THE NON-TRIVIAL ZEROS OF THE RIEMANN ZETA FUNCTION

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ABSTRACT

This paper shows why the non-trivial zeros of the Riemann zeta function ζ will always be on the critical line Re(s) = 1/2 and not anywhere else on the critical strip bounded by Re(s) = 0 and Re(s) = 1, thus affirming the validity of the Riemann hypothesis.

MSC: 11-XX (Number Theory)

Keywords: series; solutions; non-trivial zeros; critical line

The Riemann hypothesis posits that all the non-trivial zeros of the zeta function ζ (shown below) on the critical strip bounded by Re(s) = 0 and Re(s) = 1 will always be at the critical line Re(s) = 1/2:-

$$\zeta(s) = \sum_{n=1} 1/n^s = 1 + 1/2^s + 1/3^s + 1/4^s + 1/5^s + \ldots \qquad (1)$$

This has been found to be true for the 1st. 10^{13} non-trivial zeros. The locations of these non-trivial zeros on the critical strip are described by a complex number $s = 1/2 + bi$ where the real part is 1/2 and i represents the square root of -1. It should be noted that the mathematical operations and logic of the complex numbers $a + bi$, where a and b are real numbers and i is the imaginary number square root of -1, are practically the same as for the real numbers and are even more versatile. For the zeta function ζ(s) shown above to be zero, its series would have to have both the positive terms and negative terms cancelling each other out, though the positive or "+" signs in the series may indicate positive values only which is misleading. The sum of this series is calculated with a formula, e.g., the Riemann-Siegel formula, or, the Euler-Maclaurin summation formula. Is there a possibility of any non-trivial zeros being off the critical line Re(s) = 1/2 on the critical strip between Re(s) = 0 and Re(s) = 1, e.g., at Re(s) = 1/4, 1/3, 3/4, or, 4/5, etc., the presence of any of which would disprove the Riemann hypothesis?

It had already been proven that there will not be zeros at Re(s) = 0 and Re(s) = 1. The 1st. 10^{13} non-trivial zeros are found only at the critical line Re(s) = 1/2. Nature appears to dictate that these zeros must appear only at Re(s) = 1/2, exactly mid-way in the critical strip bounded by Re(s) = 0 and Re(s) = 1 wherein the symmetry is perfect. "1/2" in the complex number 1/2 + bi, which is "square root", also appears to be compatible with and work fine with "i", which is "square root of -1" – both of them are square roots. 1/2 + bi has what is called a complex conjugate 1/2 - bi so that when 1/2 + bi and 1/2 - bi are added together the terms bi in both 1/2 + bi and 1/2 - bi will cancel out one another – in this way the troublesome i which does not actually make mathematical sense will be out of the way. 1/2 is also the reciprocal of the smallest prime and the smallest even number 2, which is significant. But there is a much more compelling reason why all the non-trivial zeros must lie on the critical line Re(s) = 1/2 and it is due to some important similarity to Fermat's last theorem.

The reasoning which follows will be reasoning by analogy, with Fermat's last theorem taken as an analogue, whereby the reasoning is that if it is true for Fermat's last theorem it will be true for something comparable in the Riemann hypothesis.

Fermat's Last Theorem

2 square numbers can be added together to form a 3rd. square, e.g., $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$. Fermat's last theorem states that for any 4 whole numbers x, y, z & n, there are no solutions to the equation $x^n + y^n = z^n$ when $n > 2$. (Such an equation involving whole numbers is known as a Diophantine equation.)

Fermat's last theorem is connected with Pythagoras' theorem which states that if x, y, z represent the lengths of the 3 sides of a right-angled triangle, x & y being the adjacent sides & z being the hypotenuse (the side opposite the right angle), then $x^2 + y^2 = z^2$. (Here x, y, z need not and may not be whole numbers, i.e., this equation needs not be a Diophantine equation.)

