Electro-Magnetic Field Equation and Lorentz gauge in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime. We treat Lorentz gauge transformation in Rindler spacetime. Specially, this article say the uniqueness of the accelerated frame because the accelerated frame can treat electro-magnetic field equation.

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1. Introduction

In 2007 year, G.F.Torres del Castillio and C.I.Perez Sanchez already discover Maxwell equations in uniformly accelerated frame in vaccum(see Ref [13]). In 2011 year, J.W.Maluf and F.F.Faria discover electro-magnetic field transformation in Rindler space-time on ArXiv preprint(see also Ref[11]). But they mistake they use the transformation in gravity field. Uniformly accelerated frame has to treat in flat Minkowski space-time not in gravity space-time.

Our theory's aim is that we find electro-magnetic field equation in Rindler space-time in vaccum also not in vaccum in the general relativity theory. In Section 2, we prepare for finding electro-magnetic field equation in Rindler space-time. In this section, we discover Lorentz gauge transformation and Lorentz fixing condition, transformation of the electro-magnetic 4-vector potential in Rindler space-time. In Section 3, we define the electro-magnetic field in Rindler space-time and we find the transformation of the electro-magnetic field. In Section 4, we obtain the electro-magnetic field equation in Rindler space-time and we apply the gauge theory to Maxwell equations (discovered by us) in Rindler space-time for viewing invariant about the gauge transformation .

We think seriously electro-magnetic wave function (radiation) in Rindler space-time but we know it doesn't satisfy electro-magnetic wave equation in mathematically. We understand electro-magnetic wave function can exist in inertial frame by J.C.Maxwell or A. Einstein.

2. Transformation of the electro-magnetic 4-vector potential, Lorentz gauge condition and Lorentz fixing condition

The Rindler coordinate is

$$ct = \left(\frac{c^2}{a_0} + \xi^1\right)\sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$x = \left(\frac{c^2}{a_0} + \xi^1\right)\cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3 \tag{1}$$

In this time, the tetrad $e^a{}_\mu$ is (see Ref [12])

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2}\eta_{ab}\frac{\partial x^a}{\partial \xi^\mu}\frac{\partial x^b}{\partial \xi^\upsilon}d\xi^\mu d\xi^\upsilon$$

$$= -\frac{1}{c^2}\eta_{ab}e^a{}_\mu e^b{}_\upsilon d\xi^\mu d\xi^\upsilon = -\frac{1}{c^2}g_{\mu\upsilon}d\xi^\mu d\xi^\upsilon \quad , \quad e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \tag{2}$$

$$e^{\alpha}{}_{0}(\xi^0) = \frac{\partial x^{\alpha}}{c\partial\xi^0} = ((1+\frac{a_0}{c^2}\xi^1)\cosh(\frac{a_0\xi^0}{c}),(1+\frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c}),0,0) \quad (3)$$

About y -axis's and z -axis's orientation

$$e^{\alpha}{}_{2}(\xi^0) = \frac{\partial x^{\alpha}}{\partial\xi^2} = (0,0,1,0) \quad , \quad e^{\alpha}{}_{3}(\xi^0) = \frac{\partial x^{\alpha}}{\partial\xi^3} = (0,0,0,1) \quad (4)$$

The other unit vector $e^{\alpha}{}_{1}(\xi^0)$ is

$$e^{\alpha}{}_{1}(\xi^0) = \frac{\partial x^{\alpha}}{\partial\xi^1} = (\sinh(\frac{a_0\xi^0}{c}),\cosh(\frac{a_0\xi^0}{c}),0,0) \quad (5)$$

Therefore,

$$cdt = c\cosh(\frac{a_0\xi^0}{c})d\xi^0(1+\frac{a_0}{c^2}\xi^1) + \sinh(\frac{a_0\xi^0}{c})d\xi^1$$

$$dx = c\sinh(\frac{a_0\xi^0}{c})d\xi^0(1+\frac{a_0}{c^2}\xi^1) + \cosh(\frac{a_0\xi^0}{c})d\xi^1, dy = d\xi^2, \ dz = d\xi^3 \quad (6)$$

The vector transformation is

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}}V^{\alpha}, \qquad U'_{\mu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}U_{\alpha} \quad (7)$$

Therefore, the transformation of the electro-magnetic 4-vector potential $(\phi,\vec{A}) = A^{\alpha}$ is

$$A^{\alpha} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}A'^{\mu} = \frac{\partial x^{\alpha}}{\partial\xi^{\mu}}A_{\xi}{}^{\mu} = e^{\alpha}{}_{\mu}A_{\xi}{}^{\mu} \quad , \quad e^{\alpha}{}_{\mu} = \frac{\partial x^{\alpha}}{\partial\xi^{\mu}}$$

$$dx^{\alpha} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}dx'^{\mu} = \frac{\partial x^{\alpha}}{\partial\xi^{\mu}}d\xi^{\mu} = e^{\alpha}{}_{\mu}d\xi^{\mu} \quad , e^{\alpha}{}_{\mu} = \frac{\partial x^{\alpha}}{\partial\xi^{\mu}} \quad (8)$$

