General divisibility's criteria

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SUMMARY

This work is a study of divisibility and these criteria, in which we will give general relationships and divisibility criteria. We begin this work by answering the following question: what conditions should check the digits dialing the number to make it divisible by d? Among the most known and used criteria are the divisibility by 2, 3, 5, 11...

We know well that for D∈ $\mathbb{Z}$, It is said that it is divisible by 3 If the sum of the digits component D is divisible by 3, and D is written as $\overline{a_1 a_2, \dots a_i}$ *in base 10 D is said divisible by 3, if and only if* $a_1 + a_2 + a_3 + \cdots + a_i$ *is divisible by 3. There are also criteria of divisibility for numbers less than 20, for d = p + 10 and p < 10, we have this result:* $\overline{a_1 a_2, \dots a_i}_{(10)}$ *is divisible by d, if and only if* $p^0 a_i - p^1 a_{i-1} + p^2 a_{i-2} - p^3 a_{i-3} \cdots p^{(i-1)} a_1$ *is divisible by d. We note that for p = 1 it realizes the divisibility of 11, and for p = 2 is right and we have also, if 12 divides* $\overline{a_1 a_2, \dots a_i}_{(10)}$ *so 12 divides* $\overline{a_1 a_2, \dots a_i}_{(2)}$

We give a criteria of divisibility by 7, let D = $\overline{a_1 a_2, \dots a_i}_{(10)}$ *an integer, we say that D is divisible by 7, if* $3\overline{a_1 a_2 \dots a_{i-1}}_{(10)} - 2a_i - (i-2)$ *is divisible by 7. We can also write that if 7 divides* $\overline{a_1 a_2, \dots a_i}_{(10)}$*, it divides* $1a_i + 3a_{i-1} + 2a_{i-2} + 132\,\overline{a_1 a_2 \dots a_{i-3}}_{(10)}$*, and divides*

$$\sum_{k=0}^{i-1} (10^k - k') a_{i-k}$$

In this part we give a property for compounds three-digit numbers. Let be D an integer written as $\overline{a_1 a_2 a_3}_{(10)}$ *there are some criteria of divisibility, let d ∈* $\mathbb{Z}$ *, and t ∈* $\mathbb{N}^*$, $d = 8 - t$ *is said that* $\overline{a_1 a_2 a_3}_{(10)}$ *is divisible by d if and only if* $\overline{a_2 a_3}_{10} - (10 \times 2^{t+1} a_1)$ *is divisible by d. In this relationship it is true for 0≤d≤ 9, and we can generalize and we obtain for d ∈* $\mathbb{Z}$ *, and l ∈* $\mathbb{N}^*$, $d = 8 - t$ *we say that* $\overline{a_1 a_2, \dots a_i}_{(10)}$ *is divisible by d if and only if* $\overline{a_{i-j} a_{i-j+1} a_{i-j+2} \cdots a_{i-1} a_i}_{(10)} - 2^{t+1}(10\overline{a_1 a_2 \cdots a_{i-j-1}}_{(10)})$ *is divisible by d . Now we take an integer d equal in* $\overline{b_1 b_2}_{(10)}$ *and we study the divisibility of*

$\overline{a_1a_2}_{(10)}$ we have if $\overline{b_1b_2}_{(10)}$ divides $\overline{a_1a_2}_{(10)}$ then it divides $[(a_1 \times b_2) - (a_2 \times b_1)]$, it divides

$[(b_1 \times b_2 \times a_1) - a_2]$ now we take D consisting of three digits, we obtain if $\overline{b_1b_2}_{(10)}$ devides $\overline{a_1a_2a_3}_{(10)}$ we find that $[(b_1 \times b_2 \times a_1a_2) - a_3]$ is a multiple of $\overline{b_1b_2}_{(10)}$. In the same sense, $\overline{a_2a_3}_{(10)} + N^2a_1$ is a multiple of $\overline{b_1b_2}_{(10)}$ and N equal to $\overline{b_1b_2}_{(10)} - 10$. For a number with n digits we have to say as a generalization, $\overline{a_1a_2 \dots a_\iota}_{(10)}$ is divisible by d and d equal to $10 + N$ if d divides M. when

