Introduction to Neutrosophic BCI/BCK-Algebras

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Abstract

In this paper, we introduce the concept of neutrosophic BCI/BCK-algebras. Elementary properties of neutrosophic BCI/BCK algebras are presented.

AMS (2010): 03B60, 03G25, 06F35.

Key words: BCI/BCK-algebra, neutrosophic set, neutrosophic BCI/BCK-algebra.

1 Introduction

Logic algebras are the algebraic foundation of reasoning mechanism in many fields such as computer sciences, information sciences, cybernetics and artificial intelligence. In 1966, Imai and Iséki [8, 9] introduced the notions, called BCK-algebras and BCI-algebras. These notions are originated from two different ways: One of them is based on set theory; another is from classical and non-classical propositional calculi. As is well known, there is a close relationship between the notions of the set difference in set theory and the implication functor in logical systems. Since then many researchers worked in this area and lots of literatures had been produced about the theory of BCK/BCI-algebra. On the theory of BCK/BCI-algebras, for example see

[7, 9, 10, 11, 14]. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. MV-algebras were introduced by Chang in [6], in order to show that Lukasiewicz logic is complete with respect to evaluations of propositional variables in the real unit interval $[0,1]$. It is well known that the class of MV-algebras is a proper subclass of the class of BCK- algebras.

By a BCI-algebra we mean an algebra $(X,*,0)$ of type $(2,0)$ satisfying the following axioms, for all $x,y,z \in X$,

(1) $((x*y)*(x*z))*(z*y) = 0$,

(2) $(x*(x*y))*y = 0$,

(3) $x*x = 0$,

(4) $x*y = 0$ and $y*x = 0$ imply $x = y$.

We can define a partial ordering $\leq$ by $x \leq y$ if and only if $x*y = 0$.

If a BCI-algebra X satisfies $0*x = 0$ for all $x \in X$, then we say that X is a BCK-algebra. Any BCK-algebra X satisfies the following axioms for all $x,y,z \in X$,

(1) $(x*y)*z = (x*z)*y$,

(2) $((x*z)*(y*z))*(x*y) = 0$,

(3) $x*0 = x$,

(4) $x*y = 0 \Rightarrow (x*z)*(y*z) = 0$, $(z*y)*(z*x) = 0$.

Let $(X,*,0)$ be a BCK-algebra.

(1) X is said to be commutative if for all $x,y \in X$ we have $x*(x*y) = y*(y*x)$.

(2) X is said to be implicative if for all $x,y \in X$, we have $x = x*(y*x)$.

In 1995, Smarandache introduced the concept of neutrosophic logic as an extension of fuzzy logic, see [15, 16, 17]. In 2006, Kandasamy and Smarandache introduced the concept of neutrosophic algebraic structures, see [12, 13]. Since then, several researchers have studied the concepts and a great deal of literature has been produced. Agboola et al in [1, 2, 3, 4, 5] continued the study of some types of neutrosophic algebraic structures.

Let X be a nonempty set. A set $X(I) =< X, I >$ generated by X and I is called a neutrosophic set. The elements of $X(I)$ are of the form (x, yI) where x and y are elements of X.

In the present paper, we introduce the concept of neutrosophic BCI/BCK-algebras. Elementary properties of neutrosophic BCI/BCK-algebras are presented.

2 Main Results

Definition 2.1. Let $(X, *, 0)$ be any BCI/BCK-algebra and let $X(I) =< X, I >$ be a set generated by X and I. The triple $(X(I), *, (0, 0))$ is called a neutrosophic BCI/BCK-algebra. If (a, bI) and (c, dI) are any two elements of $X(I)$ with $a, b, c, d \in X$, we define

$$(a, bI) * (c, dI) = (a * c, (a * d \wedge b * c \wedge b * d)I) \tag{1}$$

An element $x \in X$ is represented by $(x, 0) \in X(I)$ and $(0, 0)$ represents the constant element in $X(I)$. For all $(x, 0), (y, 0) \in X$, we define

$$(x, 0) * (y, 0) = (x * y, 0) = (x \wedge \neg y, 0) \tag{2}$$

where $\neg y$ is the negation of y in X.

