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SMARANDACHE – R-MODULE AND COMMUTATIVE AND BOUNDED BE-ALGEBRAS

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ABSTRACT

In this paper we introduced Smarandache – 2 – algebraic structure of R-Module namely Smarandache – R-Module. A Smarandache – 2 – algebraic structure on a set N means a weak algebraic structure A_0 on N such that there exist a proper subset M of N, which is embedded with a stronger algebraic structure A_1, stronger algebraic structure means satisfying more axioms, by proper subset one understands a subset different from the empty set, from the unit element if any, from the whole set. We define Smarandache – R-Module and obtain some of its characterization through Commutative and Bounded BE-Algebras. For basic concepts we refers to Florentin smarandache[2] and Raul Padilla[9].

Keyword: R-Module, Smarandache – R-Module, BE-Algebras.

1.INTRODUCTION

New notions are introduced in algebra to study more about the congruence in number theory by Florentin smarandache[2]. By <proper subset> of a set A, We consider a set P included in A and different from A, different from the empty set, and from the unit element in A – if any they rank the algebraic structures using an order relationship.

The algebraic structures $S_1 << S_2$ if :both are defined on the same set :: all S_1 laws are also S_2 laws; all axioms of S_1 law are accomplished by the corresponding S_2 law; S_2 law strictly accomplishes more axioms than S_1 laws, or in other words S_2 laws has more laws than S_1.

For example : semi group << monoid << group << ring << field, or Semi group << commutative semi group, ring << unitary ring, etc. they define a General special structure to be a structure SM on a set A, different from a structure SN, such that a proper subset of A is an SN structure, where SM << SN.

2. Prerequistics

Definition 2.1: An algebra (A; *, 1) of type (2, 0) is called a BE-algebra if for all x, y and z in A,

(BE1) $x * x = 1$

(BE2) $x * 1 = 1$

(BE3) $1 * x = x$

(BE4) $x * (y * z) = y * (x * z)$.

In A, a binary relation "≤" is defined by $x \leq y$ if and only if $x * y = 1$.

Definition 2.2: A BE-algebra (X; *, 1) is said to be self-distributive if $x * (y * z) = (x * y) * (x * z)$ for all x, y and $z \in$ A.

Definition 2.3: A dual BCK-algebra is an algebra (A; *, 1) of type (2,0) satisfying (BE1) and (BE2) and the following axioms for all x, y, z $\in$ A.

(dBCK1) $x * y = y * x = 1$ implies $x = y$

(dBCK2) $(x * y) * ((y * z) * (x * z)) = 1$

(dBCK3) $x * ((x * y) * y) = 1$.

Definition 2.4: Let A be a BE-algebra or dual BCK-algebra . A is said to be commutative if the following identity holds:

$x \vee_B y = y \vee_B x$ where $x \vee_B y = (y * x) * x$ for all $x, y \in$ A.

Definition 2.5: Let A be a BE-algebra. If there exists an element 0 satisfying $0 \leq x$ (or $0 * x = 1$) for all $x \in$ A, then the element "0" is called unit of A. A BE-algebra with unit is called a bounded BE-algebra.

Note : In a bounded BE-algebra $x * 0$ denoted by xN.

Definition 2.6: In a bounded BE-algebra, the element x such that $xNN = x$ is called an involution .

Let S (A) = $\{x \in A \,;\, xNN = x\}$ where A is a bounded BE-algebra. S(A) is the set of all involutions in A. Moreover, since $1NN = (1 * 0) * 0 = 0 * 0 = 1$ and $0NN = (0 * 0) * 0 = 1 * 0 = 0$, We have $0, 1 \in S(A)$ and so $S(A) \neq \emptyset$.

Definition 2.7: Each of the elements a and b in a bounded BE-algebra is called the complement of the other if $a \vee b = 1$ and $a \wedge b = 0$.

Definition 2.8: Now we have introduced our concept smarandache – R – module : **"** Let R be a module, called R-module. If R is said to be smarandache – R – module. Then there exist a proper subset A of R which is an algebra with respect to the same induced operations of R."

3.Theorem

Theorem 3.1: Let R be a smarandache-R-module, if there exists a proper subset A of R in which (BE1) to (BE4) are hold, then the following conditions are satisfied,

(i) $1N = 0,\ 0N = 1$

(ii) $x \le xNN$

(iii) $x * yN = y * xN$

(iv) $0 \vee x = xNN,\ x \vee 0 = x.$

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (BE1) to (BE4) then A is bounded BE-algebras.

