Wave functions for the electron and positron

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Abstract

The Wave/Particle duality of particles in Physics is well known. Particles have properties that uniquely characterize them from one another, such as mass, charge and spin. Charged particles have associated Electric and Magnetic fields. Also, every moving particle has a De Broglie wavelength determined by its mass and velocity. In this paper I show that all of these properties of a particle can be derived from a single wave function equation for that particle. I present wave functions for the Electron and the Positron and provide principles that can be used to calculate the wave functions of all the fundamental particles in Physics.

1. Introduction

In order to provide the foundations of a link between Classical Physics field concepts and the wave/particle duality in Quantum Mechanics it is necessary to demonstrate how particles can be modeled both from a Classical Wave perspective while also satisfying the requirements of Quantum Mechanics, in particular the Schrödinger wave equation and the De Broglie equations.

There is already evidence of this connection in the energy sum of the Electric and Magnetic fields in the Hamiltonian function that expresses the total energy of an atomic system:

[Ref. 1] “In 1926 Schrödinger used energy conservation to obtain a quantum mechanical equation in a variable called the wave function that accurately described single-electron states such as the hydrogen atom. The wave function depended on a Hamiltonian function and the total energy of an atomic system, and was compatible with Hertz's potential formulation. The wave function depends on the sum of the squares of E- and H-fields as is seen by examining the energy density function of the electromagnetic field.”

In order to satisfy both the wave and particle natures of particles in a model of a particle, the particle’s wave function must satisfy both the Classical wave equation (which ensures that the wave function can represent a vibration of the space-time continuum) and the Schrödinger wave equation (which ensures that the wave function can represent a quantum of energy – thus a particle) (Ref. [3]).

A wave function solution to the Classical wave equation describes the motion of all points on the wave at any location in space and time. The position of a test point in space as it is affected by the wave motion can be represented as a displacement vector drawn from the starting location of the point to its current location.

In the case of Electromagnetism there is a single vector field that describes the motion of an Electromagnetic wave in this way, it is known as the Hertzian vector field [Ref. 2, 6]. The Electric and Magnetic fields can both be derived from the Hertzian vector field by differentiation with respect to space and time.

[Ref. 2] “in a vacuum a single Hertz vector written as the product of a scalar potential and a constant vector, naturally arises as consequence of the transversality of the electromagnetic fields”

Therefore, a wave function that describes a field of vectors representing Hertzian vectors can also represent a wave function describing Electric and Magnetic field vectors. If the wave function satisfies both the Classical wave equation and the Schrödinger wave equation then it can also represent a vibration of space-time and a potential solution for a Quantum particle.

This paper presents two such solutions, one representing an electron and one representing a positron. In addition, I show that the correct Classical fields are produced by them and that the Quantum Mechanical requirements of the De Broglie equations are also met by them.

2. The solutions

These are the suggested wave function equations for the Electron and the Positron. The images located in [Fig. 2-10] show graphical representations of the fields derived from these wave functions using a 3D vector modelling program I wrote to aid in the visualization and testing of proposed wave function solutions.

For the Electron:

$$\Psi_e = \frac{\sqrt{2}iQ_e\hbar}{4\pi r M_e c\varepsilon_0} exp\left[\frac{-iM_e c^2}{\hbar}\left(t - \frac{r^2}{2c}\right)\right] \quad (1)$$

For the Positron:

$$\Psi_p = \frac{\sqrt{2}iQ_p\hbar}{4\pi r M_p c\varepsilon_0} exp\left[\frac{-iM_p c^2}{\hbar}\left(t + \frac{r^2}{2c}\right)\right] \quad (2)$$

Where:

ψ_e = Electron wave function

ψ_p = Positron wave function

Q_e = Electron Charge (-)

Q_p = Positron Charge (+)

M_e = Mass of an Electron

M_p = Mass of a Positron

ε_0 = Permittivity of free space

t = Time

r = Distance from particle's centre

c = The speed of light

$\hbar$ = The Reduced Plank's Constant

3. The solutions satisfy the wave equations

The wave nature of particles is being modelled here as a vibration of the space-time continuum and the particle nature is modelled as localized quanta of this wave energy. In order to satisfy both the wave and particle natures of particles in the model, the wave function must satisfy both the Classical wave equation (which ensures that the wave function can represent a vibration of the space-time continuum) and the Schrödinger wave equation (which ensures that the wave function can represent a quantum of energy – thus a particle) (Ref. [3]).

Classical wave equation: $$\nabla^2\psi = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} \qquad (3)$$

Schrödinger wave equation: $$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$$

Where: $$\hat{H}\psi = Total\ Energy$$

$$\hat{H}\psi = Kinetic\,Energy\left(KE\right) + PotentialEnergy\left(PE\right)$$

The wave function describes a field of rotating vectors which can each be thought of as comprising two Quantum Harmonic Oscillators (Ref. [10]); one along each axis of the complex plane. The vectors trace out a circle, such that at any given time half of the energy is present as Kinetic energy and half as Potential energy depending on the phase of each of the component Quantum Harmonic Oscillators. In their simple harmonic motion oscillation, each oscillates between full KE and full PE, but when one has full KE the other has full PE and vice-versa.

Due to Equipartition of energy in a Classical wave (Ref. [4]):

$$\text{KE} = \text{PE} = -\frac{\hbar^2}{2m}\nabla^2\psi$$

So: $$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{m}\nabla^2\psi \qquad (4)$$

Testing the electron solution with the Schrödinger wave equation

Referring to Equations 1 and 4:

In Spherical coordinates the Laplacian of Ψ_e is:

$$\nabla^2\Psi_e = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi_e}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta^2}\left(sin\theta\frac{\partial\Psi_e}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial\Psi_e}{\partial\emptyset^2} \quad (5a)$$

As the wave function Ψ_e is spherically symmetrical, all the vectors at the same distance r from the origin are identical, so the terms involving θ and $\emptyset$ are zero.

So $\nabla^2\Psi_e$ reduces to:

$$\nabla^2\Psi_e = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi_e}{\partial r}\right) \quad (6a)$$

Thus, the Schrödinger wave equation (4) becomes:

$$i\hbar\frac{\partial\Psi_e}{\partial t} = -\frac{\hbar^2}{m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi_e}{\partial r}\right) \quad (7a)$$

Thus:

$$\frac{\partial\Psi_e}{\partial t} = \frac{i\hbar}{m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi_e}{\partial r}\right) \quad (8a)$$

As from Ψ_e by differentiation of Eqn 1, we can also say that:

$$\frac{\partial\Psi_e}{\partial t} = \frac{\sqrt{2}Q_e c}{4\pi r\varepsilon_0} exp\left[\frac{-iM_e c^2}{\hbar}\left(t - \frac{r^2}{2c}\right)\right] \quad (9a)$$

And

$$\frac{\partial\Psi_e}{\partial r} = \frac{-\sqrt{2}Q_e}{4\pi r\varepsilon_0} exp\left[\frac{-iM_e c^2}{\hbar}\left(t - \frac{r^2}{2c}\right)\right] \quad (10a)$$

So:

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial \Psi_e}{\partial r}\right) = \frac{-\sqrt{2}iQ_e r M_e c}{4\pi\varepsilon_0\hbar} exp\left[\frac{-iM_e c^2}{\hbar}\left(t-\frac{r^2}{2c}\right)\right] \qquad (11a)$$

Thus:

$$\frac{i\hbar}{m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \Psi_e}{\partial r}\right) = \frac{\sqrt{2}Q_e c}{4\pi r\varepsilon_0} exp\left[\frac{-iM_e c^2}{\hbar}\left(t-\frac{r^2}{2c}\right)\right] \qquad (12a)$$

So LHS = RHS of the Schrödinger wave equation (Eqn 4), so the wave function (Eqn 1) is a solution to it:

$$\frac{\partial \Psi_e}{\partial t} = \frac{i\hbar}{m}\nabla^2\Psi_e$$

Eqn 9a equals Eqn 12a:

$$\frac{\sqrt{2}Q_e c}{4\pi r\varepsilon_0} exp\left[\frac{-iM_e c^2}{\hbar}\left(t-\frac{r^2}{2c}\right)\right] = \frac{\sqrt{2}Q_e c}{4\pi r\varepsilon_0} exp\left[\frac{-iM_e c^2}{\hbar}\left(t-\frac{r^2}{2c}\right)\right]$$

Testing the electron solution with the Classical wave equation

Referring to Equations 1 and 3:

$$\frac{\partial \Psi_e}{\partial t} = \frac{\sqrt{2} Q_e c}{4\pi r \varepsilon_0} exp\left[\frac{-iM_e c^2}{\hbar}\left(t - \frac{r^2}{2c}\right)\right] \quad (13a)$$

So:

$$\frac{\partial^2 \Psi_e}{\partial t^2} = \frac{-\sqrt{2} i Q_e M_e c^3}{4\pi r \varepsilon_0 \hbar} exp\left[\frac{-iM_e c^2}{\hbar}\left(t - \frac{r^2}{2c}\right)\right] \quad (14a)$$

Thus:

$$\frac{1}{c^2}\frac{\partial^2 \Psi_e}{\partial t^2} = \frac{-\sqrt{2} i Q_e M_e c}{4\pi r \varepsilon_0 \hbar} exp\left[\frac{-iM_e c^2}{\hbar}\left(t - \frac{r^2}{2c}\right)\right] \quad (15a)$$

And, substituting Eqns 6a and 15a into Eqn 3:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Psi_e}{\partial r}\right) = \frac{-\sqrt{2} i Q_e M_e c}{4\pi r \varepsilon_0 \hbar} exp\left[\frac{-iM_e c^2}{\hbar}\left(t - \frac{r^2}{2c}\right)\right] \quad (16a)$$

So LHS = RHS of Classical Wave equation (Eqn 3) too, so the electron wave function (Eqn 1) is a solution to it also.