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\phi = 4\pi\rho$$

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\vec{A} = \frac{4\pi}{c}\vec{j}$$

4-vector $(c\rho,\vec{j}) = \rho_0\frac{dx^{\alpha}}{d\tau}$ (9)

Lorentz gauge transformation is in Rindler spacetime,

$$A^{\mu} \rightarrow A^{\mu} + \partial^{\mu}\Lambda = A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda$$

, Λ is a scalar function.

$$g^{00} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, g^{11} = g^{22} = g^{33} = 1$$

Hence,

$$\phi_\xi \to \phi_\xi - \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, \ \vec{A}_\xi \to \vec{A}_\xi + \vec{\nabla}_\xi\Lambda, \ \Lambda \text{ is a scalar function.} \qquad (10)$$

Lorentz gauge fixing condition is in Rindler spacetime,

$$A^\mu{}_{;\mu} = \frac{\partial A^\mu}{\partial\xi^\mu} + \Gamma^\mu{}_{\mu\rho}A^\rho,$$

$$\Gamma^\mu{}_{\mu\rho} = \Gamma^0{}_{01} = \frac{1}{2}g^{00}(\frac{\partial g_{00}}{\partial\xi^1}) = \frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}$$

$$g^{00} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, g^{11} = g^{22} = g^{33} = 1 \qquad (11)$$

$$0 = \frac{\partial\phi}{c\partial t} + \vec{\nabla}\cdot\vec{A} = \frac{\partial\phi_\xi}{c\partial\xi^0} + \vec{\nabla}_\xi\cdot\vec{A}_\xi + \frac{A_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}$$

$$A^\mu \to A^\mu + \partial^\mu\Lambda = A^\mu + g^{\mu\upsilon}\partial_\upsilon\Lambda$$

$$A^\mu{}_{;\mu} = \frac{\partial A^\mu}{\partial\xi^\mu} + \Gamma^\mu{}_{\mu\rho}A^\rho \to \partial_\mu(A^\mu + g^{\mu\upsilon}\partial_\upsilon\Lambda) + \Gamma^0{}_{01}(A^1 + \frac{\partial\Lambda}{\partial\xi^1})$$

$$= \partial_\mu A^\mu + (g^{\mu\upsilon}\partial_\mu\partial_\upsilon)\Lambda + \Gamma^0{}_{01}(A^1 + \frac{\partial\Lambda}{\partial\xi^1})$$

$$0 = \frac{\partial\phi_\xi}{c\partial\xi^0} + \vec{\nabla}_\xi\cdot\vec{A}_\xi + \frac{A_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}$$

$$\to \frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi\cdot\vec{A}_\xi - [\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]\Lambda$$

$$+ \frac{A_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)} + \frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})} = 0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\Lambda-\frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

(12)

Hence, the transformation of the electro-magnetic 4-vector potential $(\phi,\vec{A})$ in inertial frame and the eletro-magnetic 4-vector potential $(\phi_\xi,\vec{A}_\xi)$ in uniformly accelerated frame is

$$\phi=\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi+\sinh(\frac{a_0\xi^0}{c})A_{\xi^1}$$

$$A_x=\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi+\cosh(\frac{a_0\xi^0}{c})A_{\xi^1}$$

$$A_y=A_{\xi^2},A_z=A_{\xi^3} \quad (13)$$

$$g=\begin{pmatrix}-(1+\frac{a_0\xi^1}{c^2})^2 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix},\eta=\begin{pmatrix}-1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

(see Ref [12])

$$e^a{}_\mu e_b{}^\mu=\delta^a{}_b \ , \ e^a{}_\mu e_a{}^\upsilon=\delta_\mu{}^\upsilon$$

$$e^a{}_\mu e^b{}_\upsilon\eta_{ab}=g_{\mu\upsilon} \ \rightarrow \ A^T\eta A=g$$

$$e_a{}^\mu e_b{}^\upsilon g_{\mu\upsilon}=\eta_{ab}\rightarrow \ (A^T)^{-1}gA^{-1}=(A^T)^{-1}A^T\eta AA^{-1}=\eta$$

$$e^a{}_\mu=\eta^{ab}g_{\mu\upsilon}e_b{}^\upsilon\rightarrow\eta^{-1}(A^T)^{-1}A^T\eta A=A=\eta^{-1}(A^T)^{-1}g \quad (14)$$

$$\begin{pmatrix}cdt\\ dx\\ dy\\ dz\end{pmatrix}=\begin{pmatrix}\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \sinh(\frac{a_0\xi^0}{c}) & 0 & 0\\ \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}cd\xi^0\\ d\xi^1\\ d\xi^2\\ d\xi^3\end{pmatrix}$$

$$= A\begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ d\xi^3 \end{pmatrix}$$

$$= \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\hat{\xi}^0 \\ d\hat{\xi}^1 \\ d\hat{\xi}^2 \\ d\hat{\xi}^3 \end{pmatrix}$$

