GAUGE TRANSFORMATIONS FOR SELF/ANTI-SELF CHARGE CONJUGATE STATES

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(Received July 2, 1997)

Gauge transformations of type-II spinors are considered in the Majorana–Ahluwalia construct for self/anti-self charge conjugate states. Some speculations on the relations of this model with the earlier ones are given.

PACS numbers: 11.30. Er, 12.10. Dm, 12.60. -i, 14.60. St

Recently, new constructs in the $(1/2,0)\oplus(0,1/2)$ representation of the Lorentz group have been proposed [1–5]. One of their surprising features is the fact that dynamical equations in these formalisms take eight-component form. As shown in Refs [1, 4, 6] the Majorana–McLennan-Case construct for self/anti-self charge conjugate states leads to the equations

$$i\gamma^\mu\partial_\mu\lambda^{\rm S}(x) - m\rho^{\rm A}(x) \;=\; 0\,, \tag{1a}$$
$$i\gamma^\mu\partial_\mu\rho^{\rm A}(x) - m\lambda^{\rm S}(x) \;=\; 0\,, \tag{1b}$$

and

$$i\gamma^\mu\partial_\mu\lambda^{\rm A}(x) + m\rho^{\rm S}(x) \;=\; 0\,, \tag{2a}$$
$$i\gamma^\mu\partial_\mu\rho^{\rm S}(x) + m\lambda^{\rm A}(x) \;=\; 0\,. \tag{2b}$$

They can be written in the 8-component form as follows:

$$\left[i\varGamma^\mu\partial_\mu - m\right]\varPsi_{(+)}(x) \;=\; 0\,, \tag{3a}$$
$$\left[i\varGamma^\mu\partial_\mu + m\right]\varPsi_{(-)}(x) \;=\; 0\,, \tag{3b}$$

where

$$\varPsi_{(+)}(x) = \begin{pmatrix}\rho^{\rm A}(x)\\ \lambda^{\rm S}(x)\end{pmatrix}\,,\quad \varPsi_{(-)}(x) = \begin{pmatrix}\rho^{\rm S}(x)\\ \lambda^{\rm A}(x)\end{pmatrix}\,, \tag{4}$$

with $\lambda^{\rm S,A}(p^\mu)$, $\rho^{\rm S,A}(p^\mu)$ being the self/anti-self charge conjugate spinors in the momentum representation, which are defined in Ref. [1]. The interpretation of $\lambda^{\rm S}$ and $\rho^{\rm A}$ corresponding to positive-energy solutions and $\lambda^{\rm A}$, $\rho^{\rm S}$, to negative-energy solutions, has been used[1]. After writing those papers we became aware of similar problems which have been studied in the old papers [7–10] from various viewpoints. A group-theoretical basis for such constructs has been proposed by Bargmann, Wightman and Wigner [11].

Let us consider the question of gauge transformations for this kind of states. First of all, the possibility of the γ^5 phase transformations has been noted in [6]. The Lagrangian [6, Eq.(24)], which (like in the Dirac construct) is equal to zero on the solutions of the dynamical equations[2],

$$\begin{aligned}\mathcal{L} &= \frac{i}{2}\Big[\overline{\lambda^{\rm S}}\gamma^\mu\partial_\mu\lambda^{\rm S} - (\partial^\mu\overline{\lambda^{\rm S}})\gamma^\mu\lambda^{\rm S} + \overline{\rho^{\rm A}}\gamma^\mu\partial_\mu\rho^{\rm A} - (\partial^\mu\overline{\rho^{\rm A}})\gamma^\mu\rho^{\rm A}\\ &+ \overline{\lambda^{\rm A}}\gamma^\mu\partial_\mu\lambda^{\rm A} - (\partial^\mu\overline{\lambda^{\rm A}})\gamma^\mu\lambda^{\rm A} + \overline{\rho^{\rm S}}\gamma^\mu\partial_\mu\rho^{\rm S} - (\partial^\mu\overline{\rho^{\rm S}})\gamma^\mu\rho^{\rm S}\Big]\\ &- m\Big[\overline{\lambda^{\rm S}}\rho^{\rm A} + \overline{\rho^{\rm A}}\lambda^{\rm S} - \overline{\lambda^{\rm A}}\rho^{\rm S} - \overline{\rho^{\rm S}}\lambda^{\rm A}\Big]\end{aligned} \tag{5}$$