Example 1. Let $(X(I), +)$ be any commutative neutrosophic group. For all $(a, bI), (c, dI) \in X(I)$ define

$$(a, bI) * (c, dI) = (a, bI) - (c, dI) = (a - c, (b - d)I). \tag{3}$$

Then $(X(I), *, (0, 0))$ is a neutrosophic BCI-algebra.

Example 2. Let $X(I)$ be a neutrosophic set and let $A(I)$ and $B(I)$ be any two non-empty subsets of $X(I)$. Define

$$A(I) * B(I) = A(I) - B(I) = A(I) \cap B'(I). \tag{4}$$

Then $(X(I), *, \emptyset)$ is a neutrosophic BCK-algebra.

Theorem 2.2. *Every neutrosophic BCK-algebra $(X(I), *, (0, 0))$ is a neutrosophic BCI-algebra.*

Proof. It is straightforward. □

Theorem 2.3. *Every neutrosophic BCK-algebra $(X(I), *, (0, 0))$ is a BCK-algebra and not the converse.*

Proof. Suppose that $(X(I), *, (0,0))$ is a neutrosophic BCK-algebra. Let $x = (a, bI), y = (c, dI), z = (e, fI)$ be arbitrary elements of $X(I)$. Then

(1) We have

$$\begin{aligned} ((x*y)*(x*z))*(z*y) &= (((a,bI)*(c,dI))*((a,bI)*(e,fI)))*((e,fI)*(c,dI)) \\ &\equiv [(r,sI)*(p,qI)]*(u,vI), \end{aligned}$$

where

$$\begin{aligned} (r,sI) &= (a*c,(a*d\wedge b*c\wedge b*d)I) = (a\wedge\neg c,(a\wedge\neg d\wedge b\wedge\neg c)I), \\ (p,qI) &= (a*e,(a*f\wedge b*e\wedge b*f)I) = (a\wedge\neg e,(a\wedge\neg f\wedge b\wedge\neg e)I), \\ (u,vI) &= (e*c,(e*d\wedge f*c\wedge f*d)I) = (e\wedge\neg c,(e\wedge\neg d\wedge f\wedge\neg c)I). \end{aligned}$$

Hence,

$$\begin{aligned} (r,sI)*(p,qI) &= (r*p,(r*q\wedge s*p\wedge s*q)I) = (r\wedge\neg p,(r\wedge\neg q\wedge s\wedge\neg p)I) \\ &\equiv (m,kI), \end{aligned}$$

and

$$\begin{aligned} (m,kI)*(u,vI) &= (m*u,(m*v\wedge k*u\wedge k*v)I) = (m\wedge\neg u,(m\wedge\neg v\wedge k\wedge\neg u)I) \\ &\equiv (g,hI). \end{aligned}$$

Now, we obtain

$$\begin{aligned} g &= m\wedge\neg u = r\wedge\neg p\wedge\neg u \\ &= (a\wedge\neg c\wedge e)(\neg e\vee c) = 0. \end{aligned}$$

Also, we have

$$\begin{aligned} h &= m\wedge\neg v\wedge k\wedge\neg u \\ &= r\wedge\neg p\wedge\neg q\wedge s\wedge\neg v\wedge\neg u \\ &= a\wedge\neg c\wedge\neg d\wedge b\wedge\neg p\wedge\neg q\wedge\neg v\wedge\neg u \\ &= a\wedge\neg c\wedge\neg d\wedge b\wedge(\neg a\vee e)\wedge(\neg e\vee c)\wedge\neg q\wedge\neg v \\ &= a\wedge\neg c\wedge\neg d\wedge b\wedge e\wedge(\neg e\vee c)\wedge\neg q\wedge\neg v \\ &= 0. \end{aligned}$$

This shows that $(g, hI) = (0,0)$ and consequently,$((x*y)*(x*z))*(z*y) = 0$.