(15)

$$e_\mu{}^\alpha = \frac{\partial \xi^\alpha}{\partial x^\mu} = A^{-1} = \begin{pmatrix} \frac{c\partial\xi^0}{c\partial t} & \frac{c\partial\xi^0}{\partial x} & \frac{c\partial\xi^0}{\partial y} & \frac{c\partial\xi^0}{\partial z} \\ \frac{\partial\xi^1}{c\partial t} & \frac{\partial\xi^1}{\partial x} & \frac{\partial\xi^1}{\partial y} & \frac{\partial\xi^1}{\partial z} \\ \frac{\partial\xi^2}{c\partial t} & \frac{\partial\xi^2}{\partial x} & \frac{\partial\xi^2}{\partial y} & \frac{\partial\xi^2}{\partial z} \\ \frac{\partial\xi^3}{c\partial t} & \frac{\partial\xi^3}{\partial x} & \frac{\partial\xi^3}{\partial y} & \frac{\partial\xi^3}{\partial z} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & 0 & 0 \\ -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (16)$$

$$\begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = (A^{-1})^T \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} = (A^T)^{-1} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \hat{\xi}^0} \\ \frac{\partial}{\partial \hat{\xi}^1} \\ \frac{\partial}{\partial \hat{\xi}^2} \\ \frac{\partial}{\partial \hat{\xi}^3} \end{pmatrix} \quad (17)$$

$$\frac{1}{c}\frac{\partial}{\partial t} = \frac{c\partial \xi^0}{c\partial t}\frac{1}{c}\frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{c\partial t}\frac{\partial}{\partial \xi^1}$$

$$= \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}$$

$$\frac{\partial}{\partial x} = \frac{c\partial \xi^0}{\partial x}\frac{1}{c}\frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial x}\frac{\partial}{\partial \xi^1}$$

$$= -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial z} = \frac{\partial}{\partial\xi^3}$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2$$

$$\vec{\nabla} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}), \quad \vec{\nabla}_\xi = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3})$$

(18)

3. Electro-magnetic Field in the Rindler space-time

The electro-magnetic field $(\vec{E},\vec{B})$ is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial\vec{A}}{c\partial t}, \vec{B} = \vec{\nabla}\times\vec{A} \qquad (19)$$

We have to calculate for define electro-magnetic field in Rindler space-time.

$$E_x = -\frac{\partial\phi}{\partial x} - \frac{\partial A_x}{c\partial t}$$

$$= -[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2})\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2})\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$= -\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial A_{\xi^1}}{c\partial\xi^0} - (1+\frac{a_0\xi^1}{c^2})\frac{\partial\phi_\xi}{\partial\xi^1} - 2\phi_\xi\frac{a_0}{c^2}$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}[(1+\frac{a_0}{c^2}\xi^1)^2\phi_\xi] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial\xi^0} \qquad (20)$$

$$E_y = -\frac{\partial\phi}{\partial y} - \frac{\partial A_y}{c\partial t} = -\frac{\partial}{\partial\xi^2}[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^2}$$

$$=-(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})\frac{\partial\phi_\xi}{\partial\xi^2}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\cosh(\frac{a_0\xi^0}{c})\frac{\partial A_{\xi^2}}{c\partial\xi^0}$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial\xi^1}-\frac{\partial A_{\xi^1}}{\partial\xi^2}]$$

$$=\cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2]-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^2}}{c\partial\xi^0}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial\xi^1}-\frac{\partial A_{\xi^1}}{\partial\xi^2}] \qquad (21)$$

$$E_z=-\frac{\partial\phi}{\partial z}-\frac{\partial A_z}{c\partial t}=-\frac{\partial}{\partial\xi^3}[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi+\sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$=-(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})\frac{\partial\phi_\xi}{\partial\xi^3}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\cosh(\frac{a_0\xi^0}{c})\frac{\partial A_{\xi^3}}{c\partial\xi^0}$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^3}}{\partial\xi^1}-\frac{\partial A_{\xi^1}}{\partial\xi^3}]$$

$$=\cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2]-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^3}}{c\partial\xi^0}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^3}}{\partial\xi^1}-\frac{\partial A_{\xi^1}}{\partial\xi^3}]$$

(22)

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = \frac{\partial A_{\xi^3}}{\partial \xi^2} - \frac{\partial A_{\xi^2}}{\partial \xi^3} \tag{23}$$

$$B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = \frac{\partial A_x}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial x}$$

$$= \frac{\partial}{\partial \xi^3}[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^1}}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial \xi^1}]$$

$$-\sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^3}}{c\partial\xi^0}] \tag{24}$$

$$B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = \frac{\partial A_{\xi^2}}{\partial x} - \frac{\partial A_x}{\partial \xi^2}$$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$-\frac{\partial}{\partial \xi^2}[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^2}}{c\partial\xi^0}] \tag{25}$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler spacetime.

$$\vec{E}_\xi = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi \times \vec{A}_\xi$$

In this time, $\vec{\nabla}_\xi = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3}), \vec{A}_\xi = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$ (26)