Testing the positron solution with the Schrödinger wave equation

Referring to Equations 1 and 4:

In Spherical coordinates the Laplacian of Ψ_e is:

$$\nabla^2\Psi_{\mathrm{p}} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi_{\mathrm{p}}}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta^2}\left(sin\theta\frac{\partial\Psi_{\mathrm{p}}}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial\Psi_{\mathrm{p}}}{\partial\emptyset^2} \quad (5b)$$

As the wave function Ψ_{p} is spherically symmetrical, all the vectors at the same distance r from the origin are identical, so the terms involving θ and $\emptyset$ are zero.

So $\nabla^2\Psi_{\mathrm{p}}$ reduces to:

$$\nabla^2\Psi_{\mathrm{p}} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi_{\mathrm{p}}}{\partial r}\right) \quad (6b)$$

Thus, the Schrödinger wave equation (4) becomes:

$$i\hbar\frac{\partial\Psi_{\mathrm{p}}}{\partial t} = -\frac{\hbar^2}{m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi_{\mathrm{p}}}{\partial r}\right) \quad (7b)$$

Thus:

$$\frac{\partial\Psi_{\mathrm{p}}}{\partial t} = \frac{i\hbar}{m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi_{\mathrm{p}}}{\partial r}\right) \quad (8b)$$

As from Ψ_{p} by differentiation of Eqn 2, we can also say that:

$$\frac{\partial\Psi_{\mathrm{p}}}{\partial t} = \frac{\sqrt{2}Q_p c}{4\pi r\varepsilon_0}\exp\left[\frac{-iM_p c^2}{\hbar}\left(t + \frac{r^2}{2c}\right)\right] \quad (9b)$$

And

$$\frac{\partial\Psi_{\mathrm{p}}}{\partial r} = \frac{\sqrt{2}Q_p}{4\pi r\varepsilon_0}\exp\left[\frac{-iM_p c^2}{\hbar}\left(t + \frac{r^2}{2c}\right)\right] \quad (10b)$$

So:

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial \Psi_{\mathrm{p}}}{\partial r}\right)=\frac{-\sqrt{2}iQ_p r M_p c}{4\pi\varepsilon_0\hbar}\exp\left[\frac{-iM_pc^2}{\hbar}\left(t+\frac{r^2}{2c}\right)\right] \qquad (11b)$$

Thus:

$$\frac{i\hbar}{m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \Psi_{\mathrm{p}}}{\partial r}\right)=\frac{\sqrt{2}Q_p c}{4\pi r\varepsilon_0}\exp\left[\frac{-iM_pc^2}{\hbar}\left(t+\frac{r^2}{2c}\right)\right] \qquad (12b)$$

So LHS = RHS of the Schrödinger wave equation (Eqn 4), so the wave function (Eqn 2) is a solution to it:

$$\frac{\partial \Psi_{\mathrm{p}}}{\partial t}=\frac{i\hbar}{m}\nabla^2\Psi_{\mathrm{p}}$$

Eqn 9b equals Eqn 12b:

$$\frac{\sqrt{2}Q_p c}{4\pi r\varepsilon_0}\exp\left[\frac{-iM_pc^2}{\hbar}\left(t+\frac{r^2}{2c}\right)\right]=\frac{\sqrt{2}Q_p c}{4\pi r\varepsilon_0}\exp\left[\frac{-iM_pc^2}{\hbar}\left(t+\frac{r^2}{2c}\right)\right]$$

Testing the positron solution with the Classical wave equation

Referring to Equations 2 and 3:

$$\frac{\partial \Psi_{\mathrm{p}}}{\partial t} = \frac{\sqrt{2} Q_p c}{4\pi r \varepsilon_0} \exp\left[\frac{-iM_p c^2}{\hbar}\left(t + \frac{r^2}{2c}\right)\right] \qquad (13b)$$

So:

$$\frac{\partial^2 \Psi_{\mathrm{p}}}{\partial t^2} = \frac{-\sqrt{2} i Q_p M_p c^3}{4\pi r \varepsilon_0 \hbar} \exp\left[\frac{-iM_p c^2}{\hbar}\left(t + \frac{r^2}{2c}\right)\right] \qquad (14b)$$

Thus:

$$\frac{1}{c^2}\frac{\partial^2 \Psi_{\mathrm{p}}}{\partial t^2} = \frac{-\sqrt{2} i Q_p M_p c}{4\pi r \varepsilon_0 \hbar} \exp\left[\frac{-iM_p c^2}{\hbar}\left(t + \frac{r^2}{2c}\right)\right] \qquad (15b)$$

And, substituting Eqns 6b and 15b into Eqn 3:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Psi_{\mathrm{p}}}{\partial r}\right) = \frac{-\sqrt{2} i Q_p M_p c}{4\pi r \varepsilon_0 \hbar} \exp\left[\frac{-iM_p c^2}{\hbar}\left(t + \frac{r^2}{2c}\right)\right] \qquad (16b)$$

So LHS = RHS of Classical Wave equation (Eqn 3) too, so the positron wave function (Eqn 2) is a solution to it also.

4. The wave function and electromagnetism

Each of the measurable fields in Electromagnetic Theory (Ref. [5, 6]), and their connection back to the wave function, can be expressed quite simply by the following set of equations and illustrated by Fig (1).

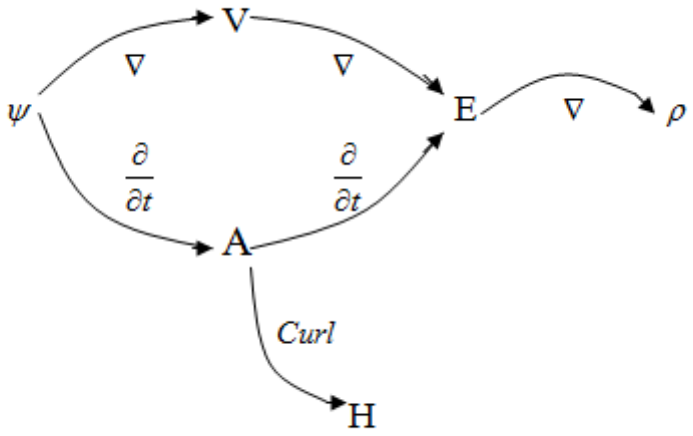

Fig. 1. The mathematical connections between the fields.

$$A = -\frac{1}{c}\frac{\partial \psi}{\partial t} \quad (17) \qquad V = \nabla.\psi \quad (18) \qquad E = -\nabla V - \frac{1}{c}\frac{\partial A}{\partial t} \quad (19)$$

$$H = \nabla \times A \quad (20) \qquad \rho = -\frac{1}{4\pi}\nabla.E \quad (21)$$

Where:

ψ = Wave function

V = Voltage (electric potential)

E = Electric field vector

A = Vector potential

H = Magnetic field vector

ρ = Charge density

5. Analysis of the wave functions

Both wave functions represent a field of rotating vectors. The pattern described by the phases of the field of rotating vectors is that of a spinning spiral wave. The phase wave flows either away from or towards the centre of the particle (Fig. [2, 3a, 3b]). The Electron spins with the phase wave flowing outward and the Positron with the phase wave flowing inwards (Ref [5]).

The angular frequency in the wave function is derived from the following three known equations.

$$E = h\nu \tag{22}$$

$$E = mc^2 \tag{23}$$

$$h = 2\pi\hbar \tag{24}$$

Substituting Eq. (23) and Eq. (24) into Eq. (22) and solving for ν we have:

$$\nu = \frac{M_e c^2}{2\pi\hbar} \tag{25}$$

Then to convert to angular frequency:

$$\omega = 2\pi \times \nu \tag{26}$$

Substituting Eq. (25) into Eq. (26) gives:

$$\omega = \frac{M_e c^2}{\hbar} \quad \text{Radians per Second} \tag{27}$$

This describes the rate of rotation of the vectors in the vector field that describes the Electron/Positron wave function.

The centre of the Electron comprises a vector that rotates around a fixed position at the particle's centre over time. As time progresses this vector propagates radially outwards away from the centre thus forming the phase wave spiral. Therefore, the phase of each vector depends not only on time, but also on distance r from the centre of the wave function.

As the waves propagate at the speed of light, the vector's dependence on distance varies in proportion to $\frac{r}{c}$, but not only this. There is also a dependence

on the area of the sphere at the distance r and the rate of change of area with distance at this location.

$$Area\ of\ Sphere = 4\pi r^2 \tag{28}$$

$$Rate\ of\ change\ in\ Area\ with\ distance = 8\pi r \tag{29}$$

As the vector propagates outwards, it must fill an ever-increasing volume of space, as each spherical shell's volume increases with distance r. So, to do this, the vector must flow through an extra distance of space, and this takes a longer time; more than simply $\frac{r}{c}$. The fractional extra amount of space the vector must propagate can be calculated by taking the area of the spherical shell at distance r which is $4\pi r^2$ and by using the rate of area change at that distance (which is $8\pi r$). Dividing the area by the rate of increase in area (a linear extrapolation at the distance r) gives the effective extra distance the vector signal has travelled for each meter it is away from the Electron's centre. So, this factor must be multiplied by the distance r in order to get the total effective distance the vector signal has travelled.

Thus, the factor that the angular frequency must be multiplied by is:

$$\frac{4\pi r^2}{8\pi r} \times \frac{r}{c} = \frac{r^2}{2c} \tag{30}$$

Every point on the wave function spiral comprises a vector that rotates on the spot as the phase waves pass through each point.

In a classical wave, each point in the medium supporting that wave (such as the water molecules in a water wave) moves in a circular motion as the wave passes. The frequency of this circular motion is the same as that of the wave. However, when two waves of equal frequency (but travelling in opposite directions) combine to form a standing wave, each point in the medium rotates at twice the angular frequency of each of the two component waves.

The spinning spiral of rotating vectors that the wave function describes can be modelled as a standing wave comprised from two interfering waves: a spherical IN wave and a spherical OUT wave. Thus, each point in the medium supporting this standing wave is rotating at twice the frequency of either the IN or OUT wave alone.