To put it another way, according to Fermat's last theorem, the following Diophantine equation which has power $n = 2$ is the only Diophantine equation with zeros or solutions (zeros and solutions are synonymous):-

$$x^2 + y^2 = z^2 \qquad (2)$$

The following is a partial list of Diophantine equations with their zeros:-

[1] $3^2 + 4^2 = 5^2$
$3^2 + 4^2 - 5^2 = 0$

[2] $5^2 + 12^2 = 13^2$
$5^2 + 12^2 - 13^2 = 0$

[3] $7^2 + 24^2 = 25^2$
$7^2 + 24^2 - 25^2 = 0$

[4] $8^2 + 15^2 = 17^2$
$8^2 + 15^2 - 17^2 = 0$

[5] $9^2 + 40^2 = 41^2$
$9^2 + 40^2 - 41^2 = 0$

[6] $11^2 + 60^2 = 61^2$
$11^2 + 60^2 - 61^2 = 0$

[7] $12^2 + 35^2 = 37^2$
$12^2 + 35^2 - 37^2 = 0$

[8] $13^2 + 84^2 = 85^2$
$13^2 + 84^2 - 85^2 = 0$

[9] $16^2 + 63^2 = 65^2$
$16^2 + 63^2 - 65^2 = 0$

[10] $20^2 + 21^2 = 29^2$
$20^2 + 21^2 - 29^2 = 0$

[11] $28^2 + 45^2 = 53^2$
$28^2 + 45^2 - 53^2 = 0$

[12] $33^2 + 56^2 = 65^2$
$33^2 + 56^2 - 65^2 = 0$

[13] $36^2 + 77^2 = 85^2$
$36^2 + 77^2 - 85^2 = 0$

[14] $39^2 + 80^2 = 89^2$
$39^2 + 80^2 - 89^2 = 0$

[15] $48^2 + 55^2 = 73^2$
$48^2 + 55^2 - 73^2 = 0$

[16] $65^2 + 72^2 = 97^2$
$65^2 + 72^2 - 97^2 = 0$

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There is some important similarity between Fermat's last theorem and the Riemann hypothesis, both of them being involved with series, which will be dealt with.

In the above list of Diophantine equations, the regularity of the powers $n = 2$ is evident. If any of these equations are raised to powers $n > 2$ the regularity will be lost, as is explained below.

We will explain why there are no zeros for the Riemann zeta function ζ for $s < 1/2$ and $s > 1/2$ by bringing up the common underlying principle behind it and Fermat's last theorem, $s = 1/2$ being evidently the optimum or equilibrium power, the only power which brings equilibrium, balance or regularity and thereby the zeros to the Riemann zeta function ζ.

For the case for $x^n + y^n = z^n$ above for Fermat's last theorem which asserts that there are no solutions for $n > 2$, we first get some mathematical insight on why there are no solutions for $n > 2$. We commence by selecting example [1] from the list of Diophantine equations above, which has the smallest odd prime number 3 and the smallest composite number 4 (which is the square of the smallest prime number 2) in the series on the left, i.e., the smallest Diophantine equation which has 2 as the power, for illustration:-

$$3^2 + 4^2 = 5^2$$

If the power of 2 in the series on the left above were increased to 3, which is the next, consecutive whole number, e.g., the sum on the right would not be a whole number anymore, which is in accordance with Fermat's last theorem:-

$$3^3 + 4^3 = 4.49795^3$$

The regularity of the power of 2 is now lost. And this is for the smallest Diophantine equation which initially had 2 as the power. For the larger Diophantine equations with initial powers of 2 the irregularity after increasing their powers to 3, which is the next, consecutive whole number, or, higher powers, could be expected to be worse.

Next we bring up the values of, say, 100, of consecutive whole number powers n, say, 2 to 5, this quantity 100 being representative of the terms of the equation $x^n + y^n = z^n$ as per Fermat's last theorem, to explain the reason for this irregularity, which is as follows:-

[1] $100^2 = 10{,}000$	(The terms of the series of Fermat's last theorem fall under this category. All zeros will be found under this category only.)
[2] $100^3 = 1{,}000{,}000$	(This quantity represents an increase of 9,900% compared to [1] above while the increase in power from $n = 2$ to $n = 3$ is only 50%.)
[3] $100^4 = 100{,}000{,}000$	(This quantity represents an increase of 999,900% compared to [1] above while the increase in power from $n = 2$ to $n = 4$ is only 100%.)
[4] $100^5 = 10{,}000{,}000{,}000$	(This quantity represents an increase of 99,999,900% compared to [1] above while the increase in power from $n = 2$ to $n = 5$ is only 150%.)

