Some single valued neutrosophic correlated aggregation operators and their applications to material selection

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Abstract

In engineering design, the decision to select an optimal material has become a challenging task for the designers, and the evaluation of alternative materials may be based on some multiple attribute group decision making (MAGDM) methods. Moreover, the attributes are often inter-dependent or correlated in the real decision making process. In this paper, with respect to the material selection problems in which the attribute values take the form of single valued neutrosophic numbers (SVNNs), a novel multiple attribute group decision making method is proposed. First, the concept and operational laws of SVNNs are briefly introduced. Then, motivated by the idea of Choquet integral, two correlated aggregation operators are proposed for aggregating single valued neutrosophic information based on the operational laws of SVNNs, such as the single valued neutrosophic correlated average (SVNCA) operator, the single valued neutrosophic correlated geometric (SVNCG) operator, and then some desirable properties of these operators and the relationships among them are investigated in detail. Furthermore, based on the proposed aggregation operators, a novel multiple attribute group decision making method is developed to select the most desirable material(s) under single valued neutrosophic environment. Finally, a numerical example of material selection is given to illustrate the application of the proposed method.

Keywords: Multiple attribute group decision making (MAGDM); Material selection; Single valued neutrosophic set (SVNS); Choquet integral; Single valued neutrosophic correlated aggregation operators

1. Introduction

Material selection is one of the most prominent activities in the process of design and development of products, which is a task normally carried out by design and materials engineers and also critical for the success and competitiveness of the producers [1, 2]. An inappropriate selection of materials may result in damage or failure of an assembly and significantly decreases the performance [3], thus negatively affecting the productivity, profitability and reputation of an organization [4]. In the process of selecting materials, there is not always a definite criterion or attribute, and the designers or engineers have to consider many attributes that influence the selection of materials for a given application simultaneously. These attributes include not only the traditional ones such as availability, production and cost, but also material impact on environment, recycling and cultural aspects and so on, which may be contradicted and even conflicting with each other. Therefore, the selection of the most desirable material is a multiple attribute decision making (MADM) problem and many traditional MADM methods have been proposed to deal with

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the material selection problem, such as Ashby approach [5], analytic hierarchy process (AHP) [6], analytic network process (ANP) [7], technique of order preference by similarity to ideal solution (TOPSIS) [8], quality function deployment (QFD)-based approach [9], gray relational analysis (GRA) [10], graph theory and matrix approach [11], ELECTRE (ELimination Et Choix Traduisant la REalite) [12], VIKOR (VIsekriterijumska optimizacija i KOmpromisno Resenje) [13-15], Preference Ranking Organization METHhod for Enrichment Evaluation (PROMETHEE) [16], DEMATEL-based ANP (DANP) [17] and COPRAS (COmplex PRoportional ASsessment) [18,19].

However, due to the increasing complexity of the material selection process and the vagueness of inherent subjective nature of human think, decision makers usually cannot express his/her preference to material alternatives by crisp numbers and some are more suitable to be denoted by fuzzy values. Since fuzzy set was introduced by Zadeh [20], many extensions of fuzzy set have been widely discussed [21-27]. Recently, a new concept called neutrosophic set (NS) has been introduced by Smarandache [28], where each element of the universe has a degree of truth (T), indeterminacy (I) and falsity (F) respectively and which lies in $]0^-, 1^+[$. Different from the intuitionistic fuzzy set where the incorporated uncertainty is dependent of the membership degree and the non-membership degree, the indeterminacy degree in neutrosophic set is independent of the truth and falsity degrees. Moreover, from the practical point of view, the neutrosophic set needs to be specified. Otherwise, it will be difficult to use in the real applications. Therefore, Wang et al. [29] introduced an instance of neutrosophic set known as single valued neutrosophic set (SVNS). By the idea of single valued neutrosophic set, we can utilize the single valued neutrosophic numbers to express the decision makers' preference to materials, such as "<0.6, 0.2, 0.3>", which means that the truth-membership, indeterminacy-membership and falsity-membership of one material alternative to a given attribute are "0.6", "0.2" and "0.3", respectively. However, if we use the intuitionistic fuzzy numbers to express the attribute preference information, we only consider the membership degree and the non-membership degree of an element to a given set, and the indeterminacy membership information is lost. Therefore, intuitionistic fuzzy set (IFS) is an instance of neutrosophic set. Since its appearance, neutrosophic set has received more and more attention from researchers and practitioners [29-32]. Majumdar and Samanta [33] defined several similarity measures between two single valued neutrosophic sets and investigated their characteristics as well as a measure of entropy of a single valued neutrosophic set was introduced. Ye [34, 35] proposed two novel multiple attribute decision making methods based on the correlation coefficient and cross-entropy of SVNSs, respectively, in which the attribute value is described by truth membership degree, indeterminacy membership degree and falsity membership degree under single valued neutrosophic environment. Hanbay and Talu [36] proposed a novel synthetic aperture radar (SAR) image segmentation algorithm based on the neutrosophic set and developed an improved artificial bee colony (I-ABC) algorithm.

In the existing research on decision making with single valued neutrosophic set, it is generally assumed that the attributes are independent of one another, which are characterized by an independent axiom [37]. However, in the real decision making problems, the attributes are often inter-dependent or correlated. Choquet integral, originally developed by Choquet [38], provides a type of operators used to process the inter-dependence or correlation among attributes [39-46]. Until now, to our best knowledge, there is not any method for solving the problem of material selection considering the inter-dependence or correlation among attributes under single valued neutrosophic environment. Hence, it is necessary to develop some new correlated aggregation operators of single valued neutrosophic information based on Choquet integral. This is the motivation of our study.

The purpose of this paper is to develop a method for solving material selection problem under single valued neutrosophic environment. Firstly, based on Choquet integral, two single valued neutrosophic correlated aggregation operators are proposed, i.e., single valued neutrosophic correlated average (SVNCA) operator and single valued neutrosophic correlated geometric (SVNCG) operator. Then, a novel MAGDM method is proposed to solve the material selection problems under single valued neutrosophic environment based on the developed operators. To do so, the remainder of this paper is organized as follows: some basic concepts of neutrosophic set and Choquet integral are introduced in Section 2; In Section 3, some new correlated aggregation operators are proposed based on Choquet integral under single valued neutrosophic environment, and then some properties and special cases of the proposed operators are examined. Section 4 develops a novel multiple attribute group decision making (MAGDM) method based on these proposed operators. In Section 5, a numerical example of material selection is given to illustrate the application of the developed method. The paper is concluded in Section 6.

2. Preliminaries

To facilitate the following discussion, some concepts related to neutrosophic set and single valued neutrosophic set are briefly introduced in this section.

2.1. Neutrosophic set and single valued neutrosophic set

Definition 1. [28]. Let X be a universe set, with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, that is $T_A(x): X \to]0^-, 1^+[$, $I_A(x): X \to]0^-, 1^+[$ and $F_A(x): X \to]0^-, 1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so

$$0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+ .$$

Definition 2. [28]. The complement of a neutrosophic set A is denoted by A^C and is defined as $T_A^c(x) = \{1^+\} \ominus T_A(x)$, $I_A^c(x) = \{1^+\} \ominus I_A(x)$, and $F_A^c(x) = \{1^+\} \ominus F_A(x)$ for every element x in X.

Definition 3. [28]. A neutrosophic set A is contained in the other neutrosophic set B, $A \subseteq B$ if and only if $\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$, $\inf I_A(x) \ge \inf I_B(x)$, $\sup I_A(x) \ge \sup I_B(x)$, $\inf F_A(x) \ge \inf F_B(x)$ and $\sup F_A(x) \ge \sup F_B(x)$ for every x in X.

Definition 4. [28]. The union of two neutrosophic sets *A* and *B* is a neutrosophic set *C*, denoted by $C = A \cup B$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of *A* and *B* by $T_C(x) = T_A(x) \oplus T_B(x) \ominus T_A(x) \odot T_B(x)$,

$I_C(x) = I_A(x) \oplus I_B(x) \ominus I_A(x) \odot I_B(x)$ and $F_C(x) = F_A(x) \oplus F_B(x) \ominus F_A(x) \odot F_B(x)$ for any *x* in *X*.

Definition 5. [28]. The intersection of two neutrosophic sets *A* and *B* is a neutrosophic set *C*, denoted by $C = A \cap B$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of *A* and *B* by $T_C(x) = T_A(x) \odot T_B(x)$,

$I_C(x) = I_A(x) \odot I_B(x)$, and $F_C(x) = F_A(x) \odot F_B(x)$ for any *x* in *X*.

2.2. Single valued neutrosophic set

A single valued neutrosophic set (SVNS) is an instance of a neutrosophic set, which can be used in real scientific and engineering applications [35].

Definition 6. [29]. Let *X* be a universe set, with a generic element in *X* denoted by *x*. A single valued neutrosophic set (SVNS) *A* in *X* is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each element *x* in *X*, $T_A(x), I_A(x), F_A(x) \in [0,1]$.

Therefore, a SVNS *A* can be written as follows [35]:

$$A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}.$$

For two SVNSs *A*, *B*, Wang et al. [29] presented the following expressions:

(1) $A \subseteq B$ if and only if $T_A(x) \le T_B(x), I_A(x) \ge I_B(x)$, and $F_A(x) \ge F_A(x)$ for every *x* in *X*.