$$M = \sum_{k=i-1}^{i} 10^{i-k}a_k + N^2 \sum_{k=1}^{i-2} 10^{i-2-K}a_k$$

In this part we give relations for the compound numbers of n figures, are d and D integers, d= $\overline{b_1b_2 \dots b_J}_{(10)}$ and D= $\overline{a_1a_2 \dots a_\iota}_{(10)}$, it is said that D is divisible by d, if only if d divides $(10 - 11d)\overline{a_1a_2 \dots a_{\iota-1}}_{10} + (1-d)a_i$, d divides $(10 - d)\overline{a_1a_2 \dots a_{\iota-1}}_{10} + a_i$ or d divides $(100 - d)\overline{a_1a_2 \dots a_{\iota-2}}_{10} + \overline{a_{\iota-1}a_\iota}_{10}$ for this relationship we can change the base 10 by the following bases $(10^2 \pm l.d)(10^2 \pm d^l)(10^2 \pm l.d^l)$. as we can generalize all this by writing, if D is divisible by d then $(10^m - d)\overline{a_1a_2 \dots a_{\iota-m}}_{10} + \overline{a_{\iota-m-1} \cdots a_\iota}_{10}$ is divisible by d we add also this generalization, let (n, N) two integers, and (m, N ') two natural numbers, we say if d divides D so d divides $\left(n10^m - Nd^{N'}\right)\overline{a_1a_2 \dots a_{\iota-m}}_{10} + n\left(\overline{a_{\iota-m-1} \cdots a_\iota}_{10}\right)$. sometimes we find numbers that check divisibility by two bases, in base 10 and base μ, like 11 and 13, we have if d divides D in base 10 it divides it in base 11 and in 13, we have also if d divides D in base 10 so it divides it in base 14 but we must add $(10 - \mu)$ we write :

$\overline{d}_{(10)} / \overline{D}_{(10)} \Rightarrow \overline{d}_{(\mu)} / (\overline{D}_{(\mu)} + (10 - \mu))$,

We have for 26 $\overline{d}_{(10)} / \overline{D}_{(10)} \Rightarrow \overline{d}_{(\mu)} / ((\overline{D}_{(d-\mu)} + (10 - \mu))$,

And for 28 we have $\overline{d}_{(10)} / \overline{D}_{(10)} \Rightarrow \overline{d}_{(\mu)} / (\overline{D}_{(\mu-d)} + (10 - \mu))$, *if $\mu - d < 0$ we use* $|\mu - d|$.

Progressive divisibility

Now we give a relationship in which we will change the base from a big value to a small one in order to facilitate the divisibility. Are D= $\overline{a_1a_2 \dots a_\iota}_{(10)}$ and d= $\overline{b_1b_2 \dots b_J}_{(10)}$, two integers related, if d divides D and j ≥ 2 it exists an integer $\xi < d$, $\xi = D \wedge d$ then d divides $(10 - \xi)\overline{a_1a_2 \dots a_{\iota-1}}_{(10)} + a_i$ and it divides $(10 \pm \xi)\overline{a_1a_2 \dots a_{\iota-1}}_{(10+\xi)} + a_i$ the same thing for $(10 \pm \xi)\overline{a_1a_2 \dots a_{\iota-1}}_{(10-\xi)} + a_i$, we can use these:

$$(10^2 \pm l.\xi)\left((10^2 \pm \xi^l)\right)(10^2 \pm l.\xi^l).$$

we want to know if $\overline{a_1a_2}_{(10)}$ *is divisible by d and so we try to minimize the number by fewer, to simplify the knowledge if it divisible or not by d, we have d divides* $\overline{a_1a_2}_{(10)}$ **so it divides** $(10-d)a_1+a_2$, *we take* $\overline{\alpha_1\alpha_2}_{(10)}=(10-d)a_1$, *and takes l an integer equal to* $\left|d.a_1(10)^{-1}\right|$ *we have* $\alpha_1=|a_1-l|$, *and* $\alpha_2=|10l-da_1|$ *We do the same operation l:* $\overline{\beta_1\beta_2}_{(10)}=\overline{\alpha_1\alpha_2}_{(10)}+a_2$ *then we obtained :* $\beta_1=|a_1-l+1|$, *and*

$\beta_2=|10l-da_1+a_2-10|$, *if* $\beta_2<0$ **we have** $10-\beta_2$ **becomes** β_1-1, *Sometimes to facilitate the calculate is whether d / D must go through several steps (operations) that is to say* $d/\overline{a_1a_2}_{(10)}\mapsto d/\overline{b_1b_2}_{(10)}\mapsto d/\overline{c_1c_2}_{(10)}\mapsto d/\overline{e_1e_2}_{(10)}$