(i) We have $1N = 1*0 = 0$ and $0N = 0 * 0 = 1.$ by using (BE1) and (BE3)

(ii) Since $x * xNN = x * ((x * 0) * 0) = (x * 0) * (x * 0) = 1$

We get $x \le x$ (by (BE1) and (BE4))

(iii) We have $x * yN = x * (y * 0)$ (by using (BE4))

$= y * (x * 0)$

$= y * xN.$

(iv) By routine operations, we have $0 \vee x = (x * 0) * 0 = xNN$ and $x \vee 0 = (0 * x) * x = 1 * x = x.$

Theorem 3.2: Let R be a smarandache-R-module, if there exists a proper subset A of R in which (BE1) to (BE4) are hold, then the following conditions are satisfied $x * y \le (y \vee x) * y$ for all $x, y \in$ A.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (BE1) to (BE4) then A is bounded BE-algebras.

Since

$(x * y) * ((y \vee x) * y) = (y \vee x) * ((x * y) * y) = (y \vee x) * (y \vee x) = 1$

We have $x * y \le (y \vee x) * y.$

Theorem 3.3: Let R be a smarandache-R-module, if there exists a proper subset A of R in which (BE1) to (BE4) are hold, In addition to that satisfy $x * (y * z) = (x * y) * (x * z)$ then the following conditions are satisfied *for all* $x, y, z \in A$

(i) $x * y \le y N * xN$

(ii) $x \le y$ *implies* $yN \le xN.$

Proof. Since R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (BE1) to (BE4) then A is bounded and Self-Distributive BE-algebras.

(i) Since $(x * y) * (yN * xN)$

$= (x * y) * ((y * 0) * (x * 0))$

$= (y * 0) * ((x * y) * (x * 0))$ (by BE4)

$= (y * 0) * (x * (y * 0))$ (by distributivity)

$= x * ((y * 0) * (y * 0))$ (by BE4)

$= x * 1$ (by BE1)

$= 1$ (by BE2) ,

We have $x * y \leq yN * xN$.

(ii) It is trivial by $x \leq y$, We have $z * x \leq z * y$

then $y * z \leq x * z$ *for all* $x, y, z \in A$.

Theorem 3.4: Let R be a smarandache-R-module, if there exists a proper subset A of R in which (BE1) to (BE4) are hold, In addition to that satisfy $x * (y * z) = (x * y) * (x * z)$, then the following conditions are satisfied

(i) $(y \vee x) * y \leq x * y$.

(ii) $x * (x * y) = x * y$.

Proof. Since R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (BE1) to (BE4) then A is a Self-Distributive BE-algebras.

(i) Since

$$x * (y \vee x) = x * ((x * y) * y)$$
$$= (x * y) * (x * y)$$
$$= 1.$$

We have $x \leq y \vee x$. By $z * x \leq z * y$

We have $(y \vee x) * y \leq x * y$ *for all* $x, y, z \in A$

(ii) By using self distributive definition, (BE1) and (BE3), we have

$$x * (x * y) = (x * x) * (x * y)$$
$$= 1 * (x * y)$$
$$= x * y.$$

Theorem 3.5: Let R be a smarandache-R-module, if there exists a proper subset A of R in which (BE1) to (BE4) are hold, In addition to that satisfy $0 \leq x$ (or $0 * x = 1$), then the following conditions are satisfied *for all* $x, y \in A$

(i) $xNN = x$

(ii) $xN \wedge yN = (x \vee y)$

(iii) $xN \vee yN = (x \wedge y)$

(iv) $xN * yN = y * x.$

Proof. Since R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (BE1) to (BE4) then A is a bounded and Commutative BE-algebras.

(i) It is obtained that

$xNN = (x * 0) * 0$ (from BE3)

$= (0 * x) * x$ (by commutativity)

$= 1*x$

$= x.$

(ii) By the definition of "$\wedge$" and (i) we have that

$xN \wedge yN = (xNN \vee yNN)N = (x \vee y)N.$

(iii)By the definition of "$\wedge$" and (i) we have that

$(x \wedge y)N = (xN \vee yN)NN = xN \vee yN.$

(iv)We have $xN * yN = (x * 0) * (y * 0)$

$= y * ((x * 0) * 0)$

$= y * (xNN) = y * x.$

Theorem 3.6: Let R be a smarandache-R-module, if there exists a proper subset A of R in which (BE1) to (BE4) are hold, In addition to that, there exists a complement of any element of A and then it is unique.

Proof. Since R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (BE1) to (BE4) then A is a bounded and Commutative BE-algebras. Let $x \in$ A and a, b be two complements of x. Then we know that $x \wedge a = x \wedge b = 0$ and $x \vee a = x \vee b = 1$. Also since $x \vee a = (x * a) * a = 1$ and $a * (x * a) = x * (a * a) = x * 1 = 1$,

We have $x*a \leq a$ and $a \leq x * a$. So we get $x * a = a$.