The spherical IN and OUT waves work together, by means of constructive and destructive interference due to a slight frequency difference between the IN and OUT waves, forming the spinning spiral structure of the particle. As each point

in this spatial structure is being influenced by both IN and OUT waves (one wave from each side), a vector at that point spins around at a rate which is the sum of the IN and OUT wave frequencies, as it receives wave crests from both sides simultaneously. The vector rotation period at each point is completed in the time is takes only half of the wavelength of an IN or OUT wave to pass each point (π Radians rather than 2π Radians). The frequency of the IN and OUT waves is the same except for a slight difference that modulates this fundamental frequency and thereby forms the spiral pattern, therefore the rotation rate of each vector is two times the fundamental frequency of an IN or OUT wave.

The frequency of the vector rotation for any point in the wave function is given by Eq. (27). Thus, the angular wave frequency of each IN/OUT wave is given by:

$$\frac{\omega}{2} \quad \text{Radians per Second} \tag{31}$$

From Eq. (26) and Eq. (31), the travelling wave fundamental frequency of an IN or OUT wave is:

$$\nu = \frac{\omega}{4\pi} \quad \text{Hertz} \tag{32}$$

So, from Eq. (27) and Eq. (32), the Electron's IN/OUT wave frequency is:

$$\nu = \quad 6.17794982 \times 10^{19} \quad \text{Hertz} \tag{33a}$$

And (as the propagation speed is c) the spatial frequency (wavenumber) is:

$$f_{electron} = 2.06074224199 \times 10^{11} \tag{33b}$$

6. Verification using the De Broglie equations

The De Broglie wavenumber for a moving particle is $\frac{mv}{h}$ (34)

This is 13747.792 for an Electron travelling at $10m\sec^{-1}$ (35)

The Classical interpretation of the De Broglie wave is that of a beat frequency of the upstream and downstream components (with respect to the particle's direction of motion) of the Electron's IN/OUT wave, so:

$$f_{up} = f_{electron} \times \frac{c}{c - v} \quad (36)$$

$$f_{down} = f_{electron} \times \frac{c}{c + v} \quad (37)$$

Again, the speed of the Electron v = $10m\sec^{-1}$

Thus from Eq. (36) and Eq. (37) the beat frequency wavenumber is:

$$f_{up} - f_{down} = 13747.792 \quad (38)$$

So, we can see that the De Broglie wavenumber matches the beat frequency wavenumber of the calculated Electron IN/OUT waves for an Electron travelling at $10m\sec^{-1}$ (Eq. (35) equals Eq. (38)).

The Energy of the Electron can be checked too, using the De Broglie relation:

$$E = \hbar\omega \quad (39)$$

Using Eq. (27):

$$E = \hbar \frac{M_e c^2}{\hbar} = M_e c^2 \quad (40)$$

Which is the Energy/Mass relationship as it should be.

7. Derivation of the classical electric potential for the Electron

For the Electron wave function, the Electric Potential (V) is $div(\psi_e)$, which in spherical coordinates is (Ref. 8]).

$$\text{div}(\Psi_e) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\Psi_e) + \frac{1}{rsin\theta}\frac{\partial}{\partial \theta}(\Psi_e sin\theta) + \frac{1}{rsin\theta}\frac{\partial \Psi_e}{\partial \phi} \quad (41a)$$

As the wave function Ψ_e is spherically symmetrical, all the vectors at the same distance r from the origin are identical, so the terms involving θ and $\emptyset$ are zero. So this reduces to:

$$\text{div}(\Psi_e) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\Psi_e) \quad (42a)$$

Recalling Eqn 1 for Ψ_e, so div(Ψ_e) is:

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\Psi_e) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\sqrt{2}iQ_e\hbar r}{4\pi M_e c\varepsilon_0} exp\left[\frac{-iM_e c^2}{\hbar}\left(t - \frac{r^2}{2c}\right)\right]\right) \quad (43a)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\Psi_e) = \frac{1}{r^2}\left(\frac{-\sqrt{2}Q_e r}{4\pi\varepsilon_0} exp\left[\frac{-iM_e c^2}{\hbar}\left(t - \frac{r^2}{2c}\right)\right]\right) \quad (44a)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\Psi_e) = \frac{-\sqrt{2}Q_e}{4\pi r\varepsilon_0} exp\left[\frac{-iM_e c^2}{\hbar}\left(t - \frac{r^2}{2c}\right)\right] \quad (45a)$$

When viewed close up, the spinning spiral and charge layers that comprise the electron/positron are clearly visible. Due to the fast spinning of the spiral (and outward or inward phase flow), or at large distance scales where the undulations of the spinning charge layers are small by comparison, the fields appear to become smooth and be of a continuous nature [Fig. 5-10]. So, for example, the Electric Potential for the Electron in this case appears to be the RMS (Root-Mean-Squared Ref. [9]) of Eq. (42a), which is equal to the classical equation:

$$\frac{-Q_e}{4\pi r\varepsilon_0} \quad (46a)$$

8. Derivation of the classical electric potential for the Positron

For the Positron wave function, the Electric Potential (V) is $div(\psi_e)$, which in spherical coordinates is (Ref. 8]).

$$\text{div}(\Psi_\text{p}) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\Psi_\text{p}\right) + \frac{1}{rsin\theta}\frac{\partial}{\partial\theta}\left(\Psi_\text{p}sin\theta\right) + \frac{1}{rsin\theta}\frac{\partial\Psi_\text{p}}{\partial\phi} \qquad (41b)$$

As the wave function Ψ_p is spherically symmetrical, all the vectors at the same distance r from the origin are identical, so the terms involving θ and $\emptyset$ are zero. So this reduces to:

$$\text{div}(\Psi_\text{p}) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\Psi_\text{p}\right) \qquad (42b)$$

Recalling Eqn 2 for Ψ_p, so div(Ψ_p) is:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\Psi_\text{p}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\sqrt{2}iQ_p\hbar r}{4\pi M_p c\varepsilon_0}\exp\left[\frac{-iM_pc^2}{\hbar}\left(t+\frac{r^2}{2c}\right)\right]\right) \qquad (43b)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\Psi_\text{p}\right) = \frac{1}{r^2}\left(\frac{\sqrt{2}Q_p r}{4\pi\varepsilon_0}\exp\left[\frac{-iM_pc^2}{\hbar}\left(t+\frac{r^2}{2c}\right)\right]\right) \qquad (44b)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\Psi_\text{p}\right) = \frac{\sqrt{2}Q_p}{4\pi r\varepsilon_0}\exp\left[\frac{-iM_pc^2}{\hbar}\left(t+\frac{r^2}{2c}\right)\right] \qquad (45b)$$

The Electric Potential for the Positron in this case appears to be the RMS (Root-Mean-Squared Ref. [9]) of Eq. (42b), which is equal to the classical equation:

$$\frac{Q_p}{4\pi r\varepsilon_0} \qquad (46b)$$

9. Conclusion

The wave functions presented here describe particles with all the correct properties for an Electron and a Positron and satisfy the requirements of both the Classical and Quantum Mechanical interpretations.

The wave function represents a field of rotating vectors. The spinning vectors form a phase wave that describes a spinning spiral. The phase wave flows either away from or towards the centre of the particle. Interactions between the phase waves of two or more particles could be the cause of the Electrical & Magnetic attraction/repulsion between charged particles due to momentum exchanges between the wave structures (Ref. [7]).

In general, the concepts use to build these two wave equations could be applied to all particles in Physics. The key principles are:

1. The frequency of the waves that comprise the three-dimensional wave structure of the particle is based on the particle's mass (via the calculation shown above).

2. A particle's charge is defined by either an outward or inward flowing phase wave. A neutral particle would have no net phase flow inward or outward, but may contain regions of either inward or outward flow, which cancel out in the region surrounding the particle.

3. The completed wave function must satisfy both the Classical and Schrödinger wave equation.

4. Particles such as Protons (or other particles containing Quarks) would contain several components to the overall wave function, which work together to form a stable particle (i.e. together they satisfy the other three principles stated here).

