frames "forward scenario"*

In this case, $c'_{K' \to K} = c + v$, and $c_{K' \to K} = c$. Applying Eq.(1) for the perceived time interval in K from the K' perspective, we get

$$t = \frac{t'}{1 - \dfrac{v}{c'_{K' \to K}}} = \frac{t'}{1 - \dfrac{v}{c + v}} = t'\left(1 + \frac{v}{c}\right). \tag{29}$$

Whereas, the same perceived time interval in K from the perspective of $K,$ is given by Eq. (2) as

$$t = t'\left(1 + \frac{v}{c_{K' \to K}}\right) = t'\left(1 + \frac{v}{c}\right). \tag{30}$$

Therefore, the perceived time interval in K is the same from the perspective of both frames.

It follows that the time interval measured at the origin of the traveling frame K' between two events will be perceived dilated in K by a factor of $(1 + v\,/\,c)$.

3.2.1.1.1 *Space alteration*

Let's suppose the time t' measured at K' origin was for an event that has taken place at a point of coordinate $x' > 0 \ (x > 0)$ on the x-x' axis. Then, t' could be replaced by $x'\,/\,(c + v)$ and t by $x\,/\,c$ in Eq. (30), yielding

$$\frac{x}{c} = \frac{x'}{c + v} + \frac{vt'}{c};$$

$$x = \frac{x'}{1 + \dfrac{v}{c}} + vt'; \tag{31}$$

$$x = \frac{1}{1+\dfrac{v}{c}}(x' + vt). \tag{32}$$

Equation (32) shows the x-coordinate contracted by the factor $(1 + v\,/\,c)^{-1}$ with respect to its value $(x' + vt)$ given by the classical Galilean transformation. In fact, Eq. (31) can be physically deduced, since by the time the light signal, emitted from a distance x' with respect to K', reaches the origin of K', K' would have moved closer to the starting point, making the distance x' shorter with respect to K. Let x'^{*} be the perceived x' in K. Then, we can write

$$x'^{*} = x' - \frac{vx'^{*}}{c};$$

$$x'^{*} = \frac{x'}{1+\dfrac{v}{c}},$$

and, since $x = x'^{*} + vt'$, Eqs. (31) and (32) would follow.

3.2.1.2 *Change of duration for events occurring at ether frame K (traveling frame) origin— Ether Theory—receding frames "backward scenario"*

The ether frame K is considered to be the "traveling" frame where an event proper time interval τ is measured.

In this case, $c_{K \to K'} = c$, and $c'_{K \to K'} = c - v$. Applying Eq. (3) for the perceived time interval τ' in K' from the K' perspective, we get

$$\tau' = \tau\left(1 + \frac{v}{c'_{K \to K'}}\right) = \tau\left(1 + \frac{v}{c - v}\right).$$

$$\tau' = \frac{\tau}{1-\dfrac{v}{c}}. \tag{33}$$

Whereas, the same perceived time interval in K' from the perspective of K, is given by Eq. (4) as

$$\tau' = \frac{\tau}{1 - \dfrac{v}{c}_{K \to K'}};$$

$$\tau' = \frac{\tau}{1 - \dfrac{v}{c}}. \tag{34}$$

Therefore, the perceived time interval in K' is the same from the perspective of both frames.

It follows that the proper time interval measured at the origin of the frame K ("backward scenario") between two events will be perceived as a time interval dilated in K' by a factor of $(1 - v \,/\, c)^{-1}$, as opposed to the time dilation factor of $(1 + v \,/\, c)$ for the "forward scenario". Hence, the time dilation is asymmetrical with respect to the reference frames in the case when the Ether Theory is adopted.

3.2.1.2.1 *Space alteration*

Let's suppose the time τ measured at K origin was for an event that has taken place initially at a point of coordinate $x < 0$ $(x' < 0)$ on the x-x' axis. Then, τ could be replaced by $-x \,/\, c$ and τ' by $-x \,/\, c - v$ in Eq. (34), yielding

$$\tau = \tau' - \frac{v\tau'}{c}$$

$$\frac{-x}{c} = \frac{-x'}{c - v} + \frac{-v\tau'}{c};$$

$$x' = x\left(1 - \frac{v}{c}\right) - v\tau. \tag{35}$$

$$x' = x\left(1 - \frac{v}{c}\right) - v\tau'\left(1 - \frac{v}{c}\right).$$

$$x' = \left(1 - \frac{v}{c}\right)(x - v\tau'). \tag{36}$$

Equation (36) shows the x'- coordinate contracted by the factor $(1 - v \,/\, c)$ with respect to its value $(x - v\tau')$ given by the classical Galilean transformation. In fact, Eq. (35) can be physically deducted, since by the time the light signal, emitted from a distance x with respect to K, reaches the origin of K, K would have moved closer to the starting point, making the distance x shorter with respect to K'. Let x^{*} be the perceived x in K'. Then, we can write

$$x^* = x - \frac{vx^*}{c-v};$$

$$x^* = x\left(1 - \frac{v}{c}\right),$$

and, since $x' = x^* - v\tau$, Eqs. (31) and (32) would follow.

3.2.1.3 Doppler Effect

3.2.1.3.1 Forward Scenario

If the proper time interval in K' represents the period of a periodic event (e.g., wave, vibration or rotation period), then the relation between the actual and perceived frequency of the event can be determined from Eq. (29) as

$$f = f'\left(1 + \frac{v}{c}\right)^{-1}. \tag{37}$$

where, f and f' are the perceived and actual frequency with respect to an observer in K and K', respectively, for a source in K'. Hence, the perceived frequency is lower than the proper frequency in the receding source frame.

Equation (37) expresses the Doppler effect for the case of a receding source from the observer at a uniform velocity v, under the assumption that the speed of light is c with respect to the observer, $c + v$ with respect to the retreating source when light travels towards the observer.

For the case of a light wave, assume λ and λ' are the perceived and actual wave length with respect to K and K'. Then, Eq.(37) leads to

$$\frac{c}{\lambda} = \frac{c+v}{\lambda'}\left(1 + \frac{v}{c}\right)^{-1};$$

$$\lambda = \lambda'.$$

Therefore, the perceived wave length of a light wave in the observer frame is the same as the emitted light wave in the moving source frame.

3.2.1.3.2 Backward Scenario

If the proper time interval in K represents the period of a periodic event, then the relation between the actual and perceived frequency of the event can be determined from Eq. (34) as

$$\phi' = \phi\left(1 - \frac{v}{c}\right). \tag{38}$$

where, ϕ' and ϕ are the perceived and actual frequency with respect to an observer in K' and K, respectively, for a source in K. Hence, the perceived frequency is lower than the proper frequency in the receding source frame. However, the decreasing factor is different from the case where the source is in the other frame. Therefore, the Doppler effect is asymmetrical in the case of the Ether Theory assumption.

Equation (38) expresses the Doppler effect for the case of a receding source from the observer at a uniform velocity v, under the assumption that the speed of light is $c - v$ with respect to the observer, c with respect to the retreating source, when light travels towards the observer.

For the case of a light wave, Eq. (38) leads to

$$\frac{c - v}{\lambda} = \frac{c}{\lambda'}\left(1 - \frac{v}{c}\right);$$

$$\lambda = \lambda'.$$

Therefore, the perceived wave length of a light wave in the observer frame is the same as the emitted light wave in the moving source frame.

3.2.2 Case between the Ether and an Approaching Frame

3.2.2.1 *Change of duration for events occurring at K' (traveling frame) origin—Ether Theory—approaching frames "forward scenario"*

In this case, $c'_{K' \to K} = c - v$, and $c_{K' \to K} = c$. Applying Eq. (6) for the perceived time interval in K from the K' perspective, we get

$$\Delta t = \frac{\Delta t'}{1 + \dfrac{v}{c'_{K' \to K}}} = \frac{\Delta t'}{1 + \dfrac{v}{c - v}};$$

$$\Delta t = \Delta t'\left(1 - \frac{v}{c}\right). \tag{39}$$

Whereas, the same perceived time interval in K from the perspective of K, is given by Eq. (7) as

$$\Delta t = \Delta t'\left(1 - \frac{v}{c_{K' \to K}}\right).$$

$$\Delta t = \Delta t'\left(1 - \frac{v}{c}\right); \tag{40}$$

Therefore, the perceived time interval in K is the same from the perspective of both frames.

It follows that the time interval measured at the origin of the traveling frame K' between two events will be perceived as a time interval contracted in K by a factor of $(1 - v / c)$, in the case of approaching frames.

3.2.2.1.1 Space alteration

If the time t' measured at K' origin was for an event that has taken place at a point of coordinate $x' > 0$ $(x > 0)$ on the x-x' axis, then $\Delta t'$ could be replaced by $\Delta x' / (c - v)$ and Δt by $\Delta x / c$ in Eq. (6), yielding

$$\frac{\Delta x}{c} = \frac{\Delta x'}{c - v} - \frac{v \Delta t'}{c};$$

$$\Delta x = \frac{\Delta x'}{1 - \dfrac{v}{c}} - v \Delta t'; \tag{41}$$

$$\Delta x = \frac{1}{1 - \dfrac{v}{c}} \left(\Delta x' - v \Delta t \right). \tag{42}$$

Equation (42) shows the x-coordinate expanded by the factor $(1 - v / c)^{-1}$ with respect to its value $(\Delta x' - v \Delta t)$ given by the classical Galilean transformation for approaching frames. In fact, Eq. (41) can be physically deduced, since by the time the light signal, emitted from a distance x' with respect to K', reaches the origin of K', K' would have moved farther from the starting point, making the distance x' longer with respect to K. Let x'^* be the perceived x' in K. Then, we can write

$$x'^* = x' + \frac{v x'^*}{c};$$

$$x'^* = \frac{x'}{1 - \dfrac{v}{c}},$$

and, since $\Delta x = \Delta x'^* - v \Delta t'$, Eqs. (41) and (42) would follow.