Lorentz gauge transformation is in Rindler spacetime,

$$\phi_\xi \to \phi_\xi - \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, \quad \vec{A}_\xi \to \vec{A}_\xi + \vec{\nabla}_\xi\Lambda, \quad \Lambda \text{ is a scalar function.} \quad (27)$$

$$\vec{E}_\xi = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\frac{\partial\Lambda}{c\partial\xi^0}$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}\vec{\nabla}_\xi\Lambda$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi \times \vec{A}_\xi + \vec{\nabla}_\xi \times \vec{\nabla}_\xi\Lambda = \vec{\nabla}_\xi \times \vec{A}_\xi \quad (28)$$

Lorentz gauge fixing condition is in Rindler spacetime,

$$0 = \frac{\partial\phi_\xi}{c\partial\xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi + \frac{A_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}$$

$$\to \frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi - [\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]\Lambda$$

$$+\frac{A_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}+\frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\Lambda-\frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

$$\phi-\frac{1}{c}\frac{\partial\Lambda}{\partial t}=\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\{\phi_\xi-\frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\}$$

$$+\sinh(\frac{a_0\xi^0}{c})(A_{\xi^1}+\frac{\partial\Lambda}{\partial\xi^1})$$

$$A_x+\frac{\partial\Lambda}{\partial x}=\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\{\phi_\xi-\frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\}$$

$$+\cosh(\frac{a_0\xi^0}{c})(A_{\xi^1}+\frac{\partial\Lambda}{\partial\xi^1})$$

$$A_y+\frac{\partial\Lambda}{\partial y}=A_{\xi^2}+\frac{\partial\Lambda}{\partial\xi^2},A_z+\frac{\partial\Lambda}{\partial z}=A_{\xi^3}+\frac{\partial\Lambda}{\partial\xi^3} \tag{29}$$

We obtain the transformation of the electro-magnetic field.

$$E_x=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial\xi^0}=E_{\xi^1}\ ,$$

$$E_y=E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z=E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$B_x=B_{\xi^1},$$

$$B_y=B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z=B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}) \tag{30}$$

Hence,

$$E_x = E_{\xi^1}, B_x = B_{\xi^1},$$

$$\begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix} = H \begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix}$$

$$H = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 & \sinh(\frac{a_0\xi^0}{c}) \\ 0 & \cosh(\frac{a_0\xi^0}{c}) & -\sinh(\frac{a_0\xi^0}{c}) & 0 \\ 0 & -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 \\ \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 & \cosh(\frac{a_0\xi^0}{c}) \end{pmatrix} \quad (31)$$

The inverse-transformation of the electro-magnetic field is

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$\begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix} = H^{-1} \begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix}$$

$$H^{-1} = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 & -\sinh(\frac{a_0\xi^0}{c}) \\ 0 & \cosh(\frac{a_0\xi^0}{c}) & \sinh(\frac{a_0\xi^0}{c}) & 0 \\ 0 & \sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 \\ -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 & \cosh(\frac{a_0\xi^0}{c}) \end{pmatrix} \quad (32)$$

(see also Ref [11])

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$E_{\xi^2} = E_y \cosh(\frac{a_0\xi^0}{c}) - B_z \sinh(\frac{a_0\xi^0}{c}),$$

$$B_{\xi^2} = B_y \cosh(\frac{a_0\xi^0}{c}) + E_z \sinh(\frac{a_0\xi^0}{c})$$

$$E_{\xi^3} = E_z \cosh(\frac{a_0\xi^0}{c}) + B_y \sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^3} = B_z \cosh(\frac{a_0\xi^0}{c}) - E_y \sinh(\frac{a_0\xi^0}{c}) \qquad (33)$$

4. Electro-magnetic Field Equation(Maxwell Equation) in the Rindler space-time

Maxwell equation is

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \qquad (34\text{-i})$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j} \qquad (34\text{-ii})$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad (34\text{-iii})$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{c\partial t} \qquad (34\text{-iv})$$

We continue boring calculation for discovering Maxwell equations in Rindler space-time..

1. $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$

$$E_x = E_{\xi^1} \quad ,$$

$$E_y = E_{\xi^2} \cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3} \cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0\xi^0}{c})$$

$$4\pi\rho = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]E_{\xi^1}$$

$$+\frac{\partial}{\partial\xi^2}[E_{\xi^2} \cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{\partial}{\partial\xi^3}[E_{\xi^3} \cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)(\vec{\nabla}_\xi \cdot \vec{E}_\xi) + \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial E_{\xi^1}}{c\partial \xi^0}] \qquad (35)$$

2. $\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j}$

$$B_x = B_{\xi^1}$$

$$B_y = B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z = B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

X-component) $\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$

$$= \frac{\partial}{\partial \xi^2}[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$- \frac{\partial}{\partial \xi^3}[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3}] + \sinh(\frac{a_0\xi^0}{c})[\frac{\partial E_{\xi^2}}{\partial \xi^2} + \frac{\partial E_{\xi^3}}{\partial \xi^3}]$$

$$= \frac{\partial E_x}{c\partial t} + \frac{4\pi}{c}j_x$$

$$= [\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]E_{\xi^1} + \frac{4\pi}{c}j_x$$