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The quantities from the consecutive whole number powers $n > 2$ above increase progressively compared to [1], the larger the power n is the larger the percentage of increase in the quantity is. The increases in the respective quantities and powers are also disproportionate when compared to one another, with the increases in the respective quantities being evidently much too quick. All this implies that the equilibrium, balance or regularity of $x^n + y^n = z^n$ when $n = 2$ as per Fermat's last theorem cannot be maintained when $n > 2$, when disproportionateness between the increases in the respective quantities and powers sets in as is described above, as the increase in quantity is too quick, and, when $n < 2$, e.g., $n = 5/4$, $3/2$, $7/4$, etc., as the increase in quantity is too slow as could be extrapolated from the above example. (Refer to Appendix for analogous example.) For Fermat's last theorem, $n = 2$ can be regarded as the optimum or equilibrium power, the only power wherein $x^n + y^n = z^n$ is possible. There is also the question of the easier solubility of equations with whole number powers $n = 2$ as compared to equations with powers $n > 2$, e.g., $n = 3, 4, 5$, etc., and $n < 2$, e.g., $n = 5/4$, $3/2$, $7/4$, etc., which will be explained below.

For the case of the Riemann zeta function ζ wherein there are no zeros for powers $s < 1/2$ and $s > 1/2$, we bring up the values of the reciprocals of, say, 100, with consecutive fractional powers s, say, 1/2 to 1/5, these reciprocals being representative of the terms of the Riemann zeta function ζ, to explain the reason for the irregularity for powers $s < 1/2$ and $s > 1/2$, which is as follows:-

[1] $1/100^{1/2} = 1/10 = 0.100$ (The terms of the series of the Riemann zeta function ζ as per the Riemann hypothesis fall under this category. 10^{13} zeros have been found under this category only.)

[2] $1/100^{1/3} = 1/4.6416 = 0.215$ (This quantity represents an increase of 115% compared to [1] above while the decrease in power from $s = 1/2$ to $s = 1/3$ is only 33.33%.)

[3] $1/100^{1/4} = 1/3.1623 = 0.316$ (This quantity represents an increase of 216% compared to [1] above while the decrease in power from $s = 1/2$ to $s = 1/4$ is only 50%.)

[4] $1/100^{1/5} = 1/2.5119 = 0.398$ (This quantity represents an increase of 298% compared to [1] above while the decrease in power from $s = 1/2$ to $s = 1/5$ is only 60%.)

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As can be seen above, the smaller the power of the reciprocal/denominator is the larger will be the result after division with 1 (or, the larger the power of the reciprocal/denominator is the smaller will be the result after division with 1). The quantities from the reciprocals with consecutive fractional powers $s < 1/2$ above increase progressively compared to [1], the smaller the power s is the larger the percentage of increase in the quantity is, the increases in the quantities being similar to the case above for Fermat's last theorem – this indicates a similarity between Fermat's last theorem and the Riemann hypothesis. The increases in the respective quantities and the decreases in the respective powers are also disproportionate when compared to one another, with the increases in the respective quantities being evidently much too quick, which is similar to the case above for Fermat's last theorem – this indicates another similarity between Fermat's last theorem and the Riemann hypothesis. All this implies that the equilibrium, balance or regularity of the Riemann zeta function ζ when $s = 1/2$ cannot be maintained when $s < 1/2$, when disproportionateness between the increases and decreases in the respective quantities and powers sets in as is described above, as the increase in quantity is too quick, and, when $s > 1/2$, e.g., $s = 3/4$, 4/5, 5/6, etc., as the increase in quantity is too slow as could be extrapolated from the above example. (Refer to Appendix for full details.) For these reciprocals, $s = 1/2$ can be regarded as the optimum or equilibrium power, the only power wherein zeros for the Riemann zeta function ζ are possible. Like the case for Fermat's last theorem above, there is also the question of the easier solubility of equations with fractional powers $s = 1/2$ as compared to equations with fractional powers $s < 1/2$, e.g., $s = 1/3$, 1/4, 1/5, etc., and $s > 1/2$, e.g., $s = 3/4$, 4/5, 5/6, etc., which will be explained below.

The following list of the 1st. 10 terms of the series of the Riemann zeta function ζ with consecutive fractional powers $s \leq 1/2$ also shows that the sums with smaller powers increase progressively, i.e., the smaller the power s is the larger the percentage of increase in the quantity is:-

[1] $\zeta(1/2) = 1 + 1/2^{1/2} + 1/3^{1/2} + 1/4^{1/2} + 1/5^{1/2} + 1/6^{1/2} + 1/7^{1/2} + 1/8^{1/2} + 1/9^{1/2} + 1/10^{1/2} + \ldots = 5.03$
(The Riemann hypothesis asserts that all zeros will be found in this series only.)