is invariant with respect to the phase transformations:

$$\lambda^\prime(x) \rightarrow (\cos\alpha - i\gamma^5\sin\alpha)\lambda(x)\,, \tag{6a}$$

$$\overline{\lambda}^\prime(x) \rightarrow \overline{\lambda}(x)(\cos\alpha - i\gamma^5\sin\alpha)\,, \tag{6b}$$

$$\rho^\prime(x) \rightarrow (\cos\alpha + i\gamma^5\sin\alpha)\rho(x)\,, \tag{6c}$$

$$\overline{\rho}^\prime(x) \rightarrow \overline{\rho}(x)(\cos\alpha + i\gamma^5\sin\alpha)\,. \tag{6d}$$

Obviously, the 4-spinors $\lambda^{\rm S,A}(p^\mu)$ and $\rho^{\rm S,A}(p^\mu)$ remain in the space of self/anti-self charge conjugate states[3]. In terms of the field functions $\Psi_{(\pm)}(x)$ the transformation formulas recast as follows ($\text{Ł}^5 = \text{diag}\,(\gamma^5 \; -\gamma^5)$)

$$\Psi^\prime_{(\pm)}(x) \rightarrow \left(\cos\alpha + i\text{Ł}^5\sin\alpha\right)\Psi_{(\pm)}(x)\,, \tag{7a}$$

$$\overline{\Psi}^\prime_{(\pm)}(x) \rightarrow \overline{\Psi}_{(\pm)}(x)\left(\cos\alpha - i\text{Ł}^5\sin\alpha\right)\,. \tag{7b}$$

It is well known that the Dirac theory for charged spin-1/2 particles does *not admit* conventional chiral transformations. In the meantime, as mentioned

[1] Let me remind that the sign of the phase in the field operator is considered to be invariant if we restrict ourselves by the proper orthochronous Poincaré group. This fact has also been used at the stage of writing the dynamical equations (1a, 1b, 2a, 2b).

[2] The overline implies the Dirac conjugation.

[3] Usual phase transformations like that applied to the Dirac field will destroy self/anti-self charge conjugacy. The origin lies in the fact that the charge conjugation operator is *not* a linear operator and it includes the operation of complex conjugation.

by Das and Hott [12] "an interacting fermion theory at high temperature develops a temperature dependent fermion mass where mass grows with temperature; ... it would appear that a massless, chiral invariant theory would have its chiral symmetry broken by the temperature dependent mass [13]; ... [on the other hand,] one conventionally believes that the dynamically broken chiral symmetry in QCD is restored beyond a critical temperature." Furthermore, they investigated this "apparent conflict" and proposed m-deformed *non-local* chiral transformations. Nevertheless, they indicated at the importance of further study of chiral transformations and their relevance to the modern physics. Thus, these matters appear to be of use not only from a viewpoint of constructing the fundamental theory for neutral particles, but regarding the constructs which *admit* the chiral invariance may also be useful for understanding the processes in QCD and other modern gauge models.

So, let us proceed further with the local gradient transformations (gauge transformations) in the Majorana–Ahluwalia construct. When we are interested in them one must introduce the compensating field of the vector potential

$$\partial_\mu \to \nabla_\mu = \partial_\mu - ig\text{Ł}^5 A_\mu \,, \tag{8a}$$

$$A'_\mu(x) \to A_\mu(x) + \frac{1}{g}\,\partial_\mu \alpha \,. \tag{8b}$$

Therefore, equations describing interactions of the λ^S and ρ^A with 4-vector potential are the following

$$i\gamma^\mu \partial_\mu \lambda^{\mathrm{S}}(x) - g\gamma^\mu\gamma^5 A_\mu \lambda^{\mathrm{S}}(x) - m\rho^{\mathrm{A}}(x) \;=\; 0\,, \tag{9a}$$

$$i\gamma^\mu \partial_\mu \rho^{\mathrm{A}}(x) + g\gamma^\mu\gamma^5 A_\mu \rho^{\mathrm{A}}(x) - m\lambda^{\mathrm{S}}(x) \;=\; 0\,. \tag{9b}$$

