The Electro-Magnetic Field Equation and the Electro-Magnetic Field Transformation in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime.

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic field equation in Rindler space-time.

The Rindler coordinate is

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3 \quad (1)$$

In this time, the tetrad $e^a{}_\mu$ is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2}\eta_{ab}\frac{\partial x^a}{\partial \xi^\mu}\frac{\partial x^b}{\partial \xi^\upsilon}d\xi^\mu d\xi^\upsilon$$

$$= -\frac{1}{c^2}\eta_{ab}e^a{}_\mu e^b{}_\upsilon d\xi^\mu d\xi^\upsilon = -\frac{1}{c^2}g_{\mu\upsilon}d\xi^\mu d\xi^\upsilon \quad , \quad e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \quad (2)$$

$$e^\alpha{}_0(\xi^0) = \frac{\partial x^\alpha}{c\partial \xi^0} = (\cosh(\frac{a_0\xi^0}{c}), \sinh(\frac{a_0\xi^0}{c}), 0, 0) \quad (3)$$

About y -axis's and z -axis's orientation

$$e^\alpha{}_2(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0,0,1,0) \quad , \quad e^\alpha{}_3(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0,0,0,1) \quad (4)$$

The other unit vector $e^\alpha{}_1(\xi^0)$ is

$$e^\alpha{}_1(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^1} = (\sinh(\frac{a_0\xi^0}{c}), \cosh(\frac{a_0\xi^0}{c}), 0, 0) \quad (5)$$

Therefore,

$$cdt = c\cosh(\frac{a_0\xi^0}{c})d\xi^0(1 + \frac{a_0}{c^2}\xi^1) + \sinh(\frac{a_0\xi^0}{c})d\xi^1$$

$$dx = c\sinh(\frac{a_0\xi^0}{c})d\xi^0(1 + \frac{a_0}{c^2}\xi^1) + \cosh(\frac{a_0\xi^0}{c})d\xi^1, dy = d\xi^2, \ dz = d\xi^3 \quad (6)$$

Hence, the transformation of the electro-magnetic 4-vector potential $(\phi, \vec{A})$ in inertial frame and the eletro-magnetic 4-vector potential $(\phi_\xi, \vec{A}_\xi)$ in uniformly accelerated frame is

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\phi = 4\pi\rho$$

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\vec{A} = \frac{4\pi}{c}\vec{j}$$

4-vector $(c\rho, \vec{j}) = \rho_0 \frac{dx^\alpha}{d\tau}$

$$\phi = \cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}$$

$$A_x = \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}$$

$$A_y = A_{\xi^2}, A_z = A_{\xi^3} \quad (7)$$

$$g = \begin{pmatrix} -(1+\frac{a_0\xi^1}{c^2})^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e^a{}_\mu e^b{}_\upsilon \eta_{ab} = g_{\mu\upsilon} \rightarrow \quad A^T\eta A = g$$

$$e_a{}^\mu e_b{}^\upsilon g_{\mu\upsilon} = \eta_{ab} \rightarrow \quad (A^T)^{-1}gA^{-1} = (A^T)^{-1}A^T\eta AA^{-1} = \eta$$

$$e^a{}_\mu = \eta^{ab}g_{\mu\upsilon}e_b{}^\upsilon \rightarrow \eta^{-1}(A^T)^{-1}A^T\eta A = A = \eta^{-1}(A^T)^{-1}g \quad (8)$$

$$\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ dz^3 \end{pmatrix}$$

$$= A\begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ dz^3 \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = (A^{-1})^T \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} = (A^T)^{-1} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} \tag{10}$$

$$\frac{1}{c}\frac{\partial}{\partial t} = \frac{c\partial\xi^0}{c\partial t}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{c\partial t}\frac{\partial}{\partial\xi^1}$$

$$= \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$\frac{\partial}{\partial x} = \frac{c\partial\xi^0}{\partial x}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{\partial x}\frac{\partial}{\partial\xi^1}$$

$$= -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$\frac{\partial}{\partial y}=\frac{\partial}{\partial \xi^2},\frac{\partial}{\partial z}=\frac{\partial}{\partial \xi^3}$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2=\frac{1}{c^2(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial \xi^0})^2-\nabla_\xi{}^2$$

$$\vec{\nabla}=(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}),\quad \vec{\nabla}_\xi=(\frac{\partial}{\partial \xi^1},\frac{\partial}{\partial \xi^2},\frac{\partial}{\partial \xi^3}) \tag{11}$$

