Electro-Magnetic Field Equation and Transformation in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime. We find the electro-magnetic wave equation and the electro-magnetic wave function in Rindler space-time. Specially, this article say the uniqueness of the accelerated frame because the accelerated frame can treat electro-magnetic field equation.

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic field equation in Rindler space-time.

The Rindler coordinate is

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3 \tag{1}$$

In this time, the tetrad $e^a{}_\mu$ is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2}\eta_{ab}\frac{\partial x^a}{\partial \xi^\mu}\frac{\partial x^b}{\partial \xi^\upsilon}d\xi^\mu d\xi^\upsilon$$

$$= -\frac{1}{c^2}\eta_{ab}e^a{}_\mu e^b{}_\upsilon d\xi^\mu d\xi^\upsilon = -\frac{1}{c^2}g_{\mu\upsilon}d\xi^\mu d\xi^\upsilon \quad , \quad e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \tag{2}$$

$$e^\alpha{}_0(\xi^0) = \frac{\partial x^\alpha}{c\partial \xi^0} = ((1 + \frac{a_0}{c^2}\xi^1)\cosh(\frac{a_0\xi^0}{c}), (1 + \frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c}), 0, 0) \tag{3}$$

About y -axis's and z -axis's orientation

$$e^\alpha{}_2(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0,0,1,0) \quad , \quad e^\alpha{}_3(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0,0,0,1) \tag{4}$$

The other unit vector $e^\alpha{}_1(\xi^0)$ is

$$e^\alpha{}_1(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^1} = (\sinh(\frac{a_0\xi^0}{c}), \cosh(\frac{a_0\xi^0}{c}), 0, 0) \tag{5}$$

Therefore,

$$cdt = c\cosh(\frac{a_0\xi^0}{c})d\xi^0(1 + \frac{a_0}{c^2}\xi^1) + \sinh(\frac{a_0\xi^0}{c})d\xi^1$$

$$dx = c\sinh(\frac{a_0\xi^0}{c})d\xi^0(1 + \frac{a_0}{c^2}\xi^1) + \cosh(\frac{a_0\xi^0}{c})d\xi^1, dy = d\xi^2, \ dz = d\xi^3 \tag{6}$$

The vector transformation is

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} V^{\alpha}, \qquad U'_{\mu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} U_{\alpha} \tag{7}$$

Therefore, the transformation of the electro-magnetic 4-vector potential $(\phi, \vec{A}) = A^{\alpha}$ is

$$A^{\alpha} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} A'^{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} A_{\xi}^{\ \mu} = e^{\alpha}{}_{\mu} A_{\xi}^{\ \mu} \ , \ e^{\alpha}{}_{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}}$$

$$dx^{\alpha} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} dx'^{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} d\xi^{\mu} = e^{\alpha}{}_{\mu} d\xi^{\mu} \ , e^{\alpha}{}_{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} \tag{8}$$

Hence, the transformation of the electro-magnetic 4-vector potential $(\phi, \vec{A})$ in inertial frame and the eletro-magnetic 4-vector potential $(\phi_{\xi}, \vec{A}_{\xi})$ in uniformly accelerated frame is

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\phi = 4\pi\rho$$

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\vec{A} = \frac{4\pi}{c}\vec{j}$$

4-vector $(c\rho, \vec{j}) = \rho_0 \dfrac{dx^{\alpha}}{d\tau}$

$$\phi = \cosh(\frac{a_0 \xi^0}{c})(1 + \frac{a_0}{c^2}\xi^1)\phi_{\xi} + \sinh(\frac{a_0 \xi^0}{c}) A_{\xi^1}$$

$$A_x = \sinh(\frac{a_0 \xi^0}{c})(1 + \frac{a_0}{c^2}\xi^1)\phi_{\xi} + \cosh(\frac{a_0 \xi^0}{c}) A_{\xi^1}$$

$$A_y = A_{\xi^2}, A_z = A_{\xi^3} \tag{9}$$

$$g = \begin{pmatrix} -(1+\frac{a_0 \xi^1}{c^2})^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e^{a}{}_{\mu} e_{b}^{\ \mu} = \delta^{a}{}_{b} \ , \ e^{a}{}_{\mu} e_{a}^{\ \upsilon} = \delta_{\mu}^{\ \upsilon}$$

$$e^{a}{}_{\mu} e^{b}{}_{\upsilon} \eta_{ab} = g_{\mu\upsilon} \ \rightarrow \ A^{T} \eta A = g$$

