REAL MULTIPLICATION NATURE AND ITS SCALING FACTORS

ANDREA PIGNATARO

ABSTRACT. The goal of this paper is to explain that the real nature of the multiplication operation is based on its scaling factors and why is a common mistake to understand the multiplication as a repeated addition.

1. INTRODUCTION

What we learn from the standard school teaching is that the four basic mathematical operations are:

-addition;
-subtraction;
-multiplication;
-division.

To make us memorizing their usage, we have been taught that there are two close relationship between them, specifically:

-addition and subtraction are opposites like multiplication and division;
-addition and multiplication share the same concept like subtraction and division.

These assumptions are indelibly written in our brains, but they are absolutely wrong.

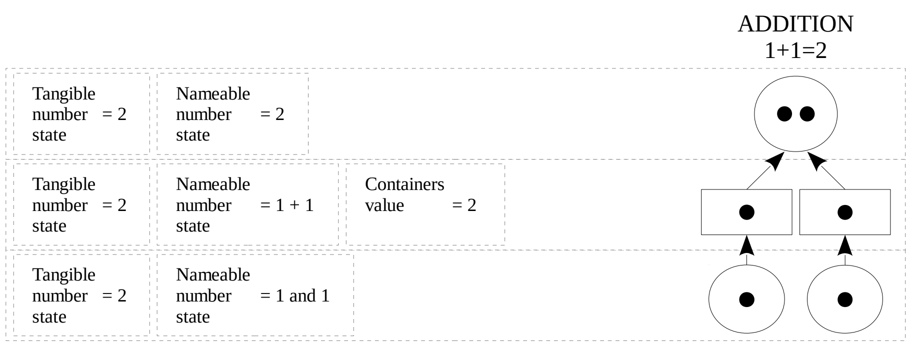
Let's see why.

The key to understand correctly what are the real relationship between the four basic operations is to relate them with the **reality** field. Every single operation can be represented with a tangible case of usage where its concept is explained in the frame of our dimension.

There are three main **number states**: **nameable**, **tangible** and **hypothetical**. The nameable state represents every nameable single number in the current section of the operation's process. The tangible state represents the real tangible sum of elements in the current section of the operation's process. We will examine the hypothetical state later since that in the addition and division operations this state is not present.

In the following graphical examples the **black dots** represent the tangible **elements** of the number, the **squares** represent the **elements containers** and the **circles** are the **static elements states** before and after the operations.

(1.1)



This reality related analysis shows us clearly the **stacking** nature of the addition operation. Thus, contrary to the common belief, the **opposite operation** of addition is not the subtraction but the **division**. The following example will possibly clear out any doubt about the correct definition of opposite as **inverse order of operations**.

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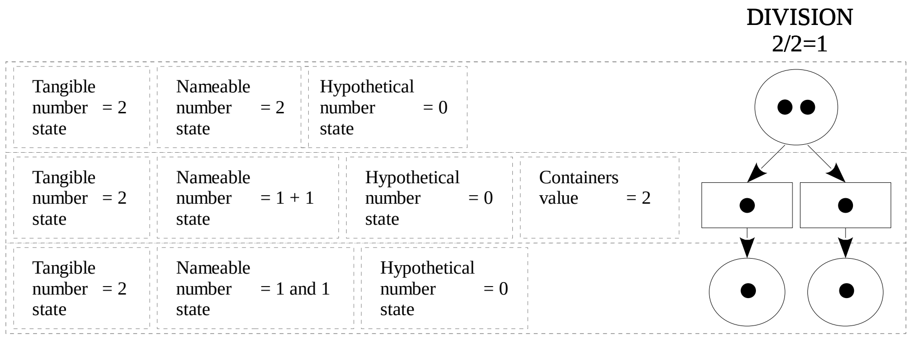
(1.2)