3.2.2.1.2   Receding–approaching frames

We note from Eqs. (30) and (40) that if the frames receded for a certain proper time interval $\Delta t_o'$ and then approached for an equal proper time interval, then the total perceived time interval in K would become

$$\Delta t = \Delta t_o'\left(1+\frac{v}{c}\right) + \Delta t_o'\left(1-\frac{v}{c}\right);$$

$$\Delta t = 2\Delta t_o' = \Delta t',$$

exhibiting an invariant net time interval.

3.2.2.2   Change of duration for events occurring at the ether frame K (traveling frame) origin— Ether Theory—approaching frames "backward scenario"

The ether frame K is considered to be the "traveling" frame where an event proper time interval τ is measured.

In this case, $c_{K\to K'} = c$, and $c_{K\to K'}' = c + v.$ Applying Eq. (8) for the perceived time interval $\Delta\tau'$ in K' from the K' perspective, we get

$$\Delta\tau' = \Delta\tau\left(1-\frac{v}{c_{K\to K'}'}\right) = \Delta\tau\left(1-\frac{v}{c+v}\right);$$

$$\Delta\tau' = \frac{\Delta\tau}{1+\dfrac{v}{c}}. \tag{43}$$

Whereas, the same perceived time interval in K' from the perspective of $K,$ is given by Eq. (9) as

$$\Delta\tau' = \frac{\Delta\tau}{1+\dfrac{v}{c_{K\to K'}}};$$

$$\Delta\tau' = \frac{\Delta\tau}{1+\dfrac{v}{c}}. \tag{44}$$

Therefore, the perceived time interval in K' is the same from the perspective of both frames.

It follows that the proper time interval measured at the origin of the frame K ("backward scenario") between two events will be perceived as a time interval contracted in K' by a factor of

$(1 + v / c)^{-1}$, as opposed to the time contraction factor of $(1 - v / c)$ in K for the "forward scenario". Hence, the time dilation is asymmetrical with respect to the reference frames in the case when the Ether Theory is adopted.

3.2.2.2.1   *Space alteration*

Let's suppose the time τ measured at K origin was for an event that has taken place initially at a point of coordinate $x < 0$ $(x' < 0)$ on the x-x' axis. Then, τ could be replaced by $-x / c$ and τ' by $-x / (c + v)$ in Eq. (44), yielding

$$\Delta x' = \left(1 + \frac{v}{c}\right)\Delta x + v\Delta\tau; \tag{45}$$

$$\Delta x' = \left(1 + \frac{v}{c}\right)\left(\Delta x + v\Delta\tau'\right). \tag{46}$$

Equation (46) shows the x'- coordinate expanded by the factor $(1 + v / c)$ with respect to its value $(\Delta x + v\Delta\tau')$ given by the classical Galilean transformation for approaching frames. In fact, Eq. (45) can be physically deducted, since by the time the light signal, emitted from a distance x with respect to K, reaches the origin of K, K would have moved farther from the starting point, making the distance x longer with respect to K'. Let x^* be the perceived x in K'. Then, we can write

$$x^* = x + \frac{vx^*}{c + v};$$

$$x^* = x\left(1 + \frac{v}{c}\right),$$

and, since $\Delta x' = \Delta x^* + v\Delta\tau$, Eqs. (45) and (46) would follow.

3.2.2.2.2   *Receding–approaching frames*

We note from Eqs. (33) and (44) that if the frames receded for a certain proper time interval $\Delta\tau_o$ and then approached for an equal proper time interval, then the total perceived time interval in K' would become

$$\Delta\tau' = \frac{2\Delta\tau_o}{1 - \dfrac{v^2}{c^2}} = \frac{\Delta\tau}{1 - \dfrac{v^2}{c^2}},$$

with a net dilation factor of $\left(1 - v^2 / c^2\right)^{-1}$, asymmetrical with respect to the "forward scenario", for which no net time variation is exhibited when K' oscillated back and forth with respect to K for equal proper time intervals.

3.2.2.3   Doppler Effect

3.2.2.3.1   Forward Scenario

If the proper time interval in K' represents the period of a periodic event (e.g., wave, vibration or rotation period), then the relation between the actual and perceived frequency of the event can be determined from Eq. (39) as

$$f = f'\left(1 - \frac{v}{c}\right)^{-1},$$
(47)

where, f and f' are the perceived and actual frequency with respect to an observer in K and K', respectively. Hence, the perceived frequency is higher than the proper frequency in the approaching source frame.

Equation (47) expresses the Doppler effect for the case of a source approaching the observer at a uniform velocity v, under the assumption that the speed of light is c with respect to the observer, $c - v$ with respect to the approaching source, when light travels towards the observer.

For the case of a light wave, assume λ and λ' are the perceived and actual wave length with respect to K and K'. Then, Eq. (47) leads to

$$\frac{c}{\lambda} = \frac{c - v}{\lambda'}\left(1 - \frac{v}{c}\right)^{-1};$$

$$\lambda = \lambda'.$$

Therefore, the perceived wave length of a light wave in the observer frame is the same as the emitted light wave in the moving source frame.

3.2.2.3.2   Backward Scenario

If the proper time interval in K represents the period of a periodic event, then the relation between the actual and perceived frequency of the event can be determined from Eq. (44) as

$$\phi' = \phi\left(1 + \frac{v}{c}\right).$$
(48)

where, ϕ' and ϕ are the perceived and actual frequency with respect to an observer in K' and K, respectively, for a source in K. Hence, the perceived frequency is higher than the proper frequency in the approaching source frame. However, the increasing factor is different from the case where the

source is in the other frame. Therefore, the Doppler effect is asymmetrical with respect to the approaching frames in the case of the Ether Theory assumption.

Equation (48) expresses the Doppler effect for the case of an approaching source to the observer at a uniform velocity v, under the assumption that the speed of light is $c + v$ with respect to the observer, c with respect to the approaching source, when light travels towards the observer.

It can be shown, for this case as well, that the perceived wave length of a light wave in the observer frame is the same as the emitted light wave in the moving source frame.

3.2.3   Case between Two Reference Frames Receding From the Ether Frame K_o

Here, our two inertial frames of reference are taken to be $K\left(x, y, z\right)$ and $K'\left(x', y', z'\right)$ in relative translational motion. Let their origins be aligned along the overlapped x- and x'- axes, and let v be the velocity between K' and the ether frame K_o, and w the velocity between K' and K_o. Therefore, the velocity between K and K' is $w - v$. Both frames are receding from the ether frame.

Assume that K and K' are overlapping at the time $t = t' = 0$. The event coordinates can then be considered as space and time intervals measured from the initial zero coordinates of the overlapped-frames event.

3.2.3.1   *Change of duration for events occurring at K' (traveling frame) origin—Ether Theory— "forward scenario"*

Suppose a signal of an event $E'(0,0,0)$ is emitted from K' origin at time t' with respect to K', which will be perceived at time interval t in K.

The speed of the light signal traveling from K' to K would be $c + w$ with respect to K', and also $c + v$ with respect to K.

K' perspective

From the perspective of K', the origin of K at the event occurring time is at a distance of $(w - v)t'$ from that of K'. Let $t^{*'}$ be the time interval it takes the event signal to reach the origin of K, from K' perspective. By that time, K' will have moved a further distance of $vt^{*'}$ away from K, bringing the distance traveled by the signal to $(w - v)t' + (w - v)t^{*'}$ with respect to K'. The time interval $t^{*'}$ can then be expressed as

$$t^{*'} = \frac{(w - v)t' + (w - v)t^{*'}}{c + w},$$

leading to

$$t^{*'} = \frac{(w - v)t'}{c + v}.$$

It follows that the event perception time interval t in K with respect to K' will be given by

$$t = t' + t^{*\prime};$$

$$t = t' + \frac{(w-v)t'}{c+v};$$

$$t = t'\left(\frac{c+w}{c+v}\right) = t'\left(\frac{1+\dfrac{w}{c}}{1+\dfrac{v}{c}}\right). \tag{49}$$

K perspective

Now, from the perspective of K, the origin of K' at the event occurring time is at a distance of $(w-v)t'$ from that of K. The signal will have traveled a distance of $(w-v)t''$ at the speed of $c+v$ with respect to K', when it reaches K' origin. Therefore, the event will be perceived at time t in K, given by

$$t = t' + \frac{(w-v)t'}{c+v};$$

$$t = t'\left(\frac{c+w}{c+v}\right) = t'\left(\frac{1+\dfrac{w}{c}}{1+\dfrac{v}{c}}\right),$$

the same as from the perspective of K'.

It follows that the proper time interval measured at the origin of the traveling frame K' between two events will be perceived as a time interval dilated in K by a factor of $(c+w)/(c+v)$, when $w > v$, i.e. when the frames are receding (for frames traveling in the same direction) from each other.

In case K' was traveling towards K, so as the frames were approaching each other, w would be replaced by $-w$ in Eq. (49), resulting in time contraction by a factor of $(c-w)/(c+v)$.