$$e_a{}^{\mu} e_b{}^{\upsilon} g_{\mu\upsilon} = \eta_{ab} \rightarrow \quad (A^T)^{-1} g A^{-1} = (A^T)^{-1} A^T \eta A A^{-1} = \eta$$

$$e^a{}_{\mu} = \eta^{ab} g_{\mu\upsilon} e_b{}^{\upsilon} \rightarrow \eta^{-1}(A^T)^{-1} A^T \eta A = A = \eta^{-1}(A^T)^{-1} g \tag{10}$$

$$\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ dz^3 \end{pmatrix}$$

$$= A \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ dz^3 \end{pmatrix} \tag{11}$$

$$e_{\mu}{}^{\alpha} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} = A^{-1} = \begin{pmatrix} \frac{c\partial\xi^0}{c\partial t} & \frac{c\partial\xi^0}{\partial x} & \frac{c\partial\xi^0}{\partial y} & \frac{c\partial\xi^0}{\partial z} \\ \frac{\partial\xi^1}{c\partial t} & \frac{\partial\xi^1}{\partial x} & \frac{\partial\xi^1}{\partial y} & \frac{\partial\xi^1}{\partial z} \\ \frac{\partial\xi^2}{c\partial t} & \frac{\partial\xi^2}{\partial x} & \frac{\partial\xi^2}{\partial y} & \frac{\partial\xi^2}{\partial z} \\ \frac{\partial\xi^3}{c\partial t} & \frac{\partial\xi^3}{\partial x} & \frac{\partial\xi^3}{\partial y} & \frac{\partial\xi^3}{\partial z} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & 0 & 0 \\ -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = (A^{-1})^{T} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} = (A^{T})^{-1} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} \tag{13}$$

$$\frac{1}{c}\frac{\partial}{\partial t} = \frac{c\partial\xi^0}{c\partial t}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{c\partial t}\frac{\partial}{\partial\xi^1}$$

$$= \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$\frac{\partial}{\partial x} = \frac{c\partial\xi^0}{\partial x}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{\partial x}\frac{\partial}{\partial\xi^1}$$

$$= -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial z} = \frac{\partial}{\partial\xi^3}$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_{\xi}{}^2$$

$$\vec{\nabla} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}), \quad \vec{\nabla}_\xi = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3}) \tag{14}$$

2. Electro-magnetic Field in the Rindler space-time

The electro-magnetic field $(\vec{E}, \vec{B})$ is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{c\partial t}, \vec{B} = \vec{\nabla} \times \vec{A} \tag{15}$$

$$E_x = -\frac{\partial \phi}{\partial x} - \frac{\partial A_x}{c\partial t}$$

$$= -[-\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}] \cdot [\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2})\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}] \cdot [\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2})\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$= -\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial A_{\xi^1}}{c\partial \xi^0} - (1+\frac{a_0\xi^1}{c^2})\frac{\partial \phi_\xi}{\partial \xi^1} - 2\phi_\xi \frac{a_0}{c^2}$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial \xi^1}[(1+\frac{a_0}{c^2}\xi^1)^2\phi_\xi] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial \xi^0} \tag{16}$$

$$E_y = -\frac{\partial \phi}{\partial y} - \frac{\partial A_y}{c\partial t} = -\frac{\partial}{\partial \xi^2}[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]A_{\xi^2}$$

$$= -(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})\frac{\partial \phi_\xi}{\partial \xi^2} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\cosh(\frac{a_0\xi^0}{c})\frac{\partial A_{\xi^2}}{c\partial \xi^0}$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^2}}{c\partial\xi^0}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial\xi^1} - \frac{\partial A_{\xi^1}}{\partial\xi^2}] \tag{17}$$

$$E_z = -\frac{\partial\phi}{\partial z} - \frac{\partial A_z}{c\partial t} = -\frac{\partial}{\partial\xi^3}[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$= -(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})\frac{\partial\phi_\xi}{\partial\xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\cosh(\frac{a_0\xi^0}{c})\frac{\partial A_{\xi^3}}{c\partial\xi^0}$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^3}}{\partial\xi^1} - \frac{\partial A_{\xi^1}}{\partial\xi^3}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^3}}{c\partial\xi^0}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^3}}{\partial\xi^1} - \frac{\partial A_{\xi^1}}{\partial\xi^3}] \tag{18}$$