10. Supplementary material

9.1 Images from the Model

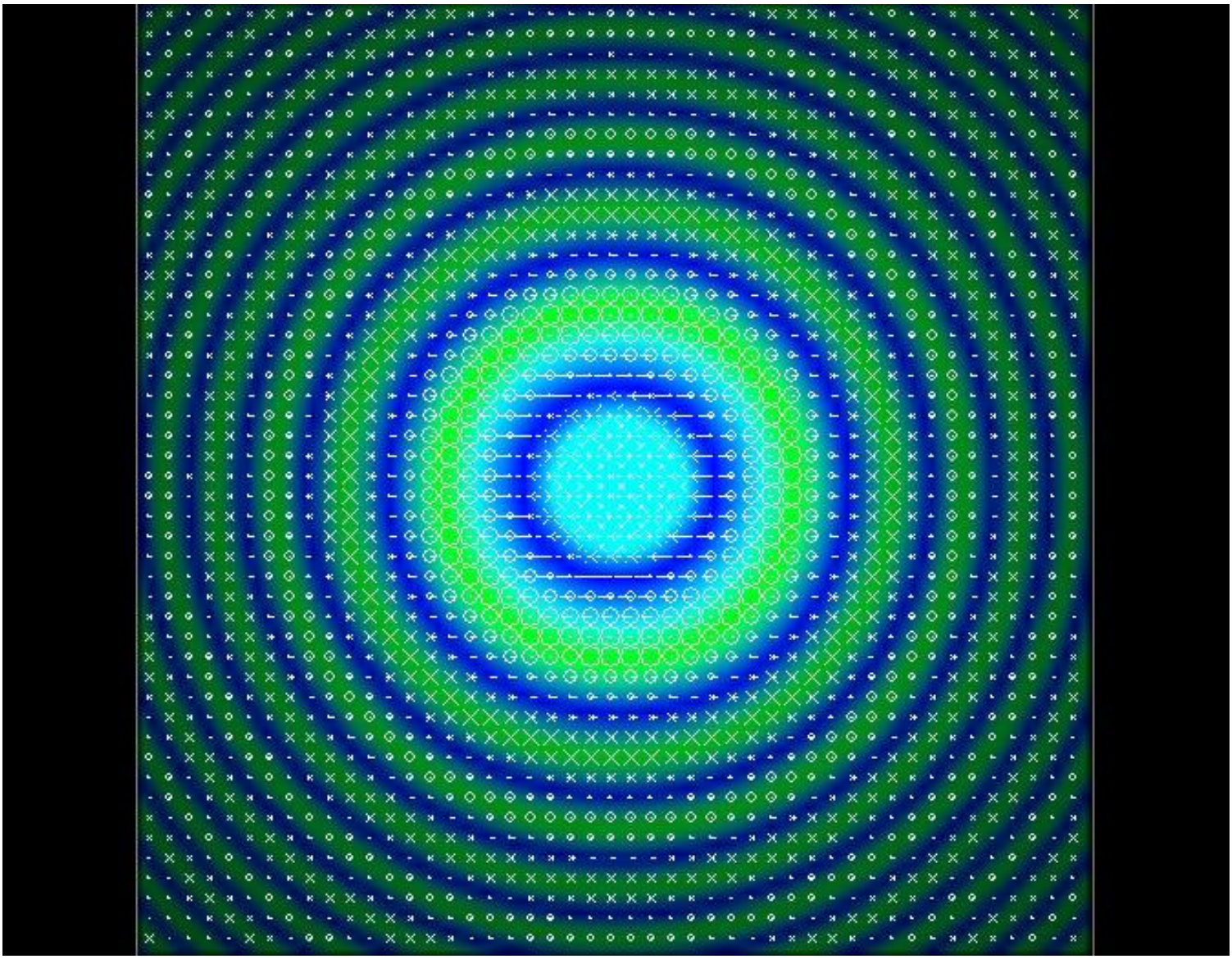

Fig. 2. The electron wave function from the side (the spin axis is vertical).

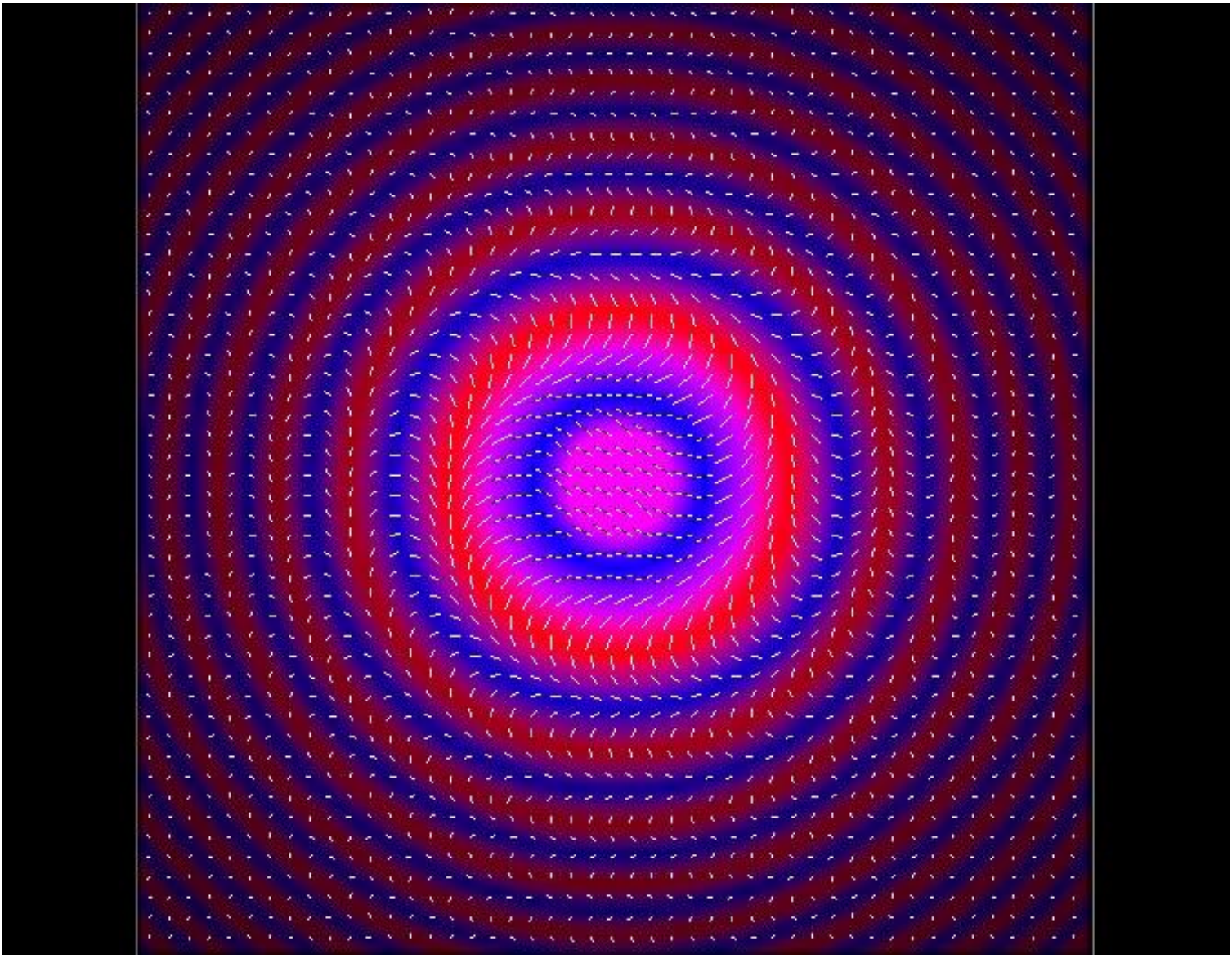

Fig. 3(a). The electron wave function viewed from the top (looking down the spin axis).

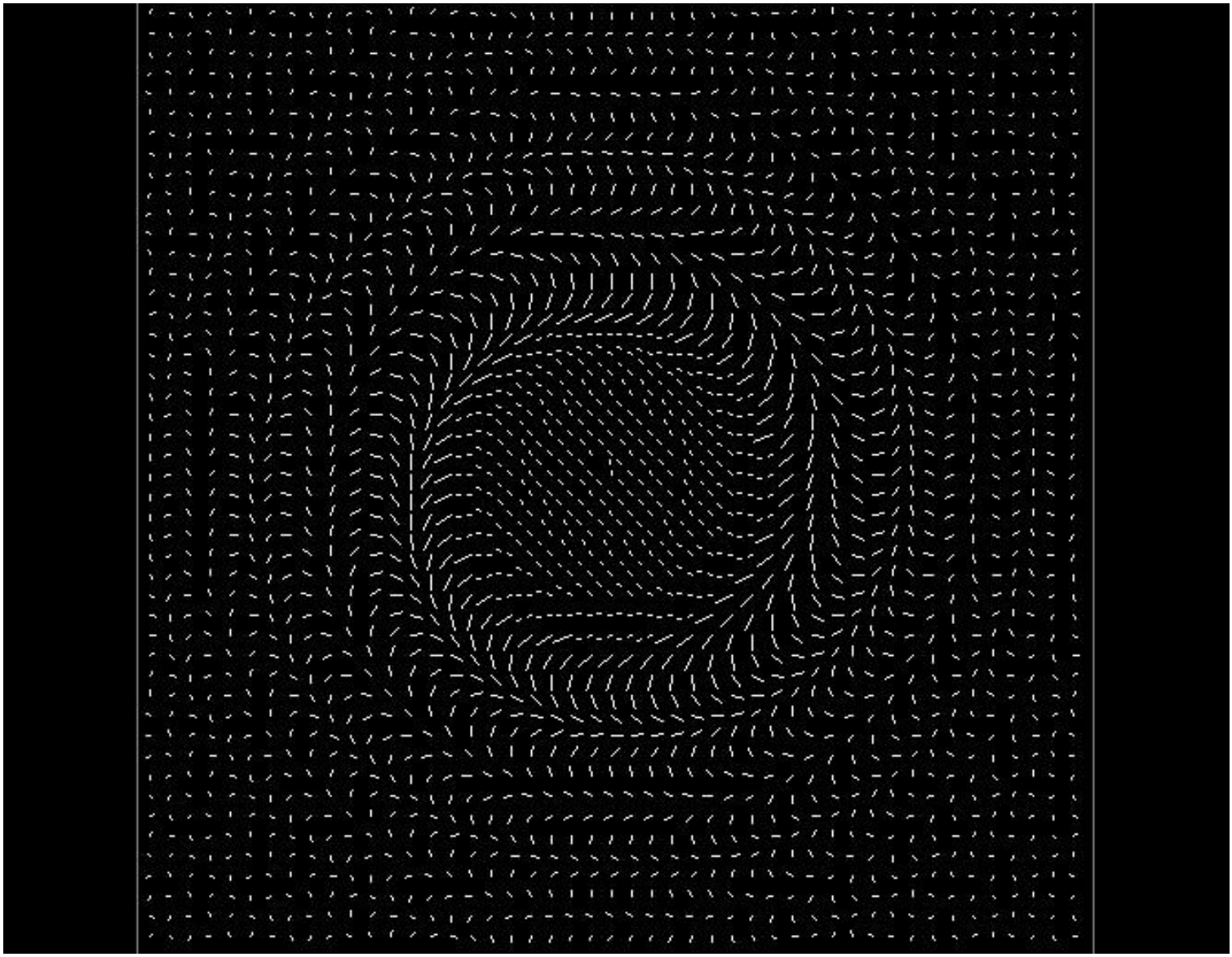

Fig. 3(b). The electron wave function (vector arrows only) viewed from the top (looking down the spin axis).