Hence,

$$\frac{4\pi}{c}j_x$$

$$= \sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi \cdot \vec{E}_\xi) + \cosh(\frac{a_0\xi^0}{c})[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial E_{\xi^1}}{c\partial \xi^0}] \qquad (36)$$

Y-component) $\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}$

$$=\frac{\partial B_{\xi^1}}{\partial \xi^3}$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$=\frac{\partial E_y}{c\partial t}+\frac{4\pi}{c}j_y$$

$$=[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{4\pi}{c}j_y$$

$$\frac{4\pi}{c}j_y=\frac{\partial B_{\xi^1}}{\partial\xi^3}-\frac{\partial B_{\xi^3}}{\partial\xi^1}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}B_{\xi^3}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^2}}{c\partial\xi^0}$$

$$=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}\{B_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{B_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^2}}{c\partial\xi^0}$$

(37)

Z-component) $\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}$

$$=[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial B_{\xi^1}}{\partial\xi^2}$$

$$=\frac{\partial E_z}{c\partial t}+\frac{4\pi}{c}j_z$$

$$=[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{4\pi}{c}j_z$$

$$\frac{4\pi}{c}j_z=\frac{\partial B_{\xi^2}}{\partial\xi^1}-\frac{\partial B_{\xi^1}}{\partial\xi^2}+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}B_{\xi^2}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^3}}{c\partial\xi^0}$$

$$=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{B_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}\{B_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^3}}{c\partial\xi^0}$$

(38)

3. $\vec{\nabla}\cdot\vec{B}=0$

$$\vec{\nabla}\cdot\vec{B}=\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}$$

$$=[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]B_{\xi^1}$$

$$+\frac{\partial}{\partial\xi^2}[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{\partial}{\partial\xi^3}[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$=\cosh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi\cdot\vec{B}_\xi)+\sinh(\frac{a_0\xi^0}{c})[-(-\frac{\partial E_{\xi^2}}{\partial\xi^3}+\frac{\partial E_{\xi^3}}{\partial\xi^2})-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^1}}{c\partial\xi^0}]=0$$

(39)

4. $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{c\partial t}$

$$E_x = E_{\xi^1} \quad ,$$

$$E_y = E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

X-component) $\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}$

$$= \frac{\partial}{\partial \xi^2}[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$- \frac{\partial}{\partial \xi^3}[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3}] - \sinh(\frac{a_0\xi^0}{c})[\frac{\partial B_{\xi^2}}{\partial \xi^2} + \frac{\partial B_{\xi^3}}{\partial \xi^3}]$$

$$= -\frac{\partial B_x}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]B_{\xi^1}$$

Hence,

$$-\sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi \cdot \vec{B}_\xi) + \cosh(\frac{a_0\xi^0}{c})\,[(\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3}) + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial B_{\xi^1}}{c\partial \xi^0}] = 0 \qquad (40)$$

Y-component) $\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}$

$$= \frac{\partial E_{\xi^1}}{\partial \xi^3}$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]\cdot[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= -\frac{\partial B_y}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$\frac{\partial E_{\xi^1}}{\partial\xi^3} - \frac{\partial E_{\xi^3}}{\partial\xi^1} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}E_{\xi^3} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^2}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}\{E_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{E_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^2}}{c\partial\xi^0}$$

$$= 0$$

(41)

Z-component) $\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial E_{\xi^1}}{\partial\xi^2}$$

$$= -\frac{\partial B_z}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$\frac{\partial E_{\xi^2}}{\partial\xi^1} - \frac{\partial E_{\xi^1}}{\partial\xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}E_{\xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^3}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{E_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}\{E_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^3}}{c\partial\xi^0}$$

$$= 0$$

(42)

Therefore, we obtain the electro-magnetic field equation by Eq (35)-Eq(42) in Rindler spacetime. (see also Ref[13])

$$\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi} = 4\pi\rho_{\xi}(1+\frac{a_0\xi^1}{c^2}) \qquad \text{(43-i)}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\xi} \times \{\vec{B}_{\xi}(1+\frac{a_0\xi^1}{c^2})\} = \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial \vec{E}_{\xi}}{c\partial\xi^0} + \frac{4\pi}{c}\vec{j}_{\xi} \qquad \text{(43-ii)}$$

$$\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi} = 0 \qquad \text{(43-iii)}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\xi} \times \{\vec{E}_{\xi}(1+\frac{a_0\xi^1}{c^2})\} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial \vec{B}_{\xi}}{c\partial\xi^0} \qquad \text{(43-iv)}$$

$$\vec{E}_{\xi} = (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}), \vec{B}_{\xi} = (B_{\xi^1}, B_{\xi^2}, B_{\xi^3}),$$

$$\vec{\nabla}_{\xi} = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3})$$

Hence, the transformation of 4-vector $(c\rho, \vec{j}) = \rho_0 \dfrac{dx^{\alpha}}{d\tau}$ is

$$\rho = \rho_{\xi}(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c}) + \frac{j_{\xi^1}}{c}\sinh(\frac{a_0\xi^0}{c})$$

$$j_x = j_{\xi^1}\cosh(\frac{a_0\xi^0}{c}) + c\rho_{\xi}(1+\frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c}), \quad j_y = j_{\xi^2}, j_z = j_{\xi^3}$$