The second-order equations follow immediately from the set (9a), (9b)

$$\left\{\left(i\widehat{\partial} + g\widehat{A}\gamma^5\right)\left(i\widehat{\partial} - g\widehat{A}\gamma^5\right) - m^2\right\}\lambda^{\mathrm{S}}(x) \;=\; 0\,, \tag{10a}$$

$$\left\{\left(i\widehat{\partial} - g\widehat{A}\gamma^5\right)\left(i\widehat{\partial} + g\widehat{A}\gamma^5\right) - m^2\right\}\rho^{\mathrm{A}}(x) \;=\; 0\,, \tag{10b}$$

with the notation being used: $\widehat{a} \equiv \gamma^\mu a_\mu = \gamma^0 a^0 - (\boldsymbol{\gamma}\cdot\boldsymbol{a})$. After algebraic transformations in the spirit of [14, 15] one obtains

$$\left\{\mathit{\Pi}_\mu^+ \mathit{\Pi}^{\mu\,+} - m^2 - \frac{g}{2}\gamma^5 \Sigma^{\mu\nu} F_{\mu\nu}\right\}\lambda^{\mathrm{S}}(x) \;=\; 0\,, \tag{11a}$$

$$\left\{\mathit{\Pi}_\mu^- \mathit{\Pi}^{\mu\,-} - m^2 + \frac{g}{2}\gamma^5 \Sigma^{\mu\nu} F_{\mu\nu}\right\}\rho^{\mathrm{A}}(x) \;=\; 0\,, \tag{11b}$$

where the "covariant derivative" operators acting in the $(1/2,0)\oplus(0,1/2)$ representation are defined as

$$\mathit{\Pi}_\mu^\pm = \frac{1}{i}\partial_\mu \pm g\gamma^5 A_\mu\,, \tag{12}$$

and

$$\Sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu\,,\,\gamma^\nu\right]_-\,. \tag{13}$$

The case of $\lambda^{\rm A}$ and $\rho^{\rm S}$ is very similar and we shall give below the final result only:

$$\left\{\mathit{\Pi}_\mu^+\mathit{\Pi}^{\mu\,+} - m^2 - \frac{g}{2}\gamma^5\Sigma^{\mu\nu}F_{\mu\nu}\right\}\lambda^{\rm A}(x) \;=\; 0\,, \tag{14a}$$

$$\left\{\mathit{\Pi}_\mu^-\mathit{\Pi}^{\mu\,-} - m^2 + \frac{g}{2}\gamma^5\Sigma^{\mu\nu}F_{\mu\nu}\right\}\rho^{\rm S}(x) \;=\; 0\,. \tag{14b}$$

Thus, the equations for the particles described by the field operator (Eq. (46) in [1c])

$$\begin{aligned}\nu^{DL}(x) \;\equiv\; \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\,\frac{1}{2p_0}\sum_\eta \Big[\lambda^{\rm S}_\eta(p^\mu)a_\eta(p^\mu)\exp(-ip\cdot x) \\ +\lambda^{\rm A}_\eta(p^\mu)b^\dagger_\eta(p^\mu)\exp(+ip\cdot x)\Big]\,,\end{aligned} \tag{15}$$

which interact with the 4-vector potential, have the same form for positive- and negative-energy parts. The same is true in the case of the use of the field operator composed from $\rho^{\rm A}$ and $\rho^{\rm S}$. One can see the difference with the Dirac case, namely, the presence of γ^5 matrix in the "Pauli term" and in the lengthening derivatives. Next, we are able to decouple the set (11a), (11b), (14a), (14b) for the up- and down-components of the bispinors in the coordinate representation. For instance, the up- and the down-parts of the $\nu^{DL}(x) = \mathrm{column}(\chi\;\phi)$ interact with the vector potential in the following manner:

$$\left\{\begin{array}{l}\left[\pi_\mu^-\pi^{\mu\,-} - m^2 - \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}\right]\chi(x) = 0\,,\\ \left[\pi_\mu^+\pi^{\mu\,+} - m^2 + \frac{g}{2}\widetilde{\sigma}^{\mu\nu}F_{\mu\nu}\right]\phi(x) = 0\,,\end{array}\right. \tag{16}$$

where already one has $\pi_\mu^\pm = i\partial_\mu \pm gA_\mu$, $\sigma^{0i} = -\widetilde{\sigma}^{0i} = i\sigma^i$, $\sigma^{ij} = \widetilde{\sigma}^{ij} = \varepsilon_{ijk}\sigma^k$. Of course, introducing the operator composed of the ρ states one can write corresponding equations for its up- and down- components and, hence, restore the Feynman–Gell–Mann equation [16, Eq.(3)] and its charge conjugate ($g=-e$; A^μ and $F^{\mu\nu}$ are assumed to be real fields). In fact, this way would lead us to the consideration which is identical to the recent papers [3]. It was based on the linearization procedure for 2-spinors, which is similar to that used by Feshbach and Villars [17] in order to deduce the Hamiltonian form of the Klein–Gordon equation. Some insights in the interaction issues with the 4-vector potential in the eight-component equation have been made there: for instance, while explicit form of the wave functions slightly differ from the Dirac case, the hydrogen atom spectrum is the same to that in the usual Dirac theory [14, p.66, 74-75]. Next, like in the paper [18] the equations of [3] presume a non-CP-violating[4] electric dipole moment of the corresponding states.

We are also interested in finding other forms of gauge interactions for spinors of the $(1/2,0)\oplus(0,1/2)$ representation. Indeed, one can propose other kinds of phase transformations and, hence, other compensating fields for fermion functions composed of $\lambda^{\rm S,A}(p^\mu)$ and $\rho^{\rm S,A}(p^\mu)$ spinors. First of all, one may wish to introduce the 2×2 matrix Ξ, which is defined

$$\Xi = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}\,, \tag{17}$$

where ϕ is the azimuthal angle associated with $\boldsymbol{p}\to\boldsymbol{0}$. This matrix has been used in the generalized Ryder–Burgard relation connecting 2-spinor and its complex conjugate in the zero-momentum frame (Eq. (26), (27) of [1c]). Using the relation $\Xi\Lambda_{R,L}(\overset{\circ}{p}{}^\mu\leftarrow p^\mu)\Xi^{-1}=\Lambda^*_{R,L}(\overset{\circ}{p}{}^\mu\leftarrow p^\mu)$ it is easy to check that under the phase transformations

$$\lambda'_S(p^\mu) = \begin{pmatrix} \Xi & 0 \\ 0 & \Xi \end{pmatrix}\lambda_S(p^\mu) \equiv \lambda^*_A(p^\mu)\,, \tag{18a}$$

$$\lambda''_S(p^\mu) = \begin{pmatrix} 0 & i\Xi \\ i\Xi & 0 \end{pmatrix}\lambda_S(p^\mu) \equiv i\gamma^0\lambda^*_A(p^\mu)\,, \tag{18b}$$

$$\lambda'''_S(p^\mu) = \begin{pmatrix} 0 & \Xi \\ -\Xi & 0 \end{pmatrix}\lambda_S(p^\mu) \equiv \gamma^0\lambda^*_S(p^\mu)\,, \tag{18c}$$

$$\lambda^{IV}_S(p^\mu) = \begin{pmatrix} i\Xi & 0 \\ 0 & -i\Xi \end{pmatrix}\lambda_S(p^\mu) \equiv -i\lambda^*_S(p^\mu) \tag{18d}$$