If the frames receded for a certain proper time interval $\Delta t_o'$ and then approached for an equal proper time interval, then the total perceived time interval in the stationary frame K would become

$$\Delta t = \Delta t_o'\left(\frac{c+w}{c+v}\right) + \Delta t_o'\left(\frac{c-w}{c+v}\right);$$

$$\Delta t = \frac{2\Delta t'_o c}{c + v} = \frac{\Delta t'}{1 + \dfrac{v}{c}},$$

with a net contraction factor of $\left(1 + v\,/\,c\right)^{-1}$.

3.2.3.1.1   Space alteration

Let's suppose the time t' measured at K' origin was for an event that has taken place at a point of coordinate $x' > 0$ $(x > 0)$ on the x-x' axis. Then, t' could be replaced by $x'\,/\,(c + w)$ and t by $x\,/\,(c + v)$ in Eq. (49), yielding

$$\frac{x}{c + v} + \frac{vt}{c} = \frac{x'}{c + w} + \frac{wt'}{c};$$

$$\frac{x}{1 + \dfrac{v}{c}} = \frac{x'}{1 + \dfrac{w}{c}} + wt' - vt;$$

$$\frac{x}{1 + \dfrac{v}{c}} = \frac{x'}{1 + \dfrac{w}{c}} + wt\left(\frac{1 + \dfrac{v}{c}}{1 + \dfrac{w}{c}}\right) - vt;$$

$$\frac{x}{1 + \dfrac{v}{c}} = \frac{x'}{1 + \dfrac{w}{c}} + \frac{t(w - v)}{1 + \dfrac{w}{c}};$$

$$x = x'\left(\frac{c + v}{c + w}\right) + t(w - v)\left(\frac{c + v}{c + w}\right);$$

$$x_+ = \left(\frac{c + v}{c + w}\right)x'_+ + (w - v)t'; \tag{50}$$

$$x_+ = \left(\frac{c + v}{c + w}\right)\left(x'_+ + (w - v)t\right); \tag{51}$$

Equation (51) shows the x-coordinate contracted by the factor $(c + v)\,/\,(c + w)$ with respect to its value given by the classical Galilean transformation.

3.2.3.2   Change of duration for events occurring at K (traveling frame) origin—Ether Theory—"backward scenario"

Now, consider the case where the signal of an event $E(0,0,0)$ is emitted from K origin at time τ with respect to K, which will be perceived at time τ' in K'.

The speed of the light signal traveling from K to K' would be $c - w$ with respect to K', and also $c - v$ with respect to K.

Using the above methodology, it can be shown that the perceived event duration in K' from the perspective of either frame would be given by

$$\tau' = \tau + \frac{(w - v)\tau}{c - v};$$

$$\tau' = \tau \left(\frac{c - v}{c - w} \right) = \tau \left(\frac{1 - \dfrac{v}{c}}{1 - \dfrac{w}{c}} \right). \tag{52}$$

It follows that the time interval measured at the origin of the traveling frame K between two events will be perceived dilated in K' by a factor of $(c - v)/(c - w)$, when $w > v$, i.e. when the frames are receding.

Consequently, for the case of receding reference frames, the perceived time dilated would not be the same for the two cases when the traveling frame is considered to be K' in the first and K in the second case. Thus, the perceived time dilation is not symmetrical in the case the Ether Theory is adopted.

In case K' was traveling towards K, so as the frames were approaching each other, w would be replaced by $-w$ in Eq. (52), resulting in time contraction by a factor of $(c - v)/(c + w)$.

If the frames receded for a certain proper time interval $\Delta\tau_o$ and then approached for an equal proper time interval, then the total perceived time interval in K' would become

$$\Delta\tau' = \Delta\tau_o \left(\frac{c - v}{c - w} \right) + \Delta\tau_o \left(\frac{c - v}{c + w} \right);$$

$$\Delta\tau' = 2\Delta\tau_o c \left(\frac{c - v}{c^2 - w^2} \right) = \Delta\tau \left(\frac{1 - v/c}{1 - w^2/c^2} \right),$$

asymmetrical with respect to the "forward scenario", which exhibited a net contraction factor of $(1 + v/c)^{-1}$ when the traveling frame K' oscillated back and forth with respect to the stationary frame K for equal proper time intervals.

3.2.3.2.1   *Space alteration*

Let's suppose the time τ measured at K origin was for an event that has taken place at a point of coordinate $x < 0$ $(x' < 0)$ on the x-x' axis. Then, τ could be replaced by $x / (c - v)$ and τ' by $x' / (c - w)$ in Eq.(52), yielding

$$x'_- = \left(\frac{c-w}{c-v}\right)x_- - (w-v)\tau; \tag{53}$$

$$x'_- = \left(\frac{c-w}{c-v}\right)(x_- - (w-v)\tau'). \tag{54}$$

Equation (54) shows the x-coordinate contracted by the factor $(c-w) / (c-v)$ with respect to its value given by the classical Galilean transformation.

3.2.3.3   **Doppler Effect**

3.2.3.3.1   **Forward Scenario**

If the time interval represents the period of a periodic event (e.g., wave, vibration or rotation period) in K', then the relation between the actual and perceived frequency of the event can be determined from Eq. (49) as

$$f = f'\left(\frac{c+v}{c+w}\right), \tag{55}$$

where, f and f' are the perceived and actual frequency with respect to an observer in K and K', respectively. Hence, the perceived frequency is lower than the proper frequency in the receding source frame (i.e. when $w > v$).

Equation (55) expresses the Doppler effect for the case of both the source and observer are receding from the ether frame at the speeds of w and v, respectively, and the source is receding from the observer at a uniform velocity $w - v$, under the assumption that the speed of light is $c + v$ with respect to the observer, $c + w$ with respect to the receding source, when light travels towards the observer.

For the case of a light wave, assume λ and λ' are the perceived and actual wave length with respect to K and K'. Then, Eq. (55) leads to

$$\frac{c+v}{\lambda} = \frac{c+w}{\lambda'}\left(\frac{c+v}{c+w}\right);$$

$$\lambda = \lambda'.$$

Therefore, the perceived wave length of a light wave in the observer frame is the same as the emitted light wave in the moving source frame.

3.2.3.3.2   Backward Scenario

If the proper time interval in K represents the period of a periodic event, then the relation between the actual and perceived frequency of the event can be determined from Eq. (52) as

$$\phi' = \phi\left(\frac{c-w}{c-v}\right). \tag{56}$$

where, ϕ' and ϕ are the perceived and actual frequency with respect to an observer in K' and K, respectively, for a source in K. Hence, the perceived frequency is lower than the proper frequency in the receding (when $w > v$) source frame. However, the decreasing factor is different from the case where the source is in the other frame. Therefore, the Doppler effect is asymmetrical in the case of the Ether Theory assumption.

Equation (56) expresses the Doppler effect for the case of both the source and observer are receding from the ether frame at the speeds of v and w, respectively, and the source is receding from the observer at a uniform velocity $w - v$, under the assumption that the speed of light is $c - w$ with respect to the observer, $c - v$ with respect to the receding source, when light travels towards the observer.

For the case of a light wave, assume λ' and λ are the perceived and actual wave length with respect to K' and K. Then, Eq. (56) leads to

$$\frac{c-w}{\lambda'} = \frac{c-v}{\lambda}\left(\frac{c-w}{c-v}\right);$$

$$\lambda' = \lambda.$$

Therefore, the perceived wave length of a light wave in the observer frame is the same as the emitted light wave in the moving source frame.

3.3   SPECIAL RELATIVITY APPROACH

In Special Relativity, light travels in the absence of a propagating medium at a constant speed with respect to all inertial reference frames. Let c be the absolute speed of light with respect to both frames, K and K'.

3.3.1   Case of Receding Reference Frames—Special Relativity

3.3.1.1 *Change of duration for events occurring at K' (traveling frame) origin—Special Relativity—receding frames "forward scenario"*

The speed $c'_{K' \to K}$ or $c_{K' \to K}$ of a light signal traveling from K' to K with respect to K' or K, would be c.

Applying Eq. (1) for the perceived time interval in K from the K' perspective, we get

$$t = \frac{t'}{1 - \dfrac{v}{c'_{K' \to K}}} = \frac{t'}{1 - \dfrac{v}{c}}. \tag{57}$$

Whereas, the same perceived time interval in K from the perspective of K, is given by Eq. (2) as

$$t = t'\left(1 + \frac{v}{c_{K' \to K}}\right) = t'\left(1 + \frac{v}{c}\right). \tag{58}$$

Each of the parameters t' and t represents the same entity in Eqs. (57) and (58) (i.e., t' represents the time interval between two successive events, measured in the traveling frame K', and t the corresponding time interval as perceived in the stationary frame K. It follows that

$$t = \frac{t'}{1 - \dfrac{v}{c}} = t'(1 + \frac{v}{c}),$$

leading to $v = 0$, unless an ad hoc assumption is made such that t is transformed by a certain factor (say γ) with respect to K, and by another factor (say β) with respect to K', leading to the equation

$$t = \frac{\beta t'}{\left(1 - \dfrac{v}{c}\right)} = \gamma t'\left(1 + \frac{v}{c}\right);$$

which can be satisfied only if $\beta = 1 / \gamma$, resulting in

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}.$$

Therefore,

$$t = \frac{t'}{\gamma\left(1 - \dfrac{v}{c}\right)};$$

$$t = \gamma t' \left(1 + \frac{v}{c} \right); \tag{59}$$

$$t' = \frac{t}{\gamma \left(1 + \dfrac{v}{c} \right)};$$

$$t' = \gamma t \left(1 - \frac{v}{c} \right). \tag{60}$$

It follows that the proper time interval measured at the origin of the traveling frame K' between two events will be perceived as a time interval dilated in K by a factor of $\gamma(1 + v/c)$, or $[\gamma(1 - v/c)]^{-1}$.