(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(3) $A^c = \{< x, F_A(x), 1 - I_A(x), T_A(x) > | x \in X\}$.

A SVNS *A* is usually denoted by the simplified symbol $A =< T_A(x), I_A(x), F_A(x) >$ for any *x* in *X*. For any two SVNSs *A* and *B*, the operational relations are defined by Wang et al. [29].

(1) $A \cup B =< \max(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) >$ for every *x* in *X*.

(2) $A \cap B =< \min(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) >$ for every *x* in *X*.

(3) $A \times B =< T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x) >$ for every *x* in *X*.

For a SVNS *A* in *X*, Ye [47] called the triplet $< T_A(x), I_A(x), F_A(x) >$ single valued neutrosophic number (SVNN), which is denoted by $\alpha = < T_A, I_A, F_A >$.

Definition 7. [48]. Let $\tilde{a} = \langle T, I, F \rangle$ be a SVNN, then the score function and the accuracy function of *A* are determined by Eqs. (1) and (2), respectively.

$$S(\tilde{a}) = (T + 1 - I + 1 - F)/3 \tag{1}$$

$$V(\tilde{a}) = (T + F + 1 - I)/3 \tag{2}$$

Theorem 1. [48]. Let $\tilde{a} = \langle T_a, I_a, F_a \rangle$ and $\tilde{b} = \langle T_b, I_b, F_b \rangle$ be two SVNNs, then the comparison laws between them are shown as follows:

If $S(\tilde{a}) > S(\tilde{b})$, then $\tilde{a} > \tilde{b}$;

If $S(\tilde{a}) < S(\tilde{b})$, then $\tilde{a} < \tilde{b}$;

If $S(\tilde{a}) = S(\tilde{b})$, then:

(1) If $V(\tilde{a}) > V(\tilde{b})$, then $\tilde{a} > \tilde{b}$;

(2) If $V(\tilde{a}) < V(\tilde{b})$, then $\tilde{a} < \tilde{b}$;

(3) If $V(\tilde{a}) = V(\tilde{b})$, then $\tilde{a} = \tilde{b}$.

Definition 8 [49]. Let $\tilde{a} = \langle T, I, F \rangle$, $\tilde{a}_1 = \langle T_1, I_1, F_1 \rangle$ and $\tilde{a}_2 = \langle T_2, I_2, F_2 \rangle$ be any three single valued neutrosophic numbers, and $\lambda > 0$, then some operational laws of the SVNNs are defined as follows.

(1) $\tilde{a}_1 \oplus \tilde{a}_2 = \langle T_1 + T_2 \text{-} T_1 \times T_2, I_1 \times I_2, F_1 \times F_2 \rangle$;

(2) $\tilde{a}_1 \otimes \tilde{a}_2 = \langle T_1 \times T_2, I_1 + I_2 \text{-} I_1 \times I_2, F_1 + F_2 \text{-} F_1 \times F_2 \rangle$;

(3) $\lambda \tilde{a} = \left\langle 1 - (1 - T)^{\lambda}, I^{\lambda}, F^{\lambda} \right\rangle$, $\lambda > 0$;

(4) $\tilde{a}^{\lambda} = \left\langle T^{\lambda}, 1 - (1 - I)^{\lambda}, 1 - (1 - F)^{\lambda} \right\rangle$, $\lambda > 0$.

Obviously, the above operational results are still SVNNs. Some relationships can be further established for these operations on SVNNs.

2.2. Choquet integral

Definition 9. [50]. Let $X = \{x_1, x_2, ..., x_n\}$ be a finite set and *P*(*X*) be the power set of *X*. The set function $\mu : P(X) \rightarrow [0,1]$ is called a fuzzy measure satisfying the following axioms:

(1) $\mu(\phi) = 0$, $\mu(X) = 1$;

(2) If $A, B \in P(X)$ and $A \subseteq B$, then $\mu(A) \leq \mu(B)$;

(3) If $F_n \in P(X)$ for $1 \leq n \leq \infty$ and a sequence $\{F_n\}$ is monotone, then $\lim_{n \to \infty} \mu(F_n) = \mu(\lim_{n \to \infty} F_n)$.

To avoid the problems with computational complexity and practical estimation, λ-fuzzy measure μ, a special kind of fuzzy measure, was proposed by Sugeno [51], which satisfies the following additional properties [52]:

$$\mu(A\cup B)=\mu(A)+\mu(B)+\lambda\mu(A)\mu(B) \tag{3}$$

where $\lambda\in(-1,\infty)$, for all $A,B\in P(X)$ and $A\cap B=\phi$. For the interaction between A and B, if $\lambda>0$, then there exists the multiplicative effect; if $\lambda<0$, then there exists the substitutive effect. If $\lambda=0$, then A and B are independent of each other and the Eq.(3) reduces to the following additive measure:

$$\mu(A\cup B)=\mu(A)+\mu(B)\text{, for all } A,B\in P(X) \text{ and } A\cap B=\phi. \tag{4}$$

If the elements of A in X are independent, we have

$$\mu(A)=\sum_{x_i\in A}\mu(x_i)\text{, for all } A\in P(X). \tag{5}$$

If X is a finite set, then $\bigcup_{i=1}^{n}x_i=X$. The λ-fuzzy measure μ satisfies the following Eq.(6):

$$\mu(X)=\mu(\bigcup_{i=1}^{n}x_i)=\begin{cases}\frac{1}{\lambda}\left[\prod_{i=1}^{n}(1+\lambda\mu(x_i))-1\right], & \lambda\neq 0\\ \sum_{i=1}^{n}\mu(x_i), & \lambda=0\end{cases} \tag{6}$$

where $x_i\cap x_j=\phi$, for all $i,j=1,2,\ldots,n$ and $i\neq j$. It can be noted that $\mu(x_i)$ for a subset with a single element x_i is called a fuzzy density and can be denoted as $\mu_i=\mu(x_i)$.

Definition 10. [53]. Let μ be a fuzzy measure of $(X, P(X))$, $X=\{x_1,x_2,\ldots,x_n\}$ be a finite set. The Choquet integral of a function $h:X\rightarrow[0,1]$ with respect to the fuzzy measure μ is expressed as follows:

$$\int hd\mu=\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))\cdot h(x_{\sigma(i)}) \tag{7}$$

where $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$ such that $h(x_{\sigma(1)})\geq h(x_{\sigma(2)})\geq\cdots\geq h(x_{\sigma(n)})$, $H_{\sigma(i)}=\{x_{\sigma(1)},x_{\sigma(2)},\ldots,x_{\sigma(i)}\}$ and $H_{\sigma(0)}=\phi$.

3. Some single valued neutrosophic correlated aggregation operators

In this section, we shall develop some correlated aggregation operators to aggregate single valued neutrosophic information based on the operations of single valued neutrosophic numbers.

Definition 11. Let $\tilde{a}_j=\langle T_j,I_j,F_j\rangle$ (j=1,2,…,n) be a collection of SVNNs on X, μ be a fuzzy measure on X, then the single valued neutrosophic correlated average (SVNCA) operator is defined as follows:

$$\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\mathop{\oplus}_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))\tilde{a}_{\sigma(i)} \tag{8}$$

where $(\sigma(1),\sigma(2),...,\sigma(n))$ is a permutation of $(1,2,...,n)$ such that $\tilde{a}_{\sigma(1)}\geq\tilde{a}_{\sigma(2)}\geq\ldots\geq\tilde{a}_{\sigma(n)}$, $x_{\sigma(i)}$ is the attribute corresponding to $\tilde{a}_{\sigma(i)}$, $H_{\sigma(i)}=\{x_{\sigma(k)}|k\leq i\}$, for $i\geq 1$, $H_{\sigma(0)}=\phi$.

Based on the operational laws of SVNNs, we get Theorem 2.

Theorem 2. Let $\tilde{a}_i=\langle T_i,I_i,F_i\rangle\ (i=1,2,\ldots,n)$ be a collection of SVNNs on X, μ be a fuzzy measure on X, then their aggregated value obtained by the SVNCA operator is still a SVNN, and

$$\begin{aligned}&\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)\\&=\left\langle 1-\prod_{i=1}^{n}(1-T_{\sigma(i)})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{n}(I_{\sigma(i)})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{n}(F_{\sigma(i)})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right\rangle\end{aligned} \tag{9}$$

Proof. The first result follows quickly from Definition 11. In what follows, we prove Eq. (9) using the mathematical induction on n.