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = \frac{\partial A_{\xi^3}}{\partial\xi^2} - \frac{\partial A_{\xi^2}}{\partial\xi^3} \tag{19}$$

$$B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = \frac{\partial A_x}{\partial\xi^3} - \frac{\partial A_{\xi^3}}{\partial x}$$

$$= \frac{\partial}{\partial\xi^3}[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$=\cosh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^1}}{\partial\xi^3}-\frac{\partial A_{\xi^3}}{\partial\xi^1}]$$

$$-\sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2]-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^3}}{c\partial\xi^0}] \qquad (20)$$

$$B_z=\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}=\frac{\partial A_{\xi^2}}{\partial x}-\frac{\partial A_x}{\partial\xi^2}$$

$$=[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$-\frac{\partial}{\partial\xi^2}[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi+\cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$=\cosh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial\xi^1}-\frac{\partial A_{\xi^1}}{\partial\xi^2}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2]-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^2}}{c\partial\xi^0}] \qquad (21)$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi,\vec{B}_\xi)$ in Rindler spacetime.

$$\vec{E}_\xi=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0}$$

$$\vec{B}_\xi=\vec{\nabla}_\xi\times\vec{A}_\xi$$

In this time, $\vec{\nabla}_\xi=(\frac{\partial}{\partial\xi^1},\frac{\partial}{\partial\xi^2},\frac{\partial}{\partial\xi^3}),\vec{A}_\xi=(A_{\xi^1},A_{\xi^2},A_{\xi^3})$ (22)

We obtain the transformation of the electro-magnetic field.

$$E_x = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial\xi^0} = E_{\xi^1} \ ,$$

$$E_y = E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$B_x = B_{\xi^1},$$

$$B_y = B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z = B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}) \qquad (23)$$

Hence,

$$E_x = E_{\xi^1}, B_x = B_{\xi^1},$$

$$\begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix} = H \begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix}$$

$$H = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 & \sinh(\frac{a_0\xi^0}{c}) \\ 0 & \cosh(\frac{a_0\xi^0}{c}) & -\sinh(\frac{a_0\xi^0}{c}) & 0 \\ 0 & -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 \\ \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 & \cosh(\frac{a_0\xi^0}{c}) \end{pmatrix} \qquad (24)$$

The inverse-transformation of the electro-magnetic field is

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$\begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix} = H^{-1} \begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix}$$

$$H^{-1} = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 & -\sinh(\frac{a_0\xi^0}{c}) \\ 0 & \cosh(\frac{a_0\xi^0}{c}) & \sinh(\frac{a_0\xi^0}{c}) & 0 \\ 0 & \sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 \\ -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 & \cosh(\frac{a_0\xi^0}{c}) \end{pmatrix} \quad (25)$$

$$E_{\xi^1} = E_x \, , B_{\xi^1} = B_x$$

$$E_{\xi^2} = E_y \cosh(\frac{a_0\xi^0}{c}) - B_z \sinh(\frac{a_0\xi^0}{c}) \, ,$$

$$B_{\xi^2} = B_y \cosh(\frac{a_0\xi^0}{c}) + E_z \sinh(\frac{a_0\xi^0}{c})$$

$$E_{\xi^3} = E_z \cosh(\frac{a_0\xi^0}{c}) + B_y \sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^3} = B_z \cosh(\frac{a_0\xi^0}{c}) - E_y \sinh(\frac{a_0\xi^0}{c}) \quad (26)$$

3. Electro-magnetic Field Equation(Maxwell Equation) in the Rindler space-time

Maxwell equation is

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad (27\text{-i})$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j} \quad (27\text{-ii})$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (27\text{-iii})$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{c\partial t} \quad (27\text{-iv})$$

1. $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$

$E_x = E_{\xi^1}$,

$$E_y = E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$4\pi\rho = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]E_{\xi^1}$$

$$+\frac{\partial}{\partial\xi^2}[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{\partial}{\partial\xi^3}[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)(\vec{\nabla}_\xi\cdot\vec{E}_\xi) + \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial\xi^2} - \frac{\partial B_{\xi^2}}{\partial\xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial E_{\xi^1}}{c\partial\xi^0}] \qquad (28)$$

2. $\vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j}$

$$B_x = B_{\xi^1}$$

$$B_y = B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z = B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

