# Comment on "*The generalized Newton's law of gravitation*"

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**Abstract:** In a relatively recent article by Arbab I. Arbab (Astrophys. Space Sci. **325**, 37, 2010), the author claimed the generalization of Newton's law of gravitation *via* the introduction of the gravitomagnetic field. However, the present comment proves more conclusively that the proposed *formulae* and the so-called *generalized Newton's law of gravitation* are physico-mathematically erroneous. Consequently, the alleged generalization cannot be a law of physics at all.

**Keywords:** generalized Newton's law of gravitation, gravitomagnetism

## 1. Introduction

In his original article entitled '*The generalized Newton's law of gravitation*' [1], Arbab I. Arbab claimed the generalization of Newton's law of gravitation by means of the introduction of the gravitomagnetic field; and step by step, he derived five formulae, namely, (7), (8), (9), (10) and (11) in Ref.[1], which are explicitly or implicitly supposed by the author as a generalization of the classical (Newtonian) gravitational physics. The main reason that eventually led the author to the erroneous formulae has been identified – it is the deliberate confusion between gravitation and electromagnetism *through* the superfluous idea of gravitomagnetism.

Before tackling the paper under discussion, it is judged important to recall the following epistemological considerations. As we know, epistemologically speaking, any scientific theory should be characterized by its own *strong points* and *weak points* in its proper area of applications both theoretically and practically. However, if the weak points are largely exceeded the strong points in such a situation the whole theory should be radically revised or rejected. In view of the fact that the notion of *generalization* is one of the corner-stones of the whole edifice of the paper in question, hence in order to understand this notion correctly, let $S = \{L_1, L_2, L_3 \ldots L_n\}$ to be a set of physical generalized laws established to study a set of physical phenomena $A = \{P_1, P_2, P_3 \ldots P_n\}$ and $s = \{\ell_1, \ell_2, \ell_3 \ldots \ell_n\}$ to be a set of non-generalized laws of physics, which are established to study a certain subset of physical phenomena $B = \{P_1, P_2, P_3 \ldots P_i\}$ with $i \le n$. So, it is clear that phenomenologically $s \subset S$ if $\ell_i$ is a special case of $L_i$ or equivalently if $L_i$ is a generalization of $\ell_i$ and $B \subseteq A$ if $\mathrm{card}(B) \le \mathrm{card}(A)$. Therefore, according to these axioms, if $L_i$ is conceptually and phenomenological a generalization of $\ell_i$ which is destined to study the physical phenomenon $P_i$, this fact should imply, among other things, that $L_i$ is reducible to $\ell_i$ under certain conditions. Thus, if $\ell_i$ represents the Newton's law of universal gravitation in the vector form

$$\mathbf{F} = -\frac{k}{r^3}\mathbf{r}, \quad k = GMm, \tag{1.1}$$

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then, this deliberation implies that the generalized law $L_i$ should be reducible to (1.1),*i.e.*, phenomenologically (1.1) is a special case of $L_i$. However, as we shall see soon, in the aforementioned paper, there is no any such general law $L_i$ satisfying the above requirements.

## 2. Proofs of fatal errors

Now, we arrive at our main subject namely the scrutiny of the paper under consideration *'The generalized Newton's law of gravitation'* [1]. Recall that our first major objection is that the author failed to derive the correct formulae supposed to be a generalization of classical (Newtonian) gravitational physics. To be more objective and credible ,we split the proofs of fatal errors in two parts: mathematical proof and physical proof.

In order to make our scrutiny more comprehensible, we are obliged to rewrite the author's central claims, word by word. In his Section2 (page38) entitled "The gravitomagnetic force and the modified Newton's law of gravitation", the author wrote « ... *The gravitational Lorentz force takes the general form (Arbab 2009b)*

$$\vec{F}_{gm} = m\left(\vec{a} + \vec{v} \times \vec{B}_g\right) = \vec{F}_g + \vec{F}_m , \tag{4}$$