It is critical to note that both equations (59) and (60) describe the time dilation in K of a proper time interval $t' > 0$ measured in K' origin $(x' = 0)$ and perceived as t (time interval) in K origin. In one equation, the perceived time interval t is written in terms of the proper time interval t'; in the other one, t' in terms of t. In other words, the time interval t' in Eq. (60) should not be interpreted as the perceived time interval in K' of a proper time interval t in K, which must be dilated as well (discussed in next section), yet Eq. (60) exhibits time contraction for the proper time interval t' (with respect to the dilated time interval t). Equations (59) and (60) are equivalent (the same equation written in two different forms), so proper time in one remains proper in the other, so as the perceived time. One should not be misled by the swapped primed variables and the sign change of the velocity, in going from Eq. (59) to Eq. (60), which is merely a property of the expression $\gamma \left(1 - v/c \right)$ or $\gamma \left(1 + v/c \right)$, the inverse of which would be simply obtained by reversing the sign of the v term!

3.3.1.2 Change of duration for events occurring at K (traveling frame) origin—Special Relativity—receding frames "backward scenario"

Using the above methodology in reaching Eqs. (59) and (60), it can be easily verified that a proper time interval τ in K will be perceived as a dilated time interval τ' in K', according to the equations

$$\tau' = \frac{\tau}{\gamma \left(1 - \dfrac{v}{c} \right)} = \gamma \tau \left(1 + \frac{v}{c} \right); \tag{61}$$

$$\tau = \frac{\tau'}{\gamma \left(1 + \dfrac{v}{c} \right)} = \gamma \tau' \left(1 - \frac{v}{c} \right). \tag{62}$$

In the case of receding frames, the perceived time interval is always dilated with respect to the proper time interval.

It follows that the proper time interval measured at the origin of the traveling frame K ("backward scenario") between two events will be perceived as a time interval dilated in K' by a factor of $\gamma(1 + v\,/\,c)$, or $[\gamma(1 - v\,/\,c)]^{-1}$, the same as the time dilation factor in K for the "forward scenario" (Eq.(59)). Hence the Special Relativity results in symmetry with respect to the reference frames as to the extent of the time dilation between the receding reference frames.

3.3.1.3   *Lorentz Transformation*

3.3.1.3.1   *Forward Scenario*

Now, back to Eq. (59), if the time t' measured at K' origin was for an event that has initially taken place at a point of coordinate $x' > 0\ (x > 0)$ on the x-x' axis, then t' could be replaced by $x'\,/\,c$, in the last term of Eq. (59), and t by $x\,/\,c$ in the last term of Eq. (60), to yield

$$t = \gamma\left(t' + \frac{vx'_+}{c^2}\right). \tag{63}$$

$$t' = \gamma\left(t - \frac{vx_+}{c^2}\right). \tag{64}$$

Equations (63) and (64) are limited to K' proper events with positive x and x' coordinates, and not applicable for events having $x' = 0$ when $t' > 0\ (x = 0$ when $t > 0)$—in which case Eqs. (59) and (60) should be used—since they were obtained under $x' = ct'$ and $x = ct$, which returns $t' = 0$ and $t = 0$ for $x' = 0$ and $x = 0$, respectively. Letting $x' = 0$ in Eq. (63) (or $x = 0$ in Eq. (64)) returns the wrong result $t = \gamma t'$ (or $t' = \gamma t$).

Now, replacing t and t' with $x\,/\,c$ and $x'\,/\,c$ in Eqs. (59) and (60), yields

$$x_+ = \gamma(x'_+ + vt'), \tag{65}$$

$$x'_+ = \gamma(x_+ - vt), \tag{66}$$

exhibiting perceived length expansion in K.

Equations (63) to (66) are the Lorentz transformation equations, the basis of the special relativity theory.[2,3] However, these equations are derived under the assumption $x' = ct'$ and $x = ct$. i.e. they are limited to K' proper events with positive x and x' coordinates. Indeed, it has been shown in related studies that the Lorentz transformation equations result in mathematical contradictions when applied for co-local or simultaneous events.[4][5][6]

It should be emphasized again that equation (63) and its converse (64) describe the time dilation in K of a proper time interval t' measured in the traveling frame K' for an event having $x' > 0$,

and perceived as t (perceived time interval) in K at a corresponding $x > 0$. Eq. (65) and its converse (66) are the corresponding space transformation equation, applicable in the same space coordinate domain.

3.3.1.3.2 Backward Scenario

Similarly, if the "reversed" proper time interval τ measured at the traveling frame K origin was for an event that has initially taken place at a point of coordinate $x < 0$ $(x' < 0)$ on the x-x' axis, then τ and τ' may be replaced by $-x\,/\,c$ and $-x'\,/\,c$ in Eqs. (61) and (62), respectively, leading to

$$\tau' = \gamma\left(\tau - \frac{vx_-}{c^2}\right), \tag{67}$$

$$\tau = \gamma\left(\tau' + \frac{vx'_-}{c^2}\right), \tag{68}$$

and consequently,

$$x'_- = \gamma(x_- - v\tau), \tag{69}$$

$$x_- = \gamma(x'_- + v\tau'). \tag{70}$$

exhibiting perceived length expanded in K'.

Equations (67) to (70) are limited to the traveling frame K proper events with negative x and x' coordinates, and not applicable for events having $x = 0$ when $\tau > 0$ $(x' = 0$ when $\tau' > 0)$— in which case Eqs. (61) and (62) should be used—since they were obtained under $x = -c\tau$ and $x' = -c\tau'$, which returns $\tau = 0$ and $\tau' = 0$ for $x = 0$ and $x' = 0$, respectively. Letting $x = 0$ in Eq. (64) (or $x' = 0$ in Eq.(63)) returns the wrong result $\tau' = \gamma\tau$ (or $\tau = \gamma\tau'$).

Lorentz transformation Eqs. (63) – (66) and its reversed Eqs. (67) – (70) take the same form, but with different domains, i.e. with the difference being that in the first set, where the proper time interval is considered as t' in the traveling frame K', perceived in the stationary frame K as t, the terms $x = ct$ and $x' = ct'$ are positive and embedded in the equations, whereas $x = -c\tau$ and $x' = -c\tau'$ are negative and embedded in the second set, where the proper time interval is considered as τ in the traveling frame K and perceived in the stationary frame K' as τ'. Therefore, the sets of Lorentz transformation equations given by Eqs. (63) – (66) and Eqs. (67) – (70) may be replaced with the former set with eliminating the "+" and "-" indices, keeping in mind the above coordinate application domains. Hence, the Lorentz Transformation equations interpretation changes depending on which frame is considered as the "traveling" one (i.e., the "traveling" frame in which the proper time is measured). Lorentz Transformation is restricted to positive x and x' coordinates when K' is taken as the traveling frame (Eqs. (63) and (65) become the principle transformation equations for this case; Eqs. (64) and (66) are their equivalent equations in the reversed form), and to negative coordinates when K is taken as the traveling frame (Eqs. (64) and (66) become the principle

"reversed" transformation equations for this case; Eqs. (63) and (65) are their equivalent equations in the reversed form). This can be easily checked by plugging numerical values in the Lorentz Transformation time equations. For instance, assume $c = 1$, $v = 0.5$, hence $\gamma = 1.15$, and let an event take place in K' at $x' = 1$, $t' = 1$. Using Eq. (63), the corresponding time in K would be $t = 1.73$, dilated in agreement with Eq. (59). However, if the same event took place at $x' = -1$, Eq. (63) yields $t = 0.58$, an apparent time contraction, in disagreement with the basic Eq. (59). In fact, this apparent time contraction is justified by the fact this event's point location $(x' = -1)$ in K' is approaching the origin of K that would receive signals from this approaching point with contracted time intervals, as demonstrated in the later section related to approaching frames. The contracted time $t = 0.58$ is in agreement with the respective apparent time contraction equation (Eq. (75)) for approaching frames given in the subsequent related section. This constitutes a clear self-contradiction within the Special Relativity that claims time dilation should always occur for a moving reference whether it was approaching or receding from the observer.

Similar checking can be done using Eq. (67) with an event taking place in K at $x = -1$, $\tau = 1$, to obtain a dilated time $\tau' = 1.73$, in agreement with Eq. (61) whereas an erroneous contracted time of $\tau' = 0.58$ would be obtained, had the event taken place at $x = 1$.

3.3.1.3.3   Symmetry

Considering the direct time transformation Eq. (59), and its reversed Eq. (61):

$$t = \gamma t'\left(1 + \frac{v}{c}\right), \quad \tau' = \gamma\tau\left(1 + \frac{v}{c}\right),$$

we see they exhibit a total symmetry in regard to the perceived time dilation factor. When rewriting these equations in their reversed form,

$$t' = \gamma t\left(1 - \frac{v}{c}\right), \quad \tau = \gamma\tau'\left(1 - \frac{v}{c}\right),$$

the primed and unprimed term are swapped, and the sign of the v term is reversed, yet they do not represent the "reversed" transformation equations.