(2) We have

$$\begin{aligned} (x*(x*y))*y &= ((a,bI)*((a,bI)*(c,dI))*(c,dI) \\ &= ((a,bI)*(a*c,(a*d\wedge b*c\wedge b*d)I)*(c,dI) \\ &= ((a,bI)*(r,sI))*(c,dI), \end{aligned}$$

where

$$\begin{aligned}(r, sI) &= (a, bI) * (c, dI) = (a * c, (a * d \wedge b * c \wedge b * d)I) \\ &= (a \wedge \neg c, (a \wedge \neg d \wedge b \wedge \neg c)I).\end{aligned}$$

Then,

$$\begin{aligned}(a, bI) * (r, sI) &= (a * r, (a * s \wedge b * r \wedge b * s)I) = (a \wedge \neg r, (a \wedge \neg s \wedge b \wedge \neg r)I) \\ &\equiv (u, vI)\end{aligned}$$

Therefore, we obtain

$$\begin{aligned}(u, vI) * (c, dI) &= (u * c, (u * d \wedge v * c \wedge v * d)I) = (u \wedge \neg c, (u \wedge \neg d \wedge v \wedge \neg c) \\ &\equiv (p, qI),\end{aligned}$$

where

$$\begin{aligned}p &= u \wedge \neg c = a \wedge \neg r \wedge \neg c \\ &= a \wedge (\neg a \vee c) \wedge \neg c = a \wedge c \wedge \neg c = 0\end{aligned}$$

and

$$\begin{aligned}q &= u \wedge \neg d \wedge v \wedge \neg c = a \wedge \neg r \wedge \neg d \wedge v \wedge \neg c \\ &= a \wedge (\neg a \vee c) \wedge \neg d \wedge v \wedge \neg c = a \wedge c \wedge \neg d \wedge v \wedge \neg c = 0.\end{aligned}$$

Since $(p, qI) = (0, 0)$, it follows that $(x * (x * y)) * y = 0$.

(3) We have

$$\begin{aligned}x * x &= (a, bI) * (a, bI) \\ &= (a * a, (a * b \wedge b * a \wedge b * b)I) \\ &= (a \wedge \neg a, (a \wedge \neg b \wedge b \wedge \neg a \wedge b \wedge \neg b)I) \\ &= (0, 0).\end{aligned}$$

(4) Suppose that $x * y = 0$ and $y * x = 0$. Then $(a, bI) * (c, dI) = (0, 0)$ and $(c, dI) * (a, bI) = (0, 0)$ from which we obtain $(a * c, (a * d \wedge b * c \wedge b * d)I) = (0, 0)$ and $(c * a, (c * b \wedge d * a \wedge d * b)I) = (0, 0)$. These imply that $(a \wedge \neg c, (a \wedge \neg d \wedge b \neg c)I) = (0, 0)$ and $(c \wedge \neg a, (c \wedge \neg b \wedge d \neg a)I) = (0, 0)$ and therefore, $a \wedge \neg c = 0$, $a \wedge \neg d \wedge b \neg c = 0$, $c \wedge \neg a = 0$ and $c \wedge \neg b \wedge d \neg a = 0$ from which we obtain $a = c$ and $b = d$. Hence, $(a, bI) = (c, dI)$ that is $x = y$.

(5) We have

$$\begin{aligned}0 * x &= (0, 0) * (a, bI) = (0 * a, (0 * b \wedge 0 * a)I) \\ &= (0, (0 \wedge 0)I) = (0, 0).\end{aligned}$$

Items (1)-(5) show that $(X(I), *, (0, 0))$ is a BCK-algebra. $\square$

Lemma 2.4. *Let $(X(I), *, (0,0))$ be a neutrosophic BCK-algebra. Then $(a, bI) * (0,0) = (a, bI)$ if and only if $a = b$.*

Proof. Suppose that $(a, bI) * (0,0) = (a, bI)$. Then $(a * 0, (a * 0 \wedge b * 0)I) = (a, bI)$ which implies that $(a, (a \wedge b)I) = (a, bI)$ from which we obtain $a = b$. The converse is obvious. $\square$

Lemma 2.5. *Let $(X(I), *, (0,0))$ be a neutrosophic BCI-algebra. Then for all $(a, bI), (c, dI) \in X(I)$:*

(1) $(0,0) * ((a, bI) * (c, dI)) = ((0,0) * (a, bI)) * ((0,0) * (c, dI))$.

(2) $(0,0) * ((0,0) * ((a, bI) * (c, dI))) = (0,0) * ((a, bI)) * (c, dI))$.