(1) When n=2, it is easy to conclude that Eq. (9) holds according to the operational law (1) in Definition 8:

$$\begin{aligned}&\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2)\\&=\mathop{\oplus}_{i=1}^{2}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))\tilde{a}_{\sigma(i)}\\&=\left((\mu(H_{\sigma(1)})-\mu(H_{\sigma(0)}))\tilde{a}_{\sigma(1)}\right)\oplus\left((\mu(H_{\sigma(2)})-\mu(H_{\sigma(1)}))\tilde{a}_{\sigma(2)}\right)\\&=\left\langle 1-\left(1-T_{\sigma(1)}\right)^{\mu(H_{\sigma(1)})-\mu(H_{\sigma(0)})},\left(I_{\sigma(1)}\right)^{\mu(H_{\sigma(1)})-\mu(H_{\sigma(0)})},\left(F_{\sigma(1)}\right)^{\mu(H_{\sigma(1)})-\mu(H_{\sigma(0)})}\right\rangle\oplus\\&\left\langle 1-\left(1-T_{\sigma(2)}\right)^{\mu(H_{\sigma(2)})-\mu(H_{\sigma(1)})},\left(I_{\sigma(2)}\right)^{(\mu(H_{\sigma(2)})-\mu(H_{\sigma(1)})},\left(F_{\sigma(2)}\right)^{\mu(H_{\sigma(2)})-\mu(H_{\sigma(1)})}\right\rangle\\&=\left\langle 1-\prod_{i=1}^{2}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{2}\left(I_{\sigma(i)}\right)^{(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{2}\left(F_{\sigma(i)}\right)^{(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right\rangle\end{aligned}$$

(2) Assume that Eq. (9) holds for $n=k\,(k\geq 2)$, namely,

$$\begin{aligned}&\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)\\&=\left\langle 1-\prod_{i=1}^{k}(1-T_{\sigma(i)})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{k}(I_{\sigma(i)})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{k}(F_{\sigma(i)})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right\rangle\end{aligned}$$

When $n=k+1$, we get

$$\begin{aligned}
&\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\dots,\tilde{a}_{k+1}) \\
&= \mathop{\oplus}_{i=1}^{k+1}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))\tilde{a}_{\sigma(i)} \\
&= \left(\mathop{\oplus}_{i=1}^{k}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))\tilde{a}_{\sigma(i)}\right)\oplus\left((\mu(H_{\sigma(k+1)})-\mu(H_{\sigma(k)}))\tilde{a}_{k+1}\right) \\
&= \text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\dots,\tilde{a}_k)\oplus\left((\mu(H_{\sigma(k+1)})-\mu(H_{\sigma(k)}))\tilde{a}_{k+1}\right) \\
&= \left\langle 1-\prod_{i=1}^{k}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{k}\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{k}\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right\rangle\oplus \\
&\left\langle 1-\left(1-T_{\sigma(k+1)}\right)^{\mu(H_{\sigma(k+1)})-\mu(H_{\sigma(k)})},\left(I_{\sigma(k+1)}\right)^{\mu(H_{\sigma(k+1)})-\mu(H_{\sigma(k)})},\left(F_{\sigma(k+1)}\right)^{\mu(H_{\sigma(k+1)})-\mu(H_{\sigma(k)})}\right\rangle
\end{aligned}$$

Let $a_1=\prod_{i=1}^{k}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}$, $b_1=\prod_{i=1}^{k}\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}$, $c_1=\prod_{i=1}^{k}\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}$, $a_2=\left(1-T_{\sigma(k+1)}\right)^{\mu(H_{\sigma(k+1)})-\mu(H_{\sigma(k)})}$, $b_2=\left(I_{\sigma(k+1)}\right)^{\mu(H_{\sigma(k+1)})-\mu(H_{\sigma(k)})}$, $c_2=\left(F_{\sigma(k+1)}\right)^{\mu(H_{\sigma(k+1)})-\mu(H_{\sigma(k)})}$,

According to the operational law (1) in Definition 8, we have

$$\begin{aligned}
&\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\dots,\tilde{a}_{k+1}) \\
&= \langle 1-a_1,b_1,c_1\rangle\oplus\langle 1-a_2,b_2,c_2\rangle \\
&= \langle 1-a_1a_2,b_1b_2,c_1c_2\rangle \\
&= \left\langle 1-\prod_{i=1}^{k+1}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{k+1}\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{k+1}\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right\rangle
\end{aligned}$$

i.e., Eq. (9) holds for $n=k+1$.

According to steps (1) and (2), we know that Eq. (9) holds for any positive integer *n*.

Some special cases of the SVNCA operator are considered as follows. Let $\tilde{a}_i=\langle T_i,I_i,F_i\rangle$ $(i=1,2,\dots,n)$ be a collection of SVNNs on *X*, and μ be a fuzzy measure on *X*.

(1) If $\mu(H)=1$ for any $H\in P(x)$, then

$$\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\dots,\tilde{a}_n)=\langle T_{\sigma(1)},I_{\sigma(1)},I_{\sigma(1)}\rangle.$$

(2) If $\mu(H)=0$ for any $H\in P(x)$ and $H\neq X$, then

$$\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\dots,\tilde{a}_n)=\langle T_{\sigma(n)},I_{\sigma(n)},I_{\sigma(n)}\rangle.$$

(3) If the independent condition (5) holds, then

$$\mu(x_{\sigma(i)})=\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}),i=1,2,...,n \tag{10}$$

In this case, the SVNCA operator reduces to the following single valued neutrosophic weighted average (SVNWA) operator:

$$\text{SVNWA}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\mathop{\oplus}_{i=1}^{n}\left(\mu(x_i)\tilde{a}_i\right)=\left\langle 1-\prod_{i=1}^{n}(1-T_i)^{\mu(x_i)},\prod_{i=1}^{n}(I_i)^{\mu(x_i)},\prod_{i=1}^{n}(F_i)^{\mu(x_i)}\right\rangle \tag{11}$$

In particular, if $\mu(x_i)=\dfrac{1}{n}$, for $i=1,2,...,n$, then the SVNCA operator in Eq.(8) reduces to the single valued neutrosophic arithmetic average (SVNAA) operator.

$$\mathrm{SVNAA}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\overset{n}{\underset{i=1}{\oplus}}(\mu(x_i)\tilde{a}_i)=\left\langle 1-\prod_{i=1}^{n}(1-T_i)^{1/n},\prod_{i=1}^{n}(I_i)^{1/n},\prod_{i=1}^{n}(F_i)^{1/n}\right\rangle \tag{12}$$

(4) If

$$\mu(H)=\sum_{i=1}^{|H|}w_i, \text{for all } H\subseteq X \tag{13}$$

where $|H|$ is the number of the elements in *H*, then

$$\omega_i=\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}), i=1,2,...,n \tag{14}$$

where $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ such that $\omega_i\geq 0, i=1,2,...,n,$ and $\sum_{i=1}^{n}\omega_i=1$. In this case, the the SVNCA operator reduces to the following single valued neutrosophic ordered weighted average (SVNOWA) operator:

$$\mathrm{SVNOWA}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\overset{n}{\underset{j=1}{\oplus}}(\omega_i\tilde{a}_{\sigma(i)})=\left\langle 1-\prod_{i=1}^{n}(1-T_{\sigma(i)})^{\omega_i},\prod_{i=1}^{n}(I_{\sigma(i)})^{\omega_i},\prod_{i1}^{n}(F_{\sigma(i)})^{\omega_i}\right\rangle$$

In particular, if $\mu(H)=\frac{|H|}{n}$, for all $H\subseteq X$, then the SVNCA operator in Eq.(8) reduces to the SVNAA operator in Eq. (12).

(5) If I_i=0 and $T_i+F_i\leq 1$, then SVNNs $\tilde{a}_i=\langle T_i,I_i,F_i\rangle$ (*i*=1,2,…,*n*) are reduced to intuitionistic fuzzy numbers (IFNs), and we can obtain the following intuitionistic fuzzy correlated average (IFCA) operators proposed by Tan and chen [39, 54].

$$\begin{aligned}&\mathrm{IFCA}_\mu(\tilde{b}_1,\tilde{b}_2,\cdots,\tilde{b}_n)\\&=\overset{n}{\underset{i=1}{\oplus}}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))\tilde{b}_{\sigma(i)}\\&=\left\langle 1-\prod_{i=1}^{n}(1-T_{\sigma(i)})^{(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))},\prod_{i=1}^{n}(F_{\sigma(i)})^{(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}\right\rangle\end{aligned}$$

where $\tilde{b}_i=\langle T_i,F_i\rangle$ (*i*=1,2,…,*n*) be a collection of intuitionistic fuzzy values on *X*, and μ be a fuzzy measure on *X*.

It can be proved that the SVNCA operator has the following properties.

Theorem 3. Let $\tilde{a}_i=\langle T_i,I_i,F_i\rangle$ (*i*=1,2,…,*n*) be a collection of SVNNs on *X*, μ be a fuzzy measure on X, then we have the following properties.