Now, for co-local events not occurring at the origin of K' and K for the forward and backward scenario, respectively, the above "direct" and "reversed" transformations would hold with $x = ct$; $x' = ct'$, and $x = -c\tau$; $x' = -c\tau'$, respectively, yielding the Lorentz transformation and its reversed,

$$t = \gamma\left(t' + \frac{vx'_+}{c^2}\right), \quad \tau' = \gamma\left(\tau - \frac{vx_-}{c^2}\right),$$

and in reversed form:

$$t' = \gamma\left(t - \frac{vx_+}{c^2}\right), \quad \tau = \gamma\left(\tau' + \frac{vx'_-}{c^2}\right).$$

Therefore, the equations of t and t' represent the same transformation and can be obtained from each other by swapping the primed and unprimed terms and reversing the v term sign, due merely to the property of the expression $\gamma\left(1 - v \,/\, c\right)$ or $\gamma\left(1 + v \,/\, c\right)$, the inverse of which would be simply obtained by reversing the sign of the v term! Taking account of the spatial coordinate sign, the equations of t and τ' have total symmetry, and the sign change of the v term in τ' equation is merely due to the replacement of x with $-c\tau$, and not to the reversed direction of v.

3.3.1.4   Doppler Effect

3.3.1.4.1   Forward Scenario

If the time interval represents the period of a periodic event (e.g., wave, vibration or rotation period) in K' (source), then the relation between the actual and perceived frequency of the event can be determined from Eq. (59) as

$$f = \gamma f'\left(1 - \frac{v}{c}\right); \tag{71}$$

$$f' = \gamma f\left(1 + \frac{v}{c}\right).$$

where, f and f' are the perceived and actual (proper) frequency with respect to an observer in K and K', respectively. Hence, the perceived frequency is lower than the proper frequency in the receding source frame.

Equation (71) expresses the relativistic Doppler Effect[2] for the case of a receding source from the observer at a uniform velocity v, under the assumption that the speed of light is c with respect to both the source and the observer when light travels towards the observer.

For the case of a light wave, assume λ and λ' are the perceived and actual wave length with respect to K and K'. Then, Eq. (71) leads to

$$\frac{c}{\lambda} = \frac{\gamma c}{\lambda'}\left(1 - \frac{v}{c}\right);$$

$$\lambda = \frac{\lambda'}{\gamma\left(1 - \dfrac{v}{c}\right)};$$

$$\lambda = \gamma\lambda'\left(1 + \frac{v}{c}\right).$$

Therefore, the perceived wave length of a light wave in the observer frame is longer than the light wave emitted in the moving receding frame.

3.3.1.4.2   Backward Scenario

If the time interval represents the period of a periodic event in K (source), then the relation between the actual and perceived frequency of the event can be determined from Eq. (61) as

$$\phi' = \gamma\phi\left(1 - \frac{v}{c}\right);$$

$$\phi = \gamma\phi'\left(1 + \frac{v}{c}\right). \tag{72}$$

where, ϕ' and ϕ are the perceived and actual (proper) frequency with respect to an observer in K' and K, respectively. Hence, the perceived frequency is lower than the proper frequency in the receding source frame, with same decreasing factor as for the reversed case where the source is in K'. Hence, the Doppler Effect is symmetrical under the Special Relativity assumptions.

Equation (72) expresses the relativistic Doppler effect for the case of a receding source from the observer at a uniform velocity v, under the assumption that the speed of light is c with respect to both the source and the observer when light travels towards the observer.

3.3.2   Case of Approaching Reference Frames—Special Relativity

3.3.2.1   Change of duration for events occurring at K' (traveling frame) origin—Special Relativity—approaching frames "forward scenario"

In this case, $c'_{K'\to K} = c_{K'\to K} = c$. Applying Eq. (6) for the perceived time interval in K from the K' perspective, we get

$$\Delta t = \frac{\Delta t'}{1 + \dfrac{v}{c'_{K'\to K}}} = \frac{\Delta t'}{1 + \dfrac{v}{c}}; \tag{73}$$

Whereas, the same perceived time interval in K from the perspective of K, is given by Eq. (7) as

$$\Delta t = \Delta t'\left(1 - \frac{v}{c_{K'\to K}}\right) = \Delta t'\left(1 - \frac{v}{c}\right). \tag{74}$$

Each of the parameters $\Delta t'$ and Δt represent the same entities in Eqs. (73) and (74) (i.e., $\Delta t'$ represents the proper time interval between two successive events, measured in the traveling frame K', and Δt the corresponding time interval as perceived in K. It follows that

$$\Delta t = \Delta t'\left(1 - \frac{v}{c}\right) = \frac{\Delta t'}{\left(1 + \dfrac{v}{c}\right)},$$

leading to $v = 0$, unless an ad hoc assumption is made such that Δt is transformed by a certain factor (say γ) with respect to K, and by another factor (say β) with respect to K', which leads to the equation

$$\Delta t = \gamma\Delta t'\left(1 - \frac{v}{c}\right) = \frac{\beta\Delta t'}{\left(1 + \dfrac{v}{c}\right)},$$

which can be satisfied only if $\beta = 1 / \gamma$, resulting in

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}.$$

Therefore,

$$\Delta t = \gamma\Delta t'\left(1 - \frac{v}{c}\right); \tag{75}$$

$$\Delta t = \frac{\Delta t'}{\gamma\left(1 + \dfrac{v}{c}\right)};$$

$$\Delta t' = \gamma\Delta t\left(1 + \frac{v}{c}\right); \tag{76}$$

$$\Delta t' = \frac{\Delta t}{\gamma\left(1 - \dfrac{v}{c}\right)}.$$

It follows that the time interval measured at the origin of the traveling frame K' between two events will be perceived as a time interval contracted in K by a factor of $\gamma(1 - v / c)$, or $[\gamma(1 + v / c)]^{-1}$, in the case of approaching frames.

It is critical to note that both equations (75) and (76) describe the time contracted in K of a proper time interval $\Delta t' > 0$ measured in the traveling frame K' origin $(x' = 0)$ and perceived as Δt (time interval) in K origin. In one equation, Δt is written in terms of $\Delta t'$; in the other one, $\Delta t'$ in terms of Δt. In other words, the proper time interval $\Delta t'$ in Eq. (76) should not be interpreted as the perceived time interval in K' of a proper time interval Δt in K. Equations (75) and (76) are equivalent (the same equation written in two different forms), so proper time in one remains proper in the other, so as the perceived time. One should not be misled by the swapped primed variables and the sign change of the velocity, in going from Eq. (75) to Eq. (76), which is merely a property of the expression $\gamma\left(1 - v / c\right)$ or $\gamma\left(1 + v / c\right)$, the inverse of which would be simply obtained by reversing the sign of the v term!

3.3.2.1.1 Receding–approaching frames
We note from Eqs. (59) and (75) that if the frames receded for a certain proper time interval $\Delta t_o{}'$ and then approached for an equal proper time interval, then the total perceived time interval in K would become

$$\Delta t = \gamma \Delta t_o{}'\left(1 + \frac{v}{c}\right) + \gamma \Delta t_o{}'\left(1 - \frac{v}{c}\right);$$

$$\Delta t = 2\gamma \Delta t_o{}' = \frac{\Delta t'}{\sqrt{1 - \dfrac{v^2}{c^2}}},$$

with a net dilation factor of $\left(1 - v^2 / c^2\right)^{-1/2}$.

3.3.2.2 Change of duration for events occurring at K (traveling frame) origin—Special Relativity—approaching frames "backward scenario"

Using the above methodology in reaching Eqs. (75) and (76), it can be easily verified that a proper time interval $\Delta\tau$ in the traveling frame K will be perceived as a contracted time interval $\Delta\tau'$ in K', according to the equations

$$\Delta\tau' = \gamma\Delta\tau\left(1 - \frac{v}{c}\right); \tag{77}$$

$$\Delta\tau' = \frac{\Delta\tau}{\gamma\left(1 + \dfrac{v}{c}\right)};$$

$$\Delta\tau = \gamma\Delta\tau'\left(1 + \frac{v}{c}\right); \tag{78}$$

$$\Delta\tau = \frac{\Delta\tau'}{\gamma\left(1-\dfrac{v}{c}\right)}.$$

In the case of approaching frames, the perceived time interval is always contracted with respect to the proper time interval.

It follows that the proper time interval measured at the origin of the traveling frame K ("backward scenario") between two events will be perceived contracted in the stationary frame K' by a factor of $\gamma(1-v\,/\,c)$, or $[\gamma(1+v\,/\,c)]^{-1}$, the same as the time contraction factor in K for the "forward scenario" (Eq. (75)). Hence the Special Relativity results in symmetry with respect to the reference frames as to the extent of the perceived time contraction between the approaching reference frames.

3.3.2.2.1 Receding–approaching frames

We note from Eqs. (61) and (77) that if the frames receded for a certain proper time interval $\Delta\tau_o$ and then approached for an equal proper time interval, then the total perceived time interval in K' would become

$$\Delta\tau' = 2\gamma\Delta\tau_o = \frac{\Delta\tau}{\sqrt{1-\dfrac{v^2}{c^2}}},$$

with a net dilation factor of $\left(1-v^2\,/\,c^2\right)^{-1/2}$,

symmetrical with respect to the "forward scenario".