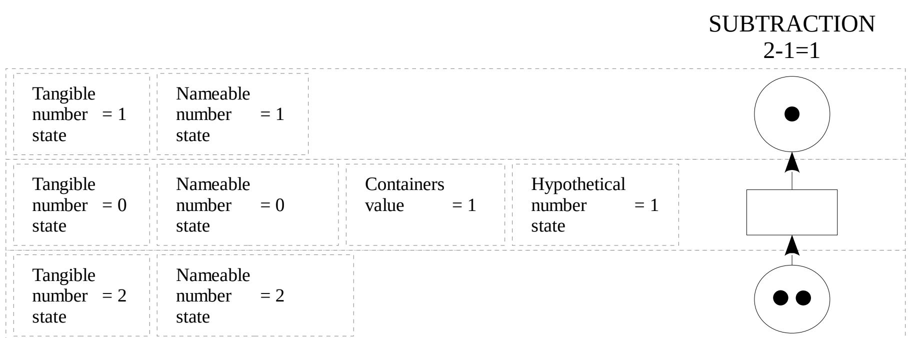
As we can see, the only variation between the division and the addition process is the inverted order of their operations. This happens due to the fact that both are **stackable operations**. In reality there is not an adding or dividing act, but rather a stacking and unstacking process.

The reality approach leads us to carefully examine the last two operations: multiplication and subtraction.

A fundamental law that regulates the dimension in which we live is that **nothing can be created or destroyed but everything can be transformed or rearranged**. Under this light the multiplication and the subtraction operations may seem utterly wrong and unrealistic. And this may be even true if we consider these two operation under the wrong teaching given to us at school. In reality they are not. The multiplication and the subtraction are perfectly **real** and **tangible** if explained as what they really are. In their process is present a number state which in the addition and the division is always set to 0, that is non-existent: the **hypothetical number state**. This state represents the **imaginary value** for which in a multiplication or subtraction operation the elements are **scaled by**. More specifically, the scaling in the subtraction is linear and it does not imply any variation from a **funnel like extrusion**. In fact, the exclusion of an element can be executed only with a rescaled version of its former value by referencing the hypothetical number state during the subtraction process.

Let' examine a graphical example of the subtraction, keeping in mind that it is the opposite operation of the multiplication. This will help us to better understand the multiplication in a later analysis.

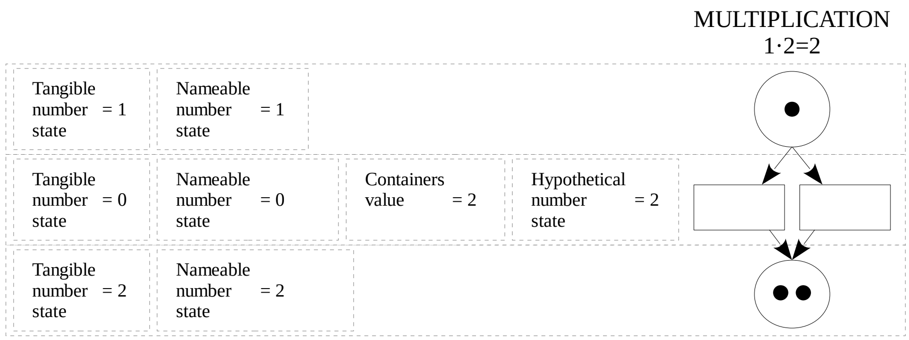
(1.3)



The empty container share its value (the **quantity** of containers) with the hypothetical number state. They may seem basically the same thing but the hypothetical number state is a state which bases its imaginary value on the tangible containers quantity.

In the following example we will see why the subtraction is the opposite operation of multiplication.

(1.4)



The only difference between the multiplication and the subtraction process is their scalable part.

Even if the container value may change, accordingly to the hypothetical number state, the scaling process of the multiplication is based on the **scaling factors** of the operation. The imaginary value is in a **wild card zone**. In reality the hypothetical number state does not exist but in our mind, so it's easy to manage this value without affecting the whole operation process. The really important and tangible part, that makes the multiplication stands out among the other operations, happens right after the choice of the hypothetical number state: **the calculation of the scaling factors**.