(1) *(Idempotency)* If $\tilde{a}_i=\langle T_i,I_i,F_i\rangle$ (*i*=1,2,…,*n*) are equal, i.e., $\tilde{a}_i=\tilde{a}=\langle T_a,I_a,F_a\rangle$, then

$$\mathrm{SVNCA}_\mu(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\tilde{a}.$$

(2) (*Boundedness*) Let $T_{\min}=\min_{1\le i\le n}\{T_i\}$, $T_{\max}=\max_{1\le i\le n}\{T_i\}$, $I_{\min}=\min_{1\le i\le n}\{I_i\}$, $I_{\max}=\max_{1\le i\le n}\{I_i\}$, $F_{\min}=\min_{1\le i\le n}\{F_i\}$, $F_{\max}=\max_{1\le i\le n}\{F_i\}$. Then we can obtain

$$\langle T_{\min},I_{\max},F_{\max}\rangle\le \mathrm{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\le\langle T_{\max},I_{\min},F_{\min}\rangle. \tag{15}$$

(3) (*Monotonicity*) If $T_i\le T_i^{'}$, $I_i\ge I_i^{'}$ and $F_i\ge F_i^{'}$ for all *i*, then

$$\mathrm{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\le \mathrm{SVNCA}_{\mu}(\tilde{a}_1^{'},\tilde{a}_2^{'},\ldots,\tilde{a}_n^{'}).$$

(4) (*Commutativity*) If $\tilde{a}_i^{'}=\langle T_i^{'},I_i^{'},F_i^{'}\rangle$ $(i=1,2,\ldots,n)$ is any permutation of $\tilde{a}_i=\langle T_i,I_i,F_i\rangle$ $(i=1,2,\ldots,n)$, then

$$\mathrm{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\mathrm{SVNCA}_{\mu}(\tilde{a}_1^{'},\tilde{a}_2^{'},\ldots,\tilde{a}_n^{'})$$

Proof. (1) Since $\tilde{a}_i=\langle T_a,I_a,F_a\rangle$ for all *i*, we have

$$\begin{aligned}
&\mathrm{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)\\
&=\left\langle 1-\prod_{i=1}^{n}(1-T_a)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{n}(I_a)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{n}(F_a)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right\rangle\\
&=\left\langle 1-(1-T_a)^{\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))},(I_a)^{\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))},(F_a)^{\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}\right\rangle\\
&=\langle T_a,I_a,F_a\rangle
\end{aligned}$$

Thus, we have $\mathrm{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\tilde{a}$.

(2) Since $T_{\min}\le T_i\le T_{\max}$, $I_{\min}\le I_i\le I_{\max}$, $F_{\min}\le F_i\le F_{\max}$ for all *i*, then we have

$$1-\prod_{i=1}^{n}(1-T_{\sigma(i)})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\ge 1-\prod_{i=1}^{n}(1-T_{\min})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}=1-(1-T_{\min})^{\sum_{i=1}^{n}\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}=T_{\min},$$

$$1-\prod_{i=1}^{n}(1-T_{\sigma(i)})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\le 1-\prod_{i=1}^{n}(1-T_{\max})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}=1-(1-T_{\max})^{\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}=T_{\max},$$

i.e.,

$$T_{\min}\le 1-\prod_{i=1}^{n}(1-T_{\sigma(i)})^{(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}\le T_{\max}. \tag{16}$$

Similarly, we have

$$I_{\min}\le\prod_{i=1}^{n}(I_{\sigma(i)})^{(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}\le I_{\max}, \tag{17}$$

$$F_{\min}\le\prod_{i=1}^{n}(F_{\sigma(i)})^{(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}\le F_{\max}. \tag{18}$$

Let $\mathrm{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\langle T_a,I_a,F_a\rangle=\tilde{a}$, $\langle T_{\max},I_{\min},F_{\min}\rangle=\tilde{a}^{*}$ and $\langle T_{\min},I_{\max},F_{\max}\rangle=\tilde{a}_{*}$, then Eqs. (16), (17) and (18) are transformed into the following forms, respectively:

$$T_{\min}\le T_a\le T_{\max} \tag{19}$$

$$I_{\min} \le I_{a} \le I_{\max} \qquad (20)$$

$$F_{\min} \le F_{a} \le F_{\max} \qquad (21)$$

Thus, we have

$$S(\tilde{a}) = \frac{1}{3}\left(T_{a} + 1 - I_{a} + 1 - F_{a}\right) \le \frac{1}{3}\left(T_{\max} + 1 - I_{\min} + 1 - F_{\min}\right) = S(\tilde{a}^{*}).$$

<1> If $S(\tilde{a}) < S(\tilde{a}^{*})$, then by Theorem 1, we have

$$\mathrm{SVNCA}_{\mu}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) < \langle T_{\max}, I_{\min}, F_{\min} \rangle. \qquad (22)$$

<2> If $S(\tilde{a}) = S(\tilde{a}^{*})$, then by the following conditions:

$T_{a} \le T_{\max}$, $1 - I_{a} \le 1 - I_{\min}$, and $1 - F_{a} \le 1 - F_{\min}$,

we have

$T_{a} = T_{\max}$, $1 - I_{a} = 1 - I_{\min}$, and $1 - F_{a} = 1 - F_{\min}$,

thus,

$$V(\tilde{a}) = \frac{1}{3}\left(T_{a} + F_{a} + 1 - I_{a}\right) = \frac{1}{3}\left(T_{\max} + F_{\min} + 1 - I_{\min}\right) = V(\tilde{a}^{*}).$$

In this case, by Theorem 1, we have

$$\mathrm{SVNCA}_{\mu}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \langle T_{\max}, I_{\min}, F_{\min} \rangle. \qquad (23)$$

From Eqs.(22) and (23), we have

$$\mathrm{SVNCA}_{\mu}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \le \langle T_{\max}, I_{\min}, F_{\min} \rangle. \qquad (24)$$

Similarly, we have

$$\langle T_{\min}, I_{\max}, F_{\max} \rangle \le \mathrm{SVNCA}_{\mu}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n). \qquad (25)$$

From Eqs.(24) and (25), we know that Eq.(15) always holds, i.e.,

$$\langle T_{\min}, I_{\max}, F_{\max} \rangle \le \mathrm{SVNCA}_{\mu}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \le \langle T_{\max}, I_{\min}, F_{\min} \rangle.$$

(3). Since $T_i \le T_i^{'}$, $I_i \ge I_i^{'}$ and $F_i \ge F_i^{'}$ for all i, then we have

$$T_{\sigma(i)} \le T_{\sigma(i)}^{'}, \quad I_{\sigma(i)} \ge I_{\sigma(i)}^{'} \quad \text{and} \quad F_{\sigma(i)} \ge F_{\sigma(i)}^{'}.$$

So,

$$\left(1 - T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})} \ge \left(1 - T_{\sigma(i)}^{'}\right)^{\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})},$$

$$\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})} \ge \left(I_{\sigma(i)}^{'}\right)^{\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})},$$

$$\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})} \ge \left(F_{\sigma(i)}^{'}\right)^{\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})}.$$

Furthermore, we have

$$1 - \prod_{i=1}^{n}\left(1 - T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})} \le 1 - \prod_{i=1}^{n}\left(1 - T_{\sigma(i)}^{'}\right)^{\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})}, \qquad (26)$$

$$1 - \prod_{i=1}^{n}\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})} \le 1 - \prod_{i=1}^{n}\left(I_{\sigma(i)}^{'}\right)^{\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})}, \qquad (27)$$

$$1-\prod_{i=1}^{n}\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \leq 1-\prod_{i=1}^{n}\left(F'_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}. \quad (28)$$

Therefore, we have

$$\frac{1}{3}\left(1-\prod_{i=1}^{n}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}+1-\prod_{i=1}^{n}\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}+1-\prod_{i=1}^{n}\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right)$$
$$\leq\frac{1}{3}\left(1-\prod_{i=1}^{n}\left(1-T'_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}+1-\prod_{i=1}^{n}\left(I'_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}+1-\prod_{i=1}^{n}\left(F'_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right). \quad (29)$$

Let $\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\langle T_a,I_a,F_a\rangle=\tilde{a}$ and $\text{SVNCA}_{\mu}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n)=\langle T_{a'},I_{a'},F_{a'}\rangle=\tilde{a}'$, then Eq. (29) is transformed into the following forms:

$$S(\tilde{a})\leq S(\tilde{a}')$$

<1> If $S(\tilde{a})<S(\tilde{a}')$, then by Theorem 1, we have

$$\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)<\text{SVNCA}_{\mu}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n). \quad (30)$$

<2> If $S(\tilde{a})=S(\tilde{a}')$, then by Eqs.(26) to (28), we have

$$1-\prod_{i=1}^{n}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}=1-\prod_{i=1}^{n}\left(1-T'_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},$$
$$1-\prod_{i=1}^{n}\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}=1-\prod_{i=1}^{n}\left(I'_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},$$
$$1-\prod_{i=1}^{n}\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}=1-\prod_{i=1}^{n}\left(F'_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}.$$

Thus, we have

$$\frac{1}{3}\left(1-\prod_{i=1}^{n}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}+\prod_{i=1}^{n}\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}+1-\prod_{i=1}^{n}\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right)$$
$$=\frac{1}{3}\left(1-\prod_{i=1}^{n}\left(1-T'_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}+\prod_{i=1}^{n}\left(F'_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}+1-\prod_{i=1}^{n}\left(I'_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right)$$

i.e.,

$$V(\tilde{a})=V(\tilde{a}').$$

In this case, by Theorem 1, we have

$$\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\text{SVNCA}_{\mu}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n) \quad (31)$$

Therefore, from Eqs.(30) and (31), we have

$$\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\leq\text{SVNCA}_{\mu}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n).$$

(4). Since $\tilde{a}'_i=\langle T'_i,I'_i,F'_i\rangle$ (i=1,2,…,n) is a permutation of $\tilde{a}_i=\langle T_i,I_i,F_i\rangle$ (i=1,2,…,n), we have $\tilde{a}'_{\sigma(i)}=\tilde{a}_{\sigma(i)}$, for all i=1,2,…,n. Then, based on Definition 11, we obtain

$$\text{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\text{SVNCA}_{\mu}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n).$$

Theorem 4. Let $\tilde{a}_i = \langle T_i, I_i, F_i \rangle$ (i=1,2,…,n) be a set of SVNNs on X, μ be a fuzzy measure on X.