[2] $\zeta(1/3) = 1 + 1/2^{1/3} + 1/3^{1/3} + 1/4^{1/3} + 1/5^{1/3} + 1/6^{1/3} + 1/7^{1/3} + 1/8^{1/3} + 1/9^{1/3} + 1/10^{1/3} + \ldots = 6.20$
(The sum 6.20 here represents an increase of 23.26% compared to the sum 5.03 in [1] above while the percentage of decrease in power from $s = 1/2$ to $s = 1/3$ is 33.33%.)

[3] $\zeta(1/4) = 1 + 1/2^{1/4} + 1/3^{1/4} + 1/4^{1/4} + 1/5^{1/4} + 1/6^{1/4} + 1/7^{1/4} + 1/8^{1/4} + 1/9^{1/4} + 1/10^{1/4} + \ldots = 6.97$
(The sum 6.97 here represents an increase of 38.57% compared to the sum 5.03 in [1] above while the percentage of decrease in power from $s = 1/2$ to $s = 1/4$ is 50%.)

[4] $\zeta(1/5) = 1 + 1/2^{1/5} + 1/3^{1/5} + 1/4^{1/5} + 1/5^{1/5} + 1/6^{1/5} + 1/7^{1/5} + 1/8^{1/5} + 1/9^{1/5} + 1/10^{1/5} + \ldots = 7.46$
(The sum 7.46 here represents an increase of 48.31% compared to the sum 5.03 in [1] above while the percentage of decrease in power from $s = 1/2$ to $s = 1/5$ is 60%.)

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Note: Though the respective percentages of increase in quantity above, namely, 23.26%, 38.57% & 48.31%, are disproportionate with and lower than the respective percentages of decrease in power, namely, 33.33%, 50% & 60%, at a later stage when there are more and more terms in the series, there being an infinitude of terms, when the sums get larger and larger, the percentages of increase in quantity will all be infinitely higher than the percentages of decrease in power, as is evident from the tabulation below. The same will apply for the quantities when the powers $s > 1/2$, e.g., $s = 3/4, 4/5, 5/6$, etc., as could be extrapolated from the above list.

(The series of the Riemann zeta function ζ with powers $s > 1/2$, e.g., $s = 3/4, 4/5, 5/6$, etc., will have sums which are all smaller than the sums shown in the above list for powers $s \leq 1/2$ as could be extrapolated from the above list. For the largest power in the critical strip $s = 1$, which has no zeros, the sum of the 1st. 10 terms is a mere 2.93. Refer to Appendix for analogous example.)

It is clear from all the above that when the sum of the series in the Riemann zeta function ζ increases too quickly as is the case when the powers $s < 1/2$, when disproportionateness between the increases and decreases in the respective quantities and powers sets in as is described above, or, too slowly as is the case when the powers $s > 1/2$, e.g., $s = 3/4, 4/5, 5/6$, etc., as could be extrapolated from the above list, the equilibrium, balance or regularity will be lost and there will not be zeros. (Refer to Appendix for analogous example.) As is in the case of Fermat's last theorem wherein all the zeros will be at the optimum or equilibrium power $n = 2$ only, all the zeros of the Riemann zeta function ζ will be at the optimum or equilibrium power $s = 1/2$ only. (The analogue of this optimum or equilibrium power could be that of a shirt or pants that exactly fits a person, e.g., size A could be too small for the person, size C too large, while size B fits just fine.) At least 10^{13} zeros have been found at $s = 1/2$ while none has been found for $s < 1/2$ and $s > 1/2$.

An important point will be added here. If more and more terms are added to the series in the list of the sums of the Riemann zeta function ζ above where the consecutive fractional powers $s \leq 1/2$, which presently have 10 terms each, the differences in the sums between that for power $s = 1/2$ and that for powers $s < 1/2$, e.g., $s = 1/3$, 1/4, 1/5, etc., and, that for power $s = 1/2$ and that for powers $s > 1/2$, e.g., $s = 3/4$, 4/5, 5/6, etc., will be greater and greater, i.e., the differences between these sums will be more pronounced the more terms are added to the series. We can see this point by comparing, e.g., the sums of the 1st. 5 terms of the Riemann zeta function ζ for consecutive fractional powers $s \leq 1/2$ and the sums of the 1st. 10 terms of the Riemann zeta function ζ for consecutive fractional powers $s \leq 1/2$, which is as follows, and extrapolating from there:-

[1] $\zeta(1/2) = 1 + 1/2^{1/2} + 1/3^{1/2} + 1/4^{1/2} + 1/5^{1/2} + \ldots = 3.24$
(The Riemann hypothesis asserts that all zeros will be found in this series only.)