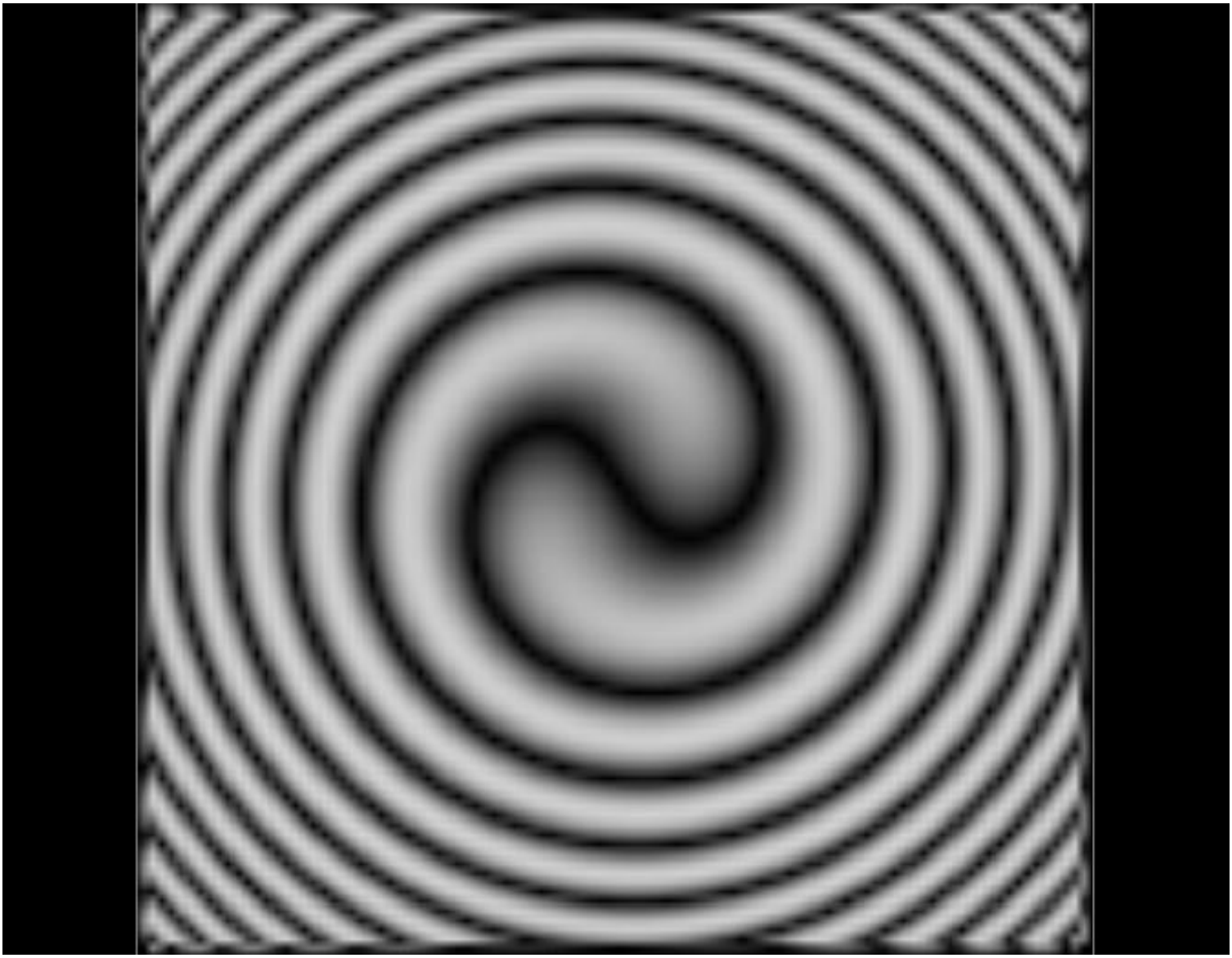

Fig. 4. The electric potential of the electron showing the double spiral of charge layers.

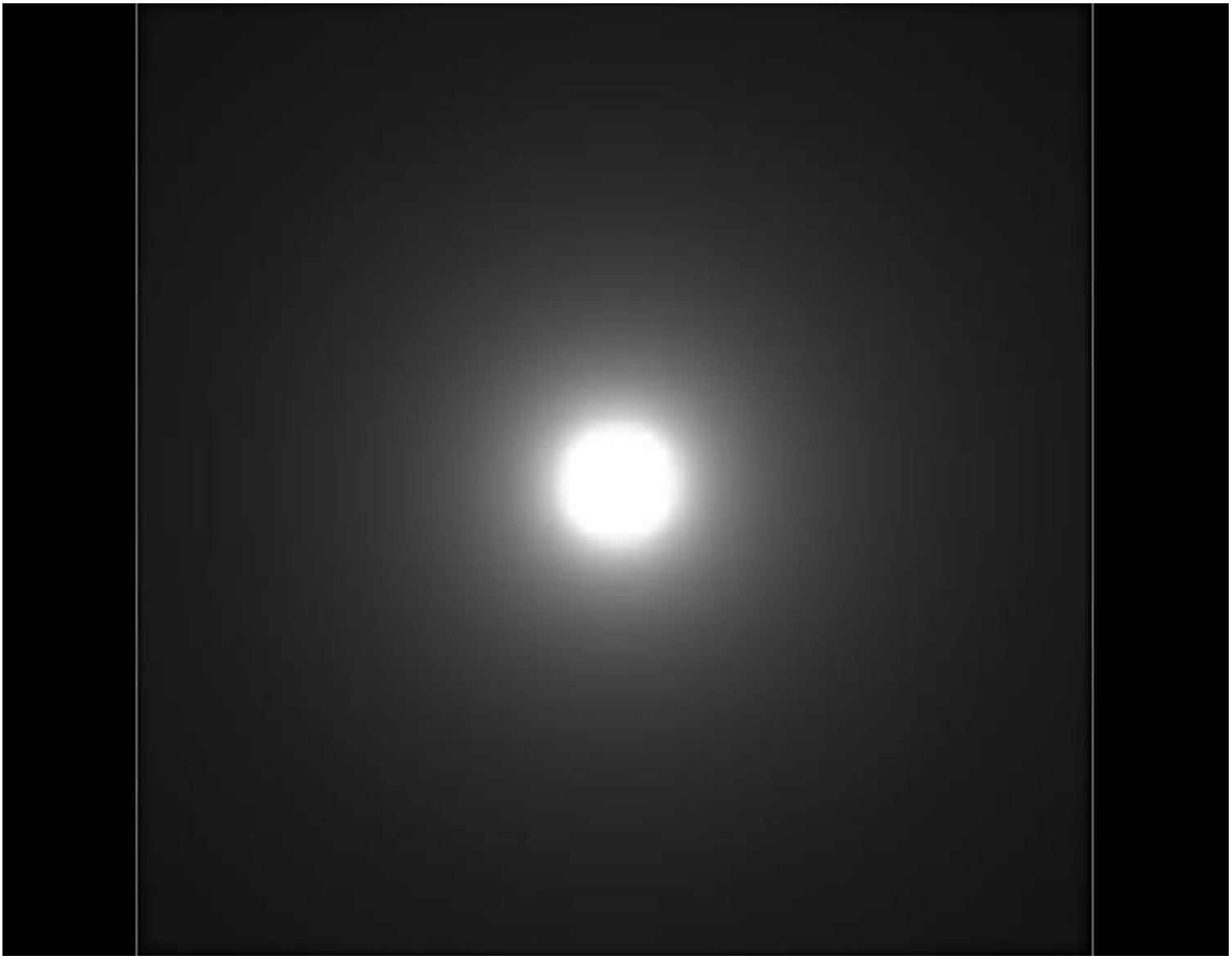

Fig. 5. The electric potential of the electron with the small scale wave function undulations smoothed out (the individual charge layers are not visible).

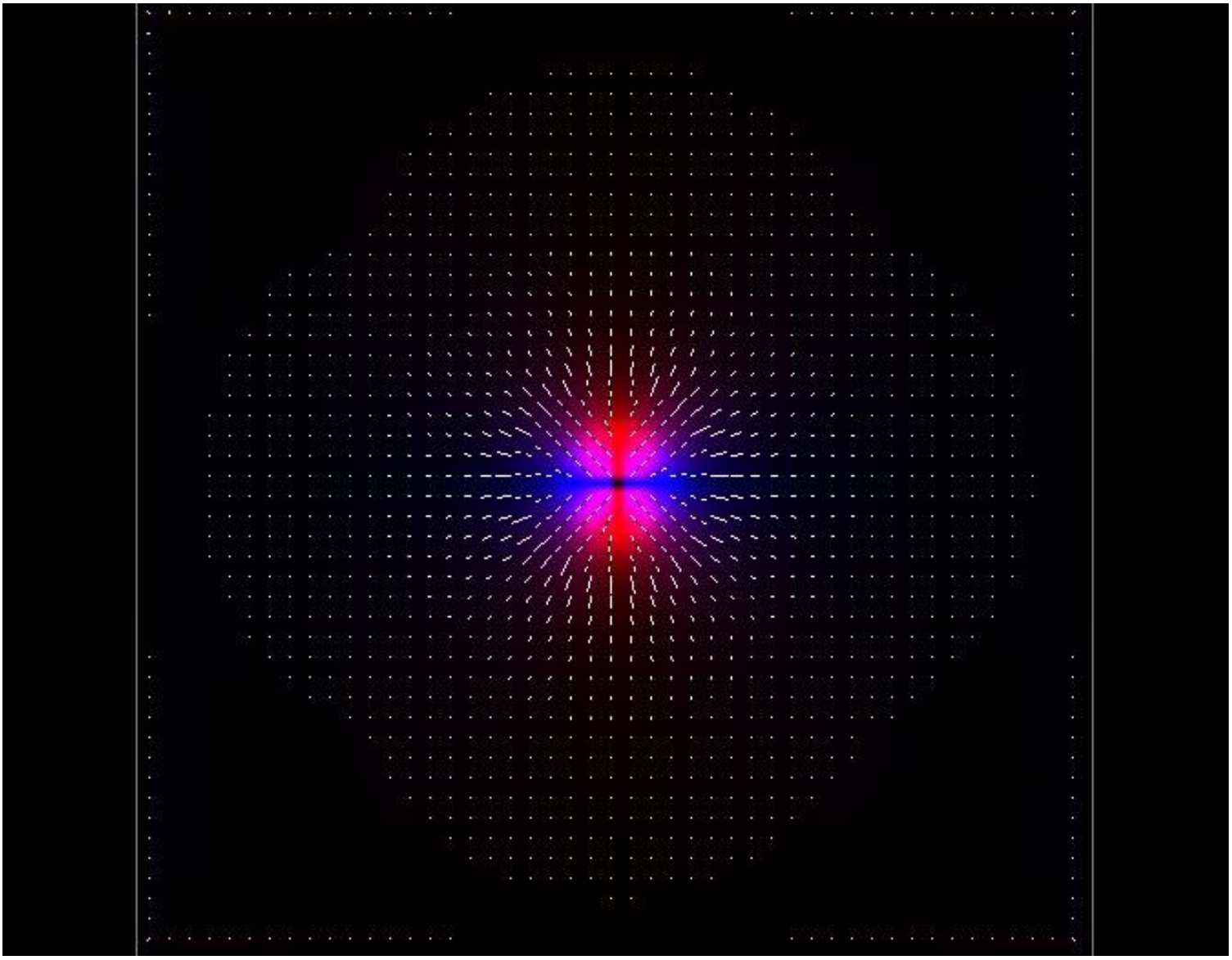

Fig. 6. The electric field of the electron with the small scale wave function undulations smoothed out.