If $\tilde{s} = \langle T_s, I_s, F_s \rangle$ is a SVNN on X, then

$$\mathrm{SVNCA}_\mu(\tilde{a}_1 \oplus s, \tilde{a}_2 \oplus s, \ldots, \tilde{a}_n \oplus s) = \mathrm{SVNCA}_\mu(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \oplus s$$

Proof. According to the operational law (1) in Definition 8, for all i=1,2,…,n, we have

$$\tilde{a}_i \oplus s = \langle T_i + T_s \text{-} T_i \times T_s, I_i \times I_s, F_i \times F_s \rangle = \langle 1-(1-T_i)(1\text{-}T_s), I_i \times I_s, F_i \times F_s \rangle.$$

According to Theorem 2, we have

$$\begin{aligned}
&\mathrm{SVNCA}_\mu(\tilde{a}_1 \oplus s, \tilde{a}_2 \oplus s, \ldots, \tilde{a}_n \oplus s) \\
&= \left\langle 1-\prod_{i=1}^{n}\left((1-T_{\sigma(i)})(1-T_s)\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, \prod_{i=1}^{n}\left(I_{\sigma(i)}I_s\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, \prod_{i=1}^{n}\left(F_{\sigma(i)}F_s\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \right\rangle \\
&= \left\langle 1-(1-T_s)^{\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}\prod_{i=1}^{n}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, (I_s)^{\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}\prod_{i=1}^{n}\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, \right. \\
&\left. (F_s)^{\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}\prod_{i=1}^{n}\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \right\rangle \\
&= \left\langle 1-(1-T_s)\prod_{i=1}^{n}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, I_s\prod_{i=1}^{n}\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, F_s\prod_{i=1}^{n}\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \right\rangle.
\end{aligned}$$

On the other hand, according to the operational law (1) in Definition 8, we have

$$\begin{aligned}
&\mathrm{SVNCA}_\mu(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \oplus s \\
&= \left\langle 1-\prod_{i=1}^{n}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, \prod_{i=1}^{n}\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, \prod_{i=1}^{n}\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \right\rangle \oplus \langle T_s, I_s, F_s \rangle \\
&= \left\langle 1-(1-T_s)\prod_{i=1}^{n}\left(1-T_{\sigma(i)}\right)^{(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}, I_s\prod_{i=1}^{n}\left(I_{\sigma(i)}\right)^{(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}, F_s\prod_{i=1}^{n}\left(F_{\sigma(i)}\right)^{(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))} \right\rangle.
\end{aligned}$$

Thus,

$$\mathrm{SVNCA}_\mu(\tilde{a}_1 \oplus s, \tilde{a}_2 \oplus s, \ldots, \tilde{a}_n \oplus s) = \mathrm{SVNCA}_\mu(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \oplus s.$$

Theorem 5. Let $\tilde{a}_i = \langle T_i, I_i, F_i \rangle$ (i=1,2,…,n) be a set of SVNNs on X, μ be a fuzzy measure on X.

If $r > 0$, then

$$\mathrm{SVNCA}_\mu(r\tilde{a}_1, r\tilde{a}_2, \ldots, r\tilde{a}_n) = r \times \mathrm{SVNCA}_\mu(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$$

Proof. According to the operational law (3) in Definition 8, for all i (i=1,2,…,n) and $r > 0$, we have

$$r\tilde{a}_i = \left\langle 1-\left(1-T_i\right)^r, I_i^r, F_i^r \right\rangle.$$

According to Theorem 2, we have

$$\mathrm{SVNCA}_{\mu}(r\tilde{a}_1,r\tilde{a}_2,\ldots,r\tilde{a}_n)$$

$$=\left\langle 1-\prod_{i=1}^{n}\left((1-T_{\sigma(i)})^{r}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{n}\left((I_{\sigma(i)})^{r}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{n}\left((F_{\sigma(i)})^{r}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right\rangle$$

$$=\left\langle 1-\prod_{i=1}^{n}\left(1-T_{\sigma(i)}\right)^{r(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))},\prod_{i=1}^{n}\left(I_{\sigma(i)}\right)^{r(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))},\prod_{i=1}^{n}\left(F_{\sigma(i)}\right)^{r(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}\right\rangle.$$

On the other hand, according to the operational law (3) in Definition 8, we have

$$r\times\mathrm{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)$$

$$=r\times\left\langle 1-\prod_{i=1}^{n}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{n}\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},\prod_{i=1}^{n}\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right\rangle$$

$$=\left\langle 1-\prod_{i=1}^{n}\left(1-T_{\sigma(i)}\right)^{r(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))},\prod_{i=1}^{n}\left(I_{\sigma(i)}\right)^{r(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))},\prod_{i=1}^{n}\left(F_{\sigma(i)}\right)^{r(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}\right\rangle$$

Thus,

$$\mathrm{SVNCA}_{\mu}(r\tilde{a}_1,r\tilde{a}_2,\ldots,r\tilde{a}_n)=r\times\mathrm{SVNCA}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n).$$

Definition 12. Let $\tilde{a}_i=\langle T_i,I_i,F_i\rangle$ $(i=1,2,\ldots,n)$ be a collection of SVNNs on X, μ be a fuzzy measure on X, then the single valued neutrosophic correlated geometric (SVNCG) operator is defined as follows:

$$\mathrm{SVNCG}_{\mu}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\bigotimes_{i=1}^{n}\left(\tilde{a}_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \tag{32}$$

where $(\sigma(1),\sigma(2),...,\sigma(n))$ is a permutation of $(1,2,...,n)$ such that $\tilde{a}_{\sigma(1)}\geq\tilde{a}_{\sigma(2)}\geq\ldots\geq\tilde{a}_{\sigma(n)}$, $x_{\sigma(i)}$ is the attribute corresponding to $\tilde{a}_{\sigma(i)}$, $H_{\sigma(i)}=\{x_{\sigma(k)}|k\leq i\}$, for $i\geq 1$, $H_{\sigma(0)}=\phi$.

Based on the operational laws of SVNNs, we get Theorem 6.

Theorem 6. Let $\tilde{a}_i=\langle T_i,I_i,F_i\rangle$ $(i=1,2,\ldots,n)$ be a collection of SVNNs on X, μ be a fuzzy measure on X, then their aggregated value obtained by the SVNCG operator is still a SVNN, and

$$\mathrm{SVNCG}_{\mu}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)$$

$$=\left\langle\prod_{i=1}^{n}\left(T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},1-\prod_{i=1}^{n}\left(1-I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},1-\prod_{i=1}^{n}\left(1-F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right\rangle \tag{33}$$

This theorem can be proved similar to Theorem 2.

Some special cases of the SVNCG operator are considered as follows. Let $\tilde{a}_i=\langle T_i,I_i,F_i\rangle$ $(i=1,2,\ldots,n)$ be a collection of SVNNs on X, and μ be a fuzzy measure on X.

(1) If $\mu(H)=1$ for any $H\in P(x)$, then

$$\text{SVNCG}_\mu(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\langle T_{\sigma(1)}, I_{\sigma(1)}, I_{\sigma(1)}\rangle .$$

(2) If $\mu(H)=0$ for any $H \in P(x)$ and $H \neq X$, then

$$\text{SVNCG}_\mu(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\langle T_{\sigma(n)}, I_{\sigma(n)}, I_{\sigma(n)}\rangle .$$

(3) If Eqs. (5) and (10) hold, then the SVNCG operator reduces to the following single valued neutrosophic weighted geometric (SVNWG) operator:

$$\text{SVNWG}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\bigotimes_{i=1}^{n}(\tilde{a}_i)^{\mu(x_i)}=\left\langle \prod_{i=1}^{n}(T_i)^{\mu(x_i)}, 1-\prod_{i=1}^{n}(1-I_i)^{\mu(x_i)}, 1-\prod_{i=1}^{n}(1-F_i)^{\mu(x_i)} \right\rangle \tag{34}$$

In particular, if $\mu(x_i)=\dfrac{1}{n}$, for $i=1,2,\ldots,n$, then the SVNCG operator in Eq.(32) reduces to the single valued neutrosophic geometric average (SVNGA) operator.

$$\text{SVNGA}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\bigotimes_{i=1}^{n}(\tilde{a}_i)^{1/n}=\left\langle \prod_{i=1}^{n}(T_i)^{1/n}, 1-\prod_{i=1}^{n}(1-I_i)^{1/n}, 1-\prod_{i=1}^{n}(1-F_i)^{1/n} \right\rangle \tag{35}$$

(4) If Eqs. (13) and (14) hold, then the SVNCG operator reduces to the following single valued neutrosophic ordered weighted geometric (SVNOWG) operator:

$$\text{SVNOWG}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\bigotimes_{i=1}^{n}(\tilde{a}_{\sigma(i)})^{\omega_i}=\left\langle \prod_{i=1}^{n}(T_{\sigma(i)})^{\omega_i}, 1-\prod_{i=1}^{n}(1-I_{\sigma(i)})^{\omega_i}, 1-\prod_{i=1}^{n}(1-F_{\sigma(i)})^{\omega_i} \right\rangle$$

In particular, if $\mu(H)=\dfrac{|H|}{n}$, for all $H \subseteq X$, then the SVNCA operator in Eq.(8) reduces to the SVNGA operator in Eq. (33).