2. Scaling factors

The scaling factor is a fundamental element of the *multiplication is not a repeated addition* (briefly *MI¬RA*) theory [1], [2], [3], [4], [5]. The scaling factors concept can be introduced through the use of this algebraic expression, where the sum of two consecutive numbers is divided by their scaling factor.
Let x be equal to $y+1$ ($x = y+1$).

(2.1)
$$\frac{x+y}{\frac{x}{y}-\frac{y}{x}}$$

This expression has been extracted from the following identity.

Let $x = y+1$.

(2.2)
$$\frac{x+y}{\frac{x}{y}-\frac{y}{x}}=x\cdot y$$

Here is an example of the above identity to demonstrate its left-hand side equivalence to a classic multiplication operation.

Let $x = 3$ and $y = 2$.

(2.3)
$$\frac{3+2}{\frac{3}{2}-\frac{2}{3}}=3\cdot 2$$

Continuing with the example.

(2.4)
$$\frac{5}{1{,}5-0{,}\overline{6}}=6$$

$$\frac{5}{0{,}8\overline{3}}=6$$

$$6=6$$

What we have just examined is a simplified variation of a more generalized identity in which we can begin to see the true scaling factor nature of multiplication, disproving in this way, with tangible evidence, the wrong theory stating that the *multiplication is a repeated addition* (briefly *MIRA*) [5].
Let x be not equal to y ($x \neq y$).

(2.5)

$$\frac{\left(\frac{\left(\frac{x+y}{x-y}\right)}{\left(\frac{x}{y}-\frac{y}{x}\right)}\right)}{\left(\frac{\left(\frac{\left(\frac{\left(\frac{x+y}{x-y}\right)}{\left(\frac{x}{y}-\frac{y}{x}\right)}\right)}{x+y}\right)}{x}\right)+\left(\frac{\left(\frac{\left(\frac{\left(\frac{x+y}{x-y}\right)}{\left(\frac{x}{y}-\frac{y}{x}\right)}\right)}{x+y}\right)}{y}\right)}=x\cdot y$$

In this identity the scaling factors are again playing the main role.
Given the generalized nature of this identity, the scaling factors, now that $x - y$ may not be equal to 1, need to be defined in every main numerator and denominator, so that we are able to use every type of number allowed in the classic multiplication operation.
The left-hand side of this identity must not be mistaken as a direct multiplication replacement, but rather as a deep explanation of its inner functioning.

Let's see an example of the identity assigning two different numbers to its x and y.

Let $x = 50$ and $y = 60$.

(2.6)

$$\frac{\left(\frac{\left(\frac{50+60}{50-60}\right)}{\left(\frac{50}{60}-\frac{60}{50}\right)}\right)}{\left(\frac{\left(\frac{\left(\frac{\left(\frac{50+60}{50-60}\right)}{\left(\frac{50}{60}-\frac{60}{50}\right)}\right)}{50+60}\right)}{50}\right)+\left(\frac{\left(\frac{\left(\frac{\left(\frac{50+60}{50-60}\right)}{\left(\frac{50}{60}-\frac{60}{50}\right)}\right)}{50+60}\right)}{60}\right)}=50\cdot 60$$

Let's solve the identity.

(2.7)
$$\frac{\left(\frac{\left(\frac{110}{-10}\right)}{(0{,}8\overline{3}-1{,}2)}\right)}{\left(\frac{\left(\frac{\left(\frac{110}{-10}\right)}{(0{,}8\overline{3}-1{,}2)}\right)}{110}\right)\Big/50 + \left(\frac{\left(\frac{\left(\frac{110}{-10}\right)}{(0{,}8\overline{3}-1{,}2)}\right)}{110}\right)\Big/60}=3000$$

$$\frac{\left(\frac{-11}{-0{,}3\overline{6}}\right)}{\left(\frac{\left(\frac{\left(\frac{-11}{-0{,}3\overline{6}}\right)}{110}\right)}{50}\right)+\left(\frac{\left(\frac{\left(\frac{-11}{-0{,}3\overline{6}}\right)}{110}\right)}{60}\right)}=3000$$

$$\frac{30}{\left(\frac{\left(\frac{30}{110}\right)}{50}\right)+\left(\frac{\left(\frac{30}{110}\right)}{60}\right)}=3000$$

$$\frac{30}{\left(\frac{0{,}2\overline{7}}{50}\right)+\left(\frac{0{,}2\overline{7}}{60}\right)}=3000$$

$$\frac{30}{0{,}00\overline{54}+0{,}00\overline{45}}=3000$$

$$\frac{30}{0{,}01}=3000$$

$$3000=3000$$

As we can see, the most important part of these identities is $\frac{x}{y}-\frac{y}{x}$, that is the scaling factor.