*where* $\vec{E}_g = \vec{a}$ *is the gravitational field. Using the vector identity,* $\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{B}\left(\vec{A}\cdot\vec{C}\right) - \vec{C}\left(\vec{A}\cdot\vec{B}\right)$, (2) *and the fact that* $\left(\vec{a}\cdot\vec{v}\right) = 0$, (4) *can be written as*

$$F_{gm} = \frac{GmM}{r^2} - \frac{mv^4}{c^2 r} . \tag{5}$$

*This shows that the force is not central, but depends on the object velocity as well. This is the modified Newton's law of gravitation. It must be applied when we study the motion of all gravitating objects. For circular motion one has*

$$\frac{mv^2}{r} = \frac{GmM}{r^2} - \frac{mv^4}{c^2 r} , \tag{6}$$

*which can be solved to find the velocity of the object in terms of its orbital distance. To this end, one has*

$$v^2 = \frac{c^2}{2}\left[-1 + \sqrt{1 + \frac{4GM}{rc^2}}\right]. \tag{7}$$

*Substituting* (7) *in* (5) *yields the generalized gravitational force on the mass, viz.,*

$$F_{gm} = -\frac{mc^2}{2}\left[\frac{1}{r} - \frac{1}{r}\sqrt{1 + \frac{4GM}{rc^2}}\right]. \tag{8}$$

*This is the generalized Newton's law of gravitation that should be used in studying any gravitational interaction of gravitating bodies. This force can be associated with a central potential energy* $\left(U_{gm}\right)$ *of the form*

$$\vec{F}_{gm} = -\vec{\nabla} U_{gm}, \tag{9}$$

*which suggests that (see the Appendix)*

$$U_{gm} = -\frac{mc^2}{2}\left(\ln r + 2\sqrt{1+\frac{4GM}{rc^2}} + \ln\frac{\sqrt{1+\frac{4GM}{rc^2}}-1}{\sqrt{1+\frac{4GM}{rc^2}}+1}\right). \tag{10}$$

*This is an effective potential describing the motion of the gravitating object. This support the assertion made by Wild (1996) that a central gravitational field can equally well be described by a modified Newtonian theory as by general relativity theory. Such a modification will satisfy the critical tests of general relativity. The curvature of space is a consequence of the force field and the Newton's equation determines this field. Hence, the two approaches are compliment to each other.*

*Employing* (6) *and* (7) *the gravitomagnetic force is given by*

$$F_m = -\frac{mc^2}{2r}\left[1+\frac{2GM}{rc^2}-\sqrt{1+\frac{4GM}{rc^2}}\right]. \tag{11}$$

*The gravitomagnetic force vanishes for first and second order terms in* $1/r$. *We remark here that this potential energy is not a correction to the Newtonian potential energy but reduces to it in some particular case. It is evident from* (6) *that the gravitomagnetic force is opposite (repulsive force) to gravitational force. This equation is found to give the correct advance of perihelion of planets and binary pulsars (Arbab 2009a).»*

## 2.1. Mathematical Proof of fatal errors

The author supposed his derived formulae (7), (8), (9), (10) and (11) a generalization of classical (Newtonian) gravitational physics. At first glance, these formulae seemed legitimate, but on closer inspection we shall find that these formulae are fatally incorrect. Our scrutiny reveals the presence of the same dimensionless quantity $\sqrt{1+\frac{4GM}{rc^2}}$ in each formula. Since, according to the author, the derived formulae should be used in studying any gravitational interaction of gravitating bodies and describing the motion of the gravitating objects, therefore the dimensionless quantity $\left(4GM/rc^2\right)$ should be always extremely small compared to unity and practically speaking may be completely neglected/omitted. For example, for the system {Earth, Moon} we have $\left(4GM_{\mathrm{E}}/rc^2\right)=4.617\times10^{-11}$ and for the system {Sun, Earth} we have $\left(4GM_{\mathrm{S}}/rc^2\right)=3.95\times10^{-8}$ . Thus, mathematically, when $\left(4GM/rc^2\right)<<1$ we get

$$\sqrt{1+\frac{4GM}{rc^2}} \cong 1, \tag{2.2}$$

and after a direct substitution in each formula, we find:

formula (7): $$v^2 = \frac{c^2}{2}\left[-1+1\right]=0,$$

formula (8): $$F_{gm} = -\frac{mc^2}{2}\left[\frac{1}{r} - \frac{1}{r}\right] = 0,$$

formula (10): $$U_{gm} = -\frac{mc^2}{2}\left(\ln r + 2 + \ln\frac{1-1}{1+1}\right),$$

and for the case $\sqrt{1+\frac{4GM}{rc^2}} \cong 1+\frac{2GM}{rc^2}$, the formula (11) becomes:

$$F_m = -\frac{mc^2}{2r}\left[1+\frac{2GM}{rc^2} - \left(1+\frac{2GM}{rc^2}\right)\right] = 0$$

It is completely clear from the above proof, no supplementary words are needed to expose such fatal errors.