(5) If $I_i=0$ and $T_i+F_i\leq 1$, then SVNNs $\tilde{a}_i=\langle T_i, I_i, F_i\rangle$ $(i=1,2,\ldots,n)$ are reduced to intuitionistic fuzzy numbers (IFNs), and we can obtain the following intuitionistic fuzzy correlated geometric (IFCG) operators proposed by Xu [54].

$$\begin{aligned}&\text{IFCG}_\mu(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)\\&=\bigotimes_{i=1}^{n}(\tilde{a}_{\sigma(i)})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\\&=\left\langle \prod_{i=1}^{n}(T_{\sigma(i)})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, 1-\prod_{i=1}^{n}(1-F_{\sigma(i)})^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \right\rangle\end{aligned}$$

where $\tilde{a}_i=\langle T_i, F_i\rangle$ $(i=1,2,\ldots,n)$ be a collection of intuitionistic fuzzy values on X, and μ be a fuzzy measure on X.

Similar to the SVNCA operator, we can prove that the SVNCG operator has the following properties.

Theorem 7. Let $\tilde{a}_i=\langle T_i, I_i, F_i\rangle$ $(i=1,2,\ldots,n)$ be a collection of SVNNs on X, μ be a fuzzy measure on X, then we have the following properties.

(1) If $\tilde{a}_i = \langle T_i, I_i, F_i \rangle$ $(i=1,2,\ldots,n)$ are equal, i.e., $\tilde{a}_i = \tilde{a} = \langle T_a, I_a, F_a \rangle$, then

$$\text{SVNCG}_\mu(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) = \tilde{a}.$$

(2) (*Boundedness*) Let $T_{\min} = \min_{1\le i\le n}\{T_i\}$, $T_{\max} = \max_{1\le i\le n}\{T_i\}$, $I_{\min} = \min_{1\le i\le n}\{I_i\}$, $I_{\max} = \max_{1\le i\le n}\{I_i\}$, $F_{\min} = \min_{1\le i\le n}\{F_i\}$, $F_{\max} = \max_{1\le i\le n}\{F_i\}$. Then we can obtain

$$\langle T_{\min}, I_{\max}, F_{\max} \rangle \le \text{SVNCG}_\mu(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \le \langle T_{\max}, I_{\min}, F_{\min} \rangle.$$

(3) (*Monotonicity*) If $T_i \le T_i^{'}$, $I_i \ge I_i^{'}$ and $F_i \ge F_i^{'}$ for all *i*, then

$$\text{SVNCG}_\mu(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \le \text{SVNCG}_\mu(\tilde{a}_1^{'}, \tilde{a}_2^{'}, \ldots, \tilde{a}_n^{'}).$$

(4) (*Commutativity*) If $\tilde{a}_i^{'} = \langle T_i^{'}, I_i^{'}, F_i^{'} \rangle$ $(i=1,2,\ldots,n)$ is any permutation of $\tilde{a}_i = \langle T_i, I_i, F_i \rangle$ $(i=1,2,\ldots,n)$, then

$$\text{SVNCG}_\mu(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \text{SVNCG}_\mu(\tilde{a}_1^{'}, \tilde{a}_2^{'}, \ldots, \tilde{a}_n^{'})$$

Theorem 8. Let $\tilde{a}_i = \langle T_i, I_i, F_i \rangle$ $(i=1,2,\ldots,n)$ be a set of SVNNs on *X*, μ be a fuzzy measure on X. If $\tilde{s} = \langle T_s, I_s, F_s \rangle$ is a SVNN on *X*, then

$$\text{SVNCG}_\mu(\tilde{a}_1 \otimes s, \tilde{a}_2 \otimes s, \ldots, \tilde{a}_n \otimes s) = \text{SVNCG}_\mu(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \otimes s$$

Proof. According to the operational law (2) in Definition 8, for all *i*=1,2,…,*n*, we have

$$\tilde{a}_i \otimes s = \langle T_i \times T_s, I_i + I_s - I_i \times I_s, F_i + F_s - F_i \times F_s \rangle = \langle T_i \times T_s, 1-(1-I_i)(1\text{-}I_s), 1-(1-F_i)(1\text{-}F_s) \rangle.$$

According to Theorem 6, we have

$$\begin{aligned}
&\text{SVNCG}_\mu(\tilde{a}_1 \otimes s, \tilde{a}_2 \otimes s, \ldots, \tilde{a}_n \otimes s) \\
&= \left\langle \prod_{i=1}^{n} \left(T_{\sigma(i)} T_s\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, 1-\prod_{i=1}^{n}\left((1-I_{\sigma(i)})(1-I_s)\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, 1-\prod_{i=1}^{n}\left((1-F_{\sigma(i)})(1-F_s)\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \right\rangle \\
&= \Bigg\langle (T_s)^{\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))} \prod_{i=1}^{n}\left(T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, 1-(1-I_s)^{\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))} \prod_{i=1}^{n}\left(1-I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, \\
&\qquad 1-(1-F_s)^{\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))} \prod_{i=1}^{n}\left(1-F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \Bigg\rangle \\
&= \left\langle T_s \prod_{i=1}^{n}\left(T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, 1-(1-I_s)\prod_{i=1}^{n}\left(1-I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}, 1-(1-F_s)\prod_{i=1}^{n}\left(1-F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \right\rangle.
\end{aligned}$$

On the other hand, according to the operational law (2) in Definition 8, we have

$$\mathrm{SVNCG}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\otimes s$$
$$=\left\langle \prod_{i=1}^{n}\left(T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},1-\prod_{i=1}^{n}\left(1-I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},1-\prod_{i=1}^{n}\left(1-F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right\rangle\otimes\left\langle T_s,I_s,F_s\right\rangle$$
$$=\left\langle T_s\prod_{i=1}^{n}\left(T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},1-(1-I_s)\prod_{i=1}^{n}\left(1-I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},1-(1-F_s)\prod_{i=1}^{n}\left(1-F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right\rangle.$$

Thus,

$$\mathrm{SVNCG}_{\mu}(\tilde{a}_1\otimes s,\tilde{a}_2\otimes s,\ldots,\tilde{a}_n\otimes s)=\mathrm{SVNCG}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\otimes s.$$

Theorem 9. Let $\tilde{a}_i=\langle T_i,I_i,F_i\rangle$ (i=1,2,…,n) be a set of SVNNs on X, μ be a fuzzy measure on X. If $r>0$, then

$$\mathrm{SVNCG}_{\mu}(\tilde{a}_1^r,\tilde{a}_2^r,\ldots,\tilde{a}_n^r)=\left(\mathrm{SVNCG}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\right)^r$$

Proof. According to the operational law (4) in Definition 8, for all i (i=1,2,…,n) and $r>0$, we have

$$\tilde{a}_i^r=\left\langle T_i^r,1-\left(1-I_i\right)^r,1-\left(1-F_i\right)^r\right\rangle.$$

According to Theorem 6, we have

$$\mathrm{SVNCG}_{\mu}(\tilde{a}_1^r,\tilde{a}_2^r,\ldots,\tilde{a}_n^r)$$
$$=\left\langle \prod_{i=1}^{n}\left((T_{\sigma(i)})^r\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},1-\prod_{i=1}^{n}\left((1-I_{\sigma(i)})^r\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})},1-\prod_{i=1}^{n}\left((1-F_{\sigma(i)})^r\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}\right\rangle$$
$$=\left\langle \prod_{i=1}^{n}\left(T_{\sigma(i)}\right)^{r(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))},1-\prod_{i=1}^{n}\left(1-I_{\sigma(i)}\right)^{r(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))},1-\prod_{i=1}^{n}\left(1-F_{\sigma(i)}\right)^{r(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}\right\rangle.$$

On the other hand, according to the operational law (4) in Definition 8, we have

$$\left(\mathrm{SVNCG}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\right)^r$$
$$=\left\langle \prod_{i=1}^{n}\left(T_{\sigma(i)}\right)^{r(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))},1-\prod_{i=1}^{n}\left(1-I_{\sigma(i)}\right)^{r(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))},1-\prod_{i=1}^{n}\left(1-F_{\sigma(i)}\right)^{r(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))}\right\rangle$$

Thus,

$$\mathrm{SVNCG}_{\mu}(\tilde{a}_1^r,\tilde{a}_2^r,\ldots,\tilde{a}_n^r)=\left(\mathrm{SVNCG}_{\mu}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\right)^r$$

Lemma 1. [55]. Let $a_j>0$, $w_j>0$, $j=1,2,\ldots,n$ and $\sum_{j=1}^{n}w_j=1$, then

$$\prod_{j=1}^{n}a_j^{w_j}\leq\sum_{j=1}^{n}w_ja_j \qquad (36)$$

with equality if and only if $a_1=a_2=\cdots=a_n$.

To compare the aggregated values between the SVNCA and SVNCG operators, we give the following theorem.