In this time, 4-vector $(c\rho_{\xi}, \vec{j}_{\xi}) = \rho_0 \dfrac{d\xi^{\alpha}}{d\tau}$ (44)

Generally, the continuity equation is in Rindler spacetime,

$$0 = j^{\mu}{}_{;\mu} = \frac{\partial j^{\mu}}{\partial\xi^{\mu}} + \Gamma^{\mu}{}_{\mu\rho}j^{\rho},$$

$$\Gamma^{\mu}{}_{\mu\rho} = \Gamma^{0}{}_{01} = \frac{1}{2}g^{00}(\frac{\partial g_{00}}{\partial\xi^1}) = \frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}$$

$$g^{00} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, g^{11} = g^{22} = g^{33} = 1$$

$$0 = \frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = \frac{\partial\rho_\xi}{\partial\xi^0} + \vec{\nabla}_\xi\cdot\vec{j}_\xi + \frac{j_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})} \tag{45}$$

We treat Lorentz gauge transformation about the electro-magnetic field equation in Rindler spacetime. Eq(43-i) is

$$\vec{\nabla}_\xi\cdot\vec{E}_\xi = \vec{\nabla}_\xi\cdot\{-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0}\}$$

$$= -\vec{\nabla}_\xi\{\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\}\cdot[\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} + \frac{\partial\vec{A}_\xi}{c\partial\xi^0}]$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi^{\ 2}\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} + \frac{\partial}{c\partial\xi^0}(\vec{\nabla}_\xi\cdot\vec{A}_\xi)]$$

$$= \frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}[\frac{\partial}{\partial\xi^1}\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} + \frac{\partial A_{\xi^1}}{c\partial\xi^0}]$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi^{\ 2} - \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}[-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{A_{\xi^1}a_0}{c^2}] \quad ,$$

$$\frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi\cdot\vec{A}_\xi = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1}$$

$$= -\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}E_{\xi^1} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi^{\ 2} - \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial A_{\xi^1}}{c\partial\xi^0}\frac{a_0}{c^2}$$

$$=4\pi\rho_\xi(1+\frac{a_0\xi^1}{c^2}) \tag{46}$$

If we apply Lorentz gauge transformation to Eq (46),

$\phi_\xi \to \phi_\xi - \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}$, $\vec{A}_\xi \to \vec{A}_\xi + \vec{\nabla}_\xi\Lambda$, Λ is a scalar function.

$$=-\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}E_{\xi^1}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial A_{\xi^1}}{c\partial\xi^0}\frac{a_0}{c^2}+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{a_0}{c^2}\frac{\partial}{c\partial\xi^0}\frac{\partial\Lambda}{\partial\xi^1}$$

$$=-\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}E_{\xi^1}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}\{[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\Lambda+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial\Lambda}{\partial\xi^1}\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial A_{\xi^1}}{c\partial\xi^0}\frac{a_0}{c^2} \tag{47}$$

In this time,

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\Lambda-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial\Lambda}{\partial\xi^1}=0 \tag{48}$$

Hence, Eq(43-i) is

$$\vec{\nabla}_\xi\cdot\vec{E}_\xi$$

$$= -\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}E_{\xi^1} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2 - \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial A_{\xi^1}}{c\partial\xi^0}\frac{a_0}{c^2}$$

$$= 4\pi\rho_\xi(1+\frac{a_0\xi^1}{c^2}) \qquad (49)$$

Eq(43-i) is invariant about Lorentz gauge transformation in Rindler spacetime.

Eq (43-ii) is

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{B}_\xi(1+\frac{a_0\xi^1}{c^2})\}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{\nabla}_\xi \times \vec{A}_\xi(1+\frac{a_0\xi^1}{c^2})\}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\{\vec{\nabla}_\xi \times \vec{A}_\xi\} + \vec{\nabla}_\xi \times \vec{\nabla}_\xi \times \vec{A}_\xi$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}(1,0,0)\times\vec{B}_\xi + \{-\nabla_\xi{}^2\vec{A}_\xi + \vec{\nabla}_\xi(\vec{\nabla}_\xi\cdot\vec{A}_\xi)\}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}(0,-B_{\xi^3},B_{\xi^2}) + \{-\nabla_\xi{}^2\vec{A}_\xi + \vec{\nabla}_\xi(\vec{\nabla}_\xi\cdot\vec{A}_\xi)\}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{E}_\xi}{c\partial\xi^0} + \frac{4\pi}{c}\vec{j}_\xi$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial}{c\partial\xi^0}[\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{A}_\xi + \frac{4\pi\vec{j}_\xi}{c}$$

$$= -\frac{\partial}{c\partial\xi^0}\vec{\nabla}_\xi\phi_\xi - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0) - \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{A}_\xi + \frac{4\pi\vec{j}_\xi}{c}$$

(50)