X-component) $\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$

$$= \frac{\partial}{\partial\xi^2}[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial}{\partial\xi^3}[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3}] + \sinh(\frac{a_0\xi^0}{c})[\frac{\partial E_{\xi^2}}{\partial \xi^2} + \frac{\partial E_{\xi^3}}{\partial \xi^3}]$$

$$= \frac{\partial E_x}{c\partial t} + \frac{4\pi}{c} j_x$$

$$= [\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]E_{\xi^1} + \frac{4\pi}{c} j_x$$

Hence,

$$\frac{4\pi}{c} j_x$$

$$= \sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi \cdot \vec{E}_\xi) + \cosh(\frac{a_0\xi^0}{c})[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial E_{\xi^1}}{c\partial \xi^0}] \qquad (29)$$

Y-component) $\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}$

$$= \frac{\partial B_{\xi^1}}{\partial \xi^3}$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}] \cdot [B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \frac{\partial E_y}{c\partial t} + \frac{4\pi}{c} j_y$$

$$= [\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}] \cdot [E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+ \frac{4\pi}{c} j_y$$

$$\frac{4\pi}{c} j_y = \frac{\partial B_{\xi^1}}{\partial \xi^3} - \frac{\partial B_{\xi^3}}{\partial \xi^1} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}B_{\xi^3} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^2}}{c\partial \xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}\{B_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{B_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^2}}{c\partial\xi^0} \quad (30)$$

Z-component) $\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial B_{\xi^1}}{\partial\xi^2}$$

$$= \frac{\partial E_z}{c\partial t} + \frac{4\pi}{c}j_z$$

$$= [\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{4\pi}{c}j_z$$

$$\frac{4\pi}{c}j_z = \frac{\partial B_{\xi^2}}{\partial\xi^1} - \frac{\partial B_{\xi^1}}{\partial\xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}B_{\xi^2} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^3}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{B_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}\{B_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^3}}{c\partial\xi^0} \quad (31)$$

3. $\vec{\nabla}\cdot\vec{B} = 0$

$$\vec{\nabla}\cdot\vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]B_{\xi^1}$$

$$+\frac{\partial}{\partial\xi^2}[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{\partial}{\partial\xi^3}[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$=\cosh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi\cdot\vec{B}_\xi)+\sinh(\frac{a_0\xi^0}{c})[-(-\frac{\partial E_{\xi^2}}{\partial\xi^3}+\frac{\partial E_{\xi^3}}{\partial\xi^2})-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^1}}{c\partial\xi^0}]=0$$

(32)

4. $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{c\partial t}$

$E_x=E_{\xi^1}$,

$E_y=E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$,

$E_z=E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$

X-component) $\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}$

$$=\frac{\partial}{\partial\xi^2}[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial}{\partial\xi^3}[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$=\cosh(\frac{a_0}{c}\xi^0)[\frac{\partial E_{\xi^3}}{\partial\xi^2}-\frac{\partial E_{\xi^2}}{\partial\xi^3}]-\sinh(\frac{a_0\xi^0}{c})[\frac{\partial B_{\xi^2}}{\partial\xi^2}+\frac{\partial B_{\xi^3}}{\partial\xi^3}]$$

$$=-\frac{\partial B_x}{c\partial t}$$

$$=-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]B_{\xi^1}$$

Hence,

$$-\sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_{\xi}\cdot\vec{B}_{\xi})+\cosh(\frac{a_0\xi^0}{c})[(\frac{\partial E_{\xi^3}}{\partial\xi^2}-\frac{\partial E_{\xi^2}}{\partial\xi^3})+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial B_{\xi^1}}{c\partial\xi^0}]=0 \quad (33)$$

Y-component) $\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}$

$$=\frac{\partial E_{\xi^1}}{\partial\xi^3}$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$=-\frac{\partial B_y}{c\partial t}$$

$$=-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$\frac{\partial E_{\xi^1}}{\partial\xi^3}-\frac{\partial E_{\xi^3}}{\partial\xi^1}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}E_{\xi^3}+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^2}}{c\partial\xi^0}$$

$$=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}\{E_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{E_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\}+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^2}}{c\partial\xi^0}$$

$$=0 \quad (34)$$

Z-component) $\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}$

$$=[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial E_{\xi^1}}{\partial\xi^2}$$

$$= -\frac{\partial B_z}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$\frac{\partial E_{\xi^2}}{\partial\xi^1} - \frac{\partial E_{\xi^1}}{\partial\xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}E_{\xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^3}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{E_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}\{E_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^3}}{c\partial\xi^0}$$

$$= 0 \tag{35}$$

Therefore, we obtain the electro-magnetic field equation by Eq (28)-Eq(35) in Rindler spacetime .