Theorem 2.6. *Let $(X(I), *, (0,0))$ be a neutrosophic BCK-algebra. Then for all $(a, bI), (c, dI), (e, fI) \in X(I)$:*

(1) $(a, bI)*(c, dI) = (0,0)$ *implies that* $((a, bI)*(e, fI))*((c, dI)*(e, fI)) = (0,0)$ *and* $((e, fI) * (c, dI)) * ((e, fI) * (a, bI)) = (0,0)$.

(2) $((a, bI) * (c, dI)) * (e, fI) = ((a, bI) * (e, fI)) * (c, dI)$.

(3) $((a, bI) * (e, fI)) * ((c, dI) * (e, fI)) * ((a, bI) * (c, dI)) = (0,0)$.

Proof. (1) Suppose that $(a, bI) * (c, dI) = (0,0)$. Then $a * c, (a * d \wedge b * c \wedge b * d)) = (0,0)$ from which we obtain

$$a \wedge \neg c = 0, a \wedge \neg d \wedge b \wedge \neg c = 0.$$

Now,

$$\begin{aligned}
(x, yI) &= (a, bI) * (e, fI) \\
&= (a \wedge \neg e, (a \wedge \neg f \wedge b \wedge \neg e)I). \\
(p, qI) &= (c, dI) * (e, fI) \\
&= (c \wedge \neg e, (c \wedge \neg f \wedge d \wedge \neg e)I).
\end{aligned}$$

Hence,

$$\begin{aligned}
(x, yI) * (p, qI) &= (x \wedge \neg p, (x \wedge \neg q \wedge y \wedge \neg p)I) \\
&\equiv (u, vI),
\end{aligned}$$

where

$$\begin{aligned}
u &= x \wedge \neg p = a \wedge \neg e \wedge (\neg c \vee e) \\
&= a \wedge \neg e \wedge \neg c = 0
\end{aligned}$$

and

$$\begin{aligned} v &= x \wedge \neg q \wedge y \wedge \neg p \\ &= a \wedge \neg e \wedge \neg f \wedge b \wedge \neg e \wedge (\neg c \vee e) \wedge \neg q \\ &= a \wedge \neg e \wedge \neg f \wedge b \wedge \neg e \wedge \neg c \wedge (\neg c \vee f \vee \neg d \vee e) \\ &= (a \wedge \neg c \wedge \neg e \wedge \neg f \wedge b) \vee (a \wedge \neg d \wedge b \wedge \neg c \wedge \neg e \wedge \neg f) \\ &= 0 \vee 0 = 0. \end{aligned}$$

This shows that $(u, vI) = (0,0)$ and so $((a,bI)*(e,fI))*((c,dI)*(e,fI)) = (0,0)$. A similar computations show that $((e,fI)*(c,dI))*((e,fI)*(a,bI)) = (0,0)$.

(2) Put

$$LHS = ((a,bI)*(c,dI))*(e,fI) = (x,yI)*(e,fI),$$

where

$$(x,yI) = (a,bI)*(c,dI) = (a \wedge \neg c, (a \wedge \neg d \wedge b \wedge \neg c)I).$$

Therefore,

$$\begin{aligned} (x,yI)*(e,fI) &= (x \wedge \neg e, (x \wedge \neg f \wedge y \wedge \neg e)I) \\ &\equiv (u, vI) \end{aligned}$$

Now, we have

$$\begin{aligned} u &= x \wedge \neg e = a \wedge \neg c \wedge \neg e. \\ v &= x \wedge \neg f \wedge y \wedge \neg e = x \wedge \neg f \wedge y \wedge \neg e = a \wedge \neg c \wedge \neg f \wedge \neg e \wedge \neg d \wedge b. \end{aligned}$$

Thus,

$$LHS = (a \wedge \neg c \wedge \neg e, (a \wedge \neg c \wedge \neg f \wedge \neg e \wedge \neg d \wedge b)I).$$

Similarly, it can be shown that

$$\begin{aligned} RHS &= ((a,bI)*(e,fI))*(c,dI) \\ &= (a \wedge \neg c \wedge \neg e, (a \wedge \neg c \wedge \neg f \wedge \neg e \wedge \neg d \wedge b)I). \end{aligned}$$