bispinors remain in the self charge conjugate space. Analogous relations for λ_A:

$$\lambda'_A(p^\mu) = \begin{pmatrix} \Xi & 0 \\ 0 & \Xi \end{pmatrix}\lambda_A(p^\mu) \equiv \lambda^*_S(p^\mu)\,, \tag{19a}$$

$$\lambda''_A(p^\mu) = \begin{pmatrix} 0 & i\Xi \\ i\Xi & 0 \end{pmatrix}\lambda_A(p^\mu) \equiv i\gamma^0\lambda^*_S(p^\mu)\,, \tag{19b}$$

[4] This is possible due to the Wigner "doubling" of the components of the wave function.

$$\lambda_A^{'''}(p^\mu) = \begin{pmatrix} 0 & \Xi \\ -\Xi & 0 \end{pmatrix} \lambda_A(p^\mu) \equiv \gamma^0 \lambda_A^*(p^\mu)\,, \tag{19c}$$

$$\lambda_A^{IV}(p^\mu) = \begin{pmatrix} i\Xi & 0 \\ 0 & -i\Xi \end{pmatrix} \lambda_A(p^\mu) \equiv -i\lambda_A^*(p^\mu)\,, \tag{19d}$$

ensure that the latter retain their property to be in the anti-self charge conjugate space under this kind of transformations[5]. Thus, the Majorana-like field operator ($b^\dagger \equiv a^\dagger$) admits additional phase (and, in general, normalization) transformations, namely

$$\nu^{ML\,\prime}(x) = [c_0 + i(\boldsymbol{\tau}\cdot\boldsymbol{c})]\nu^{ML\,\dagger}(x)\,, \tag{20}$$

where c_α are arbitrary parameters in the superpositions of the self/anti-self charge conjugate states; the $\boldsymbol{\tau}$ matrices are defined over the field of 2×2 matrices[6]; and the Hermitian conjugation operation is implied over the field of the q-numbers, *i.e.* it acts on the c-numbers as the complex conjugation. If we want to keep the normalization of the wave functions one can make parametrization of the c_α factors in (20) as follows: $c_0 = \cos\phi$ and $\boldsymbol{c} = \boldsymbol{n}\sin\phi$ leaving only three parameters independent. This induces speculations that the SU(2)× U(1) theory can be constructed on the basis of the Weyl 2-spinors. This is not surprising, because these groups are the subgroups of the extended Poincaré group. But, of course, in order to ensure this purpose one should consider the question of invariance of some Lagrangian, which involves λ and ρ fields (*e.g.*, Eq. (24) in Ref. [6]), with respect to these transformations.

Several forms of field operators were defined in Ref. [1]; one may be interested in the one composed of $\lambda^{\rm S,A}$ spinors, and in the second one, of $\rho^{\rm S,A}$ spinors. Due to the identities (see Eqs. (6a), (6b) in Ref. [19] and [1])

$$\rho_\uparrow^{\rm S}(p^\mu) = -i\lambda_\downarrow^{\rm A}(p^\mu)\,, \quad \rho_\downarrow^{\rm S}(p^\mu) = +i\lambda_\uparrow^{\rm A}(p^\mu)\,, \tag{21a}$$

$$\rho_\uparrow^{\rm A}(p^\mu) = +i\lambda_\downarrow^{\rm S}(p^\mu)\,, \quad \rho_\downarrow^{\rm A}(p^\mu) = -i\lambda_\uparrow^{\rm S}(p^\mu)\,, \tag{21b}$$

which permits one to keep the parity invariance of the theory, we can express the ρ operator in the form:

$$\rho(x^\mu) \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2p_0}\sum_\eta [\rho_\eta^{\rm A}(p^\mu)c_\eta(p^\mu)\exp(-ip\cdot x) + \rho_\eta^{\rm S}(p^\mu)d_\eta^\dagger(p^\mu)\exp(+ip\cdot x)] = \gamma^0\nu^{DL}(x^{\mu\prime})\,. \tag{22}$$

[5] One should still note that in the meaning presented here, the γ^5 transformations ($\lambda^{\rm S}(p^\mu) \leftrightarrow \pm i\lambda^{\rm A}(p^\mu)$, see above) are also transformations with a unitary matrix and also can be regarded as phase transformations of left- (right-) spinors with respect to right- (left-) spinors.

