A conjecture stronger than Oppermann's one

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Legendre conjecture states that there is always at least a prime between two consecutive squares.

Despite it looks quite simple, and even evident, proof is not (yet?) available.

One can conjecture it could be possible, and even easier, to proof an even more restrictive conjecture, implying the Legendre's one.

Examples of such conjectures do exist in literature. Quite interesting appears Oppermann's one, implying that there are at least a prime between n^2 and n(n+1) and another one between n(n+1) and (n+1)^2

As far as we know, the following conjecture, stronger than Legendre's one, as well as stronger of many of the other literature conjectures that are themselves stronger than Legendre's one, including Oppermann's one, has not yet been proposed, and may help to find a proof:

there are at at least one prime between n^2 and n(n+1) and another one between n(n+1) and (n+1)^2; **moreover:**
either at least a pair of them are symmetrical with respect to n(n+1);
or at least two pairs of them are symmetrical with respect to the same natural close to n(n+1)

This is quite restrictive (and of course not necessary for the proof of Legendre's conjecture), but it may be of help in the search of the proof.