[2] $\zeta(1/3) = 1 + 1/2^{1/3} + 1/3^{1/3} + 1/4^{1/3} + 1/5^{1/3} + \ldots = 3.69$
(The sum 3.69 here represents an increase of **13.89%** (the increase here is **23.26%** for the 1st. 10 terms as is shown in the list above) compared to the sum 3.24 in [1] above.)

[3] $\zeta(1/4) = 1 + 1/2^{1/4} + 1/3^{1/4} + 1/4^{1/4} + 1/5^{1/4} + \ldots = 3.98$
(The sum 3.98 here represents an increase of **22.84%** (the increase here is **38.57%** for the 1st. 10 terms as is shown in the list above) compared to the sum 3.24 in [1] above.)

[4] $\zeta(1/5) = 1 + 1/2^{1/5} + 1/3^{1/5} + 1/4^{1/5} + 1/5^{1/5} + \ldots = 4.15$
(The sum 4.15 here represents an increase of **28.09%** (the increase here is **48.31%** for the 1st. 10 terms as is shown in the list above) compared to the sum 3.24 in [1] above.)

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The tabulation below of the above-mentioned percentage increases for the sums for the 1st. 2 terms to the 1st. 10 terms for ζ(1/3), ζ(1/4) & ζ(1/5) will provide a clearer picture:-

	1st. 2 Terms	1st. 3 Terms	1st. 4 Terms	1st. 5 Terms	1st. 6 Terms	1st. 7 Terms	1st. 8 Terms	1st. 9 Terms	1st. 10 Terms	1st. 11 Terms	…
[1] ζ(1/2)	-	-	-	-	-	-	-	-	-	-	
[2] ζ(1/3)	4.68%	8.30%	11.47%	**13.89%**	16.16%	18.11%	20.09%	21.87%	**23.26%**	To Be Extrapolated	
[3] ζ(1/4)	7.60%	13.54%	18.28%	**22.84%**	26.30%	29.78%	32.88%	35.88%	**38.57%**	To Be Extrapolated	
[4] ζ(1/5)	9.36%	16.59%	22.94%	**28.09%**	32.88%	37.22%	41.32%	45.01%	**48.31%**	To Be Extrapolated	

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It is evident that the percentage increases shown above will go up in value continuously to infinity with the infinitude of the terms of the Riemann zeta function ζ. All this indicates more and more bad news for the solubility of the Riemann zeta function ζ for powers $s < 1/2$, and, $s > 1/2$ (as could be extrapolated from the above; refer to Appendix) when there are more and more terms in the Riemann zeta function ζ, i.e., for powers $s < 1/2$ and $s > 1/2$, the more terms there are in the Riemann zeta function ζ the less soluble it will be. This is a serious irregularity and is another reason why there are no zeros for the Riemann zeta function ζ for powers $s < 1/2$ and $s > 1/2$.