We must define the values e_1 *et* e_2 *from primary values , but first we must know the number of steps we done .for example* $d/\overline{a_1a_2}_{(10)}\mapsto d/\overline{b_1b_2}_{(10)}\mapsto d/\overline{c_1c_2}_{(10)}$ *the number of steps we made is two when the values* c_1 *and* c_2 *are :* $c_1=|2+a_1-(l_1+l_2)|$, *and* $c_2=|10(l_1+l_2)-20-d(2a_1+l_1-1)|$, *where* $l_1=\left|d.a_1(10)^{-1}\right|, l_2=\left|d.b_1(10)^{-1}\right|$ *. And for three steps we have* $d/\overline{a_1a_2}_{(10)}\mapsto d/\overline{b_1b_2}_{(10)}\mapsto d/\overline{c_1c_2}_{(10)}\mapsto d/\overline{e_1e_2}_{(10)}$ *we have the values and* e_1 **and** e_2 *are:* $e_1=|3+a_1-(l_1+l_2+l_3)|$ *and*

$e_2=|10(l_1+l_2+l_3)-30-d(3a_1+l_2+1)|$. *We will generalize these relationships for N steps, we have: Let* $d/\overline{a_1a_2}_{(10)}$ *accepte N steps and obtained* $\overline{m_1m_2}_{(10)}$ *to define the values* m_1 *and* m_2 *were two cases: For N is odd we have* $m_1=|N+a_1-(l_1+l_2+\cdots+l_N)|$ *and* $m_2=|10(l_1+l_2+\cdots+l_N)-10N-d(Na_1+l_{N-1}+1)|$. *For N is even we obtained* $m_1=|N+a_1-(l_1+l_2+\cdots+l_N)|$ *and*

$m_2=|10(l_1+l_2+\cdots+l_N)-10N-d(Na_1+l_{N-1}+1)|$ *We will repeat this operation but with a three-digit number* $\overline{a_1a_2a_3}_{(10)}$ *we follow the same method and we obtain if* $\overline{b_1b_2b_3}_{(10)}$ *divides* $\overline{a_1a_2a_3}_{(10)}$ *we have:*

$\alpha_1=|a_1-l_1|, \alpha_2=|10l_1-(da_1+a_2-l_2)|$, $\alpha_3=|10l_2-da_2|$
and $l_1=\left|d.a_1(10)^{-1}\right|$, $l_2=\left|d.\overline{a_1a_2}_{(10)}(10)^{-2}\right|$ *and to determine* b_1, b_2 *and* b_3*:*
$b_1=|a_1+1-l_1|$, $b_2=|10l_1-da_1-a_2+l_2-9|$ *and* $b_3=|a_3+10l_2-da_2-10|$
$l_1=\left|d.a_1(10)^{-1}\right|$, $l_2=\left|d.\overline{a_1a_2}_{(10)}(10)^{-2}\right|$. *We generalize this with numbers had n digits written as* $\overline{a_1a_2\ldots a_i}_{(10)}$ *we will start with one step* $(N=1)$ *that through the values* α_h *to have the values* b_h *so we have* $\alpha_1=10l_1+l_2$ *,*
$\alpha_2=10l_2-d(\overline{a_1a_2\ldots a_8}_{(10)}+a_9)l_3$ $\quad\alpha_3=10l_3-d(\overline{a_2\ldots a_8}_{(10)}+a_9)l_4$ *and*
$\alpha_4=10l_4-d(\overline{a_3\ldots a_8}_{(10)}+a_9)l_5\ldots$ $\quad\alpha_8=10l_8-d(\overline{a_7a_8}_{(10)}+a_9)l_9$ *and*
$\alpha_9=10l_2-d(a_8+a_9)$ *It was also*

$\alpha_i=10l_i-da_{i-1}$, $\alpha_{i-1}=10l_{i-1}-da_{i-2}+a_{i-1}$. *And therefore the values of* $b_1, b_2\ldots b_N$ *we have:* $b_1=\alpha_1+1$; $b_2=\alpha_2$ *;...;* $b_N=\alpha_N-(N-2)$ *and for*