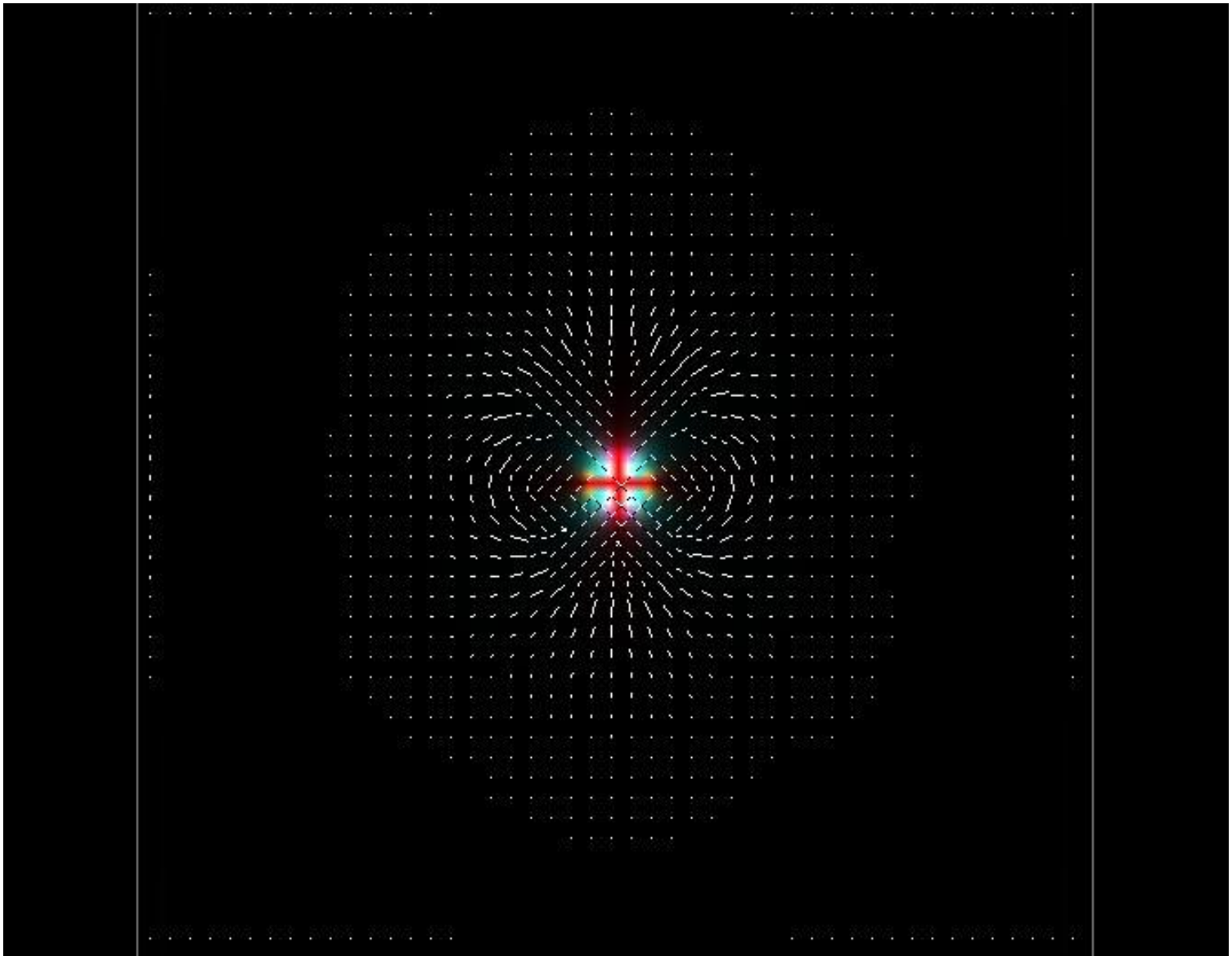

Fig. 7. The magnetic field of the electron viewed from the side (spin axis is vertical) with the small scale wave function undulations smoothed out. The vectors into/out of the page are not shown in order to reveal the nice magnetic field lines.

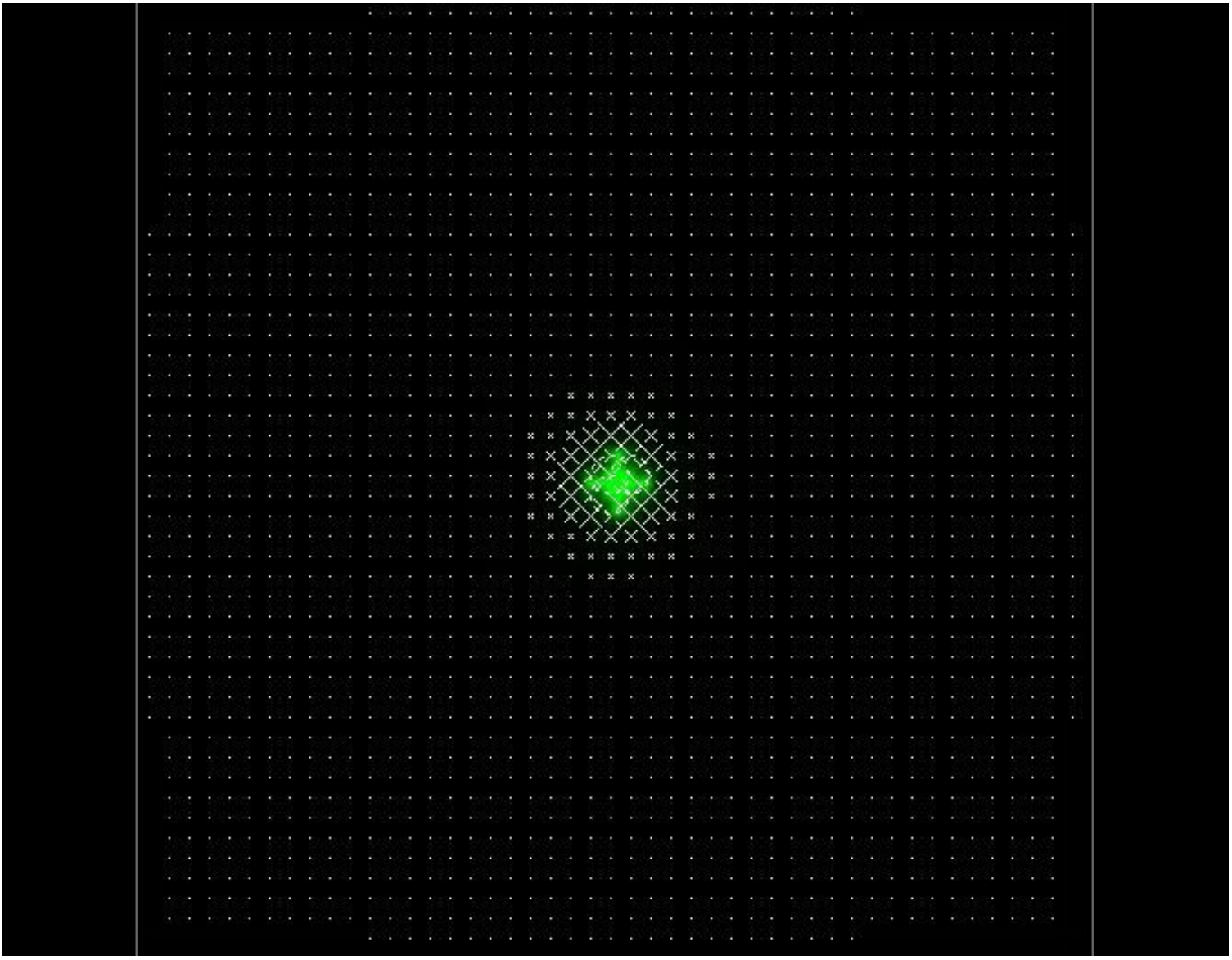

Fig. 8. The magnetic field of the electron viewed from the top (looking down the spin axis) with the small scale wave function undulations smoothed out.