The similarity between the Riemann hypothesis and Fermat's last theorem is rather striking – they each have an optimum or equilibrium power which is the only power wherein zeros are possible – $s = 1/2$ in the case of the Riemann hypothesis and $n = 2$ in the case of Fermat's last theorem, powers which are all solely responsible for all the zeros. The fact that all these optimum or equilibrium powers are either square root ($s = 1/2$ for the Riemann hypothesis) or square ($n = 2$ for Fermat's last theorem) is significant as they seem some sort of images of 2 which is the smallest prime number and the smallest even number. $s = 1/2$ is the largest root among the roots with 1 as the numerator. As such $s = 1/2$ as a fractional power with 1 as the numerator gives the largest result as compared to the fractional powers with 1 as the numerator $s < 1/2$, e.g., $s = 1/3$, 1/4, 1/5, etc. – equations with fractional powers $s = 1/2$ would evidently be easier to solve than equations with fractional powers $s < 1/2$ (e.g., in a computation $s = 1/2$ needs only 1 rooting step while $s = 1/5$ needs 4 rooting steps) and $s > 1/2$, e.g., $s = 2/3$, 3/4, 4/5, etc. (e.g., in a computation $s = 1/2$ needs only 1 rooting step, while $s = 4/5$ needs 7 steps – 3 squaring steps for $s = 4$ & 4 rooting steps for $s = 1/5$). $n = 2$ is the smallest whole number power which brings an increase in quantity. As such $n = 2$ is the whole number power which brings the smallest increase in quantity as compared to the whole number powers $n > 2$, e.g., $n = 3$, 4, 5, etc. – equations with whole number powers $n = 2$ would evidently be easier to solve than equations with powers $n > 2$ (with general equations with powers $n = 5$ having been proven unsolvable – $n = 2$ needs only 1 squaring step while $n = 5$ needs 4 squaring steps) and $n < 2$, e.g., $n = 5/4$, 3/2, 7/4, etc. (e.g., in a computation $n = 2$ needs only 1 squaring step, while $n = 7/4$ needs 9 steps – 6 squaring steps for $n = 7$ & 3 rooting steps for $n = 1/4$). $n = 2$ and its reciprocal $s = 1/2$ are the opposite of one another but despite this there appears to be complementariness and symmetry between them, as can be seen in the cases of Fermat's last theorem and the Riemann hypothesis which involve optimum or equilibrium powers $n = 2$ and its reciprocal $s = 1/2$, the only powers wherein zeros are possible for each of them. $n = 2$ and its reciprocal $s = 1/2$ are evidently important quantities which may be comparable to π (3.14159265) or e (2.71828).

It can be seen that the Riemann hypothesis is the analogue of Fermat's last theorem, which implies its validity.

Thus, for the Riemann zeta function ζ, $s = 1/2$ is the optimum or equilibrium power wherein there will be zeros. There will be no zeros in the critical strip bounded by $\mathrm{Re}(s) = 0$ and $\mathrm{Re}(s) = 1$ for $s < 1/2$ and $s > 1/2$ because if $s < 1/2$ the sum of the series in the zeta function ζ increases too quickly and if $s > 1/2$ the sum of the series in the zeta function ζ increases too slowly – $s = 1/2$ is optimum, just nice.

Hence:

Theorem Due To Riemann
All the non-trivial zeros of the Riemann zeta function ζ will always lie on the critical line $\mathrm{Re}(s) = 1/2$ only and not anywhere else on the critical strip bounded by $\mathrm{Re}(s) = 0$ and $\mathrm{Re}(s) = 1$.

Appendix
Below are the values of the reciprocals of, say, 100, with consecutive fractional powers $s \leq 4/5$, these reciprocals being representative of the terms of the Riemann zeta function ζ:-

[1] $1/100^{4/5} = 1/39.8107171 = 0.025$ (This quantity represents a decrease of 75% compared to [4] below while the increase in power from $s = 1/2$ to $s = 4/5$ is only 60%.)

[2] $1/100^{3/4} = 1/31.62278 = 0.032$ (This quantity represents a decrease of 68% compared to [4] below while the increase in power from $s = 1/2$ to $s = 3/4$ is only 50%.)

[3] $1/100^{2/3} = 1/21.5444 = 0.046$ (This quantity represents a decrease of 54% compared to [4] below while the increase in power from $s = 1/2$ to $s = 2/3$ is only 33.33%.)

[4] $1/100^{1/2} = 1/10 = 0.100$ (The terms of the series of the Riemann zeta function ζ as per the Riemann hypothesis fall under this category. 10^{13} zeros have been found under this category only.)

[5] $1/100^{1/3} = 1/4.6416 = 0.215$ (This quantity represents an increase of 115% compared to [4] above while the decrease in power from $s = 1/2$ to $s = 1/3$ is only 33.33%.)

[6] $1/100^{1/4} = 1/3.1623 = 0.316$ (This quantity represents an increase of 216% compared to [4] above while the decrease in power from $s = 1/2$ to $s = 1/4$ is only 50%.)

[7] $1/100^{1/5} = 1/2.5119 = 0.398$ (This quantity represents an increase of 298% compared to [4] above while the decrease in power from $s = 1/2$ to $s = 1/5$ is only 60%.)

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Note the disproportionateness between the respective percentages of decrease in quantity and the respective percentages of increase in power for the reciprocals with powers $s > 1/2$, and, between the respective percentages of increase in quantity and the respective percentages of decrease in power for the reciprocals with powers $s < 1/2$.

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