Theorem 10. Let $\tilde{a}_i = \langle T_i, I_i, F_i \rangle$ $(i=1,2,\ldots,n)$ be a collection of SVNNs on X, μ be a fuzzy measure on X, then

$$\mathrm{SVNCG}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) \le \mathrm{SVNCA}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n). \quad (37)$$

Proof. According to Lemma1, we have

$$\begin{aligned}
&\prod_{i=1}^{n}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \\
&\le \sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))\left(1-T_{\sigma(i)}\right) \\
&= 1-\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))T_{\sigma(i)} \\
&\le 1-\prod_{i=1}^{n}\left(T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}
\end{aligned}$$

Thus, we have

$$1-\prod_{i=1}^{n}\left(1-T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \ge \prod_{i=1}^{n}\left(T_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \quad (38)$$

Similarly, we have

$$\prod_{i=1}^{n}\left(I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \le 1-\prod_{i=1}^{n}\left(1-I_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \quad (39)$$

$$\prod_{i=1}^{n}\left(F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})} \le 1-\prod_{i=1}^{n}\left(1-F_{\sigma(i)}\right)^{\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})}. \quad (40)$$

Let $\mathrm{SVNCA}_{\mu}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) = \langle T_a, I_a, F_a \rangle = \tilde{a}$, $\mathrm{SVNCG}_{\mu}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) = \langle T_b, I_b, F_b \rangle = \tilde{b}$, then Eqs. (38), (39) and (40) are transformed into the following forms, respectively:

$$T_a \ge T_b \quad (41)$$

$$I_a \le I_b, \quad (42)$$

$$F_a \le F_b. \quad (43)$$

Thus, we have

$$S(\tilde{b}) = \frac{1}{3}(T_b + 1 - I_b + 1 - F_b) \le \frac{1}{3}(T_a + 1 - I_a + 1 - F_a) = S(\tilde{a}).$$

According to Theorem 1, we have

$$\mathrm{SVNCG}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) \le \mathrm{SVNCA}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n).$$

4. A multiple attribute group decision making method to material selection under single valued neutrosophic environment

In this section, we apply the SVNCA (SVNCG) operator to solve material selection problems with single valued neutrosophic information. For a material selection problem, let $A=\{A_1, A_2,..., A_m\}$ ($m \geq 2$) be a finite set of feasible material alternatives among which decision makers (DMs) have to choose, $C=\{C_1, C_2,..., C_n\}$ ($n \geq 2$) be a finite set of attributes with which alternative performance is measured, $DM=\{DM_1, DM_2,..., DM_t\}$ ($t \geq 2$) be a set of DMs, and $\lambda = (\lambda_1, \lambda_2,..., \lambda_t)^T$ be the weight vector of DMs, such that $\lambda_k \geq 0$, k=1,2,…,t, and $\sum_{k=1}^{t}\lambda_k = 1$. Suppose that $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{m\times n}$ is a single valued neutrosophic decision matrix given by the kth DM, where $\tilde{r}_{ij}^{(k)} = (T_{ij}^{(k)}, I_{ij}^{(k)}, F_{ij}^{(k)})$ is the assessment value on the material alternative $A_i \in A$ with respect to the attribute $C_j \in C$ provided by the kth DM, $T_{ij}^{(k)}$ indicates the degree to which the material alternative A_i satisfies the attribute C_j provided by the kth DM, $I_{ij}^{(k)}$ indicates the indeterminacy degree to which the material alternative A_i satisfies the attribute C_j provided by the kth DM, and $F_{ij}^{(k)}$ indicates the degree to which the material alternative A_i does not satisfy the attribute C_j provided by the kth DM. The proposed operators are utilized to develop a multiple attribute group decision making method for material selection with single valued neutrosophic information by the following steps:

Step 1. Aggregate all individual single valued neutrosophic decision matrices $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{m\times n} (k=1,2,...,t)$ into a collective single valued neutrosophic decision matrix $\tilde{R} = (\tilde{r}_{ij})_{m\times n}$ based on SVNWA (SVNWG) operator as follows:

$$\begin{aligned}\tilde{r}_{ij} &= \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle \\ &= \text{SVNWA}\left(\tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, \cdots, \tilde{r}_{ij}^{(k)}\right) \\ &= \left\langle 1-\prod_{k=1}^{t}\left(1-T_{ij}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{t}\left(I_{ij}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{t}\left(F_{ij}^{(k)}\right)^{\lambda_k} \right\rangle,\ i=1,2,...,m,\ j=1,2,...,n\end{aligned} \tag{44}$$

$$\begin{aligned}\tilde{r}_{ij} &= \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle \\ &= \text{SVNWG}\left(\tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, \cdots, \tilde{r}_{ij}^{(k)}\right) \\ &= \left\langle \prod_{k=1}^{t}\left(T_{ij}^{(k)}\right)^{\lambda_k}, 1-\prod_{k=1}^{t}\left(1-I_{ij}^{(k)}\right)^{\lambda_k}, 1-\prod_{k=1}^{t}\left(1-F_{ij}^{(k)}\right)^{\lambda_k} \right\rangle,\ i=1,2,...,m,\ j=1,2,...,n\end{aligned} \tag{45}$$

where $\lambda = (\lambda_1, \lambda_2,..., \lambda_t)^T$ is the weight vector of DMs.

Step 2. Confirm the fuzzy measures of the attributes C_j (j=1,2,…,n) and the attribute sets of C. The λ-fuzzy measure is used to calculate the fuzzy measure of criteria sets. Firstly, according to Eq. (6), the value of λ is obtained, and then the fuzzy measure of criteria sets of $C=\{C_1,C_2,...,C_n\}$ are calculated by Eq. (6).

Step 3. Utilize the SVNCA (or SVNCG) operator to aggregate all assessment values $\tilde{r}_{ij}$ of the alternative A_i (i=1,2,…,m) under all attributes C_j (j=1,2,…,n) and get the overall assessment values $\tilde{r}_i$ of alternatives A_i (i=1,2,…,m) by Eq.(46) or (47).

$$\begin{aligned}\tilde{r}_i &= \langle T_i, I_i, F_i\rangle \\ &= \mathrm{SVNCA}_\mu(\tilde{r}_{i1},\tilde{r}_{i2},\cdots,\tilde{r}_{in}) \\ &= \left\langle 1-\prod_{j=1}^{n}\left(1-T_{i\sigma(j)}\right)^{\mu(H_{i\sigma(j)})-\mu(H_{i\sigma(j-1)})}, \prod_{j=1}^{n}\left(I_{i\sigma(j)}\right)^{\mu(H_{i\sigma(j)})-\mu(H_{i\sigma(j-1)})}, \prod_{j=1}^{n}\left(F_{i\sigma(j)}\right)^{\mu(H_{i\sigma(j)})-\mu(H_{i\sigma(j-1)})}\right\rangle, i=1,2,...,m,\end{aligned} \tag{46}$$

or

$$\begin{aligned}\tilde{r}_i &= \langle T_i, I_i, F_i\rangle \\ &= \mathrm{SVNCG}_\mu(\tilde{r}_{i1},\tilde{r}_{i2},\cdots,\tilde{r}_{in}) \\ &= \left\langle \prod_{j=1}^{n}\left(T_{i\sigma(j)}\right)^{\mu(H_{i\sigma(j)})-\mu(H_{i\sigma(j-1)})}, 1-\prod_{j=1}^{n}\left(1-I_{i\sigma(j)}\right)^{\mu(H_{i\sigma(j)})-\mu(H_{i\sigma(j-1)})}, 1-\prod_{j=1}^{n}\left(1-F_{i\sigma(j)}\right)^{\mu(H_{i\sigma(j)})-\mu(H_{i\sigma(j-1)})}\right\rangle, i=1,2,...,m,\end{aligned} \tag{47}$$

where $\tilde{r}_{i\sigma(j)}=\langle T_{i\sigma(j)},T_{i\sigma(j)},T_{i\sigma(j)}\rangle (j=1,2,...,n)$ is a permutation of $\tilde{r}_{ij}=\langle T_{ij},T_{ij},T_{ij}\rangle (j=1,2,...,n)$ such that

$\tilde{r}_{i\sigma(1)} \ge \tilde{r}_{i\sigma(2)} \ge \ldots \ge \tilde{r}_{i\sigma(n)}$, $x_{\sigma(i)}$ is the attribute corresponding to $\tilde{r}_{i\sigma(j)}$, $H_{i\sigma(j)}=\{x_{i\sigma(k)} | k \le j\}$, for $i \ge 1$, $H_{i\sigma(0)}=\varphi$.

Step 4. Calculate the score values $V(r_i)$ of the overall assessment values $\tilde{r}_i$ (i=1,2,…,m). The score values of the alternatives A_i (i=1,2,…,m) can be calculated by Eq. (48).

$$S(\tilde{r}_i)=(T_i+1-I_i+1-F_i)/3, i=1,2,...,m. \tag{48}$$

If there is no difference between two score values $S(\tilde{r}_i)$ and $S(\tilde{r}_l)$, then we need to calculate the accuracy values $V(\tilde{r}_i)$ and $V(\tilde{r}_l)$ of the alternatives A_i and A_l (i,l=1,2,…,m), respectively, according to Eq. (49).

$$V(\tilde{r}_i)=(T_i+F_i+1-I_i)/3, i=1,2,...,m. \tag{49}$$

Step 5. Rank all feasible alternatives A_i (i=1,2,…,m) according to Theorem 1 and select the most desirable alternative(s).

Step 6. End.