(3) Put

$$\begin{aligned} LHS &= ((a,bI)*(e,fI))*((c,dI)*(e,fI))*((a,bI)*(c,dI)) \\ &\equiv ((x,yI)*(p,qI))*(u,vI), \end{aligned}$$

where

$$\begin{aligned}
(x, yI) &= (a, bI) * (e, fI) = (a \wedge \neg c, (a \wedge \neg f \wedge b \wedge \neg e)I), \\
(p, qI) &= (c, dI) * (e, fI) = (c \wedge \neg e, (c \wedge \neg f \wedge d \wedge \neg e)I), \\
(u, vI) &= (a, bI) * (c, dI) = (a \wedge \neg c, (a \wedge \neg d \wedge b \wedge \neg c)I).
\end{aligned}$$

Thus, we have

$$\begin{aligned}
(x, yI) * (p, qI) &= (x \wedge \neg p, (x \wedge \neg q \wedge y \wedge \neg p)I) \\
&\equiv (g, hI).
\end{aligned}$$

Now,

$$\begin{aligned}
(g, hI) * (u, vI) &= (g \wedge \neg u, (g \wedge \neg v \wedge h \wedge \neg u)I) \\
&\equiv (m, kI),
\end{aligned}$$

where

$$\begin{aligned}
m &= g \wedge \neg u \\
&= x \wedge \neg p \wedge (\neg a \wedge c) \\
&= a \wedge \neg e \wedge (\neg c \vee e) \wedge (\neg a \vee c) \\
&= a \wedge \neg e \wedge \neg c \wedge (\neg a \vee c) \\
&= 0 \vee 0 = 0. \\
k &= g \wedge \neg v \wedge h \wedge \neg u \\
&= x \wedge \neg p \wedge \neg v \wedge \neg q \wedge y \wedge (\neg a \vee c) \\
&= a \wedge \neg e \wedge (\neg c \vee e) \wedge (\neg a \vee c) \wedge y \wedge \neg v \wedge \neg q \\
&= a \wedge \neg e \wedge \neg c \wedge (\neg a \vee c) \wedge y \wedge \neg v \wedge \neg q \\
&= (0 \vee 0) \wedge y \wedge \neg v \wedge \neg q = 0.
\end{aligned}$$

Since $(m, kI) = (0, 0)$, it follows that $LHS = (0, 0)$. Hence this complete the proof. □

Theorem 2.7. *Let $(X(I), *, (0, 0))$ be a neutrosophic BCI/BCK-algebra. Then*

(1) *$X(I)$ is not commutative even if X is commutative.*

(2) *$X(I)$ is not implicative even if X is implicative.*

Proof. (1) Suppose that X is commutative. Let $(a, bI), (c, dI) \in X(I)$. Then

$$\begin{aligned}(a, bI) * ((a, bI) * (c, dI)) &= (a, bI) * (a * c, (a * d \wedge b * c \wedge b * d)I) \\ &= (a * (a * c), (a * (a * d \wedge b * c \wedge b * d) \wedge b * (a * c) \\ &\quad \wedge b * (a * d \wedge b * c \wedge b * d))I) \\ &\equiv (u, vI)\end{aligned}$$

where

$$\begin{aligned}u &= a * (a * c) = c * (c * a). \\ v &= a * (a * d \wedge b * c \wedge b * d) \wedge b * (a * c) \wedge b * (a * d \wedge b * c \wedge b * d) \\ &= a * (a * d) \wedge a * (b * c) \wedge a * (b * d) \wedge b * (a * c) \wedge b * (a * d) \\ &\quad \wedge b * (b * c) \wedge b * (b * d) \\ &= d * (d * a) \wedge a * (b * c) \wedge a * (b * d) \wedge b * (a * c) \wedge b * (a * d) \\ &\quad \wedge c * (c * b) \wedge d * (d * b).\end{aligned}$$