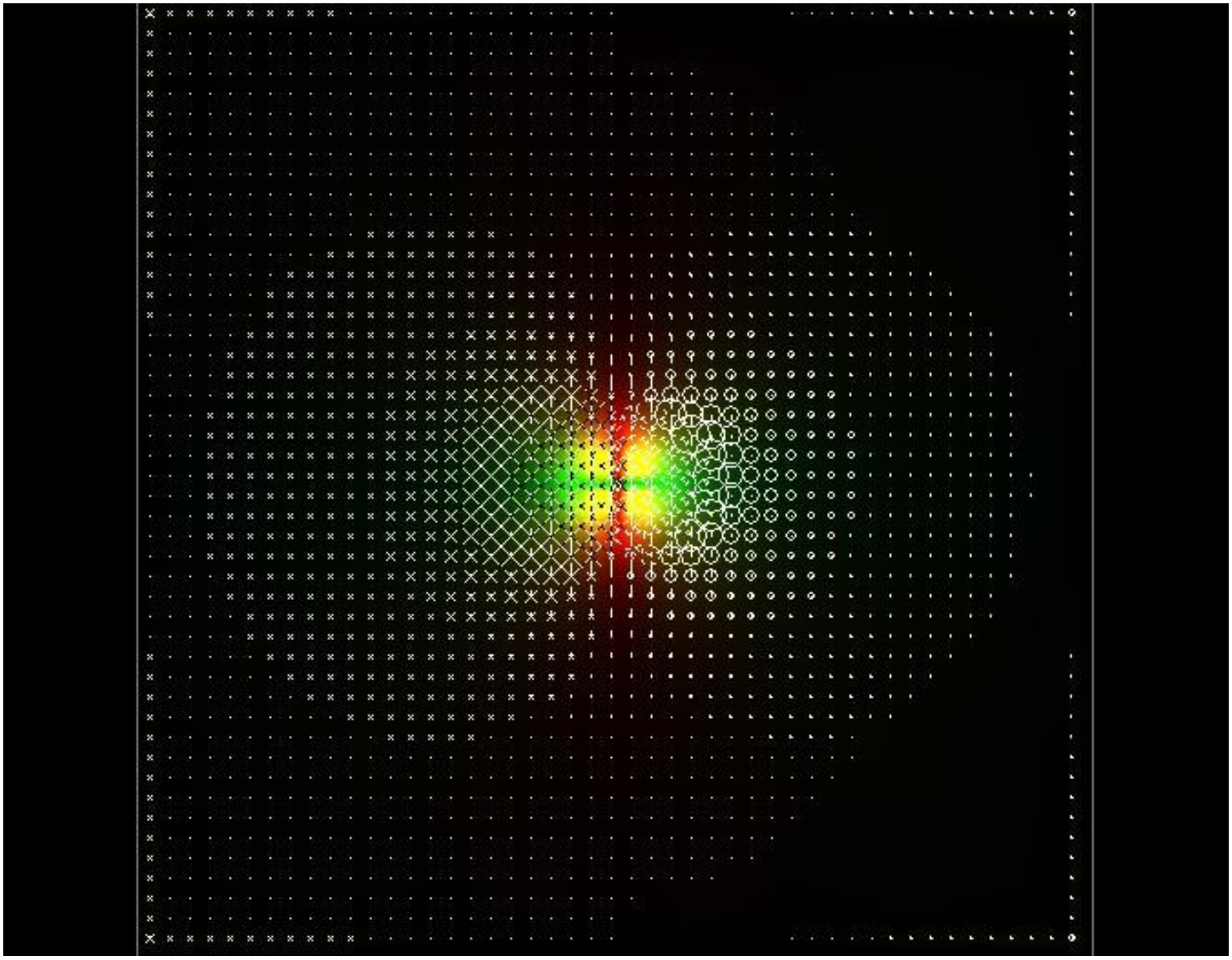

Fig. 9. The vector potential field of the electron viewed from the side (the spin axis is vertical) with the small scale wave function undulations smoothed out.

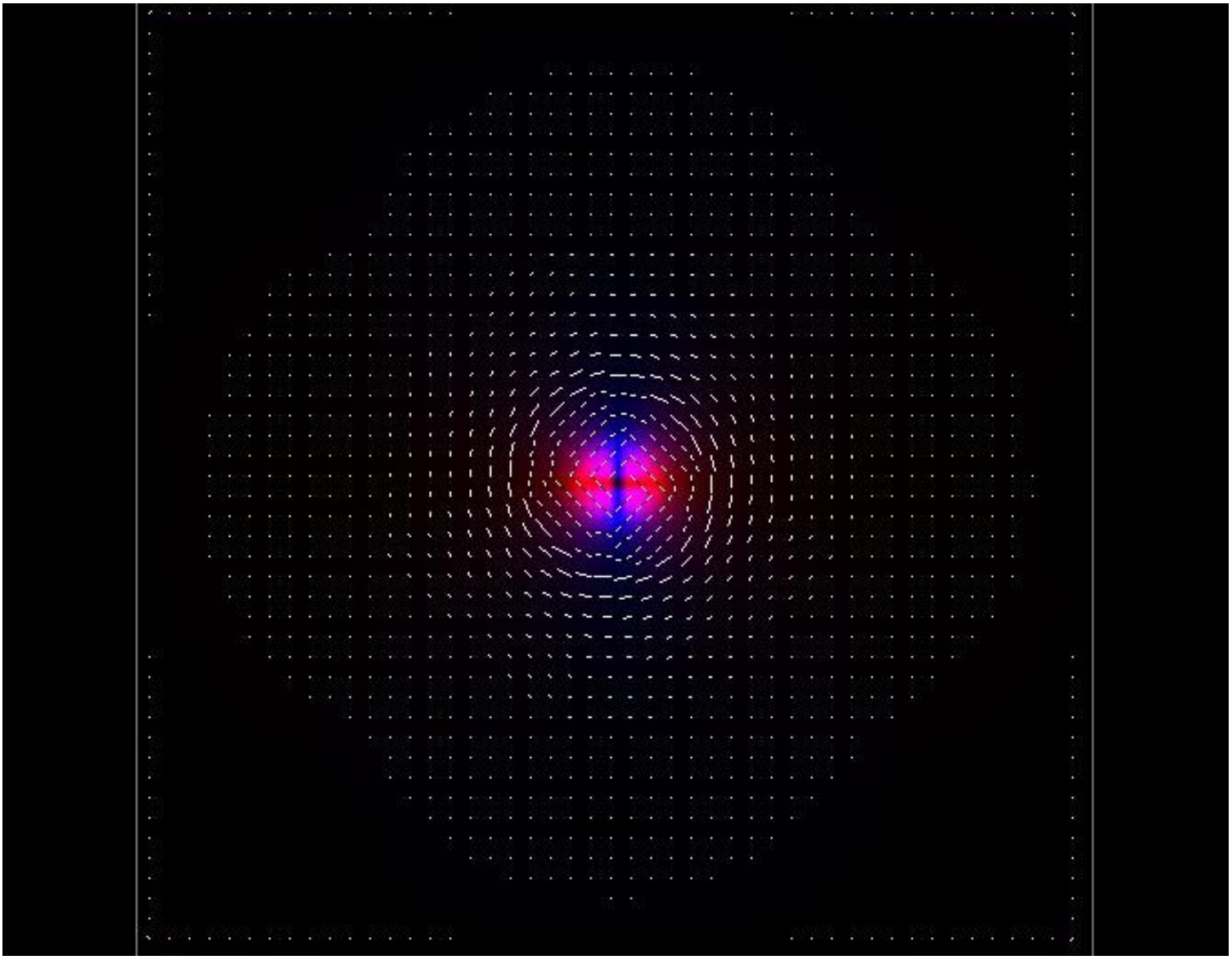

Fig. 10. The vector potential field of the electron viewed from the top (looking down the spin axis) with the small scale wave function undulations smoothed out. Note how the energy of the particle flows around the spin axis in closed loops.