2. Electro-magnetic Field in the Rindler space-time

The electro-magnetic field $(\vec{E},\vec{B})$ is in the inertial frame,

$$\vec{E}=-\vec{\nabla}\phi-\frac{\partial\vec{A}}{c\partial t},\vec{B}=\vec{\nabla}\times\vec{A} \tag{12}$$

$$E_x=-\frac{\partial\phi}{\partial x}-\frac{\partial A_x}{c\partial t}$$

$$=-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2})\phi_\xi+\sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2})\phi_\xi+\cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$=-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial A_{\xi^1}}{c\partial\xi^0}-(1+\frac{a_0\xi^1}{c^2})\frac{\partial\phi_\xi}{\partial\xi^1}-2\phi_\xi\frac{a_0}{c^2}$$

$$=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}[(1+\frac{a_0}{c^2}\xi^1)^2\phi_\xi]-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial\xi^0} \tag{13}$$

$$E_y=-\frac{\partial\phi}{\partial y}-\frac{\partial A_y}{c\partial t}=-\frac{\partial}{\partial\xi^2}[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi+\sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^2}$$

$$= -(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})\frac{\partial\phi_\xi}{\partial\xi^2} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\cosh(\frac{a_0\xi^0}{c})\frac{\partial A_{\xi^2}}{c\partial\xi^0}$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial\xi^1} - \frac{\partial A_{\xi^1}}{\partial\xi^2}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^2}}{c\partial\xi^0}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial\xi^1} - \frac{\partial A_{\xi^1}}{\partial\xi^2}] \quad (13)$$

$$E_z = -\frac{\partial\phi}{\partial z} - \frac{\partial A_z}{c\partial t} = -\frac{\partial}{\partial\xi^3}[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$= -(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})\frac{\partial\phi_\xi}{\partial\xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\cosh(\frac{a_0\xi^0}{c})\frac{\partial A_{\xi^3}}{c\partial\xi^0}$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^3}}{\partial\xi^1} - \frac{\partial A_{\xi^1}}{\partial\xi^3}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^3}}{c\partial\xi^0}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^3}}{\partial\xi^1} - \frac{\partial A_{\xi^1}}{\partial\xi^3}] \quad (14)$$

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = \frac{\partial A_{\xi^3}}{\partial\xi^2} - \frac{\partial A_{\xi^2}}{\partial\xi^3} \quad (15)$$

$$B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = \frac{\partial A_x}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial x}$$

$$= \frac{\partial}{\partial \xi^3}\left[\sinh(\frac{a_0 \xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}\right]$$

$$-\left[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}\right]A_{\xi^3}$$

$$= \cosh(\frac{a_0}{c}\xi^0)\left[\frac{\partial A_{\xi^1}}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial \xi^1}\right]$$

$$-\sinh(\frac{a_0}{c}\xi^0)\left[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}\left[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\right] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^3}}{c\partial\xi^0}\right] \quad (16)$$

$$B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = \frac{\partial A_{\xi^2}}{\partial x} - \frac{\partial A_x}{\partial \xi^2}$$

$$= \left[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}\right]A_{\xi^3}$$

$$-\frac{\partial}{\partial \xi^2}\left[\sinh(\frac{a_0 \xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}\right]$$

$$= \cosh(\frac{a_0}{c}\xi^0)\left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2}\right]$$

$$+\sinh(\frac{a_0}{c}\xi^0)\left[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}\left[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\right] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^2}}{c\partial\xi^0}\right] \quad (17)$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler spacetime.