3.3.2.3 Lorentz Transformation

3.3.2.3.1 Forward Scenario

Now, back to Eq. (75), If the time t' measured at K' origin was for an event that has initially taken place at a point of coordinate $x' > 0$ $(x > 0)$ on the x-x' axis, then $\Delta t'$ could be replaced by $\Delta x'\,/\,c$, in the last term of Eq. (75), and Δt by $\Delta x\,/\,c$ in the last term of Eq. (76), to yield

$$\Delta t = \gamma\left(\Delta t' - \frac{v\Delta x'_+}{c^2}\right). \tag{79}$$

$$\Delta t' = \gamma\left(\Delta t + \frac{v\Delta x_+}{c^2}\right). \tag{80}$$

Equations (79) and (80) are limited to K' proper events with positive x and x' coordinates, and are not applicable for $\Delta x' = 0$ with $\Delta t' > 0$ $(\Delta x = 0$ with $\Delta t > 0)$—in which case Eqs. (75)

and (76) should be used—since they were obtained under $\Delta x' = c\Delta t'$ and $\Delta x = c\Delta t$, which returns $\Delta t' = 0$ and $\Delta t = 0$ for $\Delta x' = 0$ and $\Delta x = 0$, respectively. Letting $\Delta x' = 0$ in Eq. (79) (or $\Delta x = 0$ in Eq. (80)) returns the wrong result $\Delta t = \gamma \Delta t'$ (or $\Delta t' = \gamma \Delta t$).

Now, replacing Δt and $\Delta t'$ with $\Delta x / c$ and $\Delta x' / c$ in Eqs. (75) and (76), yields

$$\Delta x_+ = \gamma(\Delta x'_+ - v\Delta t').\tag{81}$$

$$\Delta x'_+ = \gamma(\Delta x_+ + v\Delta t).\tag{82}$$

exhibiting perceived length contracted in K.

Equations (79) to (82) are the Lorentz transformation equations for the case of approaching reference frames. However, these equations are derived under the assumption $\Delta x' = c\Delta t'$ and $\Delta = c\Delta t$. i.e. they are limited to K proper events with positive x and x' coordinates.

It should be emphasized again that equation (79) and its converse (80) describe the time contraction in K of a proper time interval $\Delta t'$ measured in K' for an event having $\Delta x' > 0$, and perceived as Δt (perceived time interval) in K at a corresponding $\Delta x > 0$. Eq. (81) and its converse (82) are the corresponding space transformation equations, applicable in the same space coordinate domain.

3.3.2.3.2   Backward Scenario

Similarly, if the proper time interval τ measured at K origin was for an event that has initially taken place at a point of coordinate $x < 0$ $(x' < 0)$ on the x-x' axis, then then τ and τ' may be replaced by $-x / c$ and $-x' / c$ in Eqs. (77) and (78), respectively, leading to

$$\Delta\tau' = \gamma\left(\Delta\tau + \frac{v\Delta x_-}{c^2}\right)\tag{83}$$

$$\Delta\tau = \gamma\left(\Delta\tau' - \frac{v\Delta x'_-}{c^2}\right)\tag{84}$$

$$\Delta x'_- = \gamma(\Delta x_- + v\Delta\tau)\tag{85}$$

$$\Delta x_- = \gamma(\Delta x'_- - v\Delta\tau')\tag{86}$$

exhibiting perceived length contracted in K'.

Equations (83) to (86) are limited to K proper events with negative x and x' coordinates, and not applicable for events having $\Delta x = 0$ when $\Delta\tau > 0$ ($\Delta x' = 0$ when $\Delta\tau' > 0$)—in which case Eqs. (77) and (78) should be used—since they were obtained under $\Delta x = -c\Delta\tau$ and

$\Delta x' = -c\Delta\tau'$, which returns $\Delta\tau = 0$ and $\Delta\tau' = 0$ for $\Delta x = 0$ and $\Delta x' = 0$, respectively. Letting $\Delta x = 0$ in Eq. (83) (or $\Delta x' = 0$ in Eq. (84)) returns the wrong result $\Delta\tau' = \gamma\Delta\tau$ (or $\Delta\tau = \gamma\Delta\tau'$).

Lorentz transformation Eqs. (79) – (82) and its reversed Eqs. (83) – (86) take the same form, but with different domains, i.e. with the difference being that in the first set, where the proper time interval is considered as $\Delta t'$ in the traveling frame K', the terms $\Delta x = c\Delta t$ and $\Delta x' = c\Delta t'$ are positive and embedded in the equations, whereas $\Delta x = -c\Delta\tau$ and $\Delta x' = -c\Delta\tau'$ are negative and embedded in the second set, where the proper time interval is considered as $\Delta\tau$ in the traveling frame K. Therefore, Eqs. (79) – (82) can be used in place of Eqs(83) – (86), keeping in mind the above coordinate application domains. Hence, the Lorentz Transformation equations interpretation changes depending on which frame is considered as the "traveling" one (i.e., the frame in which the proper time is measured). Lorentz Transformation equations are restricted to positive x and x' coordinates when K' is taken as the "traveling" frame, and to negative coordinates when K is taken as the "traveling" one.

3.3.2.3.3   *Special Relativity Self-Contradiction*

Consider two receding inertial reference frames, and their Lorentz transformation time equation

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right).$$

Let an event be taking place in K' at $x' = x_o'$, satisfying $x_o' = -ct_o'$. According to the above equation, the converted time in K would be

$$t_o = \gamma t_o'\left(1 - \frac{v}{c}\right),$$

which is a time contraction. Nonconforming with the Special Relativity, the justification for this time contraction is that the point having a negative coordinate of $x_o' = -ct_o'$ in K' would be approaching the origin of K when the frames are receding.

Now, modifying the above scenario, suppose the origin of K' was shifted to the point with the original coordinate of x_o', so that the above event would occur at K' origin, now traveling towards K. This origin shifting doesn't change anything for an observer at K origin, in so far as the event time and location are concerned. In this case, however, the above Lorentz time transformation equation yields

$$t_o = \gamma t_o',$$

a time dilation with respect to the K observer, in contradiction with the time contraction obtained in the former scenario, although the two scenarios are equivalent relative to the K observer. This contradiction disappears had we used the time transformation Eq. (75) for approaching frames when the proper time is measured at the traveling frame origin, namely

$$\Delta t = \gamma \Delta t' \left(1 - \frac{v}{c} \right).$$

3.3.2.4 Doppler Effect

3.3.2.4.1 Forward Scenario

If the time interval represents the period of a periodic event (e.g., wave, vibration or rotation period), then the relation between the actual and perceived frequency of the event can be determined from Eq. (75) as

$$f = \gamma f' \left(1 + \frac{v}{c} \right);$$

(87)

$$f' = \gamma f \left(1 - \frac{v}{c} \right).$$

where, f and f' are the perceived and actual (proper) frequency with respect to an observer in K and K', respectively. Hence, the perceived frequency is higher than the proper frequency in the approaching source frame.

Equation (87) expresses the relativistic Doppler effect for the case of an approaching source to the observer at a uniform velocity v, under the assumption that the speed of light is c with respect to both the source and the observer when light travels towards the observer.

For the case of a light wave, assume λ and λ' are the perceived and actual wave length with respect to K and K'. Then, Eq. (87) leads to

$$\frac{c}{\lambda} = \frac{\gamma c}{\lambda'} \left(1 + \frac{v}{c} \right);$$

$$\lambda = \frac{\lambda'}{\gamma \left(1 + \frac{v}{c} \right)};$$

$$\lambda = \gamma \lambda' \left(1 - \frac{v}{c} \right).$$

Therefore, the perceived wave length of a light wave in the observer frame is shorter than the light wave emitted in the approaching source frame.

3.3.2.4.2 Backward Scenario

If the time interval represents the period of a periodic event in K (source), then the relation between the actual and perceived frequency of the event can be determined from Eq. (77) as

$$\phi' = \gamma\phi\left(1 + \frac{v}{c}\right); \tag{88}$$

$$\phi = \gamma\phi'\left(1 - \frac{v}{c}\right).$$

where, ϕ' and ϕ are the perceived and actual (proper) frequency with respect to an observer in K' and K, respectively. Hence, the perceived frequency is higher than the proper frequency in the approaching source frame, with same increasing factor as for the reversed case where the source is in K'. Hence, the Doppler Effect is symmetrical under the Special Relativity assumptions.

Equation (88) expresses the relativistic Doppler effect for the case of an approaching source to the observer at a uniform velocity v, under the assumption that the speed of light is c with respect to both the source and the observer when light travels towards the observer.

Equations (87) and (88), deduced from the time contraction equations, show that the perceived frequency undergoes an increase (blue shift) for the case of approaching frames, in line with the Special Relativity predictions. Hence, the Special Relativity is in contradiction between time (wave period) dilation and frequency boosting.

For the case of a light wave, assume λ' and λ are the perceived and actual wave length with respect to K' and K. Then, Eq. (88) leads to

$$\frac{c}{\lambda'} = \frac{\gamma c}{\lambda}\left(1 + \frac{v}{c}\right);$$

$$\lambda' = \frac{\lambda}{\gamma\left(1 + \frac{v}{c}\right)};$$

$$\lambda' = \gamma\lambda\left(1 - \frac{v}{c}\right).$$

Therefore, the perceived wave length of a light wave in the observer frame is shorter than the light wave emitted in the approaching source frame.