5. Numerical example

In this section, a material selection problem adopted from Venkata Rao [56] in which the alternatives are the material alternatives to be selected and the criteria are the attributes under consideration a MAGDM problem is used to illustrate the application of the proposed method with single valued neutrosophic information proposed in Section 4, and to demonstrate its feasibility and effectiveness in a realistic scenario. A company wants to select a suitable work material for a product operated in a high-temperature environment. After preliminary screening, there are four possible material alternatives A_1, A_2, A_3 and A_4 to be selected, according to the following four attributes: (1) C_1 is the tensile strength (MPa); (2) C_2 is the young's modulus (GPa); (3) C_3 is the density (gm/cm^3); (4) C_4 is the corrosion resistance. A committee of three decision makers D_k (k=1,2,3) whose weight vector is $\lambda = (0.34, 0.28, 0.38)^T$ is invited to evaluate the material alternatives A_i (i=1,2,3,4) with respect to the attributes C_j (j=1,2,3,4) and three individual single valued neutrosophic decision matrices $\tilde{R}_{ij}^{(k)} = (\tilde{r}_{ij}^{(k)})_{4\times4}$ (k=1,2,3) are constructed, which are as shown in Tables 1-3.

Table 1

Single valued neutrosophic decision matrix given by DM_1.

	C_1	C_2	C_3	C_4
A_1	<0.30, 0.40, 0.52>	<0.50, 0.65, 0.20>	<0.80, 0.24, 0.15>	<0.45, 0.32, 0.15>
A_2	<0.42, 0.90. 0.25>	<0.15, 0.45, 0.50>	<0.80, 0.21, 0.20>	<0.50, 0.36, 0.13>
A_3	<0.62, 0.34, 0.40>	<0.24, 0.22, 0.72>	<0.90, 0.35, 0.15>	<0.35, 0.40, 0.25>
A_4	<0.81, 0.23, 0.40>	<0.45, 0.42, 0.10>	<0.21, 0.52, 0.25>	<0.60, 0.40, 0.70>

Table 2

Single valued neutrosophic decision matrix given by DM_2.

	C_1	C_2	C_3	C_4
A_1	<0.57, 0.20, 0.41>	<0.25, 0.30, 0.40>	<0.35, 0.25, 0.10>	<0.75, 0.20, 0.10>
A_2	<0.67, 0.40, 0.20>	<0.40, 0.15, 0.10>	<0.28, 0.45, 0.50>	<0.50, 0.15, 0.35>
A_3	<0.45, 0.31, 0.32>	<0.70, 0.10, 0.05>	<0.55, 0.15, 0.35>	<0.53, 0.30, 0.20>
A_4	<0.45, 0.05, 0.30>	<0.70, 0.20, 0.15>	<0.90, 0.10, 0.35>	<0.52, 0.30, 0.25>

Table 3

Single valued neutrosophic decision matrix given by DM_3.

	C_1	C_2	C_3	C_4
A_1	<0.40, 0.15, 0.32>	<0.50, 0.12, 0.40>	<0.80, 0.12, 0.15>	<0.53, 0.20, 0.15>
A_2	<0.65, 0.30, 0.15>	<0.25, 0.43, 0.15>	<0.85, 0.10, 0.25>	<0.80, 0.10, 0.05>
A_3	<0.60, 0.32, 0.38>	<0.35, 0,25, 0.20>	<0.53, 0.20, 0.12>	<0.77, 0.30, 0.20>
A_4	<0.52, 0.20, 0.45>	<0.60, 0.24, 0.31>	<0.72, 0.05, 0.10>	<0.72, 0.13, 0,24>

In what follows, the proposed method with single valued neutrosophic information is utilized to get the most desirable material alternative(s), which involves the following steps:

Step 1. Utilize the individual single valued neutrosophic decision matrix $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{4\times4} (k = 1, 2, 3)$ and the SVNWA operator to derive the collective single valued neutrosophic decision matrix $\tilde{R}=(\tilde{r}_{ij})_{4\times4}$ by Eq.(44), which is shown in Table 4.

Table 4

Collective single valued neutrosophic decision matrix by using the SVNWA operator.

	C_1	C_2	C_3	C_4
A_1	<0.424, 0.227, 0.405>	<0.440, 0.276, 0.316>	<0.722, 0.187, 0.134>	<0.585, 0.235, 0.134>
A_2	<0.591, 0.472, 0.193>	<0.265, 0.325, 0.202>	<0.743, 0.196, 0.281>	<0.647, 0.173, 0.119>
A_3	<0.570, 0.324, 0.369>	<0.448, 0.185,0.210>	<0.726, 0.223, 0.175>	<0.600, 0.331, 0.330>
A_4	<0.636, 0.142, 0.386>	<0.589, 0.276, 0.172>	<0.701, 0.135, 0.194>	<0.632, 0.241, 0.349>

Step 2. Suppose that the fuzzy measures of criteria of C are given as follows:

$$\mu(C_1)=0.2,\quad \mu(C_2)=0.3,\quad \mu(C_3)=0.2,\quad \mu(C_4)=0.35$$

The λ-fuzzy measure is used to calculate the fuzzy measure of criteria sets. Firstly, according to Eq. (6), the value of λ is calculated: $\lambda=-0.2330$, and then the fuzzy measure of criteria sets of $C=\{C_1,C_2,C_3,C_4\}$ are calculated by Eq. (6), which are shown as follows:

$$\mu(C_1,C_2)=0.4860,\quad \mu(C_1,C_3)=0.4384,\quad \mu(C_1,C_4)=0.5337,\quad \mu(C_2,C_3)=0.5325,$$

$$\mu(C_2,C_4)=0.6255,\ \mu(C_3,C_4)=0.5796,\quad \mu(C_1,C_2,C_3)=0.7077,\ \mu(C_1,C_2,C_4)=0.7964,$$

$$\mu(C_1,C_3,C_4)=0.7526,\quad \mu(C_2,C_3,C_4)=0.8391,\quad \mu(C_1,C_2,C_3,C_4)=1.$$

Step 3. Utilize the SVNCA operator to calculate the overall assessments of each material alternative A_i. Take A_1 for an example: according to Theorem 1, we have

$$\tilde{r}_{1\sigma(1)}=<0.722,\ 0.187,\ 0.134>,\ \tilde{r}_{1\sigma(2)}=<0.585,\ 0.235,\ 0.134>,$$

$$\tilde{r}_{1\sigma(3)}=<0.440,\ 0.276,\ 0.316>,\quad \tilde{r}_{1\sigma(4)}=<0.424,\ 0.227,\ 0.405>.$$

Then, the overall assessments of the material alternative A_1 can be calculated as follows:

$$\begin{aligned}\tilde{r}_1 &= \mathrm{SVNCA}_\mu\left(\tilde{r}_{11},\tilde{r}_{12},\tilde{r}_{13},\tilde{r}_{14}\right)\\ &=\langle 1-(1-0.722)^{0.2-0}\times(1-0.585)^{0.5796-0.2}\times(1-0.440)^{0.8391-0.5796}\times(1-0.424)^{1-0.8391},\\ &\quad 0.187^{0.2-0}\times0.235^{0.5796-0.2}\times0.276^{0.8391-0.5796}\times0.227^{1-0.8391},0.134^{0.2-0}\times0.134^{0.5796-0.2}\times0.316^{0.8391-0.5796}\times0.405^{1-0.8391}\rangle\\ &=\langle 0.564,0.233,0.200\rangle\end{aligned}$$

Similarly,

$$\tilde{r}_2=(0.596,\ 0.248,\ 0.180),\quad \tilde{r}_3=(0.592,\ 0.262,\ 0.224),\quad \tilde{r}_4=(0.635,\ 0.205,\ 0.250).$$

Step 4. Calculate the score values $S(\tilde{r}_i)$ of the overall assessment values $\tilde{r}_i$ (i=1,2,3,4). According to Eq. (48), the score values of material alternatives A_i (i=1,2,3,4) are obtained as follows:

$$S(\tilde{r}_1)=0.7103,\quad S(\tilde{r}_2)=0.7231,\quad S(\tilde{r}_3)=0.7020,\quad S(\tilde{r}_4)=0.7267.$$

Step 5. Rank all material alternatives A_i (i=1,2,3,4) according to the descending order of corresponding score values $S(\tilde{r}_i)$ (i=1,2,3,4) and select the most desirable material alternative(s).

Since $S(\tilde{r}_4) > S(\tilde{r}_2) > S(\tilde{r}_1) > S(\tilde{r}_3)$, then the ranking of all material alternatives A_i (i=1,2,3,4) is shown as follows:

$$A_4 \succ A_2 \succ A_1 \succ A_3$$

where the symbol "$\succ$" means "superior to". Therefore, the most desirable material alternative is A_4.

6. Conclusions

In this paper, we study the material selection problems in which the attribute values take the form of single valued neutrosophic numbers. Motivated by the idea of Choquet integral, two correlated aggregation operators are proposed for aggregating the single valued neutrosophic information based on the operational laws of single valued neutrosophic numbers, such as the single valued neutrosophic correlated average (SVNCA) operator and the single valued neutrosophic correlated geometric (SVNCG) operator. The prominent characteristic of these operators is that the truth degree, indeterminacy degree and falsity degree of an element to a given set are denoted by a set of three crisp numbers. Then, some desirable properties of the proposed operators and the relationships among them are investigated in detail. Furthermore, based on the proposed operators, a novel multiple attribute group decision making method is developed to solve material selection problems under single valued neutrosophic environment, in which the attributes are often inter-dependent or correlated. Finally, a numerical example of material selection is given to illustrate the application of the proposed method. In future research, we will focus on the application of the proposed method in other real decision making problems, such as personnel evaluation and emergency management.

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