$$\vec{E}_{\xi} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\xi}\{\phi_{\xi}(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_{\xi}}{c\partial\xi^0}$$

$$\vec{B}_{\xi} = \vec{\nabla}_{\xi} \times \vec{A}_{\xi}$$

In this time, $\vec{\nabla}_{\xi} = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3}), \vec{A}_{\xi} = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$ (18)

We obtain the transformation of the electro-magnetic field.

$$E_x = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}\{\phi_{\xi}(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial\xi^0} = E_{\xi^1} \quad ,$$

$$E_y = E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$B_x = B_{\xi^1},$$

$$B_y = B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z = B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}) \tag{19}$$

Hence,

$$E_x = E_{\xi^1}, B_x = B_{\xi^1},$$

$$\begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix} = H \begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix}$$

$$H=\begin{pmatrix}\cosh(\frac{a_0\xi^0}{c}) & 0 & 0 & \sinh(\frac{a_0\xi^0}{c})\\ 0 & \cosh(\frac{a_0\xi^0}{c}) & -\sinh(\frac{a_0\xi^0}{c}) & 0\\ 0 & -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0\\ \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 & \cosh(\frac{a_0\xi^0}{c})\end{pmatrix} \quad (19\text{-i})$$

The inverse-transformation of the electro-magnetic field is

$$E_{\xi^1}=E_x\,,B_{\xi^1}=B_x$$

$$\begin{pmatrix}E_{\xi^2}\\ B_{\xi^2}\\ E_{\xi^3}\\ B_{\xi^3}\end{pmatrix}=H^{-1}\begin{pmatrix}E_y\\ B_y\\ E_z\\ B_z\end{pmatrix}$$

$$H^{-1}=\begin{pmatrix}\cosh(\frac{a_0\xi^0}{c}) & 0 & 0 & -\sinh(\frac{a_0\xi^0}{c})\\ 0 & \cosh(\frac{a_0\xi^0}{c}) & \sinh(\frac{a_0\xi^0}{c}) & 0\\ 0 & \sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0\\ -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 & \cosh(\frac{a_0\xi^0}{c})\end{pmatrix} \quad (19\text{-ii})$$

$$E_{\xi^1}=E_x\,,B_{\xi^1}=B_x$$

$$E_{\xi^2}=E_y\cosh(\frac{a_0\xi^0}{c})-B_z\sinh(\frac{a_0\xi^0}{c}),$$

$$B_{\xi^2}=B_y\cosh(\frac{a_0\xi^0}{c})+E_z\sinh(\frac{a_0\xi^0}{c})$$

$$E_{\xi^3}=E_z\cosh(\frac{a_0\xi^0}{c})+B_y\sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^3}=B_z\cosh(\frac{a_0\xi^0}{c})-E_y\sinh(\frac{a_0\xi^0}{c}) \quad (19\text{-iii})$$

3. Electro-magnetic Field Equation(Maxwell Equation) in the Rindler space-time

Maxwell equation is

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad (20\text{-i})$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j} \quad (20\text{-ii})$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (20\text{-iii})$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{c\partial t} \quad (20\text{-iv})$$

1. $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$

$$E_x = E_{\xi^1} \ ,$$

$$E_y = E_{\xi^2} \cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3} \cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0\xi^0}{c})$$

$$4\pi\rho = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]E_{\xi^1}$$

$$+ \frac{\partial}{\partial\xi^2}[E_{\xi^2} \cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0\xi^0}{c})]$$

$$+ \frac{\partial}{\partial\xi^3}[E_{\xi^3} \cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)(\vec{\nabla}_\xi \cdot \vec{E}_\xi) + \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial\xi^2} - \frac{\partial B_{\xi^2}}{\partial\xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial E_{\xi^1}}{c\partial\xi^0}] \quad (21)$$