4 CONCLUSIONS

Basic classical physical concepts of time and space were applied to analyze time and space alterations between inertial reference frames in relative motion, while considering event information

is ultimately conveyed through light—or electromagnetic—signals. As these alterations were only apparent, they were considered to be of the real time and space entities. The reference frame associated with the events proper time was labeled as traveling frame, and he events perceiving frame as stationary frame. Results were obtained for the different theories of light. Appendix A gives a tabulated summary of the results. The classical theories (Ether and Emission) showed consistency in term of alteration factors perceived from the perspective of both frames. i.e., a traveling frame proper time interval is perceived as a time interval altered by the same factor in the stationary frame from the perspective of both frames. However, with the Special Relativity second postulate being considered, the alteration factor depended on the reference frame, producing different values for the same perceived time interval, and requiring the introduction of artificial deformation factors; dilation with respect to one frame, contraction with respect to the other, in order to equate the terms $(1 - v / c)$ and $(1 + v / c)^{-1}$, where v and c are the relative motion velocity and light speed, respectively. It was found that multiplying the first term and dividing the second term by γ, where $\gamma = \left(\sqrt{1 - v^2 / c^2} \right)^{-1}$, i.e. assuming inverse deformations of the perceived time from the perspective of the reference frames, would correct the situation, and result in the same alteration factor from the perspective of either frame. The fact that the inverse of an alteration factor $\gamma(1 - v / c)$ or $\gamma(1 + v / c)$ is obtained by just reversing the v sign, $[\gamma(1 - v / c)]^{-1} = \gamma(1 + v / c)$, leads to a misconception of symmetry.

The alteration factors revealed time dilation and length contraction when the frames were receding; whereas time contraction and length expansion factors were obtained for approaching frames, in the considered classical light theories. The time alteration factors (dilation for receding and contraction for approaching frames) resulted from the Emission Theory assumptions were applicable for the Special Relativity case, after being reduced by the relativistic factor γ. Self-contradiction in the Lorentz transformation was revealed in connection with the time contraction for approaching frames.

If the frames receded and then approached during equal proper time intervals, the total perceived time interval exhibited a net time dilation for the light emission and Special Relativity theory. For the ether assumption, either time dilated or invariant was obtained depending on whether the proper-time traveling frame was taken to be the ether frame. The frame round trip apparent time dilation factor was equal to γ for the Special Relativity and γ^2 for the classical case.

The Special Relativity resulting space coordinate transformation equations showed apparent length expansion for receding frames, and contraction for approaching ones.

Interchanging the "traveling"/ "stationary" reference frames resulted in different space and time alteration factors for the Ether Theory, while the same factors were retained for the emission and Special Relativity theories. However, when the time equations were expressed in terms of both space and time coordinates, under some restricted coordinate conditions, the spatial coordinate terms in the resulted time equations had opposite sign compared to the similar terms in the "backward scenario" equations. This was due to the fact that for the "backward scenario" frames arrangement, the time equations could be expressed in terms of space coordinates in the opposite direction to the "forward scenario".

The known classical and relativistic Doppler Effects were readily derived from the established alteration factors. The relativistic Doppler Effect in the case of approaching reference frames exhibited blue shift for light, in line with the determined time (period) contraction. Thus, the Special Relativity prediction of time dilation for both receding and approaching frames was contradicted. In addition, for the Special Relativity approach, in the case of light, the wavelength exhibited an increase

in the case of receding frames, whereas it decreased when the source was approaching. For all classical approaches, and in the case of light, the wave length was invariant.

Available experimental data related to [apparent] time dilations, Doppler Effect, and non-existence of preferred reference frame for light propagation (Michelson-Morley experiment) could then be analyzed vis-à-vis the classical theories in terms of time and space transformations.

ANNEX A

Table A1 Results Summary

Relative Motion ↓	Assumed theory→ Event Scenario↓	Emission Theory, K & K' in relative motion at velocity $= v$	Ether Theory, $K =$ ether, K & K' in relative motion at velocity $= v$	Ether Theory, $K_o =$ ether, K & K' in relative motion at v & w w.r.t K_o	Special Relativity, K & K' in relative motion at velocity $= v$
Receding Frames	$t' =$ **Event proper time interval in the traveling frame** K' **(forward scenario), at origin.** $t =$ **perceived time**	$t = \dfrac{t'}{1 - \dfrac{v}{c}}$	$t = t'\left(1 + \dfrac{v}{c}\right)$	$t = t'\left(\dfrac{c+w}{c+v}\right)$	$t = \gamma t'\left(1 + \dfrac{v}{c}\right)$
		$t' = t\left(1 - \dfrac{v}{c}\right)$	$t' = \dfrac{t}{1 + \dfrac{v}{c}}$	$t' = t\left(\dfrac{c+v}{c+w}\right)$	$t' = \gamma t\left(1 - \dfrac{v}{c}\right)$
	at $x' > 0 \;\; (x > 0)$	$x_+ = \left(1 - \dfrac{v}{c}\right)(x'_+ + vt)$	$x_+ = \dfrac{1}{1 + \dfrac{v}{c}}(x'_+ + vt)$	$x_+ = \left(\dfrac{c+v}{c+w}\right)(x'_+ + (w-v)t)$	$x_+ = \gamma(x'_+ + vt')$ $t = \gamma\left(t' + \dfrac{vx'_+}{c^2}\right)$
	$x =$ perceived $x' =$ proper	$x'_+ = \dfrac{1}{1 - \dfrac{v}{c}}(x_+ - vt')$	$x'_+ = \left(1 + \dfrac{v}{c}\right)(x_+ - vt')$	$x'_+ = \left(\dfrac{c+w}{c+v}\right)(x_+ - (w-v)t')$	$x'_+ = \gamma(x_+ - vt)$ $t' = \gamma\left(t - \dfrac{vx_+}{c^2}\right)$

Relative Motion ↓	Assumed theory→ Event Scenario↓	Emission Theory, K & K' in relative motion at velocity $= v$	Ether Theory, $K =$ ether, K & K' in relative motion at velocity$= v$	Ether Theory, $K_o =$ether, K & K' in relative motion at v & w w.r.t K_o	Special Relativity, K & K' in relative motion at velocity$= v$
	Doppler Effect	$f = f'\left(1 - \dfrac{v}{c}\right)$	$f = \dfrac{f'}{1 + \dfrac{v}{c}}$	$f = f'\left(\dfrac{c+v}{c+w}\right)$	$f = \gamma f'\left(1 - \dfrac{v}{c}\right)$
		$f' = \dfrac{f}{1 - \dfrac{v}{c}}$	$f' = f\left(1 + \dfrac{v}{c}\right)$	$f' = f\left(\dfrac{c+w}{c+v}\right)$	$f' = \gamma f\left(1 + \dfrac{v}{c}\right)$
		$\lambda = \lambda'$	$\lambda = \lambda'$	$\lambda = \lambda'$	$\lambda = \gamma\lambda'\left(1 + \dfrac{v}{c}\right)$
	$\tau =$ Event proper time interval in the traveling frame K (backward scenario), at origin. $\tau' =$ perceived time	$\tau' = \dfrac{\tau}{1 - \dfrac{v}{c}}$	$\tau' = \dfrac{\tau}{1 - \dfrac{v}{c}}$	$\tau' = \tau\left(\dfrac{c-v}{c-w}\right)$	$\tau' = \gamma\tau\left(1 + \dfrac{v}{c}\right)$
		$\tau = \tau'\left(1 - \dfrac{v}{c}\right)$	$\tau = \tau'\left(1 - \dfrac{v}{c}\right)$	$\tau = \tau'\left(\dfrac{c-w}{c-v}\right)$	$\tau = \gamma\tau'\left(1 - \dfrac{v}{c}\right)$

Relative Motion ↓	Assumed theory→ Event Scenario↓	Emission Theory, K & K' in relative motion at velocity $= v$	Ether Theory, $K =$ ether, K & K' in relative motion at velocity$= v$	Ether Theory, $K_o =$ether, K & K' in relative motion at v & w w.r.t K_o	Special Relativity, K & K' in relative motion at velocity$= v$
	at $x < 0$ $(x' < 0)$ $x' = $ perceived $x = $ proper	$x'_- = \left(1 - \dfrac{v}{c}\right)(x_- - v\tau').$	$x'_- = \left(1 - \dfrac{v}{c}\right)(x_- - v\tau')$	$x'_- = \left(\dfrac{c-w}{c-v}\right)(x_- - (w-v)\tau')$	$x'_- = \gamma(x_- - v\tau)$ $\tau' = \gamma\left(\tau - \dfrac{vx_-}{c^2}\right)$
		$x_- = \dfrac{1}{1 - \dfrac{v}{c}}(x'_- + v\tau)$	$x_- = \dfrac{1}{1 - \dfrac{v}{c}}(x'_- + v\tau)$	$x_- = \left(\dfrac{c-v}{c-w}\right)(x'_- + (w-v)\tau)$	$x_- = \gamma(x'_- + v\tau')$ $\tau = \gamma\left(\tau' + \dfrac{vx'_-}{c^2}\right)$
	Doppler Effect	$\phi' = \phi\left(1 - \dfrac{v}{c}\right)$	$\phi' = \phi\left(1 - \dfrac{v}{c}\right).$	$\phi' = \phi\left(\dfrac{c-w}{c-v}\right)$	$\phi' = \gamma\phi\left(1 - \dfrac{v}{c}\right)$
		$\phi = \dfrac{\phi'}{1 - \dfrac{v}{c}}$	$\phi = \dfrac{\phi'}{1 - \dfrac{v}{c}}$	$\phi = \phi'\left(\dfrac{c-v}{c-w}\right)$	$\phi = \gamma\phi'\left(1 + \dfrac{v}{c}\right)$
		$\lambda' = \lambda$	$\lambda' = \lambda$	$\lambda' = \lambda$	$\lambda' = \gamma\lambda\left(1 + \dfrac{v}{c}\right)$