Similarly

$x * b = b.$

Hence $a * b = (x * a) * (x * b) = (aN * xN) * (bN * xN)$ by Theorem 2.5 (iv)

$= bN * ((aN * xN) * xN)$ by BE-4

$= bN * (xN \vee aN)$

$= bN * (x \wedge a)\ N$ by Theorem 2. 5 (iii)

$= (x \wedge a) * b$ by Theorem 2.5 (iii)

$= 0 * b$

$= 1.$

With similar operations, we have $b * a = 1$.

Hence we obtain $a = b$ which gives that the complement of x is unique.

Theorem 3.7: Let R be a smarandache-R-module, if there exists a proper subset A of R in which (BE1) to (BE4) are hold, In addition to that satisfy $0 \leq x$ (or $0 * x = 1$), then the following conditions are equivalent *for all* $x, y \in A$

(i) $x \wedge xN = 0$
(ii) $xN \vee x = 1$
(iii) $xN * x = x$
(iv) $x * xN = xN$
(v) $x * (x * y) = x * y$.

Proof. Since R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (BE1) to (BE4) then A is a Commutative and bounded BE-algebras.

(i) ⇒ (ii) Let $x \wedge xN = 0$. Then it follows that

$xN \vee x = (xN \vee x)$ by Theorem 2. 5 (i)
$= (xNN \wedge xN)$ by Theorem 2.5 (ii)
$= (x \wedge xN)$ by Theorem 2.5 (i)
$= 0N$
$= 1$.

(ii) ⇒ (iii) Let $xN \vee x = 1$. Then, since

$(xN * x) * x = x \vee xN = 1$ and
$x * (xN * x) = xN * (x * x) = xN * 1 = 1$

We get $xN * x = x$ by (dBCK1).

(iii)⇒ (iv) Let $xN * x = x$. Substituting xN for x and using Theorem 2.5 (i) We obtain the result.

(iv) ⇒ (v) Let $x * xN = xN$. Then

We get $yN * (x * xN) = yN * xN$.
Hence we have $x * (yN * xN) = yN * xN$. Using Theorem 2.5 (iv)
We obtain $x * (x * y) = x * y$.

(v) ⇒ (ii) Let $x * (x * y) = x * y$. Then

We have $xN \vee x = (x * (xN)) * xN$
$= (x * (x * 0)) * xN$
$= (x * 0) * (x * 0)$
$= 1$.

(ii) ⇒ (i) Let $xN \vee x = 1$. Then

We obtain $N \wedge x = xN \wedge xNN$
$= (x \vee xN)$ by Theorem 2. 5 (ii)
$= 1N$
$= 0$.

REFERENCES

[1]. Ahn S. S. and So K. S, "On generalized upper sets in BE algebras," *Bulletin of the Korean Mathematical Society*, vol. 46, no. 2, pp. 281–287, 2009.
[2]. Florentin smarandache, "Special Algebric Stuctures," University of New Mexico.MSC:06A99, 1991.
[3]. Jun Y. B., Roh E. H., and Kim H. S, "On BH-algebras," *Scientiae Mathematicae*, vol. 1, no.3, pp. 347–354, 1998.
[4]. Kannappa N and Hirudayaraj P "On some characterisation of Smarandache – R – module " International conference on Applications of Mathematics and Statistics. ISSN:978-93-81361-71-9 pp -103-106, 2012.
[5]. Kannappa N and Hirudayaraj P "Smarandache - R – module and BRK Algebras" International conference on Mathematical methods and computation. ISSN 0973-0303,pp 346-349 , 2014.
[6]. Kannappa N and Hirudayaraj P "Smarandache – R – module with BCI- Algebras and BCC Algebras", Jamal Academic Research Journal, ISSN 0973-0303, pp. 07–10, 2015.
[7]. Kim C. B. and Kim H. S, "On BM-algebras," *Scientiae Mathematicae Japonicae*, vol. 63, no. 3, pp. 421–427, 2006.
[8]. Kim H. S. and Kim Y. H, "On BE-algebras," *Scientiae Mathematicae Japonicae*, vol. 66, no. 1, pp. 113–116, 2007.
[9]. Raul Padilla "Smarandache Algebraic structures", Universidade do Minho, Portugal 1999.
[10]. Walendziak A., "On commutative BE-algebras," *ScientiaeMathematicae Japonicae*, vol. 69, no. 2, pp. 281–284, 2009.