2. $\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j}$

$$B_x = B_{\xi^1}$$

$$B_y = B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z = B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

X-component) $\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$

$$= \frac{\partial}{\partial \xi^2}[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$- \frac{\partial}{\partial \xi^3}[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3}] + \sinh(\frac{a_0\xi^0}{c})[\frac{\partial E_{\xi^2}}{\partial \xi^2} + \frac{\partial E_{\xi^3}}{\partial \xi^3}]$$

$$= \frac{\partial E_x}{c\partial t} + \frac{4\pi}{c} j_x$$

$$= [\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]E_{\xi^1} + \frac{4\pi}{c} j_x$$

Hence,

$$\frac{4\pi}{c} j_x$$

$$= \sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi \cdot \vec{E}_\xi) + \cosh(\frac{a_0\xi^0}{c})[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial E_{\xi^1}}{c\partial\xi^0}] \qquad (22)$$

Y-component) $\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}$

$$= \frac{\partial B_{\xi^1}}{\partial \xi^3}$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \frac{\partial E_y}{c\partial t} + \frac{4\pi}{c} j_y$$

$$= [\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c}) \frac{\partial}{\partial\xi^1}] \cdot [E_{\xi^2} \cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0\xi^0}{c})]$$

$$+ \frac{4\pi}{c} j_y$$

$$\frac{4\pi}{c} j_y = \frac{\partial B_{\xi^1}}{\partial\xi^3} - \frac{\partial B_{\xi^3}}{\partial\xi^1} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{a_0}{c^2} B_{\xi^3} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial E_{\xi^2}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial}{\partial\xi^3} \{B_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial}{\partial\xi^1} \{B_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial E_{\xi^2}}{c\partial\xi^0}$$

(23)

Z-component) $\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c}) \frac{\partial}{\partial\xi^1}] \cdot [B_{\xi^2} \cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3} \sinh(\frac{a_0\xi^0}{c})]$$

$$- \frac{\partial B_{\xi^1}}{\partial\xi^2}$$

$$= \frac{\partial E_z}{c\partial t} + \frac{4\pi}{c} j_z$$

$$\frac{4\pi}{c} j_z = \frac{\partial B_{\xi^2}}{\partial\xi^1} - \frac{\partial B_{\xi^1}}{\partial\xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{a_0}{c^2} B_{\xi^2} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial E_{\xi^3}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial}{\partial\xi^1} \{B_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial}{\partial\xi^2} \{B_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial E_{\xi^3}}{c\partial\xi^0}$$

(24)

3. $\vec{\nabla} \cdot \vec{B} = 0$

$$\vec{\nabla}\cdot\vec{B}=\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}$$

$$=[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]B_{\xi^1}$$

$$+\frac{\partial}{\partial\xi^2}[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{\partial}{\partial\xi^3}[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$=\cosh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi\cdot\vec{B}_\xi)+\sinh(\frac{a_0\xi^0}{c})[-(-\frac{\partial E_{\xi^2}}{\partial\xi^3}+\frac{\partial E_{\xi^3}}{\partial\xi^2})-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^1}}{c\partial\xi^0}]=0 \qquad (25)$$

4. $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{c\partial t}$

$E_x=E_{\xi^1}$,

$E_y=E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$,

$E_z=E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$

X-component) $\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}$

$$=\frac{\partial}{\partial\xi^2}[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial}{\partial\xi^3}[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$=\cosh(\frac{a_0}{c}\xi^0)[\frac{\partial E_{\xi^3}}{\partial\xi^2}-\frac{\partial E_{\xi^2}}{\partial\xi^3}]-\sinh(\frac{a_0\xi^0}{c})[\frac{\partial B_{\xi^2}}{\partial\xi^2}+\frac{\partial B_{\xi^3}}{\partial\xi^3}]$$

$$= -\frac{\partial B_x}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]B_{\xi^1}$$