Therefore,

$$\frac{4\pi}{c}\vec{j}_\xi$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}(0,-B_{\xi^3},B_{\xi^2}) + \{-\nabla_\xi{}^2\vec{A}_\xi + \vec{\nabla}_\xi(\vec{\nabla}_\xi\cdot\vec{A}_\xi)\}$$

$$+\frac{\partial}{c\partial\xi^0}\vec{\nabla}_\xi\phi_\xi + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0) + \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{A}_\xi$$

$$= \frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2}) + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$+[-\nabla_\xi{}^2 + \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi + \vec{\nabla}_\xi[-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1}]$$

$$\frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi\cdot\vec{A}_\xi = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1} \quad (51)$$

$$\frac{4\pi}{c}\vec{j}_\xi$$

$$= \frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2}) + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$+[-\nabla_\xi{}^2 + \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi$$

$$+\vec{\nabla}_\xi[-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1}] \quad (52)$$

If we apply Lorentz gauge transformation to Eq (52),

$\phi_\xi \to \phi_\xi - \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}$, $\vec{A}_\xi \to \vec{A}_\xi + \vec{\nabla}_\xi\Lambda$, Λ is a scalar function.

$$\frac{4\pi}{c}\vec{j}_\xi$$

$$= \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3}, B_{\xi^2}) + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{2a_0}{c^2} \frac{\partial \phi_\xi}{c\partial \xi^0}(1,0,0)$$

$$- \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3} \frac{2a_0}{c^2} \frac{1}{c^2} (\frac{\partial}{\partial \xi^0})^2 \Lambda(1,0,0)$$

$$+ [-\nabla_\xi{}^2 + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial \xi^0})^2] \vec{A}_\xi + [-\nabla_\xi{}^2 + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial \xi^0})^2] \vec{\nabla}_\xi \Lambda$$

$$+ \vec{\nabla}_\xi [-\frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{a_0}{c^2} A_{\xi^1}] + \vec{\nabla}_\xi [-\frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{a_0}{c^2} \frac{\partial \Lambda}{\partial \xi^1}]$$

$$= \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3}, B_{\xi^2}) + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{2a_0}{c^2} \frac{\partial \phi_\xi}{c\partial \xi^0}(1,0,0)$$

$$- \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3} \frac{2a_0}{c^2} \frac{1}{c^2} (\frac{\partial}{\partial \xi^0})^2 \Lambda(1,0,0)$$

$$+ [-\nabla_\xi{}^2 + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial \xi^0})^2] \vec{A}_\xi + [-\nabla_\xi{}^2 + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial \xi^0})^2] \vec{\nabla}_\xi \Lambda$$

$$+ \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} \frac{a_0{}^2}{c^4} A_{\xi^1}(1,0,0) - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{a_0}{c^2} \vec{\nabla}_\xi A_{\xi^1}$$

$$+ \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} \frac{a_0{}^2}{c^4} \frac{\partial \Lambda}{\partial \xi^1}(1,0,0) - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{a_0}{c^2} \frac{\partial}{\partial \xi^1} \vec{\nabla}_\xi \Lambda \qquad (53)$$

In this time,

$$[\frac{1}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} (\frac{\partial}{\partial \xi^0})^2 - \nabla_\xi{}^2] \Lambda - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{a_0}{c^2} \frac{\partial \Lambda}{\partial \xi^1} = 0$$

$$0=\vec{\nabla}_{\xi}[\{-\nabla_{\xi}{}^{2}+\frac{1}{c^{2}}\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})^{2}}(\frac{\partial}{\partial\xi^{0}})^{2}-\frac{1}{(1+\frac{a_{0}\xi^{1}}{c^{2}})}\frac{a_{0}}{c^{2}}\frac{\partial}{\partial\xi^{1}}\}\Lambda]$$

$$=\vec{\nabla}_{\xi}\{\frac{1}{c^{2}}\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})^{2}}\}(\frac{\partial}{\partial\xi^{0}})^{2}\Lambda+[-\nabla_{\xi}{}^{2}+\frac{1}{c^{2}}\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})^{2}}(\frac{\partial}{\partial\xi^{0}})^{2}]\vec{\nabla}_{\xi}\Lambda$$

$$-\vec{\nabla}_{\xi}\{\frac{a_{0}}{c^{2}}\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})}\}\frac{\partial\Lambda}{\partial\xi^{1}}-\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})}\frac{a_{0}}{c^{2}}\frac{\partial}{\partial\xi^{1}}\vec{\nabla}_{\xi}\Lambda$$

$$=-\frac{2}{c^{2}}\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})^{3}}\frac{a_{0}}{c^{2}}(\frac{\partial}{\partial\xi^{0}})^{2}\Lambda(1,0,0)+[-\nabla_{\xi}{}^{2}+\frac{1}{c^{2}}\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})^{2}}(\frac{\partial}{\partial\xi^{0}})^{2}]\vec{\nabla}_{\xi}\Lambda$$

$$+\frac{a_{0}{}^{2}}{c^{4}}\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})^{2}}\frac{\partial\Lambda}{\partial\xi^{1}}(1,0,0)-\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})}\frac{a_{0}}{c^{2}}\frac{\partial}{\partial\xi^{1}}\vec{\nabla}_{\xi}\Lambda \qquad (54)$$