Also,

$$\begin{aligned}(c, dI) * ((c, dI) * (a, bI)) &= (c, dI) * (c * a, (c * b \wedge d * a \wedge d * b)I) \\ &= (c * (c * a), (c * (c * b \wedge d * a \wedge d * b) \wedge d * (c * a) \\ &\quad \wedge d * (c * b \wedge d * a \wedge d * b))I) \\ &\equiv (p, qI),\end{aligned}$$

where

$$\begin{aligned}p &= c * (c * a) = u. \\ q &= c * (c * b \wedge d * a \wedge d * b) \wedge d * (c * a) \\ &\quad \wedge d * (c * b \wedge d * a \wedge d * b) \\ &= c * (c * b) \wedge c * (d * a) \wedge c * (d * b) \wedge d * (c * a) \\ &\quad \wedge d * (c * b) \wedge d * (d * a) \wedge d * (d * b) \\ &\neq v\end{aligned}$$

This shows that $(a, bI) * ((a, bI) * (c, dI)) \neq (c, dI) * ((c, dI) * (a, bI))$ and therefore $X(I)$ is not commutative.

(2) Suppose that X is implicative. Let $(a, bI), (c, dI) \in X(I)$. Then

$$\begin{aligned}(a, bI) * ((c, dI) * (a, bI)) &= (a, bI) * (c * a, (c * b \wedge d * a \wedge d * b)I) \\ &= (a * (c * a), (a * (c * b \wedge d * a \wedge d * b) \wedge b * (c * a) \\ &\quad \wedge b * (c * b \wedge d * a \wedge d * b))I) \\ &\equiv (u, vI)\end{aligned}$$

where

$$\begin{aligned}
u &= a*(c*a) = a. \\
v &= a*(c*b \wedge d*a \wedge d*b) \wedge b*(c*a) \\
&\quad \wedge b*(c*b \wedge d*a \wedge d*b) \\
&= a*(c*b) \wedge a*(d*a) \wedge a*(d*b) \wedge b*(c*a) \\
&\quad \wedge b*(c*b) \wedge b*(d*a) \wedge b*(d*b) \\
&\neq b.
\end{aligned}$$

Hence, $(a, bI) \neq (a, bI)*((c, dI)*(a, bI))$ and so $X(I)$ is not implicative. □

Definition 2.8. Let $(X(I), *, (0,0))$ be a neutrosophic BCI/BCK-algebra. A nonempty subset $A(I)$ is called a neutrosophic subalgebra of $X(I)$ if the following conditions hold:

(1) $(0,0) \in A(I)$.

(2) $(a, bI)*(c, dI) \in A(I)$ for all $(a, bI), (c, dI) \in A(I)$.

(3) $A(I)$ contains a proper subset which is a BCI/BCK-algebra.

If $A(I)$ does not contain a proper subset which is a BCI/BCK-algebra, then $A(I)$ is called a pseudo neutrosophic subalgebra of $X(I)$.

Example 3. Any neutrosophic subgroup of the commutative neutrosophic group $(X(I), +)$ of Example 1 is a neutrosophic BCI-subalgebra.

Theorem 2.9. *Let $(X(I), *, (0,0))$ be a neutrosophic BCK-algebra and for $a \neq 0$, let $A_{(a,aI)}(I)$ be a subset of $X(I)$ defined by*

$$A_{(a,aI)}(I) = \{(x, yI) \in X(I) : (x, yI)*(a, aI) = (0,0)\}.$$

Then,

(1) *$A_{(a,aI)}(I)$ is a neutrosophic subalgebra of $X(I)$.*

(2) *$A_{(a,aI)}(I) \subseteq A_{(0,0)}(I)$.*

Proof. (1) Obviously, $(0,0) \in A_{(a,aI)}(I)$ and $A_{(a,aI)}(I)$ contains a proper subset which is a BCK-algebra. Let $(x, yI), (p, qI) \in A_{(a,aI)}(I)$. Then

$(x, yI) * (a, aI) = (0, 0)$ and $(p, qI) * (a, aI) = (0, 0)$ from which we obtain $x * a = 0, x * a \wedge y * a = 0, p * a = 0, p * a \wedge q * a = 0$. Since $a \neq 0$, we have $x = y = p = q = a$. Now,

$$\begin{aligned}
((x, yI) * (p, qI)) * (a, aI) &= ((x * p), (x * q \wedge y * p \wedge y * q)I) * (a, aI) \\
&= ((x * p) * a, ((x * p) * a \wedge (x * q) \wedge y * p) \wedge y * q) * a)I) \\
&= ((a * a) * a, ((a * a) * a)I) \\
&= (0 * a, (0 * a)I) \\
&= (0, 0).
\end{aligned}$$

This shows that $(x, yI) * (p, qI) \in A_{(a,aI)}(I)$ and the required result follows.