$l_{10}=\left|d.\overline{a_1a_2\ldots a_9}_{(10)}.(10)^{-9}\right|$

<u>Divisibility of numbers often used</u>

In this part of work we give the criteria divisibility of numbers often used, we take d and N two integers we say that N is divisible by d if d divides $(1^2+2^2+3^2+\cdots+N^2)-[(N-1)(1+2+3+\cdots+N)]$ *we take an integer* $N=a+n$ *it said that's divisible by d if d divides* $a^2+n+2an+(4+6+8+\cdots+2n)$ *we can take N a natural number written as* $N=1+2+3+\cdots+M$ *it said that's divisible by d if d divides* $(M-1)^{-1}\big((M+1)+2(M+1)+3(M+1)+\cdots+(M-1)(M+1)\big)$ *or we write d divides* $N(M+1)(M-1)^{-1}$*, In the same purpose it said also that is divisible by d if* $M(M-1)^{-1}\big(2+2(M+1)+3(M+1)+\cdots(M-1)(M+1)\big)$ *or it divides* $M+2\big(2+4+6+8+\cdots+2(M-1)\big)$*, in the same ame we say that d divides* $4N-M^2$ *when M is even number, it divides also* $M+\big(2+4+6+8+\cdots+(2M-4)\big)$*, now let* $N'=2N$ *we have* N' *is divisible by d if* $-M+3+5+7+\cdots+(2M+1)$*. We can find numbers on forme* N^2*, so in what conserning this kind of numbers we say that is divisible by d if d divides* $\big(1+3+5+7+9+\cdots+(2N-1)\big)$*.*

Divisibility of A^N

Either A an integer and N is a positive integer we have if d divides A^N *so it divides* $A\big[(A-1)\times\big(A^2+A^3+A^3+\cdots+A^{n-1}\big)\big]+A^3$ *we can write* $A=a+1$ *and we say that if d divides* $(a+1)^N$ *so it divides* $1+Na+\binom{n}{0}A^N+\binom{n}{1}A^{N-1}+\cdots+\binom{n}{N-1}A$*.*

In this part of work we study numbers in the form $\binom{k}{n}$*, let N a natural number equal to* $N=\binom{4}{0}(a+4)^3+\binom{4}{2}(a+2)^3+\binom{4}{4}(a)^3$ *we say that d divides N if d divides* $\binom{4}{1}(a+3)^3+\binom{4}{3}(a+1)^3$ *. let D in integer equal to* $\binom{N}{1}+\binom{N}{2}+\binom{N}{3}+\cdots+\binom{N}{N}$ *it said that is divisible by d if* 2^N *is divisible by d.*
let D is an integer and (a, n) two natural numbers/ $k\in\{0,1,2,3,\cdots,n\}/\ k=2\alpha$*. Let M* $/M=\binom{n}{0}(a+n)^{n-1}+\binom{n}{1}\big(a+(n-1)\big)^{n-1}+\binom{n}{2}\big(a+(n-2)\big)^{n-1}+\cdots+\binom{n}{n}(a)^{n-1}$ *we say that d divides M if it exists* $k'\in\{1,\cdots,n\}$ **and** $k=2\alpha'+1$ *which it exists* M'
$M'=\binom{n}{0}(a+n)^{n-1}+\binom{n}{1}\big(a+(n-1)\big)^{n-1}+\binom{n}{2}\big(a+(n-2)\big)^{n-1}+\cdots+\binom{n}{n}(a)^{n-1}$ *and d* M' *is divisible by d.we take an integer D equal to* $\binom{2N}{k}+\binom{2N+1}{k}$ *we have two cases the first is* $k=2\alpha$ *so d divides D if d divides* $\binom{2N+2}{k}+1$*, the second case is* $k=2\alpha+1$ *it said that D is divisible by d if d divides* $\binom{2N+2}{k}-1$*. we give D the value* $\binom{N}{k}+\binom{n+1}{k-1}$ *so we have if* $k=2\alpha+1$ *we say that d is divisible by d if* $\binom{N+1}{k}+N$ *is divisible by d, the second case when* $k=2\alpha$ **and** $k\le 2$ *we have d divides D if* $\binom{N+1}{k}+1$ *is divisible by d, the last case if* $2<k\le 4$ *we have if d divides* $\binom{N+1}{k}+2N$ *so it divides D. for simplify the expression if we have* $D=\binom{B}{A}$*, let n and k both natural numbers, and* $A=k+1$ *,* $B=N+2$ *we have if d divides* $\binom{B}{A}$ *so it divides* $\binom{N}{k}+\binom{N+1}{k}+\binom{N}{k+1}$*.*

References

Work done without references.