Relative Motion ↓	Assumed theory→ Event Scenario↓	Emission Theory, K & K' in relative motion at velocity $= v$	Ether Theory, $K =$ ether, K & K' in relative motion at velocity $= v$	Ether Theory, $K_o =$ ether, K & K' in relative motion at v & w w.r.t K_o	Special Relativity, K & K' in relative motion at velocity $= v$
Approaching Frames	$t' =$ **Event proper time interval in the traveling frame** K' **(forward scenario), at origin.** $t =$ **perceived time**	$t = \dfrac{t'}{1 + \dfrac{v}{c}}$	$t = t'\left(1 - \dfrac{v}{c}\right)$	$t = t'\left(\dfrac{c - w}{c + v}\right)$	$t = \gamma t'\left(1 - \dfrac{v}{c}\right)$
		$t' = t\left(1 + \dfrac{v}{c}\right)$	$t' = \dfrac{t}{1 - \dfrac{v}{c}}$	$t' = t\left(\dfrac{c + v}{c - w}\right)$	$t' = \gamma t\left(1 + \dfrac{v}{c}\right)$
	at $x' > 0 \ \ (x > 0)$ $x =$ perceived $x' =$ proper	$x_+ = \left(1 + \dfrac{v}{c}\right)(x'_+ - vt)$	$x_+ = \dfrac{1}{1 - \dfrac{v}{c}}(x'_+ - vt)$	$x_+ = \left(\dfrac{c + v}{c - w}\right)(x'_+ - (w + v)t)$	$x_+ = \gamma(x'_+ - vt')$ $t = \gamma\left(t' - \dfrac{vx'_+}{c^2}\right)$
		$x'_+ = \dfrac{1}{1 + \dfrac{v}{c}}(x_+ + vt')$	$x'_+ = \left(1 - \dfrac{v}{c}\right)(x_+ + vt')$	$x'_+ = \left(\dfrac{c - w}{c + v}\right)(x_+ + (w + v)t')$	$x'_+ = \gamma(x_+ + vt)$ $t' = \gamma\left(t + \dfrac{vx_+}{c^2}\right)$

Relative Motion ↓	Assumed theory→ Event Scenario↓	Emission Theory, K & K' in relative motion at velocity $= v$	Ether Theory, K = ether, K & K' in relative motion at velocity $= v$	Ether Theory, K_o = ether, K & K' in relative motion at v & w w.r.t K_o	Special Relativity, K & K' in relative motion at velocity $= v$
	Doppler Effect	$f = f'\left(1 + \dfrac{v}{c}\right)$	$f = \dfrac{f'}{1 - \dfrac{v}{c}}$	$f = f'\left(\dfrac{c + v}{c - w}\right)$	$f = \gamma f'\left(1 + \dfrac{v}{c}\right)$
		$f' = \dfrac{f}{1 + \dfrac{v}{c}}$	$f' = f\left(1 - \dfrac{v}{c}\right)$	$f' = f\left(\dfrac{c - w}{c + v}\right)$	$f' = \gamma f\left(1 - \dfrac{v}{c}\right)$
		$\lambda = \lambda'$	$\lambda = \lambda'$	$\lambda = \lambda'$	$\lambda = \gamma \lambda'\left(1 - \dfrac{v}{c}\right)$
	τ = Event proper time interval in the traveling frame K (backward scenario), at origin. τ' = perceived time	$\tau' = \dfrac{\tau}{1 + \dfrac{v}{c}} \cdot$	$\tau' = \dfrac{\tau}{1 + \dfrac{v}{c}} \cdot$	$\tau' = \tau\left(\dfrac{c - v}{c + w}\right)$	$\tau' = \gamma\tau\left(1 - \dfrac{v}{c}\right)$
		$\tau = \tau'\left(1 + \dfrac{v}{c}\right)$	$\tau = \tau'\left(1 + \dfrac{v}{c}\right)$	$\tau = \tau'\left(\dfrac{c + w}{c - v}\right)$	$\tau = \gamma\tau'\left(1 + \dfrac{v}{c}\right)$

Relative Motion ↓	Assumed theory→ Event Scenario↓	Emission Theory, K & K' in relative motion at velocity $= v$	Ether Theory, $K =$ ether, K & K' in relative motion at velocity $= v$	Ether Theory, $K_o =$ ether, K & K' in relative motion at v & w w.r.t K_o	Special Relativity, K & K' in relative motion at velocity $= v$
	at $x < 0$ $(x' < 0)$ $x' =$ perceived $x =$ proper	$x'_- = \left(1 + \dfrac{v}{c}\right)\left(x_- + v\tau'\right)$	$x'_- = \left(1 + \dfrac{v}{c}\right)\left(x_- + v\tau'\right)$	$x'_- = \left(\dfrac{c+w}{c-v}\right)(x_- + (w+v)\tau')$	$x'_- = \gamma(x_- + v\tau)$ $\tau' = \gamma\left(\tau + \dfrac{vx_-}{c^2}\right)$
		$x_- = \dfrac{1}{1+\dfrac{v}{c}}\left(x'_- - v\tau\right)$	$x_- = \dfrac{1}{1+\dfrac{v}{c}}\left(x'_- - v\tau\right)$	$x_- = \left(\dfrac{c-v}{c+w}\right)(x'_- - (w+v)\tau)$	$x_- = \gamma(x'_- - v\tau')$ $\tau = \gamma\left(\tau' - \dfrac{vx'_-}{c^2}\right)$
	Doppler Effect	$\phi' = \phi\left(1 + \dfrac{v}{c}\right).$	$\phi' = \phi\left(1 + \dfrac{v}{c}\right)$	$\phi' = \phi\left(\dfrac{c-w}{c-v}\right)$	$\phi' = \gamma\phi\left(1 + \dfrac{v}{c}\right)$
		$\phi = \dfrac{\phi'}{1+\dfrac{v}{c}}$	$\phi = \dfrac{\phi'}{1+\dfrac{v}{c}}$	$\phi = \phi'\left(\dfrac{c-v}{c-w}\right)$	$\phi = \gamma\phi'\left(1 - \dfrac{v}{c}\right)$
		$\lambda = \lambda'$	$\lambda = \lambda'$	$\lambda = \lambda'$	$\lambda' = \gamma\lambda\left(1 - \dfrac{v}{c}\right)$

Relative Motion ↓	Assumed theory→ Event Scenario↓	Emission Theory, K & K' in relative motion at velocity $= v$	Ether Theory, $K =$ ether, K & K' in relative motion at velocity $= v$	Ether Theory, $K_o =$ ether, K & K' in relative motion at v & w w.r.t K_o	Special Relativity, K & K' in relative motion at velocity $= v$
Receding-Approaching Frames	$t' =$ Event proper time interval in the traveling frame K' (forward scenario), at origin. $t =$ perceived time	$t = \dfrac{t'}{1 - \dfrac{v^2}{c^2}}$	$t = t'$	$t = \dfrac{t'}{1 + \dfrac{v}{c}}$	$t = \dfrac{t'}{\sqrt{1 - \dfrac{v^2}{c^2}}}$
		$t' = t\left(1 - \dfrac{v^2}{c^2}\right)$	$t' = t$	$t' = t\left(1 + \dfrac{v}{c}\right)$	$t' = t\sqrt{1 - \dfrac{v^2}{c^2}}$
	$\tau =$ Event proper time interval in the traveling frame K (backward scenario), at origin. $\tau' =$ perceived time	$\tau' = \dfrac{\tau}{1 - \dfrac{v^2}{c^2}}$	$\tau' = \dfrac{\tau}{1 - \dfrac{v^2}{c^2}}$	$\tau' = \dfrac{\tau\left(1 - v/c\right)}{1 - w^2/c^2}$	$\tau' = \dfrac{\tau}{\sqrt{1 - \dfrac{v^2}{c^2}}}$
		$\tau = \tau'\left(1 - \dfrac{v^2}{c^2}\right)$	$\tau = \tau'\left(1 - \dfrac{v^2}{c^2}\right)$	$\tau = \dfrac{\tau'\left(1 - w^2/c^2\right)}{1 - v/c}$	$\tau = \tau'\sqrt{1 - \dfrac{v^2}{c^2}}$

REFERENCES

1   A.A. Michelson and E.H. Morley, "On the Relative Motion of the Earth and the Luminiferous Ether," Am. J. Sci. **34**, 333-345 (1887).
2   A. Einstein, "Zur elektrodynamik bewegter Körper," Annalen der Physik **322** (10), 891–921 (1905).
3   A. Einstein, "Einstein's comprehensive 1907 essay on relativity, part I," English translations in Am. Jour. Phys. 45 (1977), Jahrbuch der Radioaktivitat und Elektronik **4** (1907).
4   R.M. Kassir, "On Lorentz Transformation and Special Relativity: Critical Mathematical Analyses and Findings," Physics Essays **27**, 16 (2014).
5   R.M. Kassir, "On Special Relativity: Root cause of the problems with Lorentz transformation," Physics Essays **27** (2), 198-203 (2014).
6   R.M. Kassir, "The Critical Error in the Formulation of the Special Relativity," International Journal of Physics **2** (6), 197-201 (2014).