(2) Follows. □

Definition 2.10. Let $(X(I), *, (0, 0))$ and $(X'(I), \circ, (0', 0'))$ be two neutrosophic BCI/BCK-algebras. A mapping $\phi : X(I) \to X'(I)$ is called a neutrosophic homomorphism if the following conditions hold:

(1) $\phi((a, bI) * (c, dI)) = \phi((a, bI)) \circ \phi((c, dI)), \forall\ (a, bI), (c, dI) \in X(I)$.

(2) $\phi((0, I)) = (0, I)$.

In addition,

(3) If ϕ is injective, then ϕ is called a neutrosophic monomorphism.

(4) If ϕ is surjective, then ϕ is called a neutrosophic epimorphism.

(5) If ϕ is a bijection, then ϕ is called a neutrosophic isomorphism. A bijective neutrosophic homomorphism from $X(I)$ onto $X(I)$ is called a neutrosophic automorphism.

Definition 2.11. Let $\phi : X(I) \to Y(I)$ be a neutrosophic homomorphism of neutrosophic BCK/BCI-algebras.

(1) $Ker\phi = \{(a, bI) \in X(I) : \phi((a, bI)) = (0, 0)\}$.

(2) $Im\phi = \{\phi((a, bI)) \in Y(I) : (a, bI) \in X(I)\}$.

Example 4. Let $(X(I), *, (0, 0))$ be a neutrosophic BCI/BCK-algebra and let $\phi : X(I) \to X(I)$ be a mapping defined by

$$\phi((a, bI)) = (a, bI)\ \forall\ (a, bI) \in X(I).$$

Then ϕ is a neutrosophic isomorphism.

Lemma 2.12. *Let $\phi : X(I) \to X'(I)$ be a neutrosophic homomorphism from a neutrosophic BCI/BCK-algebra $X(I)$ into a neutrosophic BCI/BCK-algebra $X'(I)$. Then $\phi((0,0)) = (0',0')$.*

Proof. It is straightforward. $\square$

Theorem 2.13. *Let $\phi : X(I) \to Y(I)$ be a neutrosophic homomorphism of neutrosophic BCK/BCI-algebras. Then ϕ is a neutrosophic monomorphism if and only if $Ker\phi = \{(0,0)\}$.*

Proof. Same as the classical case and so omitted. $\square$

Theorem 2.14. *Let $X(I), Y(I), Z(I)$ be neutrosophic BCI/BCK-algebras. Let $\phi : X(I) \to Y(I)$ be a neutrosophic epimorphism and let $\psi : X(I) \to Z(I)$ be a neutrosophic homomorphism. If $Ker\phi \subseteq Ker\psi$, then there exists a unique neutrosophic homomorphism $\nu : Y(I) \to Z(I)$ such that $\nu\phi = \psi$. The following also hold:*

(1) $Ker\nu = \phi(Ker\psi)$.

(2) $Im\nu = Im\psi$.

(3) ν is a neutrosophic monomorphism if and only if $Ker\phi = Ker\psi$.

(4) ν is a neutrosophic epimorphism if and only if ψ is a neutrosophic epimorphism.

Proof. The proof is similar to the classical case and so omitted. $\square$

Theorem 2.15. *Let $X(I), Y(I), Z(I)$ be neutrosophic BCI/BCK-algebras. Let $\phi : X(I) \to Z(I)$ be a neutrosophic homomorphism and let $\psi : Y(I) \to Z(I)$ be a neutrosophic monomorphism such that $Im\phi \subseteq Im\psi$. Then there exists a unique neutrosophic homomorphism $\mu : X(I) \to Y(I)$ such that $\phi = \psi\mu$. Also:*

(1) *$Ker\mu = Ker\phi$.*

(2) *$Im\mu = \psi^{-1}(Im\phi)$.*

(3) *μ is a neutrosophic monomorphism if and only if ϕ is a neutrosophic monomorphism.*

(4) *μ is a neutrosophic epimorphism if and only if $Im\psi = Im\phi$.*

Proof. The proof is similar to the classical case and so omitted. $\square$

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