When x is equal to y (as when we want to calculate a square), the (2.5) identity can't be solved because, in this case, the denominator of $\frac{x+y}{x-y}$ is 0.

This division by 0 helps us to better understand that the scaling factor nature of the multiplication operation must include an error check for every exponentiation where the base shares the same value with its exponent (squares, cubes, etc.).

In fact, any power has a scaling factor equal to its result. This does not means that the base *scales* by the exponent, as we would have erroneously expected from such a seemingly multiplicative operation, but rather that the power is multiplied by 1. To achieve this, the identity must include a +1 in every single operation of its scaling factors. In this way, it becomes compatible with every case a multiplication operation may involve, keeping in mind that x must greater than or equal to y since that the scaling factors work in a positive and real field. If the multiplication case involve a x value smaller than the y value, then we must rearrange the factors, operation permitted by the multiplication commutative property which states that *changing the order of the factors does not change the product*.
Let's see the (2.5) identity definitive version.

(2.8)

Where $x \geq y$.

$$\frac{\left(\frac{\left(\frac{x+y+1}{x-y+1}\right)}{\left(\frac{x}{y}-\frac{y}{x}+1\right)}\right)}{\left(\frac{\left(\frac{\left(\frac{\left(\frac{x+y+1}{x-y+1}\right)}{\left(\frac{x}{y}-\frac{y}{x}+1\right)}\right)}{x+y}\right)}{x}\right)+\left(\frac{\left(\frac{\left(\frac{\left(\frac{x+y+1}{x-y+1}\right)}{\left(\frac{x}{y}-\frac{y}{x}+1\right)}\right)}{x+y}\right)}{y}\right)}=x\cdot y$$

To demonstrate its validity, let's solve the last five passages of a possible solution with x and y set both to 2.

(2.9)

$$\frac{5}{\left(\frac{\left(\frac{5}{4}\right)}{2}\right)+\left(\frac{\left(\frac{5}{4}\right)}{2}\right)}=2\cdot 2$$

$$\frac{5}{\frac{1{,}25}{2}+\frac{1{,}25}{2}}=2\cdot 2$$

$$\frac{5}{0{,}625+0{,}625}=2\cdot 2$$

$$\frac{5}{1{,}25}=2\cdot 2$$

$$4=4$$

All this endorse the demonstration that, just like the *multiplication is not a repeated addition* (*MI¬RA*), the *exponentiation is not a repeated multiplication* (*EI¬RM*) [2], [3], [4].

REFERENCES

[1] Keith Devlin, *What is conceptual understanding?,* Mathematical Association of America [**http://www.maa.org/external_archive/devlin/devlin_09_07.html**] , September 2007.

[2] __________, *It Ain't No Repeated Addition,* Mathematical Association of America [**http://www.maa.org/external_archive/devlin/devlin_06_08.html**] , June 2008.

[3] __________, *It's Still Not Repeated Addition,* Mathematical Association of America [**http://www.maa.org/external_archive/devlin/devlin_0708_08.html**] , July-August 2008.

[4] __________, *Multiplication and Those Pesky British Spellings,* Mathematical Association of America [**http://www.maa.org/external_archive/devlin/devlin_09_08.html**] , September 2008.

[5] __________, *What Exactly is Multiplication?,* Mathematical Association of America [**http://www.maa.org/external_archive/devlin/devlin_01_11.html**] , January 2011.

ANDREA PIGNATARO
E-mail address: **andreapignataro@katamail.com**
evonture@gmail.com