Therefore,

$$\frac{4\pi}{c}\vec{j}_{\xi}$$

$$=\frac{a_{0}}{c^{2}}\frac{1}{(1+\frac{a_{0}\xi^{1}}{c^{2}})}(0,-B_{\xi^{3}},B_{\xi^{2}})+\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})}\frac{2a_{0}}{c^{2}}\frac{\partial\phi_{\xi}}{c\partial\xi^{0}}(1,0,0)$$

$$-\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})^{3}}\frac{2a_{0}}{c^{2}}\frac{1}{c^{2}}(\frac{\partial}{\partial\xi^{0}})^{2}\Lambda(1,0,0)$$

$$+[-\nabla_{\xi}{}^{2}+\frac{1}{c^{2}}\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})^{2}}(\frac{\partial}{\partial\xi^{0}})^{2}]\vec{A}_{\xi}+\frac{1}{(1+\frac{a_{0}\xi^{1}}{c^{2}})^{2}}\frac{a_{0}{}^{2}}{c^{4}}A_{\xi^{1}}(1,0,0)$$

$$-\frac{1}{(1+\frac{a_{0}\xi^{1}}{c^{2}})}\frac{a_{0}}{c^{2}}\vec{\nabla}_{\xi}A_{\xi^{1}}+\frac{1}{(1+\frac{a_{0}\xi^{1}}{c^{2}})^{2}}\frac{a_{0}{}^{2}}{c^{4}}\frac{\partial\Lambda}{\partial\xi^{1}}(1,0,0)-\frac{1}{(1+\frac{a_{0}\xi^{1}}{c^{2}})}\frac{a_{0}}{c^{2}}\frac{\partial}{\partial\xi^{1}}\vec{\nabla}_{\xi}\Lambda$$

$$+\vec{\nabla}_{\xi}[\{-\nabla_{\xi}{}^{2}+\frac{1}{c^{2}}\frac{1}{(1+\frac{a_{0}}{c^{2}}\xi^{1})^{2}}(\frac{\partial}{\partial\xi^{0}})^{2}-\frac{1}{(1+\frac{a_{0}\xi^{1}}{c^{2}})}\frac{a_{0}}{c^{2}}\frac{\partial}{\partial\xi^{1}}\}\Lambda]$$

$$+\frac{2}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{a_0}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0)-\frac{a_0{}^2}{c^4}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial\Lambda}{\partial\xi^1}(1,0,0)$$

$$+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}\frac{\partial}{\partial\xi^1}\vec{\nabla}_\xi\Lambda$$

$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{a_0{}^2}{c^4}A_{\xi^1}(1,0,0)-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\vec{\nabla}_\xi A_{\xi^1} \qquad (55)$$

Hence, Eq(43-ii) is invariant about Lorentz gauge transformation in Rindler spacetime.

Eq (43-iii) is

$$\vec{\nabla}_\xi\cdot\vec{B}_\xi=\vec{\nabla}_\xi\cdot(\vec{\nabla}_\xi\times\vec{A}_\xi+\vec{\nabla}_\xi\times\vec{\nabla}_\xi\Lambda)=\vec{\nabla}_\xi\times\vec{\nabla}_\xi\cdot\vec{A}_\xi=0 \qquad (56)$$

Eq (43-iv) is

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times\{\vec{E}_\xi(1+\frac{a_0\xi^1}{c^2})\}$$

$$=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times[\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}-\vec{\nabla}_\xi(\frac{\partial\Lambda}{c\partial\xi^0})+\frac{\partial\vec{A}_\xi}{c\partial\xi^0}+\frac{\partial}{c\partial\xi^0}(\vec{\nabla}_\xi\Lambda)]$$

$$=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times\frac{\partial\vec{A}_\xi}{c\partial\xi^0}=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial(\vec{\nabla}_\xi\times\vec{A}_\xi)}{c\partial\xi^0}=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{B}_\xi}{c\partial\xi^0} \qquad (57)$$

Hence, Eq (43-iii), Eq (43-iv) are invariant about Lorentz gauge transformation in Rindler spacetime.

Hence, the electro-magnetic field equations(Maxwell Equations) in Rindler spacetime are invariant about Lorentz gauge transformation.

5. Conclusion

We find the electro-magnetic field transformation and the electro-magnetic equation in uniformly accelerated frame in one theory.

Generally, the coordinate transformation of accelerated frame is (see Ref [9])

(I) $$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3 \tag{58}$$

(II) $$ct = \frac{c^2}{a_0}\exp(\frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = \frac{c^2}{a_0}\exp(\frac{a_0}{c^2}\xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3 \tag{59}$$

If You try to use Eq(59) for make Maxwell equation in Rindler space-time, You have to fail it.

In A.Einstein's article (see Ref [10]), Einstein obtain Lorenz transformation by Maxwell equation in inertial frame, Einstein give up Galilei transformation in inertial frame. In accelerated frame, we think our article's choice is Rindler coordinate (I) can treat electro-magnetic field equation likely Einstein's election.

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