Hence,

$$-\sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi\cdot\vec{B}_\xi) + \cosh(\frac{a_0\xi^0}{c})\ [\ (\frac{\partial E_{\xi^3}}{\partial\xi^2} - \frac{\partial E_{\xi^2}}{\partial\xi^3}) + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial B_{\xi^1}}{c\partial\xi^0}] = 0 \quad (26)$$

Y-component) $\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}$

$$= \frac{\partial E_{\xi^1}}{\partial\xi^3}$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= -\frac{\partial B_y}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$\frac{\partial E_{\xi^1}}{\partial\xi^3} - \frac{\partial E_{\xi^3}}{\partial\xi^1} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}E_{\xi^3} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^2}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}\{E_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{E_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^2}}{c\partial\xi^0}$$

$$= 0 \quad (27)$$

Z-component) $\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}] \cdot [E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial E_{\xi^1}}{\partial\xi^2}$$

$$= -\frac{\partial B_z}{c\partial t}$$

$$\frac{\partial E_{\xi^2}}{\partial\xi^1} - \frac{\partial E_{\xi^1}}{\partial\xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}E_{\xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^3}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{E_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}\{E_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^3}}{c\partial\xi^0}$$

$$= 0 \tag{28}$$

Therefore, we obtain the electro-magnetic field equation by Eq (21)-Eq(28) in Rindler spacetime .

$$\vec{\nabla}_\xi \cdot \vec{E}_\xi = 4\pi\rho_\xi(1+\frac{a_0\xi^1}{c^2}) \tag{29-i}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{B}_\xi(1+\frac{a_0\xi^1}{c^2})\} = \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{E}_\xi}{c\partial\xi^0} + \frac{4\pi}{c}\vec{j}_\xi \tag{29-ii}$$

$$\vec{\nabla}_\xi \cdot \vec{B}_\xi = 0 \tag{29-iii}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{E}_\xi(1+\frac{a_0\xi^1}{c^2})\} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{B}_\xi}{c\partial\xi^0} \tag{29-iv}$$

$$\vec{E}_\xi = (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}), \vec{B}_\xi = (B_{\xi^1}, B_{\xi^2}, B_{\xi^3}), \vec{\nabla}_\xi = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3})$$

Hence, the transformation of 4-vector $(c\rho, \vec{j}) = \rho_0 \frac{dx^\alpha}{d\tau}$ is

$$\rho = \rho_\xi(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c}) + \frac{j_{\xi^1}}{c}\sinh(\frac{a_0\xi^0}{c})$$

$$j_x = j_{\xi^1}\cosh(\frac{a_0\xi^0}{c}) + c\rho_\xi(1+\frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c}), \quad j_y = j_{\xi^2}, j_z = j_{\xi^3} \qquad (30)$$

In this time, the Lorentz gauge is

$$\frac{\partial\phi}{c\partial t} + \vec{\nabla}\cdot\vec{A} = \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}\{A_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\} + \frac{\partial\phi_\xi}{c\partial\xi^0} + \frac{\partial A_{\xi^2}}{\partial\xi^2} + \frac{\partial A_{\xi^3}}{\partial\xi^3}$$

$$= \frac{\partial\phi_\xi}{c\partial\xi^0} + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\cdot\{\vec{A}_\xi(1+\frac{a_0\xi^1}{c^2})\} = 0 \quad (31)$$

4. Conclusion

We find the electro-magnetic field transformation and the electro-magnetic equation in uniformly accelerated frame.

Reference

[1]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[2]W.Rindler, Am.J.Phys.**34**.1174(1966)

[3]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[4]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[5]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[6]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)

[7]A.Miller, Albert Einstein's Special Theory of Relativity(Addison-Wesley Publishing Co., Inc., 1981)

[8]W.Rindler, Special Relativity(2nd ed., Oliver and Boyd, Edinburg,1966)

[9]S.Yi, The African Review of Physics,**8**,37(2013)