10.2 Field Calculation Code from the Model

This is a portion of the model I wrote to model the Electron/Positron and their associated fields; such as Electric, Magnetic, Vector Potential fields. It is written in the Delphi language and is the function that calculates the fields from the mathematical wave function.

```
1.procedure TForm1.RecalcFields(scr:smallint);
2.var
3.  Current_Ex,Current_Ey,Current_Ez: extended;
4.  Current_Bx,Current_By,Current_Bz : extended;
5.  r,x,y,z,unit_x,unit_y,unit_z,k : extended;
6.  theta, delta, theta_const, expTheta, lnTheta, term0, term1, term2, term3 : extended;
7.  normal_x,normal_y,normal_z,dir_x,dir_y,dir_z : extended;
8.  scalar_amp, Vector_amp, SpinConstant, E_amp : extended;
9.  NewScreen : smallint;
10. xpos,ypos,zpos,midx,midy,midz:smallint;
11. ThisGroup,NewGroup: PointGrp;
12. vect,CurlVect,DivVect: vector;
13. Scalar_Group: ScalarGrp;
14. VectGrp: VectorGrp;
15. I: Integer;
16.
17.begin
18.
19. if scr=0 then NewScreen:=1 else NewScreen:=0; {determine which data to update}
20.
21. if not Flip_YZ then begin
22.
23.   midx:=Trunc(GridWidth/2);
24.   midy:=Trunc(GridHeight/2);
25.   midz:=Trunc(GridDepth/2);
26.
27.   SpinConstant:=( Hhat / ElectronMass ); // Metres^2/(Radians*Second)
28.   delta := ( sqrt(2) * ElectronCharge * Hhat ) / ( 4 * Pi * ElectronMass * SpeedOfLight * Permittivity );
29.
30.   // theta_const is in Radians/Second ( i.e. the same as solving E = hf for f, where E=mc^2, and h=2*Pi*Hhat,
31.   // then converting f to angular frequency w, via w = 2*Pi*f )
32.   // ( theta_const could be, equivalently : - c^2/SpinConstant )
33.   theta_const:=( -ElectronMass * sqr(SpeedOfLight) ) / Hhat;
34.
35.   k:=1/SpeedOfLight; // Seconds/Metre
36.
37.   ////////////////////////////////////
38.   // Thus the Total Electron Wave Equation (Ye) is:
39.   //
```

```
40.  // Ye = ((sqrt(2) * i * Qe*Hhat) / (4*Pi*Me*c*Eo )) * Exp( ( - i * Me * c^2 / Hhat ) * ( T – r^2/2c ) )
41.  //
42.  // and the Electric Potential div(psi) in spherical coordinates is
43.  //
44.  // V = ((sqrt(2) * Qe) / ( 4 * Pi * r * Eo )) * Exp( ( - i * Me * c^2 / Hhat ) * ( T – r^2/2c ) )
45.  //
46.  // Where:
47.  // Ye is Electron Wave Function (psi)
48.  // Qe is Electron's Charge
49.  // Pi is 3.14159 etc
50.  // Eo is the Permittivity of free space
51.  // Exp is the Exponential function
52.  // i is the Complex number (square root of -1)
53.  // Me is the Mass of an Electron
54.  // c is the speed of light
55.  // Hhat is the reduced Plancks constant ( i.e. h/(2*Pi) )
56.  // T is Time
57.  // r is the radial distance from the center of the Electron
58.  //
59.  // exp(-theta) = cos(theta) - isin(theta)
60.  // using x,y,z coordinates:
61.  // x = cos(theta)
62.  // y = -sin(theta)
63.
64.  // theta:=theta_const*(Time - k*r*r/2);
65.  //
66.  // term1:=delta/r
67.  // term2:=cos(theta);
68.  // term3:=-sin(theta);
69.  //
70.  // if ( ViewTop ) then begin // Assign values to x, y, z coordinates, depending on view from the top or side.
71.  //   x:=term1 * term2;
72.  //   y:=term1 * term3;
73.  //   z:=0;
74.  // end
75.  // else begin
76.  //   x:=term1 * term2;
77.  //   y:=0;
78.  //   z:=term1 * term3;
79.  // end;
80.  ////////////////////////////////////
81.
82.  for xpos:=0 to GridWidth-1 do begin {scan grid's x coords}
83.    for ypos:=0 to GridHeight-1 do begin {scan grid's y coords}
84.      for zpos:=0 to GridDepth-1 do begin {scan grid's z coords}
85.        ThisGroup:=PointGroup(scr, xpos, ypos, zpos);
86.
87.        x:= xpos - midx;
88.        y:= ypos - midy;
```

```
89.       z:= zpos - midz;
90.
91.       r:=sqrt( sqr(x) + sqr(y) + sqr(z) );
92.       if ( r < 0.00000000001 ) then r:=0.00000000001;   // prevent divide by zero errors
93.
94.       unit_x:= x/r;
95.       unit_y:= y/r;
96.       unit_z:= z/r;
97.
98.       r:=r*(ActualWidth/GridWidth);   // get actual distance in metres
99.       if ( r < 0.00000000001 ) then r:=0.00000000001;   // prevent divide by zero errors
100.
101.      ////////////////////////////////////
102.      /// WAVE FUNCTION TO TEST
103.      ///
104.      case StartOption of
105.       1: begin
106.
107.       if ( electron ) then begin              // if electron being modelled
108.         theta:=theta_const*(Time - k*r*r/2);
109.         term1:=delta/r;
110.       end
111.       else begin                              // if positron being modelled
112.         theta:=theta_const*(Time + k*r*r/2);
113.         term1:=-delta/r;
114.       end;
115.
116.       term2:=cos(theta);
117.       term3:=-sin(theta);
118.
119.       // Assign values to x, y, z coordinates, depending on view from the top or side.
120.       with points[NewScreen,xpos,ypos,zpos].PsiVect do begin
121.         if ( ViewTop ) then begin
122.           x:=term1 * term2;
123.           y:=term1 * term3;
124.           z:=0;
125.         end
126.           else begin
127.           x:=term1 * term2;
128.           y:=0;
129.           z:=term1 * term3;
130.         end;
131.       end;
132.       points[NewScreen,xpos,ypos,zpos].Psi := term1;
133.       end;
134.     end;
135.     ///
136.     ////////////////////////////////////
137.
```

```
    end;
   end;
  end;   // end {scan grid's x coords}

  for xpos:=0 to GridWidth-1 do begin {scan grid's x coords}
   for ypos:=0 to GridHeight-1 do begin {scan grid's y coords}
    for zpos:=0 to GridDepth-1 do begin {scan grid's z coords}

     ThisGroup:=PointGroup(scr, xpos, ypos, zpos);
     NewGroup:=PointGroup(NewScreen, xpos, ypos, zpos);

     with points[NewScreen,xpos,ypos,zpos] do begin
      if (smoothing) then begin                    // If smoothing of the wave-function's undulations is enabled.
       x:= xpos - midx;
       y:= ypos - midy;
       z:= zpos - midz;

       r:=sqrt( sqr(x) + sqr(y) + sqr(z) );
       if ( r < 0.004 ) then r:=0.004;   // prevent divide by zero errors
       r:=r*(ActualWidth/GridWidth);   // get actual distance in metres
       if ( r < 0.00000000001 ) then r:=0.00000000001;   // prevent divide by zero errors

       ElectricPotential:=ElectronCharge/(4*Pi*r*Permittivity);
      end
      else begin
       VectGrp:=VectorGroup(NewGroup, PSI_VECTOR_FIELD);
       ElectricPotential:=VectDiv(VectGrp);
      end;
     end;
    end;
   end;
  end; // end {scan grid's x coords}

  for xpos:=0 to GridWidth-1 do begin {scan grid's x coords}
   for ypos:=0 to GridHeight-1 do begin {scan grid's y coords}
    for zpos:=0 to GridDepth-1 do begin {scan grid's z coords}

     ThisGroup:=PointGroup(scr, xpos, ypos, zpos);
     NewGroup:=PointGroup(NewScreen, xpos, ypos, zpos);

     { ThisGroup's points are assigned as follows: P3          P5
                                                P1 P0 P2
                                                   P4      P6
     Where P5 & P6 are in the Z plane (P5 at the back and P6 at the front) }

     x:= xpos - midx;
     y:= ypos - midy;
     z:= zpos - midz;

```

```
187.      r:=sqrt( sqr(x) + sqr(y) + sqr(z) );
188.      r:=r*(ActualWidth/GridWidth);   // get actual distance in metres
189.
190.      if ( r < 0.00000000001 ) then r:=0.00000000001;   // prevent divide by zero errors
191.
192.      // Electric Field is: -div of ElectricPotential Field - d/dt of Vector Potential field
193.      Scalar_Group:=ScalarGroup(NewGroup, ELECTRIC_POTENTIAL_FIELD);
194.
195.      // This is the negative div of ElectricPotential Field (will add the rest once the Vector Potential is known)
196.      points[NewScreen,xpos,ypos,zpos].Electric:=ScalarGrad(Scalar_Group);
197.      with points[NewScreen,xpos,ypos,zpos]. Electric do begin    // Make negative
198.        x:= -x;
199.        y:= -y;
200.        z:= -z;
201.      end;
202.
203.      // From Schrodinger's wave equation:
204.      // d(psi)/dt = i * Hhat/ElectronMass * Laplacian(psi)
205.      //
206.      // Note: div(V) = Laplacian(psi)
207.      // SpinConstant = Hhat/ElectronMass
208.      //
209.      // So…
210.      // d(psi)/dt = i*SpinConstant*div(V)
211.      //
212.      // VectorPotential = (1/c)*d(psi)/dt
213.      //
214.      // A is orthogonal to and proportional to the div(V) vector
215.      // (multiplying by i rotates the vector 90 degrees in the complex plane).
216.      // so use the Normal vector to the div(V) vector and the Static Electric field amplitude (E_amp).
217.
218.      // get amplitude of Static Electric field component
219.      E_amp:=VectSize(points[NewScreen,xpos,ypos,zpos].Electric);
220.
221.      // Calculate the Unit & Normal vectors of the div(V) vector (depending on view from top or side)
222.      with points[NewScreen,xpos,ypos,zpos]. Electric do begin
223.        unit_x:= x/E_amp;
224.        unit_y:= y/E_amp;
225.        unit_z:= z/E_amp;
226.
227.        if ( ViewTop ) then begin
228.          normal_x:=unit_y;
229.          normal_y:=-unit_x;
230.          normal_z:=unit_z;
231.        end
232.        else begin
233.          normal_x:=unit_z;
234.          normal_y:=unit_y;
235.          normal_z:=-unit_x;
```

```
236.           end;
237.         end;
238.
239.         with points[NewScreen,xpos,ypos,zpos].VectorPotential do begin
240.           if (smoothing) then begin
241.             x := normal_x*SpinConstant*E_amp/SpeedOfLight;
242.             y := normal_y*SpinConstant*E_amp/SpeedOfLight;
243.             z := normal_z*SpinConstant*E_amp/SpeedOfLight;
244.           end
245.           else begin
246.             x := -(1/TimeStep)*(points[NewScreen,xpos,ypos,zpos].PsiVect.x -
    points[scr,xpos,ypos,zpos].PsiVect.x)/SpeedOfLight;
247.             y := -(1/TimeStep)*(points[NewScreen,xpos,ypos,zpos].PsiVect.y -
    points[scr,xpos,ypos,zpos].PsiVect.y)/SpeedOfLight;
248.             z := -(1/TimeStep)*(points[NewScreen,xpos,ypos,zpos].PsiVect.z -
    points[scr,xpos,ypos,zpos].PsiVect.z)/SpeedOfLight;
249.           end;
250.         end;
251.         // Electric Field is: -div of ElectricPotential Field - d/dt of Vector Potential field
252.         // In Electric, we already have negative div of ElectricPotential Field, now subtract d/dt of Vector Potential field
253.
254.         with points[NewScreen,xpos,ypos,zpos].Electric do begin
255.           // E = -div(V) - (1/c)*dA/dt
256.           if (Time <> 0) then begin
257.             x := x - (1/TimeStep)*(points[NewScreen,xpos,ypos,zpos].VectorPotential.x-
    points[scr,xpos,ypos,zpos].VectorPotential.x)/SpeedOfLight;
258.             y := y - (1/TimeStep)*(points[NewScreen,xpos,ypos,zpos].VectorPotential.y-
    points[scr,xpos,ypos,zpos].VectorPotential.y)/SpeedOfLight;
259.             z := z - (1/TimeStep)*(points[NewScreen,xpos,ypos,zpos].VectorPotential.z-
    points[scr,xpos,ypos,zpos].VectorPotential.z)/SpeedOfLight;
260.           end;
261.         end;
262.
263.         // Magnetic Field is Curl of Vector Potential Field
264.         VectGrp:=VectorGroup(NewGroup, VECTOR_POTENTIAL_FIELD);
265.         CurlVect:=VectCurl(VectGrp);
266.
267.         // Calculate Magnetic B Field
268.         with points[NewScreen,xpos,ypos,zpos].Magnetic do begin
269.           x:=Permeability*CurlVect.x;
270.           y:=Permeability*CurlVect.y;
271.           z:=Permeability*CurlVect.z;
272.         end;
273.       end;
274.     end;
275.   end; // end {scan grid's x coords}
276. end; //if Flip_YZ
277.end;
```

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