$$\vec{\nabla}_\xi \cdot \vec{E}_\xi = 4\pi\rho_\xi(1+\frac{a_0\xi^1}{c^2}) \tag{36-i}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{B}_\xi(1+\frac{a_0\xi^1}{c^2})\} = \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{E}_\xi}{c\partial\xi^0} + \frac{4\pi}{c}\vec{j}_\xi \tag{36-ii}$$

$$\vec{\nabla}_\xi \cdot \vec{B}_\xi = 0 \tag{36-iii}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{E}_\xi(1+\frac{a_0\xi^1}{c^2})\} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{B}_\xi}{c\partial\xi^0} \tag{36-iv}$$

$$\vec{E}_\xi = (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}), \vec{B}_\xi = (B_{\xi^1}, B_{\xi^2}, B_{\xi^3}), \vec{\nabla}_\xi = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3})$$

Hence, the transformation of 4-vector $(c\rho, \vec{j}) = \rho_0\frac{dx^\alpha}{d\tau}$ is

$$\rho = \rho_\xi(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c}) + \frac{j_{\xi^1}}{c}\sinh(\frac{a_0\xi^0}{c})$$

$$j_x = j_{\xi^1}\cosh(\frac{a_0\xi^0}{c}) + c\rho_\xi(1+\frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c}), \quad j_y = j_{\xi^2}, j_z = j_{\xi^3}$$

In this time, 4-vector $(c\rho_\xi, \vec{j}_\xi) = \rho_0\frac{d\xi^\alpha}{d\tau}$ (37)

4. Electro-magnetic wave equation in Rindler space-time

The electro-magnetic wave function is

$$E_x = E_{x0}\sin\Phi, B_x = B_{x0}\sin\Phi,$$

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$E_{\xi^1} = E_X = E_{X0}\sin\Phi = E_{X0}\sin\Phi' - C_{\xi^1}, B_{\xi^1} = B_X = B_{X0}\sin\Phi = B_{X0}\sin\Phi' - C'_{\xi^1}$$

$$E_{\xi^2} = E_y\cosh(\frac{a_0\xi^0}{c}) - B_z\sinh(\frac{a_0\xi^0}{c}),$$

$$= (E_{y0}\sin\Phi)\cosh(\frac{a_0\xi^0}{c}) - (B_{z0}\sin\Phi)\sinh(\frac{a_0\xi^0}{c})$$

$$= (E_{y0}\sin\Phi' - C_{\xi^2})\cosh(\frac{a_0\xi^0}{c}) - (B_{z0}\sin\Phi' - C'_{\xi^3})\sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^2} = B_y\cosh(\frac{a_0\xi^0}{c}) + E_z\sinh(\frac{a_0\xi^0}{c})$$

$$= (B_{y0}\sin\Phi)\cosh(\frac{a_0\xi^0}{c}) + (E_{z0}\sin\Phi)\sinh(\frac{a_0\xi^0}{c})$$

$$= (B_{y0}\sin\Phi' - C'_{\xi^2})\cosh(\frac{a_0\xi^0}{c}) + (E_{z0}\sin\Phi' - C_{\xi^3})\sinh(\frac{a_0\xi^0}{c})$$

$$E_{\xi^3} = E_z\cosh(\frac{a_0\xi^0}{c}) + B_y\sinh(\frac{a_0\xi^0}{c})$$

$$= (E_{z0}\sin\Phi)\cosh(\frac{a_0\xi^0}{c}) + (B_{y0}\sin\Phi)\sinh(\frac{a_0\xi^0}{c})$$

$$= (E_{z0}\sin\Phi' - C_{\xi^3})\cosh(\frac{a_0\xi^0}{c}) + (B_{y0}\sin\Phi' - C'_{\xi^2})\sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^3} = B_z\cosh(\frac{a_0\xi^0}{c}) - E_y\sinh(\frac{a_0\xi^0}{c})$$

$$= (B_{z0}\sin\Phi)\cosh(\frac{a_0\xi^0}{c}) - (E_{y0}\sin\Phi)\sinh(\frac{a_0\xi^0}{c})$$

$$= (B_{z0}\sin\Phi' - C'_{\xi^3})\cosh(\frac{a_0\xi^0}{c}) - (E_{y0}\sin\Phi' - C_{\xi^2})\sinh(\frac{a_0\xi^0}{c})$$