[6] It is implied that $\gamma^0 \equiv \boldsymbol{\tau}_1 \otimes \mathbb{1}$, $\gamma^i \equiv -i\boldsymbol{\tau}_2 \otimes \sigma^i$ in the Weyl representation of the γ matrices.

The following notation is used: $x^{\mu'} \equiv (x^0, -\mathbf{x})$, and $c_\eta(p^\mu) \equiv a_\eta(p^{\mu'})$, $d_\eta(p^\mu) \equiv b_\eta(p^{\mu'})$. Therefore, the Lagrangian density (24) of Ref. [6] can recast

$$\mathcal{L} = \frac{i}{4}\left[\overline{\nu(x)}\gamma^\mu\partial_\mu\nu(x) - \partial_\mu\overline{\nu(x)}\gamma^\mu\nu(x) + \overline{\nu^c(x)}\gamma^\mu\partial_\mu\nu^c(x) - \partial_\mu\overline{\nu^c(x)}\gamma^\mu\nu^c(x)\right]$$
$$+ \frac{m}{2}\left[\overline{\nu(x)}\gamma^5\gamma^0\nu^c(x') + \overline{\nu^c(x)}\gamma^5\gamma^0\nu(x')\right] + (\boldsymbol{x} \to -\boldsymbol{x})\,. \quad (23)$$

The terms with $\boldsymbol{x} \to -\boldsymbol{x}$ would contribute to the action $\mathcal{S}$ in the same way as the first part of Eq. (23). Therefore, they can be disregarded. In subsequent works we shall present the properties of this Lagrangian density treated as a c-number with respect to the transformations (20). At the moment one can speculate for the q-number theory that, since Eq. (20) are indeed transformations of the inversion group, which transform to the q-Hermitian conjugate field, and the Lagrangian density usually is a scalar with respect to both q- and c- numbers, any CPT invariant theory, which accounts for two types of fields, would be intrinsically a theory admitting non-Abelian phase transformations of the components of the field operator.

In the connection with the present work one would wish to pay attention to the old papers [20]. Surprisingly, remarkable insights in the general structure of the $(1/2,0)\oplus(0,1/2)$ representation space and corresponding interactions have been made as long as thirty years ago but, unfortunately, this paper also (like several other important works which I cite here and in my previous papers) remained unnoticed. Finally, one can find probable relations between this construct and that which was used recently by Moshinsky and Smirnov [21]. The latter is based on the concept of the *sign spin* of Wigner (the generators $\boldsymbol{\tau}$, which correspond to this concept, were applied in many works until the present).

The main conclusion of the paper is: the constructs are permitted, which are based only on the 4-spinors of the Lorentz group and which admit the non-Abelian type of phase transformations and, hence, may admit interactions of corresponding fields with non-Abelian fields. If so, this assertion can serve as a basis for explanation of physical nature of isospin and weak isospin. Another non-Abelian construct has been found recently by Evans and Vigier in another representation of the group, and from very different positions [22][7].

[7] Rigorous development of the Evans–Vigier $\boldsymbol{B}^{(3)}$ construct is still required because it contains several errors and notational misunderstandings. Nevertheless, I note some interesting ideas there and think that one can work rigorously in this framework.

I greatly appreciate discussions and information from Profs D.V. Ahluwalia, M.W. Evans, Y.S. Kim, A.F. Pashkov, Yu.F. Smirnov and G. Ziino. I am grateful to Zacatecas University, México, for a professor position. This work has been partly supported by the Mexican Sistema Nacional de Investigadores, the Programa de Apoyo a la Carrera Docente and by the CONACyT, México under the research project 0270P-E.

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