## 2.2. Physical Proof of fatal errors

Now, physically, let us focus our attention on the formulae (7) and (8) and show that contrary to the author's claim, in the Section 2, formula (7) not only cannot regard as a generalization of the usual formula, *viz*,

$$v = \sqrt{GM/r}\,, \tag{3.3}$$

but also is merely wrong! To this end, let us apply the formula (7) to the system {Sun, Earth}. As we know, the Earth's orbit is almost circular since its eccentricity is $e = 0.0167$, and we have for its average orbital velocity $v \cong 3\times10^4\ \mathrm{ms}^{-1}$ and semi-major axis $r = 149.597870\times10^9\ \mathrm{m}$; the Sun's mass $M = 1.9891\times10^{30}\ \mathrm{kg}$; Newton's gravitational constant $G = 6.67384\times10^{-11}\ \mathrm{m}^3\,\mathrm{kg}^{-1}\mathrm{s}^{-2}$ and light speed in vacuum $c = 299792458\ \mathrm{ms}^{-1}$. By putting the above numerical values in the usual (classical) formula, we obtain

$$v = \sqrt{GM/r} = 2.98\times10^4\ \mathrm{ms}^{-1}. \tag{4.4}$$

Now, substituting the same numerical values in addition to the light speed in the formula (7), we find

$$v = c\sqrt{\frac{1}{2}\left[-1+\sqrt{1+\frac{4GM}{rc^2}}\right]} = c\sqrt{\frac{1}{2}\left[-1+\sqrt{1+3.95\times10^{-8}}\right]} = 0\ \mathrm{ms}^{-1}. \tag{5.5}$$

In the same Section 2, formula (8) is considered by the author as a generalization of the Newton’s law of gravitation *"that should be used in studying any gravitational interaction of gravitating bodies"*.

Firstly, the magnitude (modulus) of the gravitational force should be written without minus sign (–) that's why the modulus (magnitude) of (1.1) is:

$$F = \frac{k}{r^2}, \quad k = GMm. \tag{6.6}$$

Exactly like before, formula (8) cannot be considered as a generalization of Newton’s law of gravitation , *viz.*, formula (6.6) simply because formula (8) is completely wrong! Finally, let us calculate the intensity of gravitational force exerted by the Sun on the Earth during its orbital motion. So, in addition to the above orbital and physical quantities, we have for the Earth's mass $m = 5.9722 \times 10^{24}$ kg . Now, after substitution in (6.6), we get

$$F = 3.54255 \times 10^{22} \text{ N}. \tag{7.7}$$

Direct substitution in (8), yields

$$F_{gm} = -\frac{mc^2}{2}\left[\frac{1}{r} - \frac{1}{r}\sqrt{1+3.95\times 10^{-8}}\right] = 0 \text{ N}. \tag{8.8}$$

Again, formula (8) is conceptually, phenomenologically and physically *incorrect* and consequently cannot be a generalization of Eq.(6.6).

## 3. Conclusion

Without entering in full detail, *Gravitational Physics* is extremely hard and highly complicated domain, it is radically different from other physical areas, particularly, electromagnetism. For instance, up till now we ignore the real value of propagation speed of gravity; general relativity theory (GRT) did not consider gravity as a force of Nature but interpreted its manifestation as the curvature of space-time due to the local presence of heavy material body; and curiously GRT may be reduced to the Newton's gravity theory for low velocities and weak fields! –All that means that the *gravity* itself is only approximately understood.

In this comment, we have scrutinized the paper *'The generalized Newton's law of gravitation'* [1] and proved that this paper is physico-mathematically incorrect. The paper contains fatal errors. Consequently, the so-called *'The generalized Newton's law of gravitation'* and its extension cannot be considered as an intellectual and scientific contribution to the science in general and to the gravitational physics in particular as the paper is exceedingly questionable.

## Reference

[1] Arbab I. Arbab, Astrophys. Space Sci. **325**, 37 (2010).