In this time,

$$(\frac{\partial}{\partial\xi^0})^2\Phi' \cdot \frac{\partial(E_{X0}\sin\Phi')}{\partial\xi^0} / \frac{\partial\Phi'}{\partial\xi^0} = (\frac{\partial}{\partial\xi^0})^2 C_{\xi^1},$$

$$(\frac{\partial}{\partial\xi^0})^2\Phi' \cdot \frac{\partial(E_{y0}\sin\Phi')}{\partial\xi^0} / \frac{\partial\Phi'}{\partial\xi^0} = (\frac{\partial}{\partial\xi^0})^2 C_{\xi^2} \qquad ,$$

$$(\frac{\partial}{\partial\xi^0})^2\Phi'\cdot\frac{\partial(E_{z0}\sin\Phi')}{\partial\xi^0}/\frac{\partial\Phi'}{\partial\xi^0}=(\frac{\partial}{\partial\xi^0})^2C_{\xi^3}$$

$$(\frac{\partial}{\partial\xi^0})^2\Phi'\cdot\frac{\partial(B_{x0}\sin\Phi')}{\partial\xi^0}/\frac{\partial\Phi'}{\partial\xi^0}=(\frac{\partial}{\partial\xi^0})^2C'_{\xi^1},$$

$$(\frac{\partial}{\partial\xi^0})^2\Phi'\cdot\frac{\partial(B_{y0}\sin\Phi')}{\partial\xi^0}/\frac{\partial\Phi'}{\partial\xi^0}=(\frac{\partial}{\partial\xi^0})^2C'_{\xi^2}$$,

$$(\frac{\partial}{\partial\xi^0})^2\Phi'\cdot\frac{\partial(B_{z0}\sin\Phi')}{\partial\xi^0}/\frac{\partial\Phi'}{\partial\xi^0}=(\frac{\partial}{\partial\xi^0})^2C'_{\xi^3}$$

$$(\frac{\partial}{\partial\xi^1})^2\vec{C}_\xi=(\frac{\partial}{\partial\xi^2})^2\vec{C}_\xi=(\frac{\partial}{\partial\xi^3})^2\vec{C}_\xi=\vec{0}$$

$$,\ (\frac{\partial}{\partial\xi^1})^2\vec{C}'_\xi=(\frac{\partial}{\partial\xi^2})^2\vec{C}'_\xi=(\frac{\partial}{\partial\xi^3})^2\vec{C}'_\xi=\vec{0} \qquad (38)$$

$$\Phi=\omega(t-l\frac{x}{c}-m\frac{y}{c}-n\frac{z}{c}),$$

$$\Phi'=\omega[\frac{c}{a_0}(1+\frac{a_0\xi^1}{c^2})\sinh(\frac{a_0\xi^0}{c})-l\frac{c}{a_0}(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})-m\frac{\xi^2}{c}-n\frac{\xi^3}{c}+l\frac{c}{a_0}]$$

$$=\omega[\frac{c}{a_0}(1+\frac{a_0\xi^1}{c^2})\{\sinh(\frac{a_0\xi^0}{c})-l\cosh(\frac{a_0\xi^0}{c})\}-m\frac{\xi^2}{c}-n\frac{\xi^3}{c}+l\frac{c}{a_0}]$$

$$ct=(\frac{c^2}{a_0}+\xi^1)\sinh(\frac{a_0\xi^0}{c}),x=(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})-\frac{c^2}{a_0}\quad ,y=\xi^2,z=\xi^3$$

$$l^2+m^2+n^2=1 \qquad (39)$$

Hence,

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]E_{\xi^1}$$

$$=[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2][E_{X0}\sin\Phi'-C_{\xi^1}]=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]B_{\xi^1}$$

$$= [\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2][B_{X0}\sin\Phi' - C'_{\xi^1}] = 0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]E_y$$

$$= [\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2][E_{y0}\sin\Phi' - C_{\xi^2}] = 0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]B_y$$

$$= [\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2][B_{y0}\sin\Phi' - C'_{\xi^2}] = 0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]E_z$$

$$= [\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2][E_{z0}\sin\Phi' - C_{\xi^3}] = 0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]B_z$$

$$= [\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2][B_{z0}\sin\Phi' - C'_{\xi^3}] = 0 \qquad (40)$$

The electro-magnetic wave equation is in vacuum

$$\vec{\nabla}_\xi \times (1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi \times \{\vec{E}_\xi(1+\frac{a_0}{c^2}\xi^1)\}$$

$$= \vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1) \times \vec{\nabla}_\xi \times \{\vec{E}_\xi(1+\frac{a_0}{c^2}\xi^1)\} + (1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi \times \vec{\nabla}_\xi \times \{\vec{E}_\xi(1+\frac{a_0}{c^2}\xi^1)\}$$

$$= \vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1) \times \vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1) \times \vec{E}_\xi$$

$$+ (1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1) \times \vec{\nabla}_\xi \times \vec{E}_\xi$$

$$+(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi\times\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{E}_\xi$$

$$+(1+\frac{a_0}{c^2}\xi^1)^2\vec{\nabla}_\xi\times\vec{\nabla}_\xi\times\vec{E}_\xi$$

$$=\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{E}_\xi+(1+\frac{a_0}{c^2}\xi^1)^2\vec{\nabla}_\xi\times\vec{\nabla}_\xi\times\vec{E}_\xi$$

$$=[\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{E}_\xi]\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)-[\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)]\vec{E}_\xi$$

$$+(1+\frac{a_0}{c^2}\xi^1)^2[\vec{\nabla}_\xi(\vec{\nabla}_\xi\cdot\vec{E}_\xi)-\nabla_\xi{}^2\vec{E}_\xi]$$

$$=-\frac{1}{c}\frac{\partial}{\partial\xi^0}[\vec{\nabla}_\xi\times\{\vec{B}_\xi(1+\frac{a_0\xi^1}{c^2})\}]=-\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{E}_\xi,$$

In this time, $\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)=(\frac{a_0}{c^2},0,0)$ (41)

Hence,

$$\vec{\nabla}_\xi\times(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi\times\{\vec{E}_\xi(1+\frac{a_0}{c^2}\xi^1)\}+\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{E}_\xi$$

$$=[\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{E}_\xi]\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)-[\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)]\vec{E}_\xi$$

$$+(1+\frac{a_0}{c^2}\xi^1)^2[\vec{\nabla}_\xi(\vec{\nabla}_\xi\cdot\vec{E}_\xi)-\nabla_\xi{}^2\vec{E}_\xi]+\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{E}_\xi$$

$$=\frac{a_0{}^2}{c^4}(E_{\xi^1},0,0)-\frac{a_0{}^2}{c^4}(E_{\xi^1},E_{\xi^2},E_{\xi^3})+(1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\vec{E}_\xi$$

$$=\frac{a_0{}^2}{c^4}(0,-E_{\xi^2},-E_{\xi^3})+(1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\vec{E}_\xi$$

$$=\vec{0}$$ (42)

Hence, the magnetic wave equation is in vacuum

$$\vec{\nabla}_\xi\times(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi\times\{\vec{B}_\xi(1+\frac{a_0}{c^2}\xi^1)\}+\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{B}_\xi$$

$$=[\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{B}_\xi]\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)-[\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)]\vec{B}_\xi$$

$$+(1+\frac{a_0}{c^2}\xi^1)^2[\vec{\nabla}_\xi(\vec{\nabla}_\xi\cdot\vec{B}_\xi)-\nabla_\xi{}^2\vec{B}_\xi]+\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{B}_\xi$$

$$= \frac{a_0^{\ 2}}{c^4}(0,-B_{\xi^2},-B_{\xi^3}) + (1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial \xi^0})^2 - \nabla_\xi^{\ 2}]\vec{B}_\xi$$

$$= \vec{0} \qquad (43)$$

The electromagnetic wave function, Eq(38),Eq(39) satisfy the electromagnetic wave equation, Eq(42),Eq(43).

5. Conclusion

We find the electro-magnetic field transformation and the electro-magnetic equation in uniformly accelerated frame.

Generally, the coordinate transformation of accelerated frame is

(I) $$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0 \xi^0}{c})$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0 \xi^0}{c}) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3 \qquad (44)$$

(II) $$ct = \frac{c^2}{a_0}\exp(\frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0 \xi^0}{c})$$

$$x = \frac{c^2}{a_0}\exp(\frac{a_0}{c^2}\xi^1)\cosh(\frac{a_0 \xi^0}{c}) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3 \qquad (45)$$

Hence, this article say the accelerated frame is Rindler coordinate (I) that can treat